A Queue Network M/M/1/∞ Model

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Abstract

In this paper we model an open queueing network of cardiac treatment section in medical sector. Assume arrival of patients follows Poisson and service times at stations have exponential distribution. The performance measures of the system are evaluated. The steady state characteristics of the network are obtained and each station solved independently by using M/M/1/∞ model while blocking and non-blocking exists. Blocking occurred when at least one service center has limited queueing space or capacity before it. An illustrative example is given.

Keywords: Queueing networks, Blocking, Steady state characteristics.

I. Introduction

Collection of interactive queueing systems is known as network of queues. Queueing networks mainly classified as open queueing networks, closed queueing networks and mixed queueing networks. Open queueing networks described as customers can arrive from outside the system at any node and depart from the system from any node. At least one service center has limited waiting space or capacity, which are classified in to restricted queueing networks. Blocking may arise in a network of queues where some or all queues have finite buffer capacity [2]. Since there is restriction in waiting space between the stations, there may occur blocks.

Many relevant studies on open restricted queue systems are done by Hunt [3], Takahashi et al.[8], Perros and Atlok [6], Koizumi et al.[5], Sreekala and Manoharan [7] and Arum Helmi Manggala Putri et al [1]. Hunt [3] used a sequential series model to obtain solution for a two station series queue with limited waiting space between stations. An approximate analysis for open queueing networks with blocking done by Takahashi et al.[8] and Perros and Atlok [6]. Koizumi et al [5] analysed blocking in open restricted queueing system by decomposition method. Recently analysis of restricted queueing networks-a blocking approach with special reference to health care system studied by Sreekala and Manoharan [7].

In this paper first we study an open queueing network of Cardiac section with infinite capacity in each station. Steady state equations and performance parameters are obtained. A brief description of the model is done in section II. Diagrammatic representation and congestion types are given in Section III. Derivation of steady state equations in without blocking and analysis of each station using decomposition approach in with blocking are given in section III. Numerical analysis is section IV. Conclusions are given in last section.

II. Model description

We can consider Out Patient (OP) section of Cardiac treatment in government medical college, as an example of open queueing network. There are five stations are defined in this queueing network. In first node S_1gives token for every customer arriving to the hospital, customers arrive according to homogeneous Poisson process. Second station stands for pressure checking which follows M/M/1/∞/FCFS schedule. Doctors are available in the third and fourth node and these nodes considered as a single node. S_3 and S_4 also follow M/M/1/K/FCFS. Fifth node is for treatment. After the diagnosis, some patients in third and fourth node leave from the system with probability α_3 and $\alpha_4$and remaining patients admit for treatment with probability 1-α_3 and 1-α_4.There are some situations where usual admission procedures cannot follow. Example: - accident cases or heart attack.

Methodological Framework

Consider an open queueing network of OP section of Cardiac treatment in medical college with five single servers. LetS_i ($i = 1,2, \dots 5$) denote stations. Arrival pattern of customers to the system according to homogeneous Poisson process with rate λ. Service times are exponentially distributed with rate μ_i ($i = 1,2, \dots 5$). Queue discipline is FCFS basis. Waiting space S_5 and between stations one and two are of infinite capacity and other stations are finite. Therefore blocking happens only between $S_3 \rightarrow S_5$ and$S_4 \rightarrow S_5$. In this paper, model the flows $S_2 \rightarrow S_3$, $S_2 \rightarrow S_4$, $S_3 \rightarrow S_5$ *and* $S_4 \rightarrow S_5$. Arrival to each node is according to Poisson process. Diagrammatic representation of the model is given in figure 1.

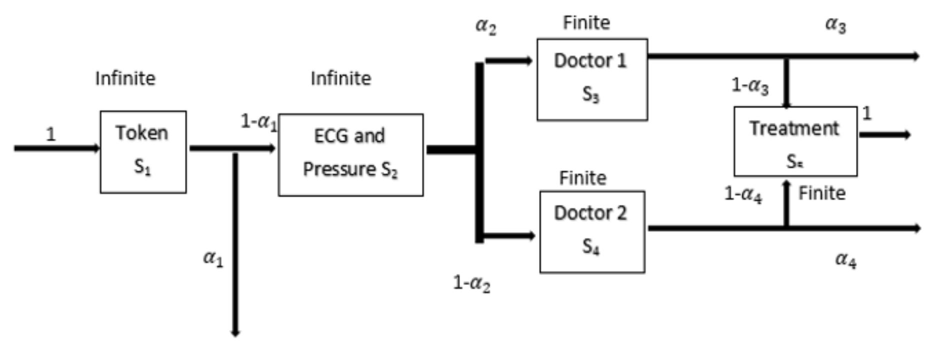

Figure 1. OP section of Cardiac treatment as queue network with blocking

Since waiting space between stations three, four and five are of finite capacity there may arise blocking between $S_3 \rightarrow S_5$ *and* $S_4 \rightarrow S_5$. We model the flows $S_3 \rightarrow S_4$and $S_4 \rightarrow S_5$. Arrival to each node is according to Poisson process. The types of congestion listed in table 1.

Table.1.Congestion types

Flow	Cause of congestion	Facing station	Congestion type
$S_1 \rightarrow S_2$	Not applicable	Not applicable	No congestion
$S_2 \rightarrow S_3$	S_3 is full	S_2	Classic Congestion
$S_2 \rightarrow S_4$	S_4 is full	S_2	Classic Congestion
$S_3 \rightarrow S_5$	S_5 is full	S_3	Blocking
$S_4 \rightarrow S_5$	S_5 is full	S_4	Blocking

III. Steady –State Analysis

In this section we first assume that every station has infinite waiting space and analyze stations without blocking. The steady state analysis of some related models can be seen in Gross and Harris [3] and Bose [2].

Steady State Analysis without blocking

The routing probability matrix generally defined as

$$P = \begin{bmatrix} r_{00} & r_{01} & r_{02} & r_{03} & r_{04} & r_{05} & r_{06} \\ r_{10} & r_{11} & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} \\ r_{20} & r_{21} & r_{22} & r_{23} & r_{24} & r_{25} & r_{26} \\ r_{30} & r_{31} & r_{32} & r_{33} & r_{34} & r_{35} & r_{36} \\ r_{40} & r_{41} & r_{42} & r_{43} & r_{44} & r_{45} & r_{46} \\ r_{50} & r_{51} & r_{52} & r_{53} & r_{54} & r_{55} & r_{56} \\ r_{60} & r_{61} & r_{62} & r_{63} & r_{64} & r_{65} & r_{66} \end{bmatrix}$$

where r_{ij} is the routing probability from station i to station j (i,j= 1,2,..,6). The routing probability matrix of our model based on figure 1 is

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \alpha_1 & 0 & 1-\alpha_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & 1-\alpha_2 & 0 \\ \alpha_3 & 0 & 0 & 0 & 1 & 1-\alpha_3 \\ \alpha_4 & 0 & 0 & 0 & 0 & 1-\alpha_4 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We can find $\lambda_i : i = 1,2,\dots 5$ (Total arrival rates) by solving the traffic equations:

$$\lambda_1 = \lambda$$
$$\lambda_2 = (1-\alpha_1)\lambda$$
$$\lambda_3 = \alpha_2(1-\alpha_1)\lambda$$
$$\lambda_4 = (1-\alpha_2)(1-\alpha_1)\lambda$$
$$\lambda_5 = (1-\alpha_2)(1-\alpha_1)\lambda[(1-\alpha_3)+(1-\alpha_4)].$$

We assume there is an infinite buffer between stations. So we can solve each station independently applying M/ M/ 1/∞ queueing model.

Average queue length and Average queue delay

Average queue length of station i is obtained from the formula [3].

$$L_i^q = \frac{\rho_i^2}{1-\rho_i}, \tag{1}$$

where $\rho_i = \lambda\_i/\mu\_i$ <1 (i=1,2,…,4) and $\rho_i = \frac{\lambda_i}{\mu_i}$>1 (i=5), in the case of non-steady state, analytical model cannot be applicable to the queueing system.

By using Little's formula[3] we can obtain the average steady state waiting time,

$$W_i^q = \frac{\rho_i^2}{\lambda_i(1-\rho_i)}, i = 1,2,\dots 5. \tag{2}$$

Steady state analysis with blocking

Blocking exists between stations when some stations are finite and congestion at any particular station could potentially affect congestion levels at all upstream stations. In order to find interactions between stations, modified Jacksons approach can be used with the help of effective service time by Takahashi et al.[9]. Here we assume effective service times follow exponential distribution. The mean effective service time at station i is denoted by $\frac{1}{\tilde{\mu}_i}$. Effective waiting time is defined as the convex combination of waiting times.

$$\frac{1}{\tilde{\mu}_i} = r_{i0}\left(\frac{1}{\mu_i}\right) + \sum_j r_{ij}\left(\frac{1}{\mu_i} + W_j\right),$$

where r_{i0} is the routing probability of patients leaving from state i without facing any wait, r_{ij} is the routing probability from station i to j. In our model stations S_3 $and,$ S_4 face blocking. The effective service time corresponding to S_3 $and,$ S_4 are

$$\frac{1}{\tilde{\mu}_3} = r_{30}\left(\frac{1}{\mu_3}\right) + r_{35}\left(\frac{1}{\mu_3} + W_5^q\right), \tag{3}$$

$$\frac{1}{\tilde{\mu}_4} = r_{40}\left(\frac{1}{\mu_4}\right) + r_{45}\left(\frac{1}{\mu_4} + W_5^q\right) \tag{4}$$

Using equations (1) and (2), we obtain steady state queue lengths and waiting times in terms of effective service times.

Analysis of Stations

By using single node decomposition approximation by Takahashi et al. [9] the steady state of every station can solve independently from last station to first station. The steady state of each finite station (M/M/1/∞ *queue*) is analyzed using this approximation.

Analysis of station six (S_5)

In our model, station five represents who needs recheck and treatment in the clinic. The downstream node S_3 and S_4 are finite. S_5 face blocking if S_3 is full. Corresponding to S_5 the queue length and queue delay obtained by solving (1) and (2) in terms of effective service times (3). The queue length corresponding to S_5 is

$$\begin{aligned} L_{35}^q &= L_5^q(\lambda_{35}/\lambda_5) \\ &= L_5^q(\mathrm{r}_{35}\,\lambda_3/\lambda_5) \end{aligned}$$

where L_{35}^q is thequeue length of blocked persons at S_3 waiting to enter S_5. S_5 face blocking if S_4 is full. Corresponding to $S_5$the queue length and queue delay obtained by solving (1) and (2) in terms of effective service times (4). The queue length corresponding to S_5 is

$$\begin{aligned} L_{45}^q &= L_5^q(\lambda_{45}/\lambda_5) \\ &= L_5^q(\mathrm{r}_{45}\,\lambda_4/\lambda_5) \end{aligned}$$

where L_{45}^q is the queue length of blocked persons at S_4 waiting to enter S_5. This method is applicable only when traffic intensity less than one.

Analysis of station four (S_4)

Station four represents patients who entered for doctors checking. The downstream node S_2 is also infinite. The queue length corresponding to S_5 is

$$L_{24}^q = L_4^q$$

where L_{24}^q is the queue length of blocked persons at S_2 waiting to enter S_4.

Analysis of station three (S_3)

Station three represents patients who entered for doctors checking. The downstream node S_2 is also infinite. The queue length corresponding to S_3 is

$$L_{23}^q = L_3^q$$

where L_{23}^q is thequeue length of blocked persons at S_2 waiting to enter S_3.

Analysis of station two (S_2)

Station two represents patients who entered for pressure checking. The downstream node S_1 is also infinite. The queue length corresponding to $S_2$is

$$L_{23}^q = L_3^q.$$

IV. Numerical Analysis

Data taken from Cardiac Section of a Medical College. To find out the Performance Parameters of with blocking and without blocking, the statistical analysis is conducted. $\lambda = 8.48$, α_1=0.85, $\alpha_2 = 0.45, \alpha_3 = 0.98, \alpha_4 = 0.97, \frac{1}{\mu_1}$=0.29, $\frac{1}{\mu_2} = 0.91$, $\frac{1}{\mu_3} = 0.83$, $\frac{1}{\mu_4} = 0.75$. Performance parameters are computed and given in table 2.

Table 2. Performance Parameters

Station	Performance Parameters	With Blocking	Without Blocking
S_2	L_2^q	1.21	1.21
	W_2^q	2.08	2.08
S_3	L_3^q	0.54	0.54
	W_3^q	0.91	0.91
S_4	L_4^q	0.66	0.66
	W_4^q	0.75	0.75

The congestion rate and waiting time of S_2 is high compared to other two stations. Blocking exists in station 5 but the traffic intensity of station 5 is greater than one. Steady state doesn't exist for this station. Normal methods cannot applicable for analyzing non-steady state queue system. Bounding of capacity of queue, Monte Carlo simulation and increase servers to the system are the possible methods for solving non-steady state queues.

V. Conclusion

We studied the Cardiac section of a Medical College as an open restricted queueing network. Blocking exists due to 5th node. Steady state equations are obtained without blocking and with blocking cases assuming traffic intensity less than one. Node to node decomposition method is used to get performance measures. But the traffic intensity of 5 th node greater than one. We cannot analyze through steady state equations. To find performance measures we can use any of these methods-Bounding of capacity of queue, Monte Carlo simulation and increase servers to the system.

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