Building the Continuous Random Process Out of The Specified Sequence of Turning Points for Fatigue Testing

Irina Gadolina[1*], Rashid Zaynetdinov[2]

•

[1] IMASH RAS
gadolina@mail.ru
[2] Independent researcher

Abstract

The method of regeneration of continuous process is intended for the first place for the calculation of spectral density of regenerated process. The main feature of this method is preserving the values and sequence of turning points (extremes) given in "saw-kind" realization. While doing so, the methods which is based on cycle-counting methods will give the exactly the same fatigue durability estimation, because the initial condition MAX-MIN-MAX … is guaranteed. To investigate the random process standard deviation (RMS) by the spectral density, the extrapolation of the original sequence is provided by continuous cosine functions. Continuality of the process and its first derivative is ensured by the condition of compatibility at the turning points. To decide on frequencies, the information from some sample realizations obtained in exploitation was employed. As one of te applications, the method is intended to be used in the analysis of comparability of two competing approaches in the evaluation of loading in the tasks of evaluation of durability, namely, those that apply cycle-counting methods and which are based on the spectral density of processes methods. Some other speculations made on the modelled process are performed.

Keywords: material fatigue, durability estimation, cosine extrapolation, cycle counting, spectral densities

I. Introduction

The problem of transforming the digital random sequences is actual in many scientific pieces of research [1]. It is especially important when comparing two main approaches in dealing with random loading in fatigue – that is the spectral approach (frequency domain) and cycle-counting approach (time-domain). A large number of standard random sequences of random process turning points are known, which can be used in testing material samples in the of high cycle fatigue. Some of them (SAE [2], TWIST [3], FALSTAFF [4] etc.) are designed for random load testing of parts for some specific industries. A method for modeling three alternative types of random sequences of load peaks for fundamental research was also developed [5]. The problem of generating the random processes with special requirements (kurtosis, for example) is discussed in [6]. In [6], a time-domain procedure for a non-Gaussian random test is introduced. [7] addresses the question of linking fatigue damage with the prescribed input kurtosis. Direct applications of these results include improved fatigue life estimations.

While the estimation of loading in fatigue, most often a special transformation is being made. Usually the transition from continuous process to "the saw". The example of "the saw" is shown in Fig.1. That particular process (FALSTAFF, [4]) is being imitated in a special way for the testing of aircraft fighters. The data for imitation had been collected during the exploitation in varied conditions. The transformation of the initial continuous process, which express the physical nature of the loading process into the turning points sequence is recommended by most of the standards for fatigue testing under random processes. Most of the methods for cycle counting deal only with "the saw". The main reason for that is the non-dependence of fatigue property on frequency and the cycle shape. This initial processing stage provides the transformation, after which only the values of peaks and trough and their original order sequence still remains. Usually, the fatigue estimation is fulfilled with taking in mind some real exploitation unit, for example, 1 km run (for transport machines), one hour of work (for technological machines) or the number of flights for the airplane. In this situation the time and frequencies does not play an important role.

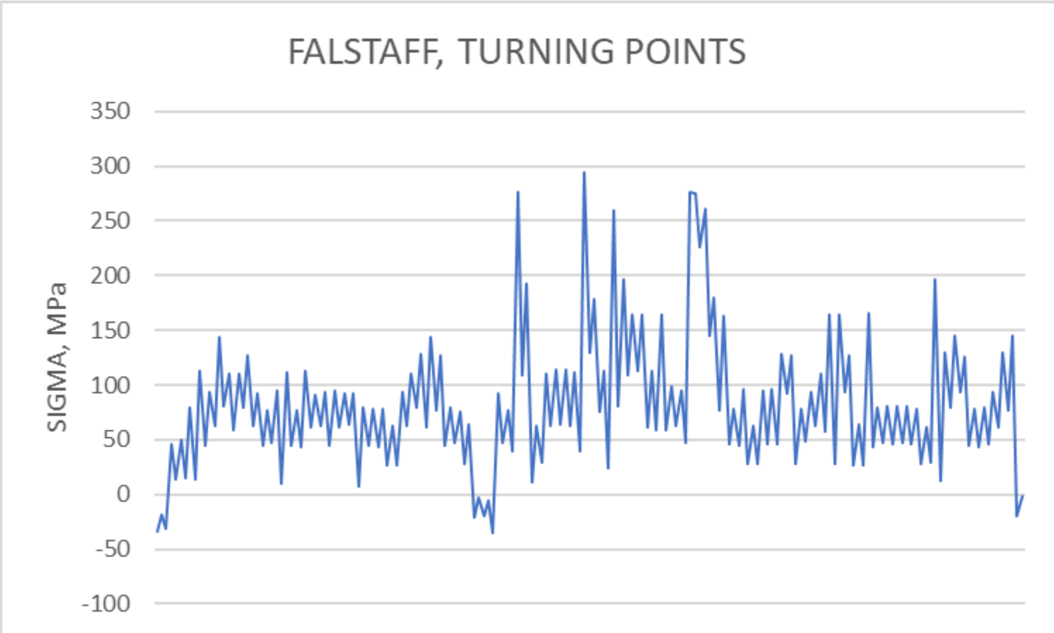


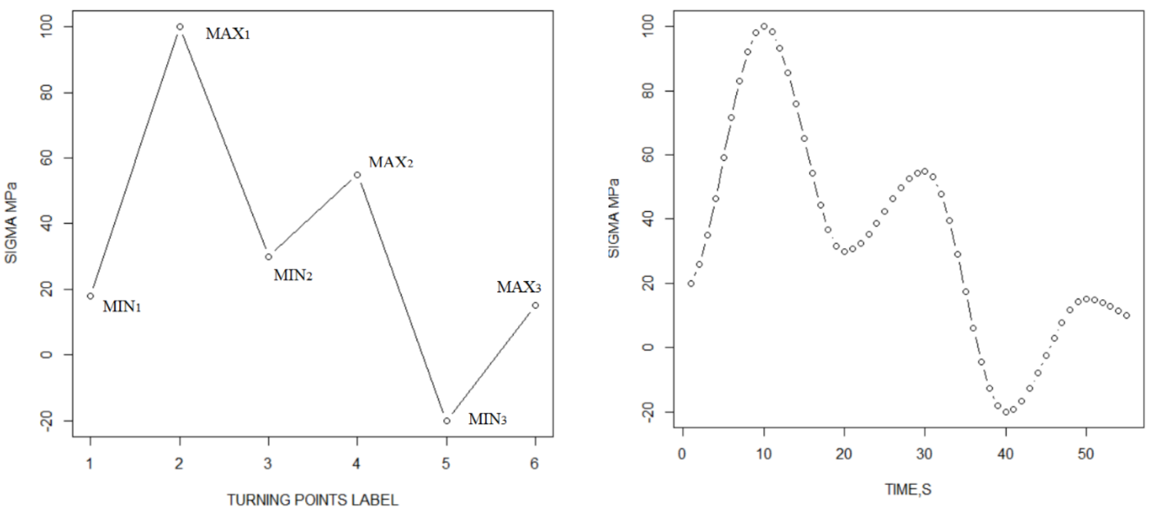
Figure 1: *Digital sequence MIN-MAX-MIN-MAX ("the saw") of representation of the part of random loading process FALSTAFF in the working parts of a fighter airplane [4 Falstaff]*

One the other hand, many applications require the necessity of the consideration of continuous process. Widespread nowadays-spectral methods for fatigue estimation [8] operate with the spectral density of the random process. Some other investigations, for example, the studying of the necessary discretization frequency, also require availability of continuous process. In problems of analyzing the strength of equipment, where there are miniature parts that do not allow registering stresses in operation using load cells or alternative measurement methods [9], it is necessary to resort to analyzing the stress distributions in the parts during vibrations by analyzing spectral densities. It is interesting to compare the results obtained using the rain method schematization with the results obtained based on the analysis of spectral densities.

While looking at the process in the Fig.1 one could easily see, that the process has the peculiarities at the peaks points and is far from being continuously smooth. It's first derivate is also non-continuous. Existing methods for functions extrapolation do not work here, because the main point here is to preserve the turning points as concerning their value as their sequence. The values of the turning points (extemes) of the process play the critical role while estimating machine parts durability under fatigue, and their sequence might as well be very important at the stage of estimation of crack propagation due to fatigue crack arrest effect after the overloading [10,11].

II. Method

To create the smooth extrapolating function with continuous derivate the cosine extrapolation has been used. Let's consider as an example a short part of "the saw" sequence, Fig.2a). Here only a few turning points are shown for the sake of simplicity.



a) *b)*

Figure 2: *Cosine extrapolation of the short peaks sequence. 1a) initial data; 2b) extrapolation*

Similarly to the shown in Fig.1, the data in Fig.2,a) is not, in reality, the process, but rather a kind of scheme, indicating the sequence of turning points MIN_1-MAX_1-MIN_2-MAX_2-MIN_3. The necessary requirement here if fulfilling the condition:

$$MIN\ i > MAX\ i > MIN\ i{+}1 \quad \& \ MAX\ i{-}1 < MIN\ i < MAX\ i \qquad \text{for } i\text{-}1,2 \ldots N \qquad (1),$$

where N is the total number of maximums (or minimums) in the sequence. It's worth mentioning, that to begin the sequence of extremes with the minimum, is a common practice among the researchers.

According to the idea [12], we consider each range in Fig.2a as a half-wave, described as a cosine function:

$$x(t) = A \cos (wt+\varphi) \qquad (2),$$

where x (t)– is the resulting continuous extrapolating function, which is described on the domain t=0… π/w, because the period of cosine function is T=2π/w, s.

For each half-wave starting from the next extremum MAX or MIN, the parameters A, w, and φ are unique. The stress amplitude A, MPa is defined as the half of modulus of successive extremes:

$$A=mod(MAXi - MIN\ i{+}1)/2 \ \vee \ A=mod(MAXi - MIN\ i)/2 \qquad (3).$$

The half-waves are appended one by one. Due to the appropriate choice of extrapolating function, the conditions of coherence and continuity in the appending points at the turning points are fulfilled:

$$x^-(t_0) = x^+(t_0) \qquad (4)$$

$$\dot{x}^-(t_0) = \dot{x}^+(t_0) \qquad (5).$$

In expressions (3) and (4), the indices – and + mean that the appending points belong to different half-waves, to the left and right of the turning points MAX i and MIN i. Condition (4) is provided by fulfilling condition (3). The condition (5) for equality of derivatives is fulfilled due to modeling principle, according to which the cosine argument is equal to $\pi/2$ at the points of extremes corresponding to $t = 0$, hence the derivative at all points of alignment is zero.

The resulting continuous process is shown in Fig.2b). Turning points MIN - MAX- MIN - MAX -MIN are the same in both processes. In this example for each half-wave 10 readings were taken.

Choice of the half-cosine frequency w

Due to the fact, that the frequency component is not defined in known standards random sequences [2-5], there is a need to define it. An analysis was performed earlier to investigate the connection between the range and the period of the oscillation [13]. Intuitively one can imagine, that the greater the oscillation, the more time will be required to it performance. That fact is well-known to the personal, who perform the testing under random loading. For the real oscillation of the torsion stresses in a shaft of caterpillar machine the regression equation has shown the existence of linear dependence.

In Fig.3 the dependence of the range *Raz* of oscillation on the half- wave period *t* of the FALSTAFF oscillation is shown. Although the scatter is significant enough (correlation factor is only K=0.6664 < 1), one could still notice a tendency of increasing *Raz* with *t* increasing. That might be used later on while modelling the random process by its turning points.

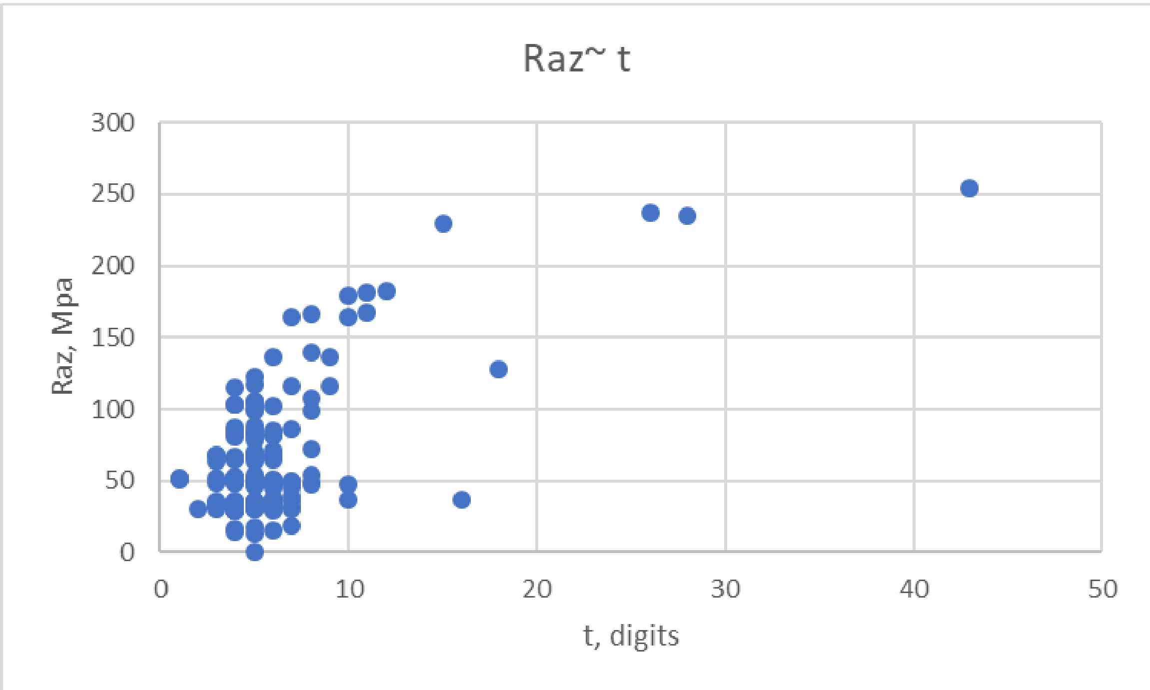


Figure 3: *Correlation of the period of oscillation t with its range Raz (FALSTAFF)*

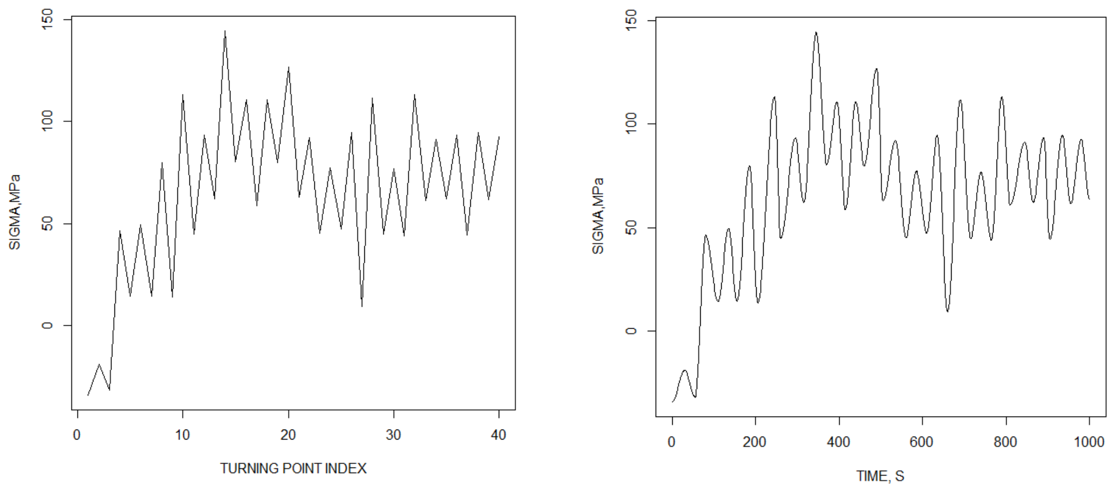
The proposed method of process imitation has been applied to the FALSTAFF sequence shown in Fig.1. [4]. Because one of the aims of this imitation is spectral density estimation for the spectral methods application [8], the number of reading in continuous process was chosen among the number:

$$2^n, \text{ where } n=9 \dots 12 \text{ (digit)} \quad (6)$$

to have the possibility to apply EXCELL for spectral density estimation.

III. Results

In Fig. 4 the part of the modelled by proposed method random process, corresponding to FALSTAFF sequence (Fig.1) is shown. Because the presence of random number generator in this algorithm this trial is not unique. Some duplications might exist. Because the aim of that modelling is the fatigue testing, the main factors for fatigue estimation were analyzed. They are: 1) irregularity factor *I;* 2) spectra fullness factor *V. V* depends on *m* – fatigue curve exponent and is always less than unity. Because of this particular modelling principle, *I* & *V* are the same for all duplications. That means, that when employing the rain-flow method for loading estimation, duplications will give the same result. The discrepancies will appear only while applying the spectral method [8] because the spectral densities function will differ due to random choice of *w* in (2).



a) *b)*

Figure 4: *FALSTAFF (the beginning part) a) digit sequence of the turning points; b) continuous process, modelled by proposed method*

Due to the fact, that for normal stationary processes a lot of tools were developed, the continuous modeled process (Fig.4, a) was investigated for its normality. To test the hypothesis that the generated process is normal, the Q-Q plot test was applied [14]. To check that the sample of random ordinates comes from a population that is Normally distributed? The Q-Q plot, shown in Fig. 5, indicates that it appears to be a fairly safe assumption. The points seem to fall about a straight line. Sample quantiles are on y-axis. The x-axis plots are the theoretical quantiles. Those are the quantiles from the standard Normal distribution with mean 0 and standard deviation 1.

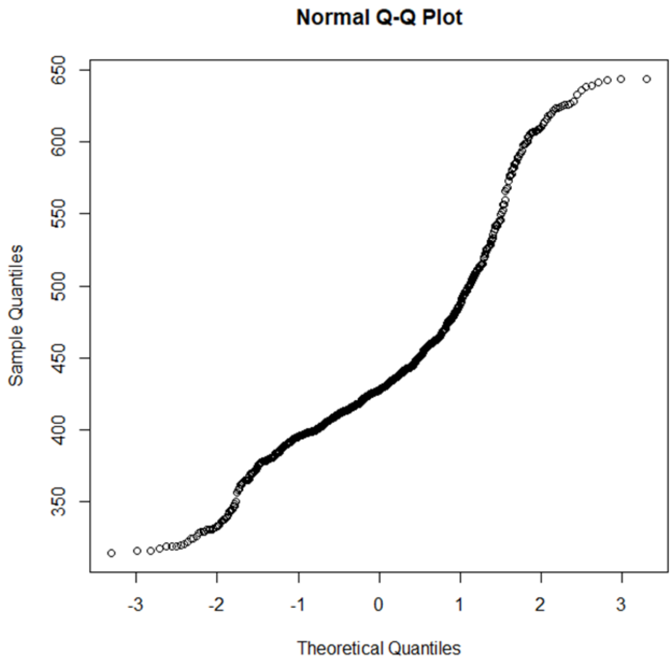


Figure 5: *Q-Q plot for random coordinates of the modeled process.*

In Fig. 6 the histogram of the selected by rain-flow cycle counting method [15] amplitudes for FALSTAFF sequence is shown. The fullness ratio *V*, which is similar to equivalent constant amplitude under the assumption of linear summation fatigue damages depends on the fatigue exponent *m*. This graph represents also the distribution of rain-flow amplitudes – the last one is exactly the same for continuous process! For the FALSTAFF example values *V* are shown in the Table 1 for some typical for construction materials *m*.

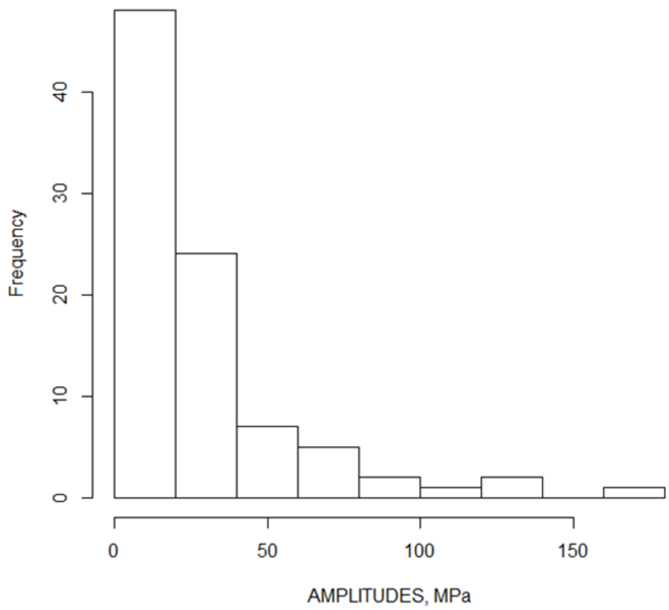


Figure 6: *Rain-flow amplitudes histogram for the selected realization*

Table 1: *Fulness ration V for FALSTAFF for some fatigue exponents m*

Fatigue exponent *m*	6	9	12
Fulness ratio *V*	0.512	0.622	0.6937

To investigate the loading characteristics of random processes sometimes the spectral densities apparatus is employed [8, 16]. For the normal stationary processes, the Rice theory [16] might be applied. It operates with the spectral densities. In particular, the irregularity factor I might be estimated through some moment, estimated on spectral density:

$$I = \left(\int_0^\infty S(w) w^2 dw\right) / \sqrt{\int_0^\infty S(w) w^4 dw \int_0^\infty S(w) w \, dw} \tag{7}$$

Here $S(w)$ is the dependence of the spectral density on frequency w.

In Fig. 7 the estimated by EXCELL $S(w)$ for continuous process is shown. The integrals (moments) in (7) were calculated numerically.

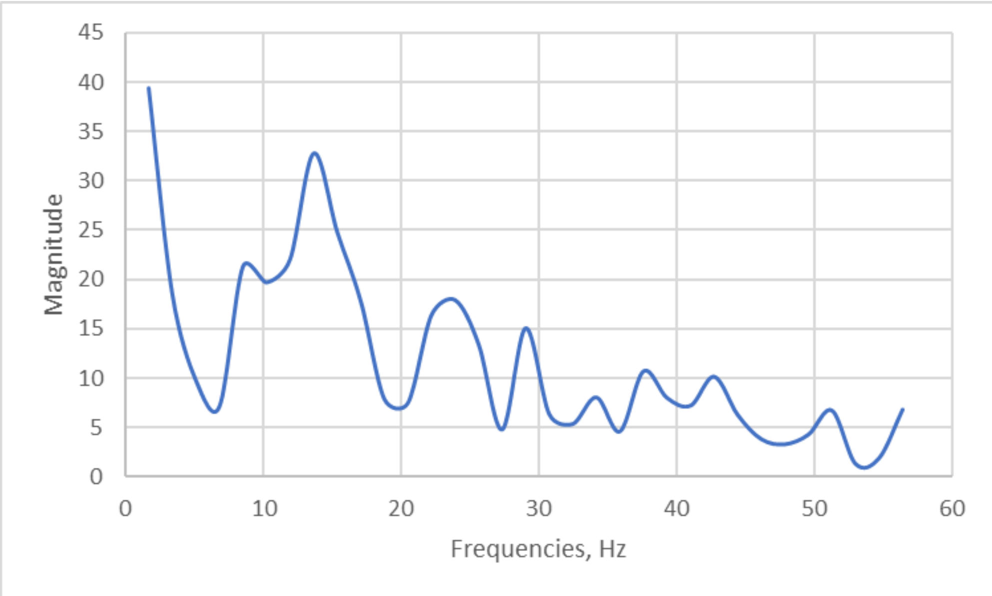


Figure 7: *Spectral density of imitated on FALSTAFF sequence process*

IV. Discussion and conclusions

The main result, which is important for fatigue estimation, is the coincidence of $V(m)$ for both representations – the turning point scheme and for the continuous modeled process (Table 1 – true for both).

The proposed method was applied for the standard sequence, recommended to fighter airplane parts testing. Before now there was no tool to generate continuous process out of sequence of digits, preserving the values and sequencing the turning points, which are critical for fatigue estimation.

The irregularity factor [15]

$$I = No/Ne \tag{8}$$

is estimated numerically by a simple manner by counting the number of crossing mean load level No and the number of extremums Ne slightly differs for two representations due to varied process mean estimation (Table 2).

On the other hand, the irregularity factor I =0.28, estimated for imitated by the proposed model process, according the Rice theory (7) differs significantly from I =0.52… 0.55, estimated numerically by the realization by formula (8). That might lead to discrepancies in fatigue estimation using the spectral approach.

Table 2: *Irregularity factor I*

Estimated numerically by turning points sequence by (8)	0.52
Estimated numerically by continuous modelled process by (8)	0.55
Estimated theoretically by spectral densities by (7)	0.28

All calculations and process imitation were performed in R [14].

References

[1] Michał Böhm. Fatigue life assessment with the use of spectral method for materials subjected to standardized wind loading spectrums. October 2018 AIP Conference Proceedings 2029(1):020005. DOI: 10.1063/1.5066467. Conference: MECHATRONICS SYSTEMS AND MATERIALS 2018At: Zakopane, Poland. Volume: AIP Conference Proceedings 2029, 020005 (2018)

[2] Tucker L., Bussa S. The SAE Cumulative Fatigue Damage Test Program / in book Fatigue under Complex Loading. Ed. R.M. Wetzel. 1977. Society of Automotive Engineering. P.1-44.

[3] D. Schütz, H. Lowak, de Jonge, J.Schijve. LBF – Report FB-106, NLR – Report TR 73, 1973. (TWIST).

[4] J. B. de Jonge et al. FALSTAFF. DESCRIPTION OF A FIGHTER AIRCRAFT LOADING STANDARD FOR FATIGUE EVALUATION (FIugzeugwerke, Emmen (Switzerland). 1976. 71 p.

[5] R. Fisher, E. Haibach. Modeling functions loading in experiments on the evaluation of materials / In the book “Behavior of steel under cyclic loads”. Edited by V. Dahl. 1983. P. 368-405.

[6] Song Cui, Enlai Zheng, Min Kang. A time-domain procedure for non-Gaussian stationary environmental testing using zero-memory nonlinear transformation// Journal of Vibration and Control · January 2020. DOI: 10.1177/1077546319894853

[7] Frédéric Kihm, Neil S. Ferguson, Jérôme Antoni. Fatigue life from kurtosis-controlled excitations//Procedia Engineering 133 (2015) 6th Fatigue Design conference, Fatigue Design 2015. P. 698 – 713.

[8] D.Benasciutti, R.Tovo. Frequency-based analysis of random fatigue loads: Models, hypotheses, reality// Mat.-Werkstofftech. 2018, 49, 345-361.

[9] Youssef M., Konstantin G. et al. Solder Joint Lifetime Modeling Under Random Vibrational Load Collectives// JOM: the journal of the Minerals, Metals & Materials Society. December 2019 DOI: 10.1007/s11837-019-03947-1

[10] S.G. Lebedinskii. Design Modeling of Propagation of the Fatigue Cracks in the Steel of Molded Parts of the Railway Structures // Journal of Machinery Manufacture and Reliability, 2018, Vol. 47, No. 1, P. 62–66. ISSN 1052-6188, © Allerton Press

[11] Irina V. Gadolina, Andrey A. Bautin, Evgenii V. Plotnikov. Studying the Crack Growth Rate Variability by Applying the Willenborg’s Model to the Markov’s Simulated Trials// In book: Advances in Intelligent Systems, Computer Science and Digital Economics. January 2020. DOI: 10.1007/978-3-030-39216-1_17

[12] Gryzlova T. P. Signal half-wave analysis (HW analysis) for recognizing the state of complex sources. // Proc of All-Russian conference "Modern signal processing technologies STOs-2018". - Moscow, RNTORES named after A. S. Popov, 2018. P. 240 – 244.

[13] Gryzlova T. P., Erpalov A.V., Gadolina I. V., Plotnikov E. V., Martynenko A. A. An analysis of the possible approaches for the consideration of loading the case of a narrow-band random loading processes. Half-wave method. //Proc. of the VI International conference " Fundamental research and innovative technologies in mechanical engineering”, Moscow, IMASH RAS, November 26-27, 2019, p. 133-135.

[14] R Core Team. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. 2017. URL https://www.R-project.org/

[15] Interstate Standard GOST 25.101-83 Strength calculation and testing. Representation of random loading of machine elements and structures and statistical evaluation of results. - M: Standartinform, 2005. – 25p. [in Russian]

[16] Erpalov, A.V., Shefer, L.A. Fatigue-based Classification of Loading Processes Procedia Engineering 2nd International Conference on Industrial Engineering (ICIE-2016) Volume 150, (2016) 2016.–Vol. 150. pp.144-149.