

Fuzzy Reliability of a System by Converting Trapezoidal Intervalued Fuzzy Number to Pentagonal Triangular Intervalued Fuzzy Number

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Abstract

In classical set theory there exist only two possibility of any element belonging to the set yes or no, that is its probability of belonging to the set either 0 or 1, but this theory is fail to predictable in many system where the possibility of an element belonging to set is not exact, that is there exist some vagueness about the element affecting the system. Therefore L. A. Zadeh gives a new theory of fuzzyness, where the belongingness of an element can except 0 or 1 and take any value between $[0, 1]$. This new approach give us much benefit to modelling the real situation and find the reliability of any system. This theory also useful to find the most critical event in any fault tree model. Fuzzy theory are applicable in many areas industrial, technical, engineering, medical etc.

Keywords: Healthcare system, Fault tree, Pentagonal-triangular intervalued fuzzy numbers, α - cut, signed distance, COG.

I. Introduction

In this article we consider a new intervalued pentagonal-triangular fuzzy number with the help of converting intervalued trapezoidal fuzzy numbers and find out reliability of mixed system. According to WHO (World Health Organization) 'Healthcare system goals are good health for citizens responsiveness to the expectation of the population and fair means of funding operation. Other dimension for the evaluation of health system include quality, efficiency, acceptability and equity. Butnariu developed a neuron model with the help of fuzzy analysis, Acoustico-vestibulary nerve as a fuzzy automation describe with this help. Similarly Rocha has been developed nervous system using fuzzy logic. The most extensive application of fuzzy theory in the area of medical diagnosis, in diagnosis process we mapped symptoms with diseases, the relation between symptom and disease are imprecise due to various stage of disease We know that in healthcare system there are many uncertainty, To determine reliability of whole system we use COG and signed distance method as defuzzification. Here we use interval valued fuzzy numbers which

belongs to (λ, ρ) , $0 < \lambda < \rho$. This study used level (λ, ρ) interval valued fuzzy numbers to determine fuzzy reliability of mixed healthcare system. Fault tree analysis (FTA) have been applied for patient safety risk modelling in healthcare [1-2],[7],[10],[12]. Fault tree analysis also extensively used as a powerful technique in health related risk analysis from both qualitative and quantitative perspectives [2],[6],[12]. Hyman and Johnson [9] present a FTA of the patient harm-related clinical alarms failures. Park and Lee [2] constructed a FTA of hand washing process to investigate the causes for faults in hygiene management, the possibility of failure of the top event is calculated from the possibilities of failure of its components according to the extension principle [3],[6].

II. Fuzzy Sets

A fuzzy set is defined by a membership function from the universal set to the interval [0,1], as given below;

$$\mu_A(x): X \rightarrow [0,1] \quad (1)$$

, here $\mu_A(x)$ gives the degree of belongingness of x in the set A. A fuzzy set A can be expressed as follows:

$$\tilde{A} = \{(x, \mu_A(x)): x \in X\} \quad (2)$$

III. Level (λ, ρ) Inter-Valued trapezoidal Fuzzy Numbers

The i-v fuzzy set \tilde{A} indicates that, when the membership grade of x belongs to the interval $[\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)]$ the largest grade is $\mu_{\tilde{A}^U}(x)$ and the smallest grade is $\mu_{\tilde{A}^L}(x)$

$$\mu_{\tilde{A}^L}(x) = \begin{cases} \frac{\lambda(x-b)}{c-b} & b \leq x \leq c \\ \lambda & c \leq x \leq d \\ \frac{\lambda(e-x)}{e-d} & d \leq x \leq e \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Therefore, $\tilde{A}^L = (b, c, d, e; \lambda) \mid b < c < d < e$

$$\mu_{\tilde{A}^U}(x) = \begin{cases} \frac{\rho(x-a)}{c-a} & a \leq x \leq c \\ \rho & c \leq x \leq d \\ \frac{\rho(f-x)}{f-d} & d \leq x \leq f \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Therefore $\tilde{A}^U = (a, c, d, f; \rho)$, $a < c < d < f$, Consider the case in which $0 < \lambda \leq \rho \leq 1$ and $a < b < c < d < e < f$.

From (3) and (4) we obtain $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] \mid [b, c, d, e; \lambda], (a, c, d, f; \rho) \mid [b, c, d, e; \lambda], (a, c, d, f; \rho)$, Which is called the level (λ, ρ) i - v trapezoidal fuzzy number. The intervalued trapezoidal fuzzy numbers shown in fig1.

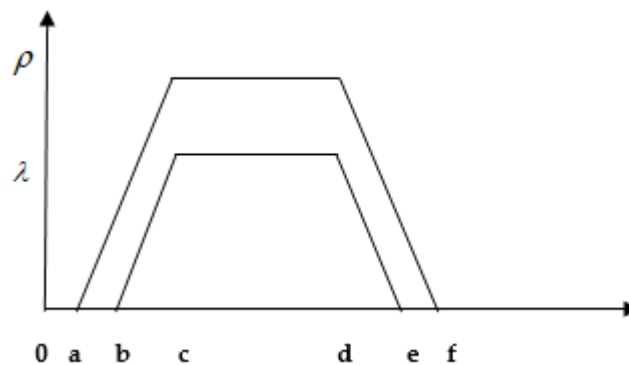


Fig1. Intervalued trapezoidal fuzzy numbers

IV. Level (λ, ρ) Inter-Valued Pentagonal-Triangular Fuzzy Numbers

The i-v pentagonal-triangular fuzzy numbers indicates that, when the membership grade of x belongs to the interval $[\mu_{\tilde{A}^L}(x), \mu_{\tilde{A}^U}(x)]$ the largest grade is $\mu_{\tilde{A}^U}(x)$ and the smallest grade is $\mu_{\tilde{A}^L}(x)$ is given by following equations.

$$\mu_{\tilde{A}^L}(x) = \begin{cases} \frac{\lambda(x-a)}{c-a} & a \leq x \leq c \\ \frac{\lambda(e-x)}{e-c} & c \leq x \leq e \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Therefore, $\tilde{A}^L = (a, c, e; \lambda) \ a < c < e$

$$\mu_{\tilde{A}^U}(x) = \begin{cases} \frac{\lambda(x-a)}{b-a} & a \leq x \leq b \\ \lambda + \frac{x-b}{c-b}(\rho - \lambda) & b \leq x \leq c \\ \lambda + \frac{x-d}{c-d}(\rho - \lambda) & c \leq x \leq d \\ \frac{\lambda(e-x)}{e-d} & d \leq x \leq e \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

Therefore $\tilde{A}^U = (a, b, c, d, e; \rho), a < b < c < d < e$. Consider the case in which $0 < \lambda \leq \rho \leq 1$ and $a < b < c < d < e$. from (5) and (6) we obtain $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a, c, e; \lambda), (a, b, c, d, e; \rho)]$, Which is called the level (λ, ρ) i - v pentagonal-triangular intervalued fuzzy numbers.

The intervalued pentagonal-triangular fuzzy numbers is shown in fig2. $A_i^U(\alpha)$ indicate left upper α - cut $A_l^L(\alpha)$ for left lower α - cut, $A_r^L(\alpha)$ for right lower α - cut and $A_r^U(\alpha)$ indicate right upper α - cut.

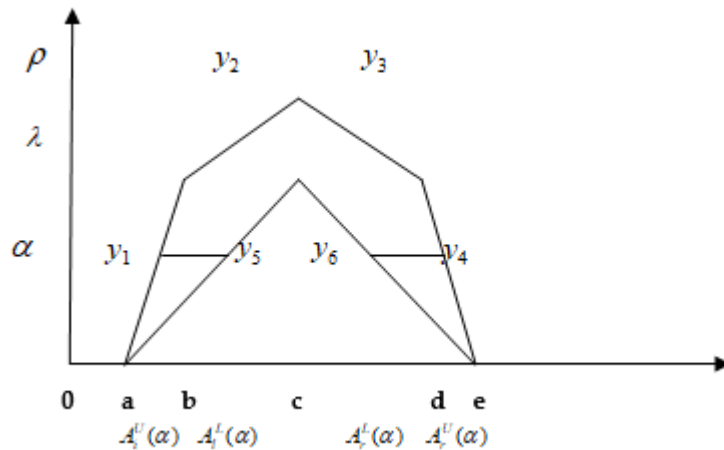


Fig2. Intervalled pentagonal-triangular fuzzy numbers

Corresponding to each curve, the x coordinate corresponding to α - cut and y coordinate given by

$$\begin{aligned}
 x_1 &= a + \frac{\alpha}{\lambda}(b - a) & y_1 &= \lambda \left(\frac{x-a}{b-a} \right) \\
 x_2 &= b + \frac{\alpha-\lambda}{\rho-\lambda}(c - b) & y_2 &= \left\{ \lambda + \frac{x-b}{c-b}(\rho - \lambda) \right\} \\
 x_3 &= d + \frac{\alpha-\lambda}{\rho-\lambda}(c - d) & y_3 &= \left\{ \lambda + \frac{x-d}{c-d}(\rho - \lambda) \right\} \\
 x_4 &= e + \frac{\alpha}{\lambda}(d - e) & y_4 &= \lambda \left(\frac{x-e}{d-e} \right) \\
 x_5 &= a + \frac{\alpha}{\lambda}(c - a) & y_5 &= \lambda \left(\frac{x-a}{c-a} \right) \\
 x_6 &= e + \frac{\alpha}{\lambda}(c - e) & y_6 &= \lambda \left(\frac{x-e}{c-e} \right)
 \end{aligned} \tag{7}$$

V. α - cut and Signed Distance of Pentagonal Triangular Intervalled Fuzzy Numbers [11]:

if $0 \leq \alpha < \lambda$ then α cut of \tilde{A} is $A(\alpha) = \{x/\mu_{\tilde{A}^U}(x) \geq \alpha\} - \{x/\mu_{\tilde{A}^L}(x) \geq \alpha\} = [A_l^U(\alpha), A_l^L(\alpha)] \cup [A_r^L(\alpha), A_r^U(\alpha)]$; Otherwise, for $\lambda \leq \alpha \leq \rho$, the α cut of \tilde{A} is $[A_l^U(\alpha), A_r^U(\alpha)]$

$$d^*(a, 0) = A_l^U(\alpha), d^*(A_l^L(\alpha), 0) = A_l^L(\alpha), d^*(A_r^L(\alpha), 0) = A_r^L(\alpha), d^*(A_r^U(\alpha), 0) = A_r^U(\alpha).$$

Therefore the signed distance [11] of the interval $[A_l^U(\alpha), A_l^L(\alpha)]$ from 0 can be defined as follows:

$$d^*([A_l^U(\alpha), A_l^L(\alpha)], 0) = \frac{1}{2}(d^*(A_l^U(\alpha), 0) + d^*(A_l^L(\alpha), 0)) = \frac{1}{2}(A_l^U(\alpha) + A_l^L(\alpha)) = \frac{1}{2} \left[a + (b - a) \frac{\alpha}{\lambda} + a + (c - a) \frac{\alpha}{\rho} \right] = \frac{1}{2} \left[2a + \frac{\alpha}{\lambda}(b + c - 2a) \right] \tag{8}$$

$$\text{Similarly } d^*([A_r^L(\alpha), A_r^U(\alpha)], 0) = \frac{1}{2} \left[e + \frac{\alpha}{\lambda}(d - e) + e + \frac{\alpha}{\lambda}(c - e) \right] = \frac{1}{2} \left[2e + \frac{\alpha}{\lambda}(d + c - 2e) \right]$$

(9)

When $[A_l^U(\alpha), A_l^L(\alpha)] \cap [A_r^L(\alpha), A_r^U(\alpha)] = \Phi$, the signed distance of $[A_l^U(\alpha), A_l^L(\alpha)] \cup [A_r^L(\alpha), A_r^U(\alpha)]$ from 0 can be defined as $d^*([A_l^U(\alpha), A_l^L(\alpha)] \cup [A_r^L(\alpha), A_r^U(\alpha)], 0) = \frac{1}{2} [d^*([A_l^U(\alpha), A_l^L(\alpha)], 0) + d^*([A_r^L(\alpha), A_r^U(\alpha)], 0)]$

$$= \frac{1}{4} \left[2(a + e) + \frac{\alpha}{\lambda}(d + 2c - 2a + b - 2e) \right]$$

(10)

For $\lambda \leq \alpha \leq \rho$, then the signed distance from \tilde{A} to 0 is

$$d([A_l^U(\alpha), A_r^U(\alpha)]: \alpha, 0) = \frac{1}{2} \left[(b + d) + \frac{\alpha-\lambda}{\rho-\lambda}(2c - b - d) \right]$$

(11)

V(I). Defintion Let $\tilde{A} = [(a, b, c, d, e; \rho), (a, c, e; \lambda)] \in F_{IV}(\lambda, \rho), \tilde{0} \in F_{\rho}$, the signed distance of \tilde{A} from $\tilde{0}$ is defined as follows:

For $0 \leq \lambda < \rho \leq 1$

$$\begin{aligned} d(\tilde{A}, \tilde{0}) &= \frac{1}{\lambda} \int_0^{\lambda} \frac{1}{4} [2(a+e) + \frac{\alpha}{\lambda}(d+2c-2a+b-2e)] d\alpha \\ &\quad + \frac{1}{\rho-\lambda} \int_{\lambda}^{\rho} \frac{1}{2} [(b+d) + \frac{\alpha-\lambda}{\rho-\lambda}(2c-b-d)] d\alpha \\ &= \frac{1}{4} [2a+2e + (d+2c+b-2a-2e)\frac{\lambda}{2}] + \frac{1}{2} [(b+d) + \frac{1}{2}(2c-b-d)(\rho-\lambda)] \\ &= \frac{1}{4} [2(a+e+b+d) + (3b+3d-2a-2e)\lambda + 2(c-b-d)\rho] \end{aligned} \tag{12}$$

Now we set $\frac{1}{2}d(\tilde{A}, \tilde{0})$ as the defuzzified value of fuzzy numbers

Now using definition we obtained the following estimate of the reliability of system is

$$= \frac{1}{8} [2(a+e+b+d) + (3b+3d-2a-2e)\lambda + 2(c-b-d)\rho] \tag{13}$$

V(II). COG method: COG method is one of the most applicable method to defuzzified the fuzzy numbers and is given by

$$x^* = \frac{\int x \mu_{\tilde{A}}(x) dx}{\int \mu_{\tilde{A}}(x) dx}$$

$$\begin{aligned} x^{*U} &= \left(\frac{\int_e^b x \cdot \lambda \left(\frac{x-a}{b-a} \right) dx + \int_b^c x \left\{ \lambda + \frac{x-b}{c-b} (\rho-\lambda) \right\} dx + \int_c^d x \left\{ \lambda + \frac{x-d}{c-d} (\rho-\lambda) \right\} dx + \int_d^e x \cdot \lambda \left(\frac{x-e}{d-e} \right) dx}{\int_e^b \lambda \left(\frac{x-a}{b-a} \right) dx + \int_b^c \left\{ \lambda + \frac{x-b}{c-b} (\rho-\lambda) \right\} dx + \int_c^d \left\{ \lambda + \frac{x-d}{c-d} (\rho-\lambda) \right\} dx + \int_d^e \lambda \left(\frac{x-e}{d-e} \right) dx} \right) \\ x^{*L} &= \left(\frac{\int_a^c x \cdot \lambda \left(\frac{x-a}{c-a} \right) dx + \int_c^e x \cdot \lambda \left(\frac{x-e}{c-e} \right) dx}{\int_a^c \lambda \left(\frac{x-a}{c-a} \right) dx + \int_c^e \lambda \left(\frac{x-e}{c-e} \right) dx} \right) \\ x^{*U} &= \frac{\frac{1}{6}[\lambda(3c^2+e^2+bc+ed-a^2-3d^2-ab-cd)+\rho(d^2+cd-b^2-bc)]}{\frac{1}{2}[\lambda(c+e-d-a)+\rho(d-b)]} \end{aligned} \tag{14}$$

$$x^{*L} = \frac{\frac{1}{6}\lambda(e^2-a^2+ec-ac)}{\frac{1}{2}\lambda(e-a)}, \text{ simplify this we obtain} \tag{15}$$

$$x^{*L} = \frac{1}{3}(a+c+e) \tag{16}$$

Then mean of both defuzzified value is the estimate failure probability and is given by

$$x^* = \frac{1}{2}(x^{*U} + x^{*L}) \tag{17}$$

$$\left(\begin{array}{l} (0.001551120, 0.00205181, 0.0030079695, 0.003964129, 0.004705971 : 1) \\ (0.001551120, 0.0030079695, 0.004705971 : 0.8) \end{array} \right)$$

TABLE 1: Fuzzy operation of two intervalued pantagonal-triangular fuzzy numbers

OPERATION **PENTAGONAL-TRIANGULAR FUZZY INTERVALUED NUMBERS**

MULTIPLICATION

$$\begin{pmatrix} a_1, b_1, c_1, d_1, e_1 : \rho \\ a_1, c_1, e_1 : \lambda \end{pmatrix} \times \begin{pmatrix} a_2, b_2, c_2, d_2, e_2 : \rho \\ a_2, c_2, e_2 : \lambda \end{pmatrix} = \begin{pmatrix} a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2, e_1 e_2 : \rho \\ a_1 a_2, c_1 c_2, e_1 e_2 : \lambda \end{pmatrix}$$

COMPLEMENT

$$1 - \begin{pmatrix} a, b, c, d, e : \rho \\ a, c, e : \lambda \end{pmatrix} = \begin{pmatrix} 1-e, 1-d, 1-c, 1-b, 1-a : \rho \\ 1-e, 1-c, 1-a : \lambda \end{pmatrix}$$

Definition 1 Let $\tilde{A} = (a_1, b_1, c_1, d_1, e_1 : \rho), (a_1, c_1, e_1 : \lambda)$ and $\tilde{B} = (a_2, b_2, c_2, d_2, e_2 : \rho), (a_2, c_2, e_2 : \lambda)$ be two i-v pentagonal-triangular fuzzy numbers then the failure possibility $F(\tilde{A} \cup \tilde{B})$ for $\tilde{A} > 0$ and $\tilde{B} > 0$ can be defined using OR operator [8] as $F(\tilde{A} \cup \tilde{B}) = 1 \ominus_{T_w} [(1 \ominus_{T_w} F(\tilde{A})) \otimes_{T_w} (1 \ominus_{T_w} F(\tilde{B}))]$

Definition 2 Let $\tilde{A} = (a_1, b_1, c_1, d_1, e_1 : \rho), (a_1, c_1, e_1 : \lambda)$ and $\tilde{B} = (a_2, b_2, c_2, d_2, e_2 : \rho), (a_2, c_2, e_2 : \lambda)$ be two i-v pentagonal-triangular fuzzy numbers then the failure possibility $F(\tilde{A} \cap \tilde{B})$ for $\tilde{A} > 0$ and $\tilde{B} > 0$ can be defined using AND operator [8] as $F(\tilde{A} \cap \tilde{B}) = F(\tilde{A}) \otimes_{T_w} F(\tilde{B})$

VI. Example

FTA of a medication pump failing to deliver medication [4]

The FTA of a medication pump failing to deliver medication to a patient is shown in Fig.3[4]. This fault tree has four combination of failures i.e. medication not delivered to patient, immediately below the top event is an OR gate meaning that any individual item below the gate is sufficient by itself to cause the next higher level failure state. For example, pump failure, clamp not removed from tube, pump not activated, and tubing kinked by patient movement are each independently work. In this example, the pump and the alarm work together. Pump failure event occurs due to two events (the pump stops and the alarm does not alert to the practitioner regarding the pump stopping) connected by an AND gate. The pump stops due to either an electrical power failure, a pump motor failure, or tubing occlusion. In this fault tree, we have considered three human errors plus one patient factor. Marx and slonim[1] considered the values of failure probabilities of all the basic events as 0.001(column 3 of table 4) However, this could not be possible for real system, and so we have considered these values as different pentagonal triangular intervalued fuzzy numbers as given in table 4(column 4).

Table 4. Failure probability in pentagonal triangular intervalued fuzzy numbers

Basic event	Failure possibility	Crisp value	TPFNs representation
A	\tilde{q}_A	0.001	$\left(\begin{array}{c} 0.0006, 0.0008, 0.001, 0.0012, 0.0015 : 1 \\ 0.0006, 0.001, 0.0015, : 0.8 \end{array} \right)$
B	\tilde{q}_B	0.001	$\left(\begin{array}{c} 0.0006, 0.0008, 0.001, 0.0012, 0.0015 : 1 \\ 0.0006, 0.001, 0.0015, : 0.8 \end{array} \right)$
C	\tilde{q}_C	0.001	$\left(\begin{array}{c} 0.00055, 0.0007, 0.001, 0.0013, 0.0014 : 1 \\ 0.00055, 0.001, 0.0014 : 0.8 \end{array} \right)$
D	\tilde{q}_D	0.001	$\left(\begin{array}{c} 0.0006, 0.0007, 0.00095, 0.0012, 0.00145 : 1 \\ 0.0006, 0.00095, 0.00145 : 0.8 \end{array} \right)$
E	\tilde{q}_E	0.001	$\left(\begin{array}{c} 0.0005, 0.0007, 0.001, 0.0013, 0.0016 : 1 \\ 0.0005, 0.001, 0.0016 : 0.8 \end{array} \right)$
F	\tilde{q}_F	0.001	$\left(\begin{array}{c} 0.0005, 0.0007, 0.001, 0.0013, 0.0016 : 1 \\ 0.0005, 0.001, 0.0016 : 0.8 \end{array} \right)$
G	\tilde{q}_G	0.001	$\left(\begin{array}{c} 0.00055, 0.00065, 0.000975, 0.0013, 0.0015 : 1 \\ 0.00055, 0.000975, 0.0015 : 0.8 \end{array} \right)$
H	\tilde{q}_H	0.001	$\left(\begin{array}{c} 0.0005, 0.0007, 0.001, 0.0013, 0.0016 : 1 \\ 0.0005, 0.001, 0.0016 : 0.8 \end{array} \right)$

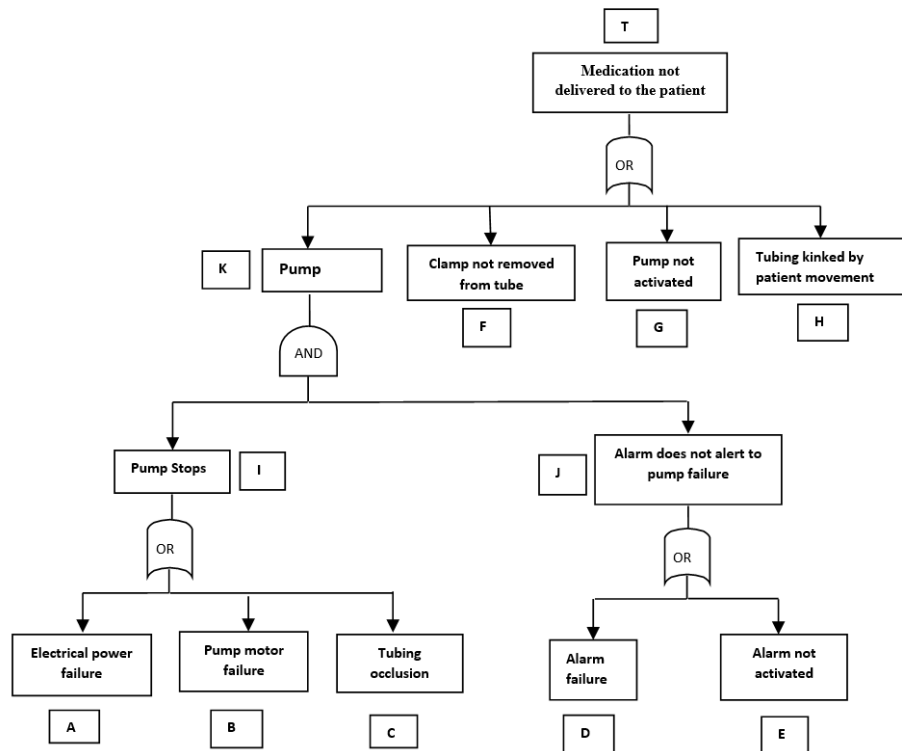


Fig.3. A medication pump fault tree with human error factor failing to deliver medication [1]

Mathematical expression of event is given by

$$\begin{aligned}
 T &= K \cup F \cup G \cup H \\
 &= (I \cap J) \cup F \cup G \cup H \\
 &= ((A \cup B \cup C) \cap (D \cup E)) \cup F \cup G \cup H
 \end{aligned} \tag{14}$$

And mathematical formula of this expression is given as :

$$\begin{aligned}
 q_T &= 1 - [(1 - q_K) \times (1 - q_F) \times (1 - q_G) \times (1 - q_H)] \\
 &= 1 - [(1 - q_I \times q_J) \times (1 - q_F) \times (1 - q_G) \times (1 - q_H)] \\
 &= 1 - [(1 - (1 - (1 - q_A) \times (1 - q_B) \times (1 - q_C)) \\
 &\quad \times (1 - (1 - q_D) \times (1 - q_E))) \times (1 - q_F) \times (1 - q_G) \times (1 - q_H)]
 \end{aligned} \tag{15}$$

VII. Result

By the fuzzy operation with the help of table 1 and table 2 we have the failure probability of top event is

$$\left(\begin{array}{l} (0.001551120, 0.00205181, 0.0030079695, 0.003964129, 0.004705971 : 1) \\ (0.001551120, 0.0030079695, 0.004705971 : 0.8) \\ (0.9955294029, 0.996035871, 0.9969920305, 0.99794819, 0.99844888; 1) \\ (0.9955294029, 0.9969920305, 0.99844888; 0.8) \end{array} \right)$$

VII(I). Conclusion1: Defuzzification by signed distance method, we obtain failure probability of top event from equation 13 is

$$x^* = 0.0028696286 \text{ and reliability of top event is } 0.97130371$$

And by COG method we obtain failure probability of top event from equation (15), (16), and (17) is

$$x^{*L} = 0.0016765622$$

$$x^{*U} = 0.0022540335$$

$$\text{Now } x^* = \frac{1}{2}(0.0016765622 + 0.0022540335) \quad (18)$$

VII(II). Conclusion2.: Therefore by COG method the failure probability of top event $x^* = 0.0039305957$ and reliability of top event is 0.996069404

VII(III). Difference Error: the difference in both method is about 0.1060967 % which imply that the COG method and Signed distance method are give similar result.

The fuzzy failure probability and fuzzy reliability in pentagonal-triangular intervalued fuzzy numbers are in fig4 and fig5 respectively.

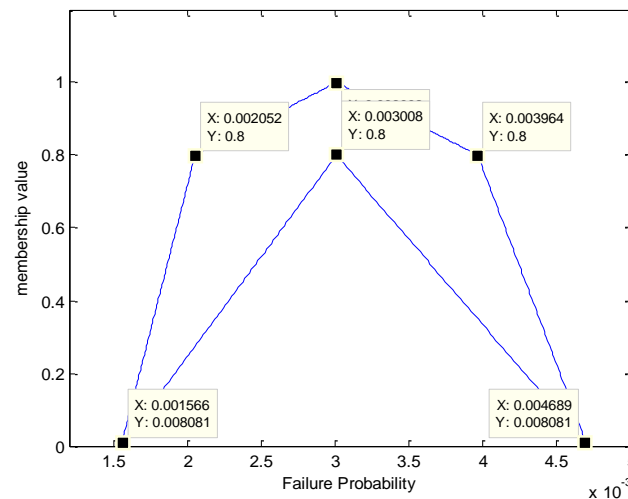


Fig 4.Fuzzy pentagonal-Triangular failure probability

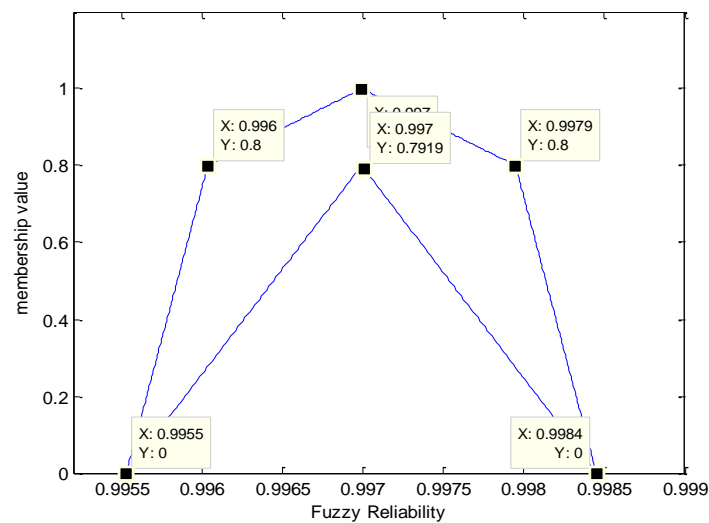


Fig 5.Fuzzy pentagonal-Triangular reliability probability

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