An Approximation to Joint System Size Distribution at Nodes in Some Multi-hop Wireless Networks

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Abstract

In this paper, we discuss a network model to study the queueing characeristics of nodes in a multi-hop wireless network, under the standard binary exponential back-off (BEB) contention resolution scheme. Based on the steady state distribution of the system size at a node, which was appeared in Sweta & Deepak [2], we compute the joint distribution of system size at all nodes in a multi-hop network, governed by some specific queue disciplines. Getting information on the joint queuing size distribution in the network will enable us to control the traffic (and hence congestion) in the whole network. In order to illustrate our theoretical results, a particular multi-hop network model is considered and analysed numerically.

Keywords: queuing networking model , multi-hop wireless network, joint system size distribution

I Introduction

Sharing data files among nodes in a network without a port, and hence by using less expensive infrastructure is one of the major attractions for switching over to wireless network from wired one. In ad hoc wireless networks, each node acts not only as a host but also as a relay of packets for forwarding packets to another nodes which are not in the direct transmission range of source nodes.

In order to illustrate the dynamics and behaviour of nodes in a wireless network, we consider a simple network, having 4 nodes with gateway GW, as shown in figure 1. Among the 4 nodes, assume that the nodes 1 and 3 are source nodes. That is, external arrivals can be generated only at these nodes. The entire route of the packets generated at each of the source nodes is also shown in figure 1.

A circle centred at a node defines the transmission range of that node. All nodes that are lying inside the transmission range of a paricular node are called the one hop neighbours of that node. All other nodes that are lying inside the circles centred at all one-hop neighbours of a node are called its two-hop neighbours. In figure 1, node 1 has only one one-hop neighbour which is node 2 but it has two two-hop neighbours namely, node 3 and 4. Node 2 has 3 one-hop neighbors 1,3 and 4, but it has no two-hop neighbour. Similarly for node 3, one-hop neighbour is node 2 and two hop neighbours are 1 and 4. If a transmission is being taken place between two nodes, all one-hop neighbours of those two nodes will sense the channel as busy, but all two-hop nodes, being not in the transmission range of source node, will not be able to sense the channel and hence there can be a chance of collision due to the possibile simultaneous transmission by these nodes.

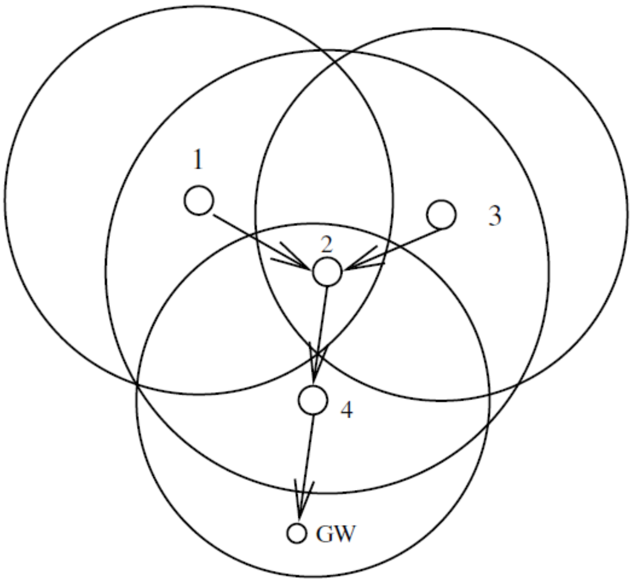

Figure 1: A general network

Many protocols have been proposed to reduce the chances of collission resulted from simultaneous transmissions by several nodes in multi-hop routing networks. Among them IEEE 802.11 [5] has been accepted as international standard, where the fundamental mechanism to access the medium is the distributed co-ordination function (DCF). According to the DCF basic access mechanism, a node with a packet for transmission monitors the channel activity and if the channel is found idle for a predetermined period called DIFS (distributed inter frame space), it transmits the packet. If the channel is found busy, the node undergoes a random back-off period- a random number of time slots- and initializes a back-off counter. At each instant at which the channel is monitored, the back-off counter is decremented if the channel is found idle for a period longer than DIFS, else it is frozen. The node, of which back-off counter expires first, begins transmission and all of its neighbouring nodes freeze their counters. Once the current transmission gets completed, back-off processes of all neighbours of transmitting node resume as explained above.

In order to minimize the possibility of collisions due to multiple simultaneous transmissions, DCF employs several contention resolution schemes namely, binary exponential back-off (BEB) rule, LIMD (linear increase multiple decrease) rule and so on. In our work (here and in our earlier problem), BEB rule is used as it is the most standard one. The rule is explained briefly as given below: If a packet is ready for transmission from a node, contention window size is chosen as W and a random value from 0, 1,2, $W-1$ is uniformly selected as its back-off counter. If the packet does not get transmittied successfully, that is, it meets with a collision in that attempt, the contention window size will be doubled so that it is set as $W_1 = 2W$. A value for back-off counter is selected uniformly from 0, 1,2, $W_1 - 1$. If it further meets with a collision on its next attempt, the contention window size will be doubled again and this will continue up to a maximum of m collisions. After m unsuccessful attempts, if it again meets with a collision, the contention window size will be fixed as $W_m = 2^m W$. If an attempt results in successful transmission, the contention window size for that node will be reset as W. Hence

$$\begin{aligned} CW_{min} &= W, \\ CW_{max} &= 2^m W. \end{aligned}$$

As an attempt to learn some major characteristics of waiting packets at an arbitrary node in a wireless network, Sweta & Deepak [2] proposed a model and analysed it by matrix theoretic approach to get some important statistical characteristics such as probability distributions of system size, waiting time of packets, number of collisions experienced by a packet at a single node, and their moments in a rigorous manner. However, Sweta & Deepak [2] couldn't take up the problem of computing the joint distribution of system size at all nodes in the entire network due to a large dimensional state space. Here, we use the theoretical approach developed by Kelly [3] to address this for a network, governed by some specific queue disciplines. A summary of the assumptions and results that appeared in Sweta & Deepak [2], and relevant to the present problem too, is given in the next section.

II Some of our earlier results

The major assumptions in Sweta & Deepak [2] were:

(i) Packets are generated at a node according to a Poisson rule of rate λ, and join a waiting line till they are being considered for transmission.

(ii) At an epoch at which a packet is considered for transmission, the back-off period for the node commences if it senses the channel as idle, and if so the node selects a back-off counter uniformly from 0, 1, 2, $W-1$. If the packet has already experienced j collisions, then the back off counter will be from 0, 1, 2, $W_j - 1$. Also, time spent on counters are assumed to be independent and identically distributed exponential variates having mean $1/\mu$.

(iii) If the channel is found busy after completion of a back off counter time, the back off timer will be frozen and will commence again only after the channel is sensed as idle. The channel idle periods and busy periods are taken as independent Phase type (PH) variates with representations (α_1, T_1) and (α_2, T_2) of order n_1 and n_2 respectively. For details on PH variates, see Neuts [4].

(iv) When the back-off counter at a particular back-off stage becomes zero, the node starts transmission. Packet transmission times are assumed to be independent and identical exponential variates having mean $1/\gamma$.

(v) A transmission results in collision with probability p and is successful with probability $1-p$.

The underlying Markov process in connection with the dynamics of a specific node could be seen as a Quasi Birth-Death (QBD) process and hence its steady state analysis could be carried out by the matrix analytical approach (See Neuts[4]).

Among major results in Sweta & Deepak [2], the one which is relevant to the present model is given below:

3.1 Distribution of the time between the instant at which a packet is considered for transmission and the instant at which it is successfully transmitted

If U represents the duration of time from the epoch at which a packet is chosen for transmission till the epoch at which it is successfully transmitted, we proved that U is a continuous phase type variate having representation (β, S). Here,

$$\beta = \begin{bmatrix} \frac{1}{W} & \frac{\alpha_1}{W} & \cdots & \frac{\alpha_1}{W} & 0 & 0 & \cdot & 0 \end{bmatrix}$$

and

$$S = \begin{bmatrix} D_0 & B_1 & 0 & \cdot & 0 & 0 & 0 & \cdot & 0 \\ 0 & D_1 & B_2 & \cdot & 0 & 0 & 0 & \cdot & 0 \\ 0 & 0 & D_2 & \cdot & 0 & 0 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & D_m + B_m & 0 & 0 & \cdot & 0 \\ F_0 & 0 & 0 & \cdot & 0 & G_0 & 0 & \cdot & 0 \\ 0 & F_1 & 0 & \cdot & 0 & 0 & G_1 & \cdot & 0 \\ \cdot & \cdot & F_2 & \cdot & 0 & 0 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & F_m & 0 & 0 & \cdot & G_m \end{bmatrix}$$

where

$$B_i = \begin{bmatrix} p\gamma/W_i & p\gamma\alpha_1/W_i & p\gamma\alpha_1/W_i & \cdots & p\gamma\alpha_1/W_i \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

for $i = 1,2, \dots m$ and

$$D_i = \begin{bmatrix} -\gamma & 0 & 0 & \cdots & 0 \\ \mu e & T_1 - \mu I & 0 & \cdots & 0 \\ 0 & \mu I & T_1 - \mu I & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & T_1 - \mu I \end{bmatrix}$$

for $i = 0,1,2, \dots m$.
Also,

$$F_i = \begin{bmatrix} 0 & T_2^0\alpha_1 & 0 & \cdots & 0 \\ 0 & 0 & T_2^0\alpha_1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & T_2^0\alpha_1 \end{bmatrix}$$

and

$$G_i = \begin{bmatrix} T_2 & 0 & 0 & \cdots & 0 \\ 0 & T_2 & 0 & \cdots & 0 \\ 0 & 0 & T_2 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ 0 & 0 & 0 & \cdots & T_2 \end{bmatrix}$$

for $i = 0,1, \dots, m$.

Hence the density of U is

$$f(u) = \beta e^{Su}(-S)e, \qquad 0 < u < \infty \tag{1}$$

and

$$E[U] = \beta(-S)^{-1}e. \tag{2}$$

Note that in the above e represents a column vector, having all entries 1, of appropriate dimension.

III Joint system size distribution

Network queues corrrespond to systems which consist of many queues with different types of customers moving from one queue to another in their routes. The route of a customer through the queues of the system may be fixed or random. Several researchers produced equalibrium system size distribution in product form for such networks based on the assumption that amounts of service required by a customer at successive queues along its route are independent and exponentially distributed. This assumption forced the said authors to demand that knowledge of the past route of a customer in a queue is of no use in predicting its future route. However, Kelly[3] conjectured that if the queues of the network were of a certain form, then even with the assumptions that the amount of service required by a customer at a queue in its route was almost arbitrarily distributed and depend on its route and the amount of service required by it at other queues along its route, the equalibrium system size distribution could be found in an analytical form. Later Barbour [1] proved this conjecture.

Kelly [3] dealt with an open system and used a customer's type to determine not only its route through the system but also the distribution of the amount of service it requires at each queue along that route. The following are the main assumptions made by Kelly [3] and Barbour [1].

- Queueing network consists of J nodes.
- Customers of type i (i=1, 2, .. , I) enter the system in a Poisson stream at rate $\nu(i)$ and pass through the sequence of queues $r(i,1), r(i,2), -., r(i,S(i))$ before leaving the system ,where S(i) denotes the number of stages a customer of type i visits along its route.

- A type i customer at its stage s (when $r(i,s) = j$) needs a random amount of service Q_{is}.
- Total service effort offered by a single server when there are n_j customers in queue j is $\phi_j(n_j)$.
- A customer in mth position of jth queue will be given a proportion $\gamma_j(m, n_j)$ of this effort, where $1 \leq m \leq n_j$.
- When a customer arrives at queue j, it moves into position m $(1 \leq m \leq n_j + 1)$ with probability $\gamma_j(m, n_j + 1)$.

Then Kelly [3] conjuctured and Barbour [1] later proved that $n(t) \equiv \{n_1(t), n_2(t), \ldots., n_J(t)\}$ has a limitting distribution $P(n)$ such that

$$P(n) \propto \prod_{j=1}^{J} \frac{a_j^{n_j}}{\prod_{m=1}^{n_j} \phi_j(m)}, \tag{3}$$

where

$$a_j = \sum_{n=1}^{I} \nu(i) \sum_{s=1}^{S(i)} I_{[r(i,s)=j]} E Q_{is}, \tag{4}$$

provided

$$M = \sum_n P(n) < \infty.$$

Note that the usage of the same fuction γ in the last two assumptions listed above is very essential, without which the existence of the equalibrium distribution of the joint system size given by eqns (3) and (4) will not be valid for network models bearing non-exponential service time distributional assumptions. For a detailed discussion on this, refer Kelly [3] and Barbour [1].

Now we use eqns (3) and (4) to determine the joint distribution of the number of packets waiting at nodes in some special type of wireless networks. Let us consider a network with nodes having identical features like the same number of one-hop and two-hop neighbours. Because of this, we can assume that the distribution of the amount of time the channel is sensed as busy by each of the nodes are identically distributed. In a similar manner, channel idle times sensed by all nodes can also be assumed to be disributed identically. Hence, the distribution of the time from the instant at which a packet is ready to the instant at which it is successfully transmitted from each node are also identically distributed. Its density and mean are defined by eqns (1) and (2) respectively. Hence EQ_{is} corresponding to our model can assumed to be the same for all i and s, and is given by

$$E[Q_{is}] = \beta(-S)^{-1}e. \tag{5}$$

Now consider the routing probability matrix as

$$R = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}.$$

As we assumed earlier, type of a customer will be decided by the route along which it may traverse. Hence, we can have a maximum of $I = n!$ types of customers. Suppose that the total external packet generation to the system obey a Poisson rule of parameter λ and a q_i proportion of these is of type i for $i = 1,2,\ldots,I$ so that $\sum_{i=1}^{I} q_i = 1$. As we assumed, average time that any type of customer takes at any node in its route is $E[Q_{is}]$, and is given by eqn (5).

As per the the two important assumptions made by Kelly [3] and Barbour [1], which are listed as the last two assumptions, given above in this section, we should also use the same fuction γ in our model due to the non-exponential variate Q_{is}. Hence, we assume two cases here namely,

case 1

Selection of packets for transmission at nodes is done by LCFS and the new packet always joins at the end of the queue.

Then we have

$$\gamma_j(m, n_j) \;\; = 1 \quad if m = n_j$$
$$= 0 \quad if m \neq n_j.$$

case 2

Selection of packets for transmission is done uniformly from the waiting line and also the customer joins a position randomly (as per uniform law) upon its arrival at a node along its route. In this case, we have

$$\gamma_j(m, n_j) \;\; = \frac{1}{n_j} \quad \text{for} m = 1,2,3, \ldots n_j.$$

In both cases, we have

$$a_j = \sum_{i=1}^{I} q_i \lambda \sum_{s=1}^{S(i)-1} p_{r(i,s),r(i,s+1)=j} \beta(-S)^{-1}e. \tag{6}$$

Hypothetically, since we have only one server at each node, $\phi_j(m) = 1$ for $m = 1,\cdots,n_j$ and $j = 1,\cdots,J$.

Therefore,

$$P(n) \equiv \prod_{j=1}^{n} \frac{a_j^{n_j}}{\prod_{m=1}^{n_j} \phi_j(m)}$$

$$\equiv \prod_{j=1}^{n} (\lambda\beta(-S)^{-1}e \sum_{i=1}^{I} q_i \sum_{s=1}^{S(i)-1} p_{r(i,s),r(i,s+1)=j})^{n_j}. \tag{7}$$

In the above, $p_{r(i,s),r(i,s+1)=j}$ represents the routing probability of a packet of type i, which is currently at the s th stage of its route, moving to node j at the next stage.

IV Numerical illustration

In order to illustrate the theoretical results established in the previous section numerically, we consider a network model with nodes having equal number of one-hop and two-hop neighbours, as shown in figure 2.

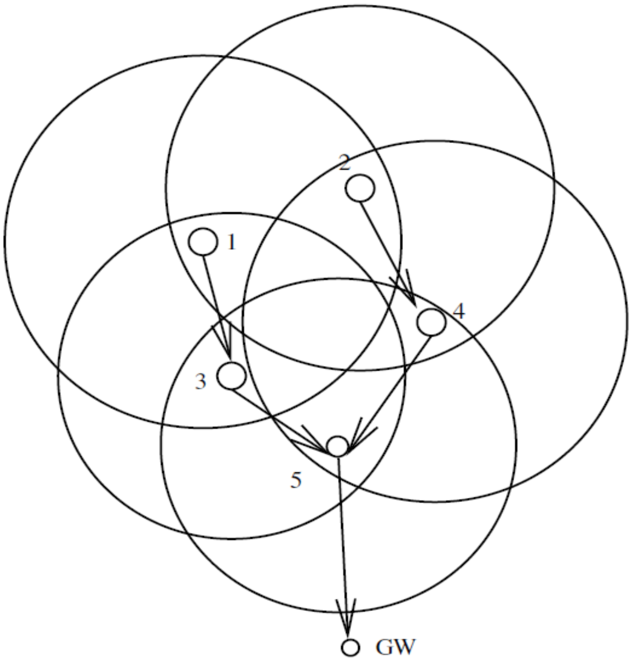

Figure 2: A particular network

Here node 1 and 2 are assumed as source nodes and GW is the gateway. The matrices R, F and N exhibit the details of routing of packets, one-hop, and two-hop neighbours of each node respectively.

$$R = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

$$F = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 \\ 4 & 0 & 1 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 & 0 \end{array}$$

$$N = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 & 0 & 1 \\ 3 & 0 & 1 & 0 & 1 & 0 \\ 4 & 1 & 0 & 1 & 0 & 0 \\ 5 & 1 & 1 & 0 & 0 & 0 \end{array}$$

As approximate Phase-type representations of the distribitions for channel busy time and idle time sensed by each node, we use the same representations that used in Sweta & Deepak [2]. These had been obtained by collecting around 300000 observations from a network which is being governed by BEB scheme under 802.11 MAC [5] specification. Activities at one of the nodes were monitored and the observations corresponding to the events like arrivals of data packets at the node and amount of time the channel was idle/busy sensed by that node were used to get an approximate phase type fit for the said variates. Also, the same observations were used for estimating the packet arrival rate. The representation thus obtained for channel busy time was

$$\alpha_1 = \begin{bmatrix} 0.7530 & 0.0766 & 0.1704 \end{bmatrix};$$

$$T_1 = \begin{bmatrix} -1.8097 & 0.2397 & 0.6790 \\ 0.1939 & -1.3306 & 0.5483 \\ 0.1847 & 0.5014 & -1.1274 \end{bmatrix},$$

and that for channel idle time was

$$\alpha_2 = \begin{bmatrix} 0.8823 & 0.0307 & 0.0870 \end{bmatrix};$$

$$T_2 = \begin{bmatrix} -7.2175 & 0.1960 & 0.5386 \\ 0.2835 & -1.5407 & 0.5142 \\ 0.2729 & 0.5297 & -1.3043 \end{bmatrix}.$$

Packet arrival rate is estimated as $\lambda = 1.0629$. Also, we have $E[Q_{is}]$=0.6034.

In the present example, there are two types of packets namely, the one that traverses the route $1 \to 3 \to 5 \to$ GW and the other having the route $2 \to 4 \to 5 \to$ GW. Suppose that the inflow of packets to the system obey Poisson rule of rate $\lambda = 1.0629$, of which both types claim the same proportion. That is, $q_i = \frac{1}{2}$ for $i = 1,2$. Table 1 presents a few values for the joint system size probabilities of packets at nodes in our model, under both case 1 and case 2 discussed in the previous section.

Table 1: Joint System Size Probabilities

n	P(n)	n	P(n)
(1,2,1,1,2)	0.000106	(1,1,1,3,2)	0.000034
(1,1,2,2,1)	0.000053	(3,1,1,2,1)	0.000017
(2,2,1,1,2)	0.000034	(1,1,2,2,3)	0.000022
(1,1,1,3,3)	0.000022	(3,1,1,1,2)	0.000034
(1,1,3,2,1)	0.000017	(1,1,1,1,2)	0.000332
(1,4,1,1,3)	0.000007	(2,1,1,2,2)	0.000034
(1,1,2,2,2)	0.000034	(3,1,1,1,3)	0.000022
(2,1,1,2,3)	0.000022	(1,2,2,1,2)	0.000034
(1,1,2,2,1)	0.000053	(1,1,1,3,3)	0.000022
(2,1,1,2,3)	0.000022	(1,1,1,3,1)	0.000053

For computing the joint system size probabilities, as displayed in table 1, the normalization constant is taken as the sum of the probabilities corresponding to state vectors $n = (n_1, n_2, n_3, n_4, n_5)$ for each n_i varies over 0 to 50.

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