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Study of Stochastic Model of a Two Unit System with Inspection and Replacement Under Multi Failure

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Abstract

The present paper studies a two non-identical units system model arranged in parallel with inspection and preparation time for replacement under multi failures. Initially, first unit (A) is in operative mode and other unit (B) is kept as warm standby. The first unit is subjected to two types of failures, i.e. minor failure and major failure. On failure of the first unit, it will be sent for inspection to check the type of failure i.e. whether minor or major failure. If some minor failure is found, it will be repaired and on major failure, the unit will be replaced by the new unit. However, the system will take some preparation time for replacement. Further, the standby unit may also fail during the standby mode. There is a single repairman which is always available with the system. Different measures of reliability have been obtained to study the effectiveness of the system such as transition probabilities, mean time to system failure, availability, busy period of repairman and net profit incurred and various system parameters are analysed graphically.

Keywords: inspection, preparation time, replacement, minor failure, major failure.

1. Introduction

Reliability is considered as important characteristic for the system design and plays a vital role in the planning of system expansion, operation and maintenance. Quality of supply can be improved by reliability. To obtain useful results from system reliability assessments, reasonable values of component reliability parameters need to be used. However, the required accuracy of the reliability depends on the system design, its performance and the failure phenomenon of the system components. However, the components failure rates may vary with component, time and the environmental conditions. Therefore, it is sometimes not accurate to assign identical failure rate to all components of a particular type. Each component is treated as an individual with a unique failure rate. Many authors had worked in the reliability modelling field with different failure rates and disciplines. Rander, Kumar and Tuteja [8] have discussed a two unit cold standby system with major and minor failure and preparation time in case of major failure. El-Damcese and Temraz[7] carried out the analysis for a parallel repairable system with different failure modes and Chander, Chand and Singh[2] has studies stochastic analysis of an operating system with two types of inspection subject to degradation. Further Bhatti, Chitkara and Bhardwaj [1] studied the profit analysis of two unit cold standby system with two types of failure under inspection policy and discrete distribution and Dhankhar and Malik[5] analyse the cost-benefit analysis of a system reliability models with server failure during inspection and repair while Chib, Joorel and Sharma

[3,4] have worked on MTSF and profit analysis of a two unit warm standby system with inspection and they also worked on the analysis of a two non-identical unit cold standby system with partial and total failure and priority and El-Damcese and Sharma [6] investigated reliability and availability analysis of a repairable system with two type of failure.
The present paper studies a two non-identical units system model arranged in parallel with inspection and preparation time for replacement under multi failures. Initially, first unit (A) is in operative mode and other unit (B) is kept as warm standby. The first unit is subjected to two types of failures, i.e. minor failure and major failure. On failure of the first unit, it will be sent for inspection to check the type of failure i.e. whether minor or major failure. If some minor failure is found, it will be repaired and on major failure, the unit will be replaced by the new unit. However, the system will take some preparation time for replacement. Further, the standby unit may also fail during the standby mode. There is a single repairman which is always available with the system. Different measures of reliability have been obtained to study the effectiveness of the system such as transition probabilities, mean time to system failure, availability, busy period of repairman and net profit incurred and various system parameters are analysed graphically.

2. Assumptions

1. All the times associated with different events are random variables and independent.
2. Failure time distribution of both the units is exponential but with different parameters.
3. Inspection time distribution is also exponential.
4. Repair time distribution of both the units is taken as general but with different cdfs and replacement time distribution of first unit is also general.
5. On failure of both the units, the system will break down.
6. Switch over time is negligible.

3. Notations

α	inspection rate for unit A
β	failure rate of unit B
α_1	rate of minor / major failure in unit A with probability p and q
α_2	rate of completion of preparation for replacement
$h_1(t)/H_1(t)$	p.d.f and c.d.f of repair time of unit A
$h_2(t)/H_2(t)$	p.d.f and c.d.f of replacement time of unit A
$g(t)/G(t)$	p.d.f and c.d.f of repair time of unit B
$\pi_I(\cdot)$	c.d.f of time to system failure when $S_i \epsilon E$.
$A_i(t)$	Pr [starting from $S_i \epsilon E$, the system is up at time t].
$B_i(t)$	Pr [Repairman is busy at time t $\backslash E_0 = S_i \epsilon E$].
$V_i(t)$	Expected number of visits by repairman in (0,t].
μ_i	Mean sojourn time in state $S_i \epsilon E$.

Following Symbols are used to study the proposed model:

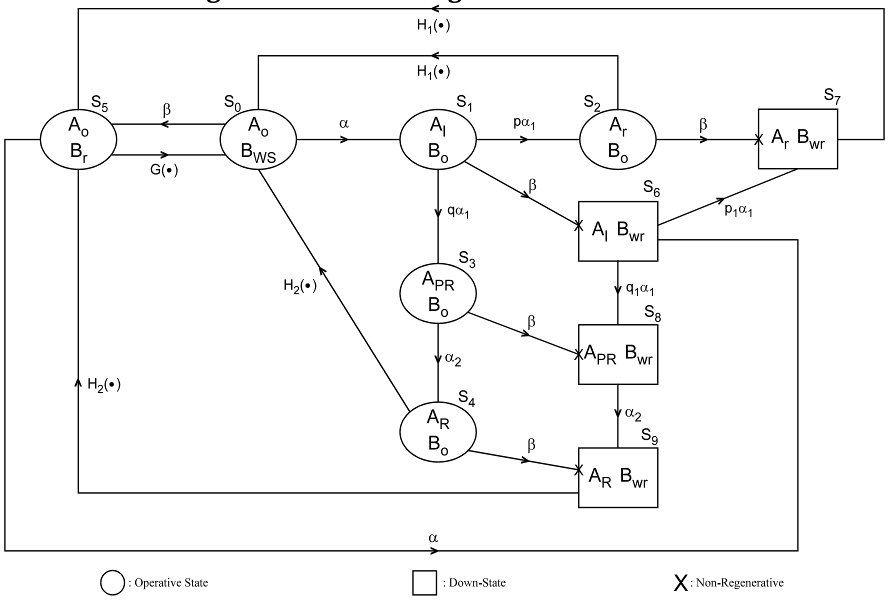
A_o/B_o	unit A/B is operative
A_r/B_r	unit A/B is under repair
A_I	unit A is under inspection and
A_{PR}/A_R	unit A is under replacement or preparation for replacement
B_{ws}	unit B is in warm standby mode
B_{wr}	unit B is waiting for repair

Different possible states of the system are described and shown by Fig.: 1

Up states
$S_0: (A_0, B_{ws})$ $S_2: (A_r, B_o)$ $S_4: (A_R, B_o)$
$S_1: (A_I, B_o)$ $S_3: (A_{PR}, B_o)$ $S_5: (A_o, B_r)$

Down states
$S_6 = (A_I, B_{wr})$ $S_8: (A_{PR}, B_{wr})$
$S_7: (A_r, B_{wr})$ $S_9: (A_R, B_{wr})$

Fig.: 1 Transition Diagram



4. Transition Probabilities and Mean Sojourn Time

If T_1, T_2, T_3 … denote the epochs at which the system enter any state and X_n denotes the state visited at point T_{n+}, i.e. just after the transition at T_n then the transient and steady state transition probabilities are defined as $Q_{ij}(t) = P[X_{n+1} = j, T_{n+1} - T_n \le t | X_n = i]$ and $p_{ij} = \lim_{t\to\infty} Q_{ij}(t)$ respectively. The following steady state transition probabilities of the system are obtained:

$p_{01} = \frac{\alpha}{\alpha+\beta}$ $p_{17}^{6} = \frac{\beta p_1}{\alpha_1+\beta}$ $p_{34} = \frac{\alpha_2}{\alpha_2+\beta}$ $p_{45}^{9} = 1 - h_2^*(\beta)$

$p_{05} = \frac{\beta}{\alpha+\beta}$ $p_{18}^{6} = \frac{\beta q_1}{\alpha_1+\beta}$ $p_{38} = \frac{\beta}{\alpha_2+\beta}$ $p_{50} = g^*(\alpha)$

$p_{12} = \frac{p\alpha_1}{\alpha_1+\beta}$ $p_{20} = h_1^*(\beta)$ $p_{39}^{8} = \frac{\beta}{\alpha_2+\beta}$ $p_{56} = 1 - g^*(\alpha)$

$p_{13} = \frac{q\alpha_1}{\alpha_1+\beta}$ $p_{27} = 1 - h_1^*(\beta)$ $p_{40} = h_2^*(\beta)$ $p_{67} = p_1$

$p_{16} = \frac{\beta}{\alpha_1+\beta}$ $p_{25}^{7} = 1 - h_1^*(\beta)$ $p_{49} = 1 - h_2^*(\beta)$ $p_{68} = q_1$

$$p_{75} = p_{89} = p_{95} = 1 \quad (1)$$

It may be noted that $\sum_j p_{ij} = 1$, *for all possible values of i* (2)

Further, if T_i denotes the sojourn time in state S_i then mean sojourn time is defined as the time of stay in state S_i before transiting to any other state and is denoted by μ_i. Following are the expressions for mean sojourn time:

$\mu_0 = \frac{1}{\alpha+\beta}$ $\mu_3 = \frac{1}{\alpha_2+\beta}$ $\mu_6 = \frac{1}{\alpha_1}$ $\mu_9 = \int_0^\infty \bar{H}_2(u)\,du$

$\mu_1 = \frac{1}{\alpha_1+\beta}$ $\mu_4 = \frac{1}{\beta}[1 - h_2^*(\beta)]$ $\mu_7 = \int_0^\infty \bar{H}_1(u)\,du$

$$\mu_2 = \frac{1}{\beta}[1 - h_1^*(\beta)] \quad \mu_5 = \frac{1}{\alpha}[1 - g^*(\alpha)] \quad \mu_8 = \frac{1}{\alpha_2} \quad (3)$$

5. Mean Time To System Failure

The distribution function of time to system failure is obtained by considering the failed states as absorbing state and the time taken by the system to reach in the failed state for the first time is known as time to system failure and is denoted by T_i and $\pi_i(t)$ denotes its expected value which is known as the mean time to system failure.
The following result for mean time to system failure is obtained by using Laplace transformation.

$$\text{MTSF}=\frac{\alpha[(\alpha_2+\beta)\beta+p\alpha_1\beta(\theta+\alpha)\{1-h_1^*(\beta)\}(\alpha_2+\beta)+q\alpha_1\{\beta+\alpha_2\{1-h_1^*(\beta)\}\}(\theta+\alpha)]+\beta(\alpha_2+\beta)\{1-g^*(\beta)\}(\alpha_1+\beta)}{(\alpha_1+\beta)(\alpha_2+\beta)(\beta+\alpha)-\alpha\{p\alpha_1h_1^*(\alpha)(\alpha_2+\beta)+q\alpha_1\alpha_2h_2^*(\beta)\}-\beta(\alpha_1+\beta)g^*(\alpha)(\alpha_2+\beta)} \quad (4)$$

6. Availability Analysis

$A_i\,(t)$ is defined as the probability that a system will be in operational service during a scheduled operating period i.e., probability that the system is up at epoch 't' given that initially it starts from state S_i without transiting to any non-regenerative state. By using simple probabilistic concepts, recurrence relations among $A_i\,(t)'s$ are obtained and on solving those equations by using the Laplace- transformation following results hold:

$$A_0^*(s)=\frac{N_2(s)}{D_2(s)}$$

where,

$$N_2(s)=[M_0^*+q_{01}^*(M_1^*+q_{12}^*M_2^*+q_{13}^*M_3^*+q_{13}^*q_{34}^*M_4^*)](1-q_{56}^*q_{67}^*q_{75}^*-q_{56}^*q_{68}^*q_{89}^*q_{95}^*)+ M_5^*[q_{05}^*+q_{01}^*(q_{12}^*q_{25}^{(7)*}+q_{13}^*q_{34}^*q_{45}^{(9)*}+q_{13}^*q_{39}^{(8)*}q_{95}^*+q_{18}^{(6)*}q_{89}^*q_{95}^*+q_{17}^{(6)*}q_{75}^* \quad (5)$$

$$D_2(s)=(1-q_{56}^*q_{67}^*q_{75}^*-q_{56}^*q_{68}^*q_{89}^*q_{95}^*)[1-q_{01}^*(q_{12}^*q_{20}^*+q_{13}^*q_{34}^*q_{40}^*)]-q_{50}^*[q_{05}^*+q_{01}^* (q_{12}^*q_{25}^{*(7)}+q_{13}^*q_{39}^{*(8)}q_{95}^*+q_{18}^{*(6)}q_{89}^*q_{95}^*+q_{17}^{*(6)}q_{75}^*)] \quad (6)$$

Steady state availability of the system starting from state s_0 is obtained as follows:

$$A_0=\lim_{t\to\infty}A_0\,(\infty)=\lim_{s\to 0}sA_0^*\,(s)=\frac{N_2(0)}{D_2'(0)} \quad (7)$$

$$A_0=\frac{\begin{array}{c}[(\alpha_1+\beta)(\alpha_2+\beta)\beta+\alpha(\alpha_1+\beta)(\alpha_2+\beta)\beta+p\alpha_1(\alpha_1+\beta)\alpha(\alpha_2+\beta)\{1-h_2^*(\beta)\}+q\alpha_1\beta(\alpha+\beta)+q\alpha_1\alpha_2(\alpha+\beta)\{1-h_2^*(\beta)\}\\ g^*(\alpha)\alpha+\{1-g^*(\alpha)\}\beta[(\alpha_1+\beta)(\alpha_2+\beta)+\alpha\{p\alpha_1\beta(\alpha_2+\beta)\{1-h_1^*(\beta)\}(\alpha+\beta)+q\alpha_1\alpha_2\beta\{1-h_2^*(\beta)\}(\alpha+\beta)+q\beta\alpha_1(\alpha+\beta)\\ +(\alpha+\beta)(\alpha_2+\beta)\beta\}]\theta_2\,\alpha_2\end{array}}{\begin{array}{c}g^*(\alpha)[(\alpha_1+\beta)(\alpha_2+\beta)\beta\alpha\alpha_2\theta_2+\alpha(\alpha_2+\beta)\alpha\beta\alpha_2\theta_2+p\alpha_1(\alpha_2+\beta)\alpha\{1-h^*(\beta)\}\alpha_2\theta_2\alpha+q\alpha\alpha_1\theta_2\alpha\beta+q\alpha\alpha_1\theta_2\alpha_2^2\theta_2\\ \alpha\{1-h_1^*(\beta)\}]+[\{1-g^*(\alpha)\}\alpha+\beta][(\alpha+\beta)\beta(\alpha_1+\beta)(\alpha_2+\beta)\alpha-\alpha\{p(\alpha_2+\beta)h_1^*(\alpha)\,\alpha_1+q\beta\alpha_1\alpha_2h_2^*(\alpha)\{1-g^*(\alpha)\}]\theta_2\alpha_2\\ +\int\bar{H}_1(u)[\alpha p_1\{1-g^*(\alpha)\}(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1h_2^*(\beta)(\alpha_2+\beta)+q\alpha_1\alpha_2\beta h_2^*(\beta)\}+\alpha g^*(\alpha)\beta p_1\alpha_1\alpha(\alpha_2+\beta)\\ +\alpha q_1\{1-g^*(\alpha)\}[(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)-p\alpha\alpha_1g^*(\alpha)(\alpha_2+\beta)+q\beta\alpha_1\alpha_2\{1-h_2^*(\beta)\}+\alpha^2q_1\beta(\alpha_2+\beta)]+\alpha^2\beta\int\bar{H}_1(u)\\ [q_1(\alpha_1+\beta)\{1-g_2^*(\alpha)\}(\alpha_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1(\alpha_2+\beta)g_2^*(\alpha)h_2^*(\beta)+q\alpha_1\alpha_2\beta\}+\alpha\{1-g^*(\alpha)\}\{q\beta\alpha_1+\beta p_1(\alpha_2+\beta)\}]\end{array}} \quad (8)$$

7. Busy Period Analysis

$B_i\,(t)$ is the probability that the repairman is busy due to repair of the unit at an instant 't' given that system entered state S_i at t=0. Now we will determine these probabilities. To illustrate the calculations we consider $B_0(t)$ and similar arguments may be employed for other probabilities.

$$B_0^*(s)=\frac{N_3(s)}{D_2(s)}$$

$$N_3(s)=q_{01}^*\big(M_1^*+q_{12}^*M_2^*+q_{13}^*q_{34}^*M_4^*+q_{13}^8q_{39}^{(8)*}M_9^*+q_{16}^{(8)*}M_8^*+q_{16}^{(8)*}q_{89}^*M_9^*\big)(1-q_{56}^*q_{67}^* q_{75}^*-q_{56}^*q_{68}^*q_{89}^*q_{95}^*)+(M_5^*+q_{56}^*M_6^*+q_{56}^*q_{68}^*M_8^*+q_{56}^*q_{68}^*q_{89}^*M_9^*)[q_{01}^*(q_{12}^*q_{25}^{*(7)}+ q_{13}^*q_{34}^*q_{45}^{*(9)}+q_{13}^*q_{39}^{*(8)}q_{95}^*+q_{18}^{*(6)}q_{89}^*q_{95}^*+q_{17}^{*(6)}q_{75}^*)+q_{05}^*] \quad (9)$$

$$D_2(s)=(1-q_{56}^*q_{67}^*q_{75}^*-q_{56}^*q_{68}^*q_{89}^*q_{95}^*)[1-q_{01}^*(q_{12}^*q_{20}^*+q_{13}^*q_{34}^*q_{40}^*)]-q_{50}^*[q_{05}^*+q_{01}^*$$

$$(q_{12}^*q_{25}^{*(7)} + q_{13}^*q_{39}^{*(8)}q_{95}^* + q_{18}^{*(6)}q_{89}^*q_{95}^* + q_{17}^{*(6)}q_{75}^*)] \quad (10)$$

Therefore, busy period analysis for repairman is given by:

$$B_0(0) = \frac{N_3(0)}{D_2'(0)} \quad (11)$$

$$B_0 = \frac{\alpha[\{(\alpha_1+\beta)\beta(\alpha_2+\beta)\alpha_2+(\alpha+\beta)(\alpha_2+\beta)p\alpha_2\alpha_1\{1-h_1^*(\beta)\}+(\alpha+\beta)\{1-h_2^*(\beta)\}q\alpha_1\alpha_2^2\}+\alpha_1\beta\alpha_2(\alpha+\beta)\int\bar{H}_2(u)q+q_1\beta(\alpha+\beta)(\alpha_2+\beta)+q_1\beta(\alpha_1+\beta)(\alpha_2+\beta)\int\bar{H}_2(u)(\alpha+\beta)]g^*(\alpha)\alpha+[\alpha[\{\alpha_1\alpha_2\theta_2+\alpha_2\alpha\theta_2+\alpha\alpha_1q_1\theta_2+\alpha q_1\}+(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)][\alpha\{p\alpha_1\beta\theta_2(\alpha_2+\beta)\{1-h_1^*(\beta)\}+q\beta\alpha_1\alpha_2\alpha_2\{1-h_2^*(\beta)\}+q\beta\alpha_1+q_1\beta(\alpha_2+\beta)\alpha_2+p_1\beta(\alpha_2+\beta)\alpha_2]+(\alpha_1+\beta)\beta(\alpha_2+\beta)]}{g^*(\alpha)[(\alpha_1+\beta)(\alpha_2+\beta)\beta\alpha\alpha_2\theta_2+\alpha(\alpha_2+\beta)\alpha\beta\alpha_2\theta_2+p\alpha_1(\alpha_2+\beta)\alpha\{1-h^*(\beta)\}\alpha_2\theta_2\alpha+q\alpha\alpha_1\theta_2\alpha\beta+q\alpha\alpha_1\theta_2\alpha_2^2\theta_2\alpha\{1-h_1^*(\beta)\}]+[\{1-g^*(\alpha)\}\alpha+\beta][(\alpha+\beta)\beta(\alpha_1+\beta)(\alpha_2+\beta)\alpha-\alpha\{p(\alpha_2+\beta)h_1^*(\alpha)\,\alpha_1+q\beta\alpha_1\alpha_2h_2^*(\alpha)\{1-g^*(\alpha)\}]\theta_2\alpha_2+\int\bar{H}_1(u)[\alpha p_1\{1-g^*(\alpha)\}(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1h_2^*(\beta)(\alpha_2+\beta)+q\alpha_1\alpha_2\beta h_2^*(\beta)\}+\alpha g^*(\alpha)\beta p_1\alpha_1\alpha(\alpha_2+\beta)+\alpha q_1\{1-g^*(\alpha)\}[(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)-p\alpha\alpha_1g^*(\alpha)(\alpha_2+\beta)+q\beta\alpha_1\alpha_2\{1-h_2^*(\beta)\}+\alpha^2q_1\beta(\alpha_2+\beta)]+\alpha^2\beta\int\bar{H}_1(u)[q_1(\alpha_1+\beta)\{1-g_2^*(\alpha)\}(\alpha_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1(\alpha_2+\beta)g_2^*(\alpha)h_2^*(\beta)+q\alpha_1\alpha_2\beta\}+\alpha\{1-g^*(\alpha)\}\{q\beta\alpha_1+\beta p_1(\alpha_2+\beta)\}]} \quad (12)$$

8. Expected Number of Visits by Repairman

$V_i(t)$ is the expected number of visits by the repairman to the system to repair the failed unit, when the system initially starts from regenerative state S_i. By probabilistic reasoning the recurrence relations for $V_i(t)$ are obtained and solving those relations by using Laplace transformation, we have,

$$V_0^*(s) = \frac{N_4(s)}{D_2(s)}$$

$$N_4(s) = [(q_{01}^* + q_{05}^*) + q_{01}^*(q_{13}^*q_{34}^* + q_{18}^{*(6)}q_{89}^*)][1 - q_{56}^*q_{67}^*q_{75}^* - q_{56}^*q_{68}^*q_{89}^*q_{95}^*] + q_{56}^* q_{67}^*q_{75}^*[q_{05}^* + q_{01}^*(q_{12}^*q_{25}^{*(7)} + q_{13}^*q_{34}^*q_{45}^{*(9)} + q_{13}^*q_{39}^{*(8)}q_{95}^* + q_{16}^{*(8)}q_{89}^*q_{95}^* + q_{17}^{*(6)}q_{75}^*)] \quad (13)$$

$$D_2(s) = (1 - q_{56}^*q_{67}^*q_{75}^* - q_{56}^*q_{68}^*q_{89}^*q_{95}^*)[1 - q_{01}^*(q_{12}^*q_{20}^* + q_{13}^*q_{34}^*q_{40}^*)] - q_{50}^*[q_{05}^* + q_{01}^* (q_{12}^*q_{25}^{*(7)} + q_{13}^*q_{39}^{*(8)}q_{95}^* + q_{18}^{*(6)}q_{89}^*q_{95}^* + q_{17}^{*(6)}q_{75}^*)] \quad (14)$$

In steady state, the number of times the repairman visits the system is given by:

$$V_0^* = \lim_{S\to 0} sV_0(s) = \frac{N_4(0)}{D_2'(0)} \quad (15)$$

$$V_0 = \frac{[(\alpha_1+\beta)(\alpha+\beta)(\alpha_2+\beta)+\alpha\alpha_1\alpha_2q+q_1\beta(\alpha+\beta)(\alpha_2+\beta)]+q_1\{1-g^*(\alpha)\}[\beta(\alpha_1+\beta)(\alpha_2+\beta)+\alpha\{(\alpha_1+\beta)(\theta_1+\beta)-p\alpha_1 g_2^*(\alpha)\}-q\alpha_1\alpha_2h_2^*(\beta)]g^*(\alpha)}{g^*(\alpha)[(\alpha_1+\beta)(\alpha_2+\beta)\beta\alpha\alpha_2\theta_2+\alpha(\alpha_2+\beta)\alpha\beta\alpha_2\theta_2+p\alpha_1(\alpha_2+\beta)\alpha\{1-h^*(\beta)\}\alpha_2\theta_2\alpha+q\alpha\alpha_1\theta_2\alpha\beta+q\alpha\alpha_1\theta_2\alpha_2^2\theta_2\alpha\{1-h_1^*(\beta)\}]+[\{1-g^*(\alpha)\}\alpha+\beta][(\alpha+\beta)\beta(\alpha_1+\beta)(\alpha_2+\beta)\alpha-\alpha\{p(\alpha_2+\beta)h_1^*(\alpha)\,\alpha_1+q\beta\alpha_1\alpha_2h_2^*(\alpha)\{1-g^*(\alpha)\}]\theta_2\alpha_2+\int\bar{H}_1(u)[\alpha p_1\{1-g^*(\alpha)\}(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1h_2^*(\beta)(\alpha_2+\beta)+q\alpha_1\alpha_2\beta h_2^*(\beta)\}+\alpha g^*(\alpha)\beta p_1\alpha_1\alpha(\alpha_2+\beta)+\alpha q_1\{1-g^*(\alpha)\}[(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)-p\alpha\alpha_1g^*(\alpha)(\alpha_2+\beta)+q\beta\alpha_1\alpha_2\{1-h_2^*(\beta)\}+\alpha^2q_1\beta(\alpha_2+\beta)]+\alpha^2\beta\int\bar{H}_1(u)[q_1(\alpha_1+\beta)\{1-g_2^*(\alpha)\}(\alpha_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1(\alpha_2+\beta)g_2^*(\alpha)h_2^*(\beta)+q\alpha_1\alpha_2\beta\}+\alpha\{1-g^*(\alpha)\}\{q\beta\alpha_1+\beta p_1(\alpha_2+\beta)\}]} \quad (16)$$

9. Profit Analysis

The profit in steady state generated by proposed model may be obtained as follows:
The expected profits incurred in (0,t] = expected total revenue in (0,t] – expected total repair in (0,t] –expected cost of visit by repairman in (0,t]
Therefore, profit analysis of the system can be written as:

$$P_1 = K_0A_0 - K_1B_0 - K_2V_0$$

where,
K_0 = revenue per unit up time of the system,
K_1 = Cost per unit time for which the repair is busy
K_2 = Cost per unit visits by the repairman
The expressions for A_0, B_0 and V_0 are given by equations (8), (12)and (16) respectively.

10. Particular cases

As we have assumed that the repair time and replacement time distribution is general, so firstly we convert it into exponential with parameters θ, θ_1 and θ_2. We have assumed
$G(t) = \theta e^{-\theta t}, H_1(t) = \theta_1 e^{-\theta_1 t}, H_2(t) = \theta_2 e^{-\theta_2 t}$
So under these assumptions the expressions for different transitions with their mean sojourn time, MTSF, availability and profit function are obtained as:

Transition probabilities

$p_{20} = \frac{\theta_1}{\theta_1+\beta}$ $\quad p_{27} = \frac{\beta}{\theta_1+\beta}$ $\quad p_{25}^{7} = \frac{\beta}{\theta_1+\beta}$ $\quad p_{40} = \frac{\theta_2}{\theta_2+\beta}$ $\quad p_{49} = \frac{\beta}{\theta_2+\beta}$

$p_{45}^{9} = \frac{\beta}{\theta_2+\beta}$ $\quad p_{50} = \frac{\theta}{\theta+\alpha}$ $\quad p_{56} = \frac{\alpha}{\theta+\alpha}$ $\quad p_{95} = 1$

Mean Sojourn Time

$\mu_2 = \frac{1}{\theta_1+\beta}$ $\quad \mu_4 = \frac{1}{\theta_2+\beta}$ $\quad \mu_5 = \frac{1}{\theta+\alpha}$ $\quad \mu_7 = \frac{1}{\theta_1}$ $\quad \mu_9 = \frac{1}{\theta_2}$

Mean time to system failure

$$MTSF=\frac{\alpha[(\theta_1+\beta)(\alpha_2+\beta)(\theta_2+\beta)+p\alpha_1\beta(\theta+\alpha)(\alpha_2+\beta)(\theta_2+\beta)+q\alpha_1\{(\theta_2+\beta)+\alpha_2\beta\}(\theta+\alpha)(\theta_2+\beta)]+\alpha\beta(\theta_1+\beta)(\alpha_2+\beta)(\alpha_1+\beta)(\theta_2+\beta)}{(\alpha_1+\beta)(\theta_1+\beta)(\alpha_2+\beta)(\theta_2+\beta)(\theta+\alpha)(\beta+\alpha)-\alpha\{p\alpha_1\theta_1(\alpha_2+\beta)(\theta_2+\beta)+q\alpha_1\alpha_2\theta_2(\theta_1+\beta)(\theta+\alpha)\}-\beta\theta(\alpha_1+\beta)(\theta_1+\beta)(\alpha_2+\beta)(\theta_2+\beta)}$$

Availability

$$A_0 = \frac{[(\alpha_1+\beta)(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)+\alpha(\alpha_1+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)\beta+p\alpha_1(\alpha_1+\beta)\alpha(\theta_2+\beta)(\alpha+\beta)+q\alpha_1(\alpha+\beta)(\theta_1+\beta)(\theta_2+\beta)\beta+q\alpha_1\alpha_2(\alpha+\beta)\alpha]\theta+[(\alpha_1+\beta)(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)+\alpha\{p\alpha_1\beta(\alpha_2+\beta)(\theta_2+\beta)(\alpha+\beta)+q\alpha_1\alpha_2\beta(\alpha+\beta)(\theta_1+\beta)+q\beta\alpha_1(\alpha+\beta)(\theta_1+\beta)(\theta_2+\beta)+(\alpha+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)\beta\}]}{\alpha_2\theta\theta_2[(\alpha_1+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)+\alpha(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)\theta_2\alpha+p\alpha_1(\alpha_2+\beta)(\theta_2+\beta)+(\theta_2+\beta)q(\theta_1+\beta)\alpha\alpha_2\theta_2\alpha+(\theta_1+\beta)q\alpha\alpha_1\theta_2\alpha_2^2\theta_2\alpha]+[2\alpha+\beta][(\alpha+\beta)\beta(\alpha_1+\beta)(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)-\alpha\{p(\theta_2+\beta)(\alpha_2+\beta)\alpha_1+q\beta\alpha_1\alpha_2(\theta_1+\beta))]\theta_2\alpha_2+[\alpha p_1(\alpha_1+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1(\alpha_2+\beta)\theta_1(\theta_2+\beta)+(\theta_2+\beta)q\alpha_1\alpha_2\beta\}+\alpha\beta p_1\alpha_1\alpha\theta(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)+\alpha q_1[(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)(\theta_1+\beta)(\theta_2+\beta)(\theta+\alpha)-p\alpha\alpha_1\theta_1(\alpha_2+\beta)(\theta_2+\beta)+q\beta\alpha_1\alpha_2(\theta_1+\beta)+\alpha^2q_1\beta(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)]+\alpha^2\beta[q_1(\alpha_1+\beta)(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1\theta_1(\alpha_2+\beta)+q\alpha_1\alpha_2\beta(\theta_1+\beta)\}+\alpha\{q\beta\alpha_1+\beta p_1(\alpha_2+\beta)\}]}$$

Busy period

$$B_0 = \frac{\alpha[(\alpha_1+\beta)\beta(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)\alpha_2+(\alpha+\beta)(\theta_2+\beta)(\alpha_2+\beta)p\alpha_2\alpha_1+(\theta_1+\beta)(\alpha+\beta)q\alpha_1\alpha_2^2+\alpha_1(\theta_1+\beta)(\theta_2+\beta)\beta\alpha_2(\alpha+\beta)q+q_1\beta(\alpha+\beta)(\alpha_2+\beta)(\theta_2+\beta)+q_1\beta(\alpha_1+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)(\alpha+\beta)]\theta+[\{\alpha_1\alpha_2\theta_2+\alpha_2\alpha\theta_2+\alpha\alpha_1q_1\theta_2+\alpha q_1\}(\theta_1+\beta)(\theta_2+\beta)(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)][\alpha\alpha_2\{p\alpha_1\beta\theta_2(\alpha_2+\beta)(\theta_2+\beta)+q\beta\alpha_1\alpha_2\alpha_2(\theta_1+\beta)+q\beta\alpha_1(\theta_1+\beta)(\theta_2+\beta)+q_1\beta(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)\alpha_2+p_1\beta(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)\alpha_2\theta_2]}{\alpha_2\theta\theta_2[(\alpha_1+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)+\alpha(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)\theta_2\alpha+p\alpha_1(\alpha_2+\beta)(\theta_2+\beta)+(\theta_2+\beta)q(\theta_1+\beta)\alpha\alpha_2\theta_2\alpha+(\theta_1+\beta)q\alpha\alpha_1\theta_2\alpha_2^2\theta_2\alpha]+[2\alpha+\beta][(\alpha+\beta)\beta(\alpha_1+\beta)(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)-\alpha\{p(\theta_2+\beta)(\alpha_2+\beta)\alpha_1+q\beta\alpha_1\alpha_2(\theta_1+\beta))]\theta_2\alpha_2+[\alpha p_1(\alpha_1+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1(\alpha_2+\beta)\theta_1(\theta_2+\beta)+(\theta_2+\beta)q\alpha_1\alpha_2\beta\}+\alpha\beta p_1\alpha_1\alpha\theta(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)+\alpha q_1[(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)(\theta_1+\beta)(\theta_2+\beta)(\theta+\alpha)-p\alpha\alpha_1\theta_1(\alpha_2+\beta)(\theta_2+\beta)+q\beta\alpha_1\alpha_2(\theta_1+\beta)+\alpha^2q_1\beta(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)]+\alpha^2\beta[q_1(\alpha_1+\beta)(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1\theta_1(\alpha_2+\beta)+q\alpha_1\alpha_2\beta(\theta_1+\beta)\}+\alpha\{q\beta\alpha_1+\beta p_1(\alpha_2+\beta)\}]}$$

Expected number of repairs

$$V_0 = \frac{\theta[(\alpha_1+\beta)(\alpha+\beta)(\alpha_2+\beta)+\alpha\alpha_1\alpha_2q+q_1\beta(\alpha+\beta)(\alpha_2+\beta)](\theta_1+\beta)(\theta_2+\beta)+q_1\alpha[\beta(\alpha_1+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)+\alpha\{(\alpha_1+\beta)(\theta_1+\beta)-p\alpha_1\theta_1\}-q\alpha_1\alpha_2\theta_2]}{\alpha_2\theta\theta_2[(\alpha_1+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)+\alpha(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)\theta_2\alpha+p\alpha_1(\alpha_2+\beta)(\theta_2+\beta)+(\theta_2+\beta)q(\theta_1+\beta)\alpha\alpha_2\theta_2\alpha+(\theta_1+\beta)q\alpha\alpha_1\theta_2\alpha_2^2\theta_2\alpha]+[2\alpha+\beta][(\alpha+\beta)\beta(\alpha_1+\beta)(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)-\alpha\{p(\theta_2+\beta)(\alpha_2+\beta)\alpha_1+q\beta\alpha_1\alpha_2(\theta_1+\beta))]\theta_2\alpha_2+[\alpha p_1(\alpha_1+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1(\alpha_2+\beta)\theta_1(\theta_2+\beta)+(\theta_2+\beta)q\alpha_1\alpha_2\beta\}+\alpha\beta p_1\alpha_1\alpha\theta(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)+\alpha q_1[(\alpha_1+\beta)(\alpha_2+\beta)(\alpha+\beta)(\theta_1+\beta)(\theta_2+\beta)(\theta+\alpha)-p\alpha\alpha_1\theta_1(\alpha_2+\beta)(\theta_2+\beta)+q\beta\alpha_1\alpha_2(\theta_1+\beta)+\alpha^2q_1\beta(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)]+\alpha^2\beta[q_1(\alpha_1+\beta)(\alpha_2+\beta)(\theta_1+\beta)(\theta_2+\beta)(\alpha+\beta)-\alpha\{p\alpha_1\theta_1(\alpha_2+\beta)+q\alpha_1\alpha_2\beta(\theta_1+\beta)\}+\alpha\{q\beta\alpha_1+\beta p_1(\alpha_2+\beta)\}]}$$

11. Graphical Study of the System Model

In order to have a graphical analysis of the above discussed model, we graphed these characteristics i.e., MTSF, availability and profit function. Firstly we have obtained the values of MTSF, availability and profit function with respect to failure and repair rates using C++ language and then we have plotted those values using STATISTICA. Firstly graphs are plotted for MTSF, Availability and Profit with respect to failure rate α_1 for different values of repair rate θ_1, θ_2 & θ keeping all other parameters constant as $\alpha = 0.5$, $\alpha_2 = 0.25, \beta = 0.35$, $k_0 = 1000, k_1 = 300, k_2 = 200, p = 0.5, q = 0.5, p_1 = 0.5, q_1 = 0.5$

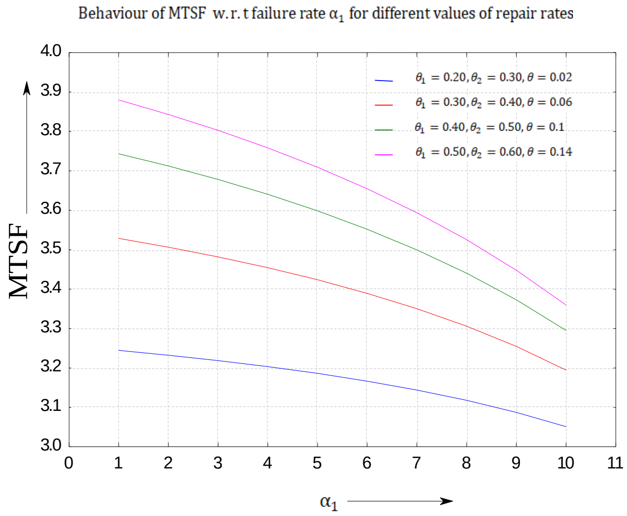


Fig.: 2

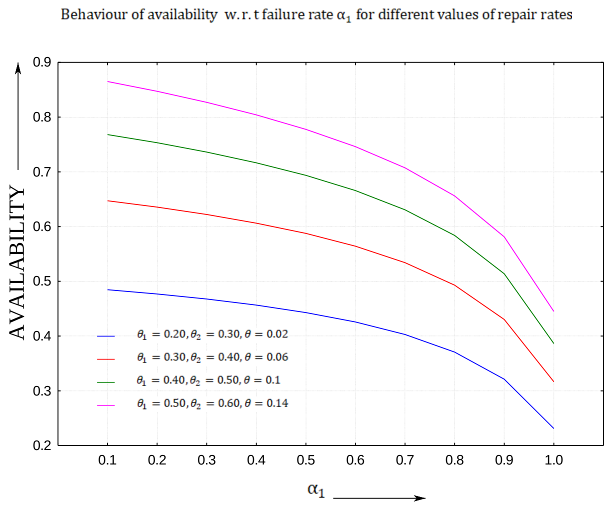


Fig.: 3

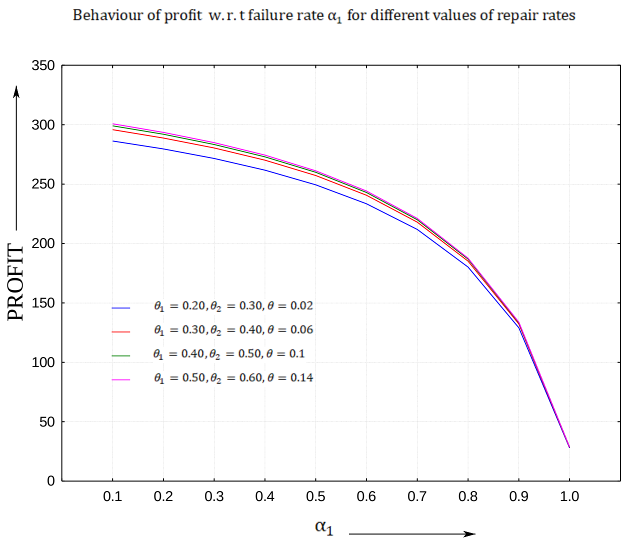


Fig.: 4

From Fig 2, 3 & 4, we have observed that MTSF, availability and profit function respectively, decreases with the increase in the failure rate of the system and these characteristics shows an increase, as we increase the repair rate of the system. Therefore, we can conclude here that the expected lifetime of the system can be increased by providing the proper repair facility to the system, as regular repair of the units improves the reliability and effectiveness of the system.

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