Reliability Modelling and Assessment of Multi Standby Hybrid System

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Abstract

Systems connected to an external supporting device for their operations viewed as hybrid systems have been manufactured to meet the demand of industries, economic growth and populace in general. Companies and organizations heavily rely on these systems to conduct their business. The paper deals with the reliability and availability characteristics of four different systems requiring external supporting device for their operation. The system consists of main unit connected to the cold standby supporting devices. The failure and time of both main unit and supporting device are assumed to be exponentially distributed. Markov models are developed and differential difference equations are derived to obtain explicit expressions for the steady-state availability and mean time to failure and perform analytical and numerical comparisons. Comparisons show that system with five cold standby supporting devices is the most reliable system.

Keywords: Availability, mean time to failure, supporting device, single unit

Mathematics Subject Classification (2000): 90B25

1. Introduction

High system reliability and availability play a vital role towards industrial growth as the profit is directly dependent on production volume which depends upon system performance. Thus the reliability and availability of a system may be enhanced by proper design, optimization at the design stage and by maintaining the same during its service life. Because of their prevalence in power plants, manufacturing systems, and industrial systems, many researchers have studied reliability and availability problem of different systems. Hajeeh (2012) deals with availability of a system with different repair options. Hu et al. (2012) presents availability analysis and design optimisation for a repairable series-parallel system with failure dependencies. Jain and Rani (2013) studied the availability analysis for repairable system with warm standby, switching failure and reboot delay. Wang et al. (2012) performed comparative analysis of availability between two systems with warm standby units and different imperfect coverage. Wang and Chen (2009) performed comparative analysis of availability between three systems with general repair times, reboot delay and switching failures.

In real-life situations we often encounter cases where the systems that cannot work without the help of external supporting devices connect to such systems. These external supporting devices are systems themselves that are bound to fail. Such systems are found in power plants, manufacturing systems, and industrial systems. Large volumes of literature exist on the issue relating to prediction of various systems performance connected to an external supporting device for their operations. Yusuf (2014) performed comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation. Yusuf et al (2016) performed reliability computation of a linear consecutive 2-out-of-3 system in the presence of supporting device. Yusuf (2016) presents reliability evaluation of a parallel system with a supporting device and two types of preventive maintenance. The problem considered in this paper is different from the work of discussed authors above. In this paper, a single unit system connected to cold standby external supporting device is considered. The objectives of this paper are: to derive the explicit expressions for the availability and mean time to failure, to determine the optimal system. The organization of the paper is as follows. Section 2 contains a description of the system under study. Section 3 presents formulations of the models. The results of our analytical comparison between the systems are presented in section 4. Numerical examples are presented in section 5. Finally, we make some concluding remarks in Section 6.

2. Description and States of the System

In this paper, a single unit system connected to an external cold standby supporting devices is considered. It is assumed that the system most work with one supporting device. System I has main unit with five cold standby supporting devices, system II has four cold standby supporting devices, system III has three cold standby supporting devices, system IV has two cold standby supporting devices. It is also assumed that the switching from standby to operation is perfect. Both the unit and supporting devices are assumed to be repairable. Each of the primary supporting devices fails independently of the state of the other and has an exponential failure distribution with parameter λ_1. Whenever a primary supporting device fails, it is immediately sent to repair with parameter μ_1 and a standby supporting device is switch to operation. System failure occurs when the unit has failed with parameter λ_0 and it is sent for repair with parameter with parameter μ_0 or the failure of all supporting device.

Table 1: Transition rate table

	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_0	0	λ_1	0	0	0	λ_0	0	0	0	0	0
S_1	μ_1	0	λ_1	0	0	0	0	λ_0	0	0	0
S_2	0	μ_1	0	λ_1	0	0	0	0	λ_0	0	0
S_3	0	0	μ_1	0	λ_1	0	0	0	0	λ_0	0
S_4	0	0	0	μ_1	0	0	λ_1	0	0	0	λ_0
S_5	μ_0	0	0	0	0	0	0	0	0	0	0
S_6	0	0	0	0	μ_1	0	0	0	0	0	0
S_7	0	μ_0	0	0	0	0	0	0	0	0	0
S_8	0	0	μ_0	0	0	0	0	0	0	0	0
S_9	0	0	0	μ_0	0	0	0	0	0	0	0
S_{10}	0	0	0	0	0	0	0	0	0	0	0

3. Formulation of the Models

In order to analyse the system availability of system I, we define $P_i(t)$ to be the probability that the system at $t \geq 0$ is in state $S_i, i = 0,1,2,3,\ldots,10$. Also let $P(t)$ be the row vector of these probabilities at time t. The initial condition for this problem is:

$$P(0) = [p_0(0), p_1(0), p_2(0), \ldots, p_{10}(0)] = [1,0,0,0,0,0,0,0,0,0,0]$$

Following Trivedi (2007), Wang et al. (2000), and Wang et al. (2006), we obtain the following differential equations from Figure 1:

$$\begin{aligned}
&p_0{}' = (\lambda_0 + \lambda_1)p_0(t) + \mu_1 p_1(t) + \mu_0 p_5(t)\\
&p_1{}'(t) = -(\lambda_0 + \lambda_1 + \mu_1)p_1(t) + \lambda_1 p_0(t) + \mu_1 p_2(t) + \mu_0 p_7(t)\\
&p_2{}'(t) = -(\lambda_0 + \lambda_1 + \mu_1)p_2(t) + \lambda_1 p_1(t) + \mu_1 p_3(t) + \mu_0 p_8(t)\\
&p_3{}'(t) = -(\lambda_0 + \lambda_1 + \mu_1)p_3(t) + \lambda_1 p_2(t) + \mu_1 p_4(t) + \mu_0 p_9(t)\\
&p_4{}'(t) = -(\lambda_0 + \lambda_1 + \mu_1)p_4(t) + \lambda_1 p_3(t) + \mu_1 p_6(t) + \mu_0 p_{10}(t)\\
&p_5{}'(t) = -\mu_0 p_1(t) + \lambda_0 p_0(t)\\
&p_6{}'(t) = -\mu_1 p_6(t) + \lambda_1 p_4(t)\\
&p_7{}'(t) = -\mu_0 p_7(t) + \lambda_0 p_1(t)\\
&p_8{}'(t) = -\mu_0 p_8(t) + \lambda_0 p_2(t)\\
&p_9{}'(t) = -\mu_0 p_9(t) + \lambda_0 p_3(t)\\
&p_{10}{}'(t) = -\mu_0 p_{10}(t) + \lambda_0 p_4(t)
\end{aligned} \tag{1}$$

This can be written in the matrix form as

$$\boldsymbol{P}' = \boldsymbol{QP}, \tag{2}$$

where

$$Q = \begin{pmatrix}
-(\lambda_0+\lambda_1) & \mu_1 & 0 & 0 & 0 & \mu_0 & 0 & 0 & 0 & 0 & 0\\
\lambda_1 & -(\lambda_0+\lambda_1+\mu_1) & \mu_1 & 0 & 0 & 0 & 0 & \mu_0 & 0 & 0 & 0\\
0 & \lambda_1 & -(\lambda_0+\lambda_1+\mu_1) & \mu_1 & 0 & 0 & 0 & 0 & \mu_0 & 0 & 0\\
0 & 0 & \lambda_1 & -(\lambda_0+\lambda_1+\mu_1) & \mu_1 & 0 & 0 & 0 & 0 & \mu_0 & 0\\
0 & 0 & 0 & \lambda_1 & -(\lambda_0+\lambda_1+\mu_1) & 0 & \mu_1 & 0 & 0 & 0 & \mu_0\\
\lambda_0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \lambda_1 & 0 & -\mu_1 & 0 & 0 & 0 & 0\\
0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0 & 0\\
0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0 & 0\\
0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0 & 0\\
0 & 0 & 0 & 0 & \lambda_0 & 0 & 0 & 0 & 0 & 0 & -\mu_0
\end{pmatrix}$$

Let T denote the time-to-failure of the system. We use the following procedure to develop the steady-state availability. The steady-state availability (the proportion of time the system is in a functioning condition or equivalently, the sum of the probabilities of operational states) is given by

$$A_{T1}(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) + p_4(\infty) = \frac{\mu_0\mu_1^5+\mu_0\mu_1^4\lambda_1+\mu_0\mu_1^3\lambda_1^2+\mu_0\mu_1^2\lambda_1^3+\mu_0\mu_1\lambda_1^4}{\mu_1\lambda_0\lambda_1^4+\mu_1^2\lambda_0\lambda_1^3+\mu_1^3\lambda_0\lambda_1^2+\mu_1^4\lambda_0\lambda_1+\mu_1^5\lambda_0+\mu_0\lambda_1^5+\mu_0\mu_1\lambda_1^4+\mu_0\mu_1^2\lambda_1^3+\mu_0\mu_1^3\lambda_1^2+\mu_0\mu_1^4\lambda_1+\mu_0\mu_1^5} \tag{3}$$

To develop the explicit expression for mean time to failure, we use the concept of Trivedi (2002), Wang and Kuo (2000) and Wang et al. (2006) as follows:

The procedures require deleting rows and columns of absorbing states of matrix Q and take the transpose to produce a new matrix, say M. The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = P(0)(-M^{-1})(1,1,1,1,1)^t \tag{4}$$

where the initial conditions are given by $P(0) = [p_0(0), p_1(0), p_2(0), p_3(0), p_4(0)] = [1,0,0,0,0]$ and

$$M = \begin{pmatrix} -(\lambda_0+\lambda_1) & \lambda_1 & 0 & 0 & 0 \\ \mu_1 & -(\lambda_0+\lambda_1+\mu_1) & \lambda_1 & 0 & 0 \\ 0 & \mu_1 & -(\lambda_0+\lambda_1+\mu_1) & \lambda_1 & 0 \\ 0 & 0 & \mu_1 & -(\lambda_0+\lambda_1+\mu_1) & \lambda_1 \\ 0 & 0 & 0 & \mu_1 & -(\lambda_0+\lambda_1+\mu_1) \end{pmatrix}$$

The explicit expression for is given by $MTTF_1$

$$MTTF_1 = \frac{N_1}{H_1} \quad (5)$$

where

$$N_1 = (\mu_1^4 + 4\lambda_0\mu_1^3 + \lambda_1\mu_1^3 + \lambda_1^2\mu_1^2 + 6\lambda_0\lambda_1\mu_1^2 + 6\lambda_0^2\mu_1^2 + 6\lambda_0\lambda_1^2\mu_1 + \lambda_1^3\mu_1 + 4\lambda_0^3\mu_1 + 9\lambda_0^2\lambda_1\mu_1 + \lambda_0^4 + 4\lambda_0\lambda_1^3 + \lambda_1^4 + 6\lambda_0^2\lambda_1^2) + \lambda_1(\lambda_0^3 + 3\lambda_0^2\lambda_1 + 3\lambda_0^2\mu_1 + 3\lambda_0\lambda_1^2 + 4\lambda_0\lambda_1\mu_1 + 3\lambda_0\mu_1^2 + \lambda_1^3 + \lambda_1^2\mu_1 + \lambda_1\mu_1^2 + \mu_1^3) + \lambda_1^2(\lambda_0^2 + 2\lambda_0\lambda_1 + 2\lambda_0\mu_1 + \lambda_1^2 + \lambda_1\mu_1 + \mu_1^2) + \lambda_1^3(\lambda_0 + \lambda_1 + \mu_1)$$

$$H_1 = (\lambda_0\mu_1^4 + 2\lambda_0\lambda_1\mu_1^3 + 4\lambda_0^2\mu_1^3 + 6\lambda_0^3\mu_1^2 + 3\lambda_0\lambda_1^2\mu_1^2 + 9\lambda_0^2\lambda_1\mu_1^2 + 4\lambda_0\lambda_1^3\mu_1 + 12\lambda_0^3\lambda_1\mu_1 + 12\lambda_0^2\lambda_1^2\mu_1 + 4\lambda_0^4\mu_1 + 10\lambda_0^3\lambda_1^2 + \lambda_0^5 + \lambda_1^5 + 5\lambda_0^4\lambda_1 + 10\lambda_0^2\lambda_1^3 + 5\lambda_0\lambda_1^4)$$

Special Cases:

Case I: Availability and mean time to failure of system requiring four cold standbys supporting devices

$$A_{T2}(\infty) = \frac{\mu_0\mu_1^4+\mu_0\mu_1^3\lambda_1+\mu_0\mu_1^2\lambda_1^2+\mu_0\mu_1\lambda_1^3}{\mu_1\lambda_0\lambda_1^3+\mu_1^2\lambda_0\lambda_1^2+\mu_1^3\lambda_0\lambda_1+\mu_1^4\lambda_0+\mu_0\lambda_1^4+\mu_0\mu_1\lambda_1^3+\mu_0\mu_1^2\lambda_1^2+\mu_0\mu_1^3\lambda_1+\mu_0\mu_1^4} \quad (6)$$

$$MTTF_2 = \frac{N_2}{H_2} \quad (7)$$

where

$$N_2 = (\lambda_0^3 + 3\lambda_0^2\lambda_1 + 3\lambda_1^2\mu_1 + 3\lambda_0\lambda_1^2 + 4\lambda_0\lambda_1\mu_1 + 3\lambda_0\mu_1^2 + \lambda_1^3 + \lambda_1^2\mu_1 + \lambda_1\mu_1^2 + \mu_1^3) + \lambda_1(\lambda_0^2 + 2\lambda_0\lambda_1 + 2\lambda_0\mu_1 + \lambda_1^2 + +\lambda_1\mu_1 + \mu_1^2) + \lambda_1^2(\lambda_0 + \lambda_1 + \mu_1) + \lambda_1^3$$

$$H_2 = \lambda_0^4 + 4\lambda_0^3\lambda_1 + 3\lambda_0^3\mu_1 + 6\lambda_0^2\lambda_1^2 + 6\lambda_0^2\lambda_1\mu_1 + 3\lambda_0^2\mu_1^2 + 4\lambda_0\lambda_1^3 + 3\lambda_0\lambda_1^2\mu_1 + 2\lambda_0\lambda_1\mu_1^2 + \lambda_0\mu_1^2 + \lambda_1^4$$

Case II: Availability and mean time to failure of system requiring three cold standbys supporting devices

$$A_{T3}(\infty) = \frac{\mu_0\mu_1^3+\mu_0\mu_1^2\lambda_1+\mu_0\mu_1\lambda_1^2}{\mu_1\lambda_0\lambda_1^2+\mu_1^2\lambda_0\lambda_1+\mu_1^3\lambda_0+\mu_0\lambda_1^3+\mu_0\mu_1\lambda_1^2+\mu_0\mu_1^2\lambda_1+\mu_0\mu_1^3} \quad (8)$$

$$MTTF_3 = \frac{\lambda_0^2+2\lambda_0\lambda_1+2\lambda_0\mu_1+2\lambda_1^2+\lambda_1\mu_1+\mu_1^2+\lambda_1(\lambda_0+\lambda_1+\mu_1)}{\lambda_0^3+3\lambda_0^2\lambda_1+2\lambda_0^2\mu_1+3\lambda_0\lambda_1^2+2\lambda_0\lambda_1\mu_1+\lambda_0\mu_1^2+\lambda_1^3} \quad (9)$$

Case III: Availability and mean time to failure of system requiring two cold standbys supporting devices

$$A_{T4}(\infty) = \frac{\mu_0\mu_1^2+\mu_0\mu_1\lambda_1}{\mu_1\lambda_0\lambda_1+\mu_1^2\lambda_0+\mu_0\lambda_1^2+\mu_0\mu_1\lambda_1+\mu_0\mu_1^2} \quad (10)$$

$$MTTF_4 = \frac{2\lambda_1+\lambda_0+\mu_1}{\lambda_0^2+2\lambda_0\lambda_1+\lambda_0\mu_1+\lambda_1^2} \quad (11)$$

4. Comparison between the systems

MAPLE software package was used to program the analytical comparison in this study. The results are presented below.

$$A_{T1}(\infty) - A_{T2}(\infty) = \frac{\mu_0^2\mu_1^5\lambda_1^4}{D_1D_2} \quad (12)$$

$$A_{T2}(\infty) - A_{T3}(\infty) = \frac{\mu_0^2\mu_1^4\lambda_1^3}{D_2D_3} \quad (13)$$

$$A_{T3}(\infty) - A_{T4}(\infty) = \frac{\mu_0^2\mu_1^3\lambda_1^2}{D_3D_4} \quad (14)$$

$$MTTF_1 - MTTF_2 = \frac{\lambda_1^4(\lambda_0^4 + 4\lambda_0^3\lambda_1 + 4\lambda_0^3\mu_1 + 6\lambda_0^2\lambda_1^2 + 9\lambda_0^2\lambda_1\mu_1 + 6\lambda_0^2\mu_1^2 + 4\lambda_0\lambda_1^3 + 6\lambda_0\lambda_1^2\mu_1 + 6\lambda_0\lambda_1\mu_1^2 + 4\lambda_0\mu_1^3 + \lambda_1^4 + \lambda_1^3\mu_1 + \lambda_1^2\mu_1^2 + \lambda_1\mu_1^3 + \mu_1^4)}{H_1H_2} \quad (15)$$

$$MTTF_2 - MTTF_3 = \frac{\left(\lambda_1^3(\lambda_0^3+3\lambda_0^2\lambda_1+3\lambda_0^2\mu_1+3\lambda_0\lambda_1^2+4\lambda_0\lambda_1\mu_1+3\lambda_0\mu_1^2+\lambda_1^3+\lambda_1^2\mu_1+\lambda_1\mu_1^2+\mu_1^3)\right)}{H_2(\lambda_0^3+3\lambda_0^2\lambda_1+2\lambda_0^2\mu_1+3\lambda_0\lambda_1^2+2\lambda_0\lambda_1\mu_1+\lambda_0\mu_1^2+\lambda_1^3)} \quad (16)$$

$$MTTF_3 - MTTF_4 = \frac{\lambda_1^2(\lambda_0^2+3\lambda_0^2\lambda_1+3\lambda_0^2\mu_1+3\lambda_0\lambda_1^2+4\lambda_0\lambda_1\mu_1+3\lambda_0\mu_1^2+\lambda_1^3+\lambda_1^2\mu_1+\lambda_1\mu_1^2+\mu_1^3)}{(\lambda_0^3+3\lambda_0^2\lambda_1+2\lambda_0^2\mu_1+3\lambda_0\lambda_1^2+2\lambda_0\lambda_1\mu_1+\lambda_0\mu_1^2+\lambda_1^3)(\lambda_0^2+2\lambda_0\lambda_1+\lambda_0\mu_1+\lambda_1^2)} \quad (17)$$

where

$$D_4 = \mu_1\lambda_0\lambda_1 + \mu_1^2\lambda_0 + \mu_0\lambda_1^2 + \mu_0\mu_1\lambda_1 + \mu_0\mu_1^2$$
$$D_3 = \mu_1\lambda_0\lambda_1^2 + \mu_1^2\lambda_0\lambda_1 + \mu_1^3\lambda_0 + \mu_0\lambda_1^3 + \mu_0\mu_1\lambda_1^2 + \mu_0\mu_1^2\lambda_1 + \mu_0\mu_1^3$$
$$D_2 = \mu_1\lambda_0\lambda_1^3 + \mu_1^2\lambda_0\lambda_1^2 + \mu_1^3\lambda_0\lambda_1 + \mu_1^4\lambda_0 + \mu_0\lambda_1^4 + \mu_0\mu_1\lambda_1^3 + \mu_0\mu_1^2\lambda_1^2 + \mu_0\mu_1^3\lambda_1 + \mu_0\mu_1^4$$
$$D_1 = \mu_1\lambda_0\lambda_1^4 + \mu_1^2\lambda_0\lambda_1^3 + \mu_1^3\lambda_0\lambda_1^2 + \mu_1^4\lambda_0\lambda_1 + \mu_1^5\lambda_0 + \mu_0\lambda_1^5 + \mu_0\mu_1\lambda_1^4 + \mu_0\mu_1^2\lambda_1^3 + \mu_0\mu_1^3\lambda_1^2 + \mu_0\mu_1^4\lambda_1 + \mu_0\mu_1^5$$

From (12) to (17)

$$A_{T1}(\infty) > A_{T2}(\infty) > A_{T3}(\infty) > A_{T4}(\infty)$$
$$MTTF_1(\infty) > MTTF_2(\infty) > MTTF_3(\infty) > MTTF_4(\infty)$$
$$\forall \lambda_0, \lambda_1, \mu_0, \mu_1 > 0$$

5. Numerical example

Numerical examples are presented to demonstrate the impact of repair and failure rates on steady-state availability and mean time to failure of the system based on given values of the parameters. MATLAB software package was used to program the numerical comparison in this study. The results are presented below. For the purpose of numerical example, the following sets of parameter values are used: $\lambda_1 = 0.3$, $\lambda_0 = 0.2$, $\mu_1 = 0.6$, $\mu_1 = 0.6$ for Figures 2 and 3 and $\lambda_1 = 0.4$, $\lambda_0 = 0.1$, $\mu_1 = 0.05$ for Figures 4 and 5.

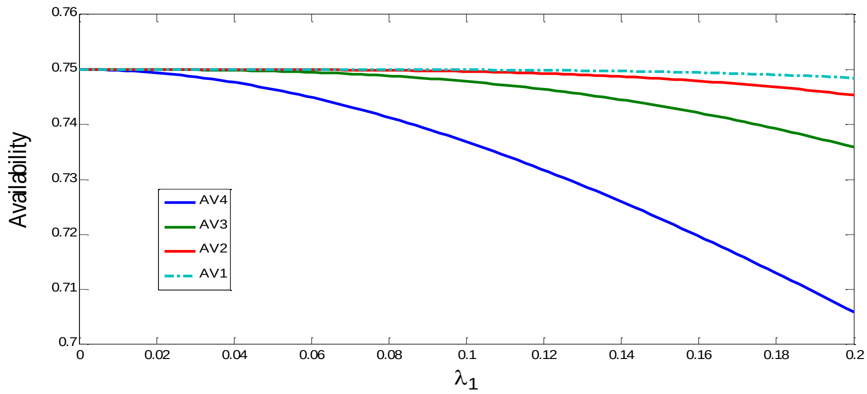

Figure 1: Availability against supporting device failure rate λ_1

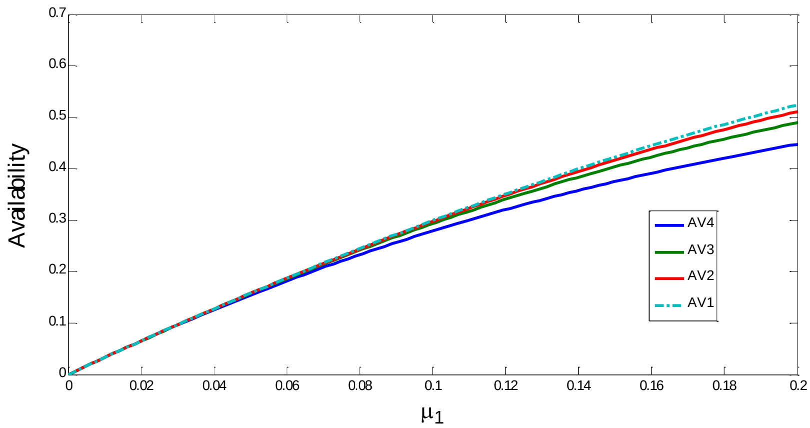

Figure 2: Availability against supporting device repair rate μ_1

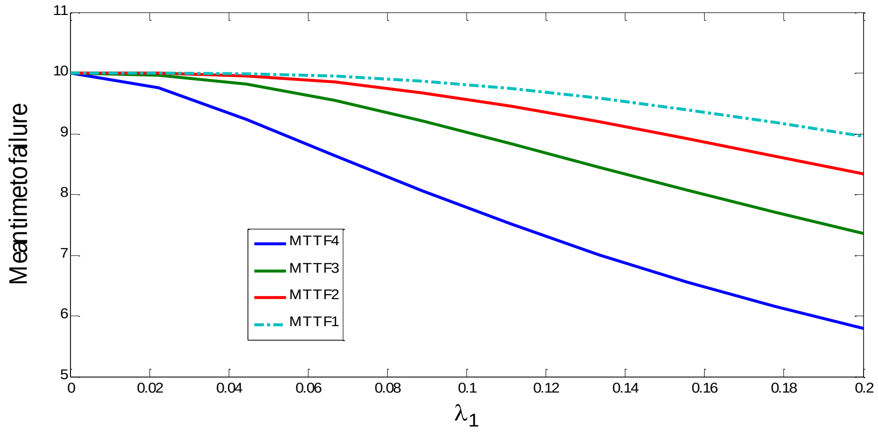

Figure 3: mean time to failure against supporting device failure rate λ_1

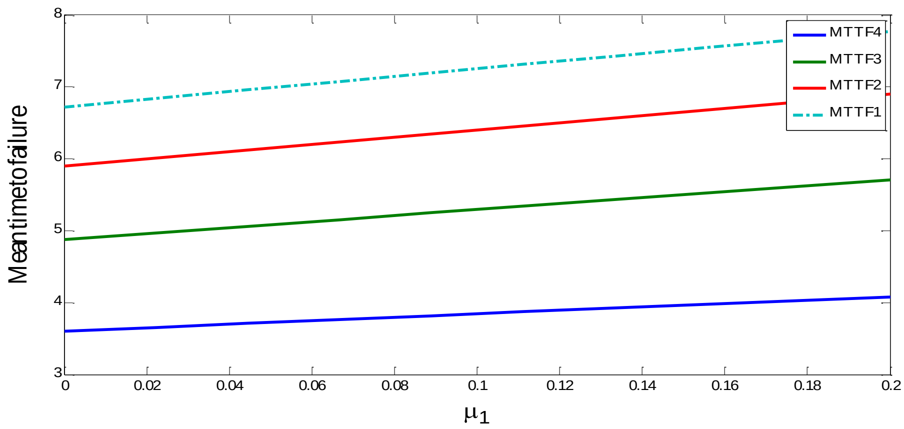

Figure 4: mean time to failure against supporting device repair rate μ_1

Figures 1 and 3 show the results availability and mean time to failure for the four systems against the failure rate λ_1. It is clear from the figures that system I (system with five standby supporting device) has higher availability and mean time to failure as compared to the other three systems. Similar observation is also depicted in Figures 2 and 4 with respect to repair rate μ_1. It is evident from these figures that system I (system with five standby supporting device) has higher availability and mean time to failure as compared to the other three systems. These tend to suggest that system I is better than the other systems.

Conclusion

This paper studied a single system connected to two types of supporting device type I and II for its operation. Explicit expression for the steady-state availability was derived. Comparative analysis was performed analytically along with numerically example in this study. It is enough to mention first that the optimal system is system with five cold standbys supporting devices.

Thus,

$$A_{T1}(\infty) > A_{T2}(\infty) > A_{T3}(\infty) > A_{T4}(\infty)$$
$$MTTF_1(\infty) > MTTF_2(\infty) > MTTF_3(\infty) > MTTF_4(\infty)$$

On the basis of the analytical and numerical results obtained f, it is suggested that the system reliability can be improved significantly by:

(i) Adding more cold standby units.
(ii) Increasing the repair rate.
(iii) Reducing the failure rate of the system by hot or cold duplication method.
(iv) Exchange the system when old with new one before failure.

The system can further be developed into system with more standbys in solving reliability and availability problems.

The present study is important to system designers, engineers, maintenance managers and plant management for proper maintenance analysis, decision and safety of the system as a whole. The study will also assist engineers, decision makers and plant management to avoid an incorrect reliability assessment and consequent erroneous decision-making, which may lead to unnecessary expenditures, incorrect maintenance scheduling and reduction of safety standards.

Conflict of Interests

The authors declare that there is no conflict of interests.

References

1. Hajeeh, M. (2012) 'Availability of a system with different repair options', International Journal of Mathematics in Operational Research, Vol. 4, No. 1, pp.41–55.
2. Hu, L., Yus, D. and Li, J. (2012) Availability analysis and design optimization for a repairable series-parallel system with failure dependencies, International Journal of Innovative Computing, Information and Control, Vol. 8, No. 10, pp.6693–6705.
3. Jain, M. and Rani, S. (2013) 'Availability analysis for repairable system with warm standby, switching failure and reboot delay', Int. J. of Mathematics in Operational Research, Vol. 5, No. 1, pp.19–39.
4. Trivedi, K. S. Probability and Statistics with Reliability, Queueing and Computer Science Applications, 2nd ed., John Wiley and Sons, New York, 2002.
5. Wang, K-H. and Chen, Y.J. (2009) 'Comparative analysis of availability between three systems with general repair times, reboot delay and switching failures', Applied Mathematics and Computation, Vol. 2, No. 15, pp.384–394.
6. Wang, K.-H. and Kuo, C.-C.(2000). Cost and probabilistic analysis of series systems with mixed standby components. Applied Mathematical Modelling, 24, 957-967.
7. Wang, K.-H. and Ke, J.-C. (2003). Probabilistic analysis of a repairable system with warm standbys plus balking and reneging. Applied Mathematical Modelling, 27, 327-336.
8. Wang, K-H., Yen, T-C. and Fang, Y-C. (2012) 'Comparison of availability between two systems with warm standby units and different imperfect coverage', Quality Technology and Quantitative Management, Vol. 9, No. 3, pp.265–282.
9. Yusuf, I. (2014). Comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation, Journal of Industrial Engineering International, 10(77), 1-9. DOI 10.1007/s40092-014-0077-3.
10. Yusuf, I., Babagana, M., Yusuf, B and Lawan, A.M. (2016). Reliability analysis of a linear consecutive 2-out-of-3 system in the presence of supporting device and repairable service station, International Journal of Operations Research Vol. 13, No. 1, 13◎24 .
11. Yusuf, I. (2016). Reliability modelling of a parallel system with a supporting device and two types of preventive maintenance, Int. J. Operational Research, Vol. 25, No. 3,269-287.