Mathematical Models of Systems with Several Lifts and Various Control Rules

Belyaev Y.K., Hajiyev A.H.
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Umeå University, Sweden, e-mail: yuri.belyaev@umu.se
Azerbaijan National Academy of Sciences, e-mail: asaf.hajiyev@gmail.com

Abstract

In the present paper, several mathematical models of lifts' systems with various control rules are constructed and simulated data are used for estimating the various parameters of the systems. For systems with a rare input flow of customers, the graphs, describing the positions of the lifts, at the preceding customer's arrival instant, are given. These graphs help analytically find, the various characteristics of the lifts' systems (a customer's average waiting time, a customer's total time, energy expenses and others). There are introduced the various control rules by the lift systems, which are investigated by simulation. Using Wolfram Mathematica, the authors have prepared several programs for simulating and estimating the various operational parameters of the lifts' systems with different control rules. Using these programs, the numerical estimates of the various parameters the considered lifts' systems, have been found. These data can be used for defining the dependence of optimal lift roominess on an intensity of input customers' flow and finding the optimal number of the lifts, during planning and construction of the buildings and skyscrapers.

Keywords: simulation, rare flow, lifts systems, customer waiting, service, total time, roominess.

1. Introduction

The development of the modern cities led to an appearance of the huge metropolises like New York, Moscow, Shanghai, Istanbul and others, with a lot of skyscrapers. It is difficult to imagine today the world metropolises and the modern cities without skyscrapers, where many lifts' systems with various control rules, are used. In the process of designing a Skyscraper, one of the important problems is to find the optimal parameters of the lift systems, e.g. to find the optimal conditions (roominess, capacity, size) of a lift cabin and reduce the customer's average waiting, the service time and to save energy expenses. The effective approach for such type of investigation is a construction of mathematical models, describing lift systems. Although there are a lot of similarities between the transportation and traffic problems with lift systems, nonetheless, it is necessary to construct new mathematical models and develop the effective approaches for investigation these systems, taking into consideration their specifics, because they have different and complicated structures. The new approaches and methods can allow estimate the main operation parameters (customers' waiting and service time, energy expenses and others) and making necessary recommendations for constructors and engineers. There are a lot of publications in this field e.g. [1-5], but complicated lift systems with various control rules, are not yet investigated widely. The mathematical models, describing a behavior of lift systems, can be applied for other systems with moving servers, for instance, Shuttle and Communication Systems,

Traffic and others. In [6] the mathematical models of the systems with one lift, different control rules were introduced. One of the effective methods is a simulation by using programs, described in Wolfram Mathematica. Then, simulated statistical data can be used to find estimates of different parameters for such lift systems.
Using the given statistical data, the estimates can demonstrate, for instance, the dependence of the lifts' roominess on an intensity of the input flow. More complicated problems appear in the investigation of the systems with several lifts. The processes, describing the behavior of the lift systems have stochastic and complicated structures. It leads to constructing and investigating the new stochastic models, approaches and programs for their simulation.

In this paper, the authors consider various lift systems with different parameters and different control rules. This paper can be regarded as a continuation of the investigations presented in [6] and hence, we will follow the notations introduced in that paper.

In contrast to [6], in this paper there are considered various lift systems with several lifts, together with the average waiting and service time and other characteristics. An important parameter such as energy expenses can also be investigated. For such issues, the analytical approaches are faced with some problems. Hence, for complicated systems, the methods of collecting the simulation data can also be used. This gives the effective results and allows draw useful conclusions for practice. Some approaches are suggested in [7-9]. The behavior of lift systems can also be described by mathematical models of moving particles. Some models are suggested in [10-12] and these methods can be applied for the lift systems.

2. Various control policies for the lift systems

As it was mentioned above, the construction of skyscrapers in the modern cities requires the creation of different control rules in lifts' systems, which reduce the customer's waiting time and energy expenses. There already exist such control policies, called Odd-Even (some lifts serve customers at the odd floors and other lifts at the even floors), or some lifts serve customers, at the floors 1, 2, …, N others at 1, N+1, N+2, …, 2N. To save time, we need new more effective control policies for the lift systems, which can minimize the expenses for their construction and optimize some parameters (waiting and service time, minimization of energy resources, increase the life-time of the lifts, working without repairs and others). There exist many control policies in the world, for instance, the FIFO service (first come, first outcome), LIFO (last come, first outcome) and others. If the lift comes to a customer at the first floor, who first called it, then this lift can serve this customer plus only other customers going to upper floors than this customer (e.g. so as in Hilton hotel in Baku). An interesting unofficial control policy was created in the seventy years of the XX-th century, by the students in the dormitory of the Lomonosov Moscow State University. There are 18 floors in the student dormitory and two lifts' halls with four lifts in each.

The first lift hall operates from the 1st to 12th, 14th, 16th and 18th floors. For the lifts work more rapidly, it was skipped the odd numbered floors, after 12th. There is also a second lift hall for serving the 1st-10th floors. If in the first hall, a lift came to the first floor and the first student yelled the word "HIGHER", then, the lift would be filled by students who are going up only to the higher floors (16th and 18th) and the next lift will be filled by students who are going to the 12th, 14th, 16th and upper. If the first call had been "LOWER", then the lift would have operated between the lower floors (12th, 14th and afterward, to the other upper floors). The students called it a Higher-Lower system. In [8, 9], a comparative analysis of simple different control policies has been done.

The comparison of the control system "Higher-Lower" with "Even-Odd" showed an advantage of "Higher-Lower" control policy (the customer's average waiting time before service is shorter). For each control policy, it is possible to construct mathematical model, which show a preference of these systems.

3. The mathematical models of the lift systems

Below, if unless otherwise agreed, it will be considered the stationary regimes for the lift systems operating. Following [6], we will construct the mathematical models of the lift systems and remind the following notations:

$L_kF_nC_{xx}$ – is the systems with k lifts, n floors and control policy xx;
i - is an ordered in time identifying number of a customer during one day simulation;
$f_a(i)$ - is the floor of appearance of the i-th customer;
$f_d(i)$ - is the floor of destination of the i-th customer.
It is necessary to note that for different i the $f_a(i)$ and $f_d(i)$ can take the same value.
$t_a(i)$ - is the instant of appearance of the i-th customer;
$t_b(i)$ - is the instant of the beginning service of the i-th customer;
$t_e(i)$ - is the instant of end service of the i-th customer;
$t_{c(j)}$ - is the instant when lift on j-th cycle is returning to the 1-st floor;
r - roominess, restriction on maximum possible number of customers who can be in the lift cabin;
h_f - time necessary for lift to move up or down between two neighbor floors;
h_d – time spending for opening and closing the floor's door;

Usually, in practice, $h_d=2h_f$. If we consider stationary input flow, then, the following parameters are used:

$l_{f_1f_2}$ –intensity of customers' flow, who appear at the f_1-th floor and want to go to f_2-th
$l_1 = \sum_{k=2}^{n} l_{1k}$ - intensity of customers flow, who appeared at the first floor and are going to upper floors;
$l_2= \sum_{k=2}^{n} l_{k1}$ –intensity of customers' flow, appeared at upper {2, 3, …, n_f} floors, who want to go down to the first floor.

We would like to remind that in [6], it was also introduced the following specific characteristics for the lift systems:
CWT(f_a, f_d) – Customer average Waiting Time (f_a, f_d) – is defined as mean waiting time for customers going from the f_a floor to the f_d floor. It is measured as the time interval $\{t_a(i), t_b(i))$ from the instant of the customers' appearance at the f_a floor, until the instant when the customer comes into the lift cabin, going (in the direction of the f_d floor) to the desired f_d floor;
CWT(f_a) – Customer average Waiting Time (f_a) is defined as the mean time for customers going from f_a floor, to any other floor (upper or lower floors). It is measured from the instant of the customers' appearance at the f_a floor, until the instant when customer comes into the lift cabin, going to desired destination floor;
CST(f_a, f_d) – Customer average Service Time (f_a, f_d), is the mean time defined for the customers coming into the cabin on the f_a floor and going to the f_d floor. It is measured from the instant when a customer comes into the cabin of the ordered lift, moving from the f_a floor (in the direction of the f_d floor), until the moment when the customer leaves the lift at the f_d floor.

Generally, if any characteristic has index i, then, it is a random variable, for instance CST ($f_a(i)$, $f_d(i)$) is a random variable.

CST (f_a) – Customer average Service Time, which is going up or down, from f_a floor to the desired floor;
CST(S) – Customer average Service Time in the system S, i.e. mean time from the instant when a customer gets the lift, until the instant when the customer gets off the lift;
CTT(S) – Customer average Total Time in the system S, which is measured as a mean time from the instant when a customer arrives into the system until the customer, gets off the lift (arrival to ordered floor).

For instance, CTT($L_kF_nC_{xx}$) is a customer average total time, for a system in a building with k lifts, n floors and control policy xx.

Finally, in the loading regime, we will be interested in customers' average total time, who are going from the first floor to the upper floors. In the unloading regime, it will be the customer's average total time, who are coming from the upper floors to the first floor. This time includes possible stops on intermediate preceded floors, ordered by other customers, being in the same cabin.
LRC – Average Lift Return Cycle time, i.e. the mean time interval between two neighbor comings of the lift, to the first floor.
In this paper, in distinction of [6] we also introduce the new parameters for the lifts systems, which describe the lift energy expenses and the single race time:
LEE_j (S) – Average value of the j-th Lift Energy Expenses in the system S, measured in Kw (kilowatt);

Note that Energy Expenses in Kw depend not only on the volume cabin weight, but also on its speed, acceleration and deceleration. Empirically, electric Energy Expenses can be shown each day, on the electric counter of each lift.
SRT(t) – Average Single Rate Time, i.e. mean time when the lift is moving without customers, during time t;
SEE(S) – average value of System Energy Expenses, i.e. mean value of energy expenses of all the lifts in the system (S)
$SEE(S)=LEE_1(S)+LEE_2(S)+\ldots+LEE_n(S)$;
k_d – coefficient, defining the energy expenses of the lift, during a unit time for opening and closing the doors;
k_f – coefficient, defining the energy expenses of the lift during a unit time necessary for covering the distance between two neighbor floors.

Remind the lift system giving service for 1, 2,…, n_f floors with k lifts and control policy xx, will be denoted as $L_kF_nC_{xx}$.

Control policy xx=IL means a system without control, i.e. all the lifts are operating independently to each other. Sometimes, such a system is denoted as $L_kF_nC_{IL}$ (Independent Lifts).
For an IL control system, if at the preceding instant of a new customer's arrival, several lifts are free (empty), then, all of them will go to this customer's call. Such systems are often used in the buildings with two lifts.
$L_kF_nC_{DL}$ is the system with k Dependent Lifts, n floors (for a customer's call, the Draws the nearest Lift is going).
$L_2F_nC_{n1,n2}$ is the system with 2 lifts, n floors and after completing the customer's service, one lift must go to the n_1 floor if there is no lift, otherwise, it should go to the n_2 floor.
Below, we consider systems with a rare input flow.

4. Unloading regime in the systems with rare flows of customers. Definition

We will say that a flow of customers is rare, if at the instant of a customers' arrival, all the lifts are available. Such situations are observing:
a) in the office buildings, between 10.00-12.00 (when offices have very few visitors);
b) in the buildings, between 10.00-12.00 and 15.00-17.00 (when old people with infant children can go for a walk);
c) in the shopping malls, if, at the instant of customers' arrival into the system,
all lifts are occupied, then, the customers use escalators.

Remark 4.1 According to the above-mentioned paragraph c), such flows or point processes can be called as the flows or point processes, transforming themselves into the point processes with rare intensity (transformed flows or point processes).

We consider IL policy ($L_2F_nC_{IL}$), which means that if both lifts are free (empty), then at the next customers' call, both lifts (perhaps from different floors) are going simultaneously. Such situations can be observed in buildings where each lift has an individual call button and when those buttons are pushed simultaneously.

Then, for that system, at the preceding of a customer's instant arrival, one lift occupies the first floor, the other k-th(k=2,3,..,n) floor,(see, Fig.4.1).Below, in all Figures, the blue color means lift is free and the red color means that lift is occupied.

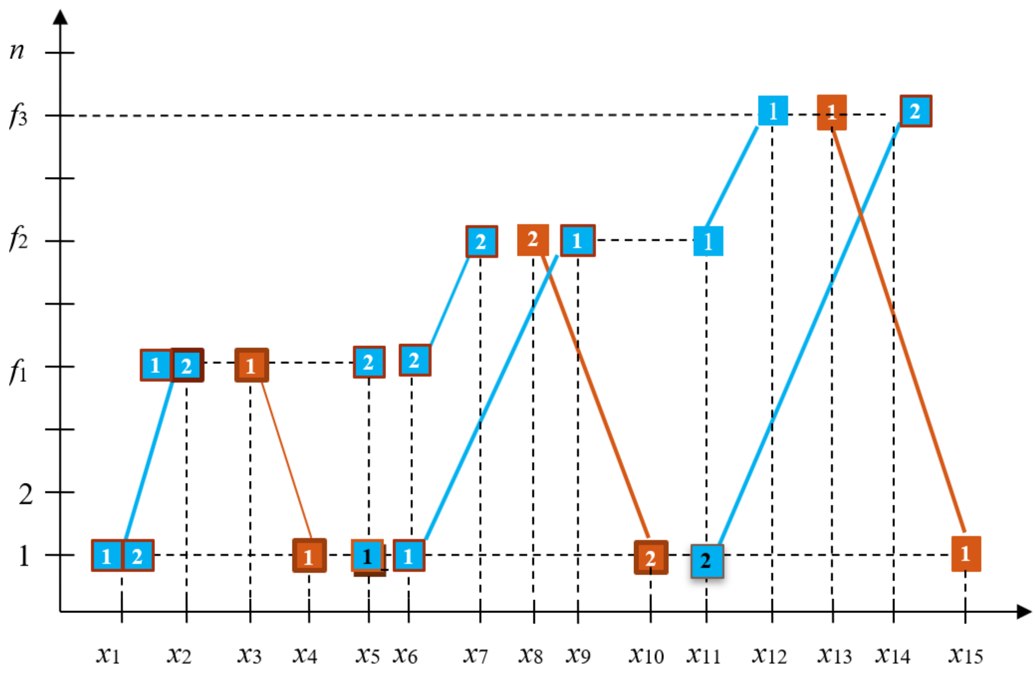

Fig.4.1

$x_1=t_a(1)$, $x_2=x_1+(f_1-1)h_f$, $x_3=x_2+h_d=t_b(1)$, $x_4=x_3+(f_1-1)h_f$, $x_5=x_4+h_d= t_e(1)$, $x_6=t_a(2)$, $x_7= x_6+(f_2-f_1)h_f$, $x_8=x_7+h_d=t_b(2)$,$x_9=x_7+(f_2-1)h_f$, $x_{10}=x_8+(f_2-1)h_f$, $x_{11}=x_{10}+h_f =t_e(2)$, $x_{12}=x_{11}+(f_2-f_1)h_f$, $x_{13}=x_{12}+h_d=t_b(3)$; $x_{14}=x_{11}+(f_3-1)h_d$, $x_{15}=x_{13}+(f_3-1)h_d$.

Thus, we get: $CWT(L_2F_nC_{IL})=h_f(n-1)/2+h_d$, $CST(L_2F_nC_{IL}) = h_f(n-1)/2+h_d$ and $CTT(L_2F_nC_{IL}) = h_f(n-1)+2h_d$; $LEE(L_2F_nC_{IL})=k_f(n-1)h_f+2k_d h_d$.

For SRT (average energy spent for one call) during the one-unit time we have $SRT=k_f h_f [(n-1)/2+(n-1)/4]+mk_d h_d = k_f h_f [3(n-1)/4]+m k_d h_d$. For Poisson input flow during the time t, an average number of arrived customers into the system, equals ◎t. Hence, SRT(t)=◎($k_f h_f$ 3(n-1)/4+m$k_d h_d$)t. As ◎T is an average number of arrivals during the time T, then ◎T$k_f h_f$ (n-1) is an average energy which lift spends for serving the customers (motion of lift), during time T. As at each arrival instant, an average number of customers equal m, then ◎Tm is average number of customers arrived during the time T into the system. For each customer's arrival, the lift spends the time h_d for opening and closing the door. If we assume that each customer spends time h_c coming in and getting off a lift, then, the mh_c is the time, which was spent for the m customers (coming in and getting off). Hence, a customer average energy spent for opening and closing the door, for customers coming in the lift and getting off equal ◎◎◎T $k_d h_d$+ 2◎◎Tm h_c=◎◎◎T($k_d h_d$+m h_c). Thus, we have $LEE(L_1F_nC_{IL})$=◎T$k_f h_f$ (n-1)+2◎T($k_d h_d$+mh_c). Below, for simplicity, we assume h_c =0, which means that during the time h_d all customers who want to come in and get off a lift, can do it.

4.1. Unloading regime for system $L_2F_nC_{DL}$

We would like to remind that DL means that for a new customers' arrival, only the draws nearest lift which will operate. Then, at the preceding of the moment of the customer 's arrival, both lifts occupy the first floor, which means that in fact, only one lift is operating (see, Fig.4.2).

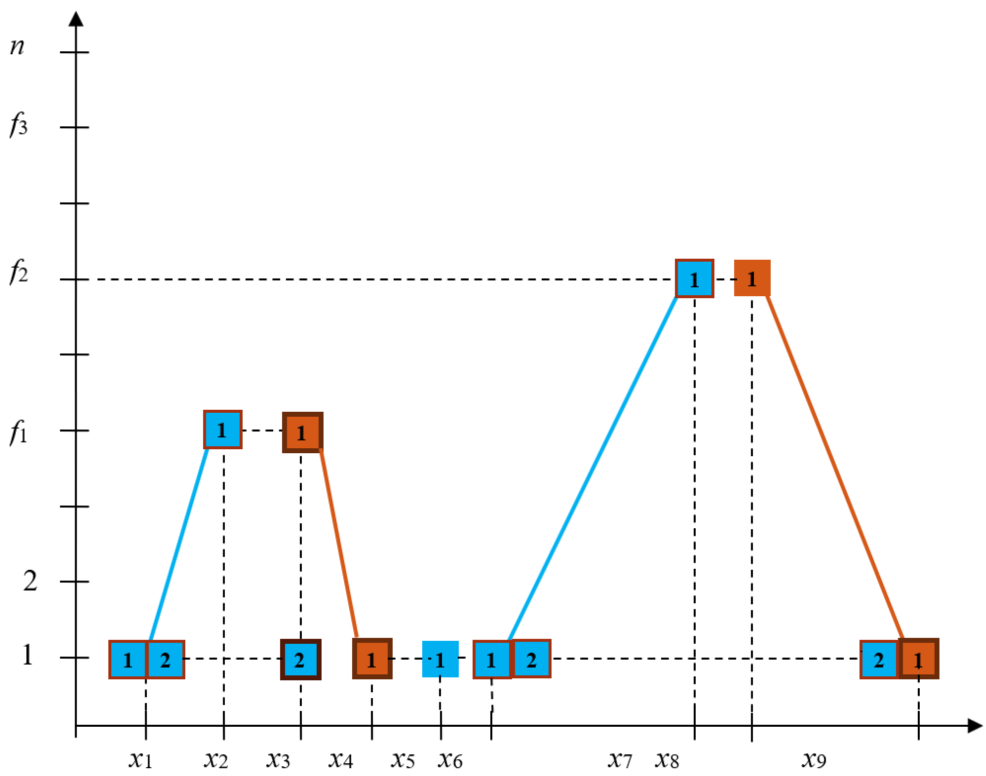

Fig.4.2

$x_1=t_a(1)$, $x_2=x_1+(f_1-1)h_f$, $x_3=x_2+h_d=t_b(1)$, $x_4=x_3+(f_1-1)h_f$, $x_5=x_4+h_d= t_e(1)$,
$x_6=t_a(2)$, $x_7= x_6+(f_2-f_1)h_f$, $x_8=x_7+h_d=t_b(2)$, $x_9=x_7+(f_2-1)h_f$.
In fact, in an unloading regime, the system $L_2F_nC_{DL}$ is operating like a system with one lift, i.e.

$L_1F_nC_{IL}$.. In Fig.4.2, the lifts' positions at different instants, for the system $L_2F_nC_{DL,}$ are shown. Hence, we can calculate
$CTT(L_2F_nC_{DL})=(n-1)h_1/2+h_2$ and $SRT(L_2F_nC_{DL})=(n-1)h_1/2h_1+h_2$

It is easy to calculate all the characteristics and we leave it to the reader. Hence, for this case of a rare input flow (unloading regime), the system $L_2F_nC_{IL}$ is preferable to the system $L_2F_nC_{IL}$ (CTT is less).

5. Loading regime for system $L_2F_nC_{IL}$.

We assume $\lambda_{12}=\lambda_{13}=\ldots=\lambda_{1n}$ and $\lambda_1=\Sigma_{k=2}{}^n\lambda_1$ represent the intensity of the transformed flow of customers (transformed point processes) and $\lambda_{21}=\lambda_{31}=\ldots=\lambda_{n1}=0$. As it was mentioned above, such a situation can be observed in the residence building, between 10.00-12.00 hours. It is clear, at the preceding of a customer's arrival instants, one lift occupies the first flow, another j-th (j=2, 3, …).

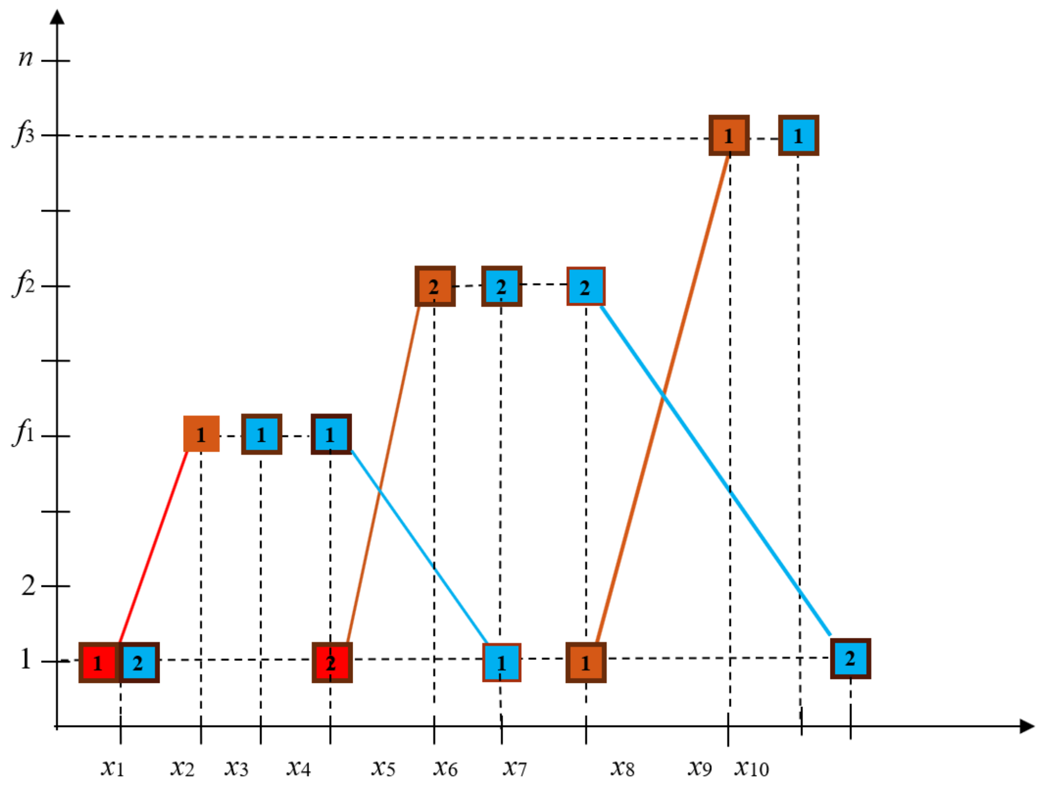

Fig.5.1

$x_1=t_a(1)=t_b(1)$, $x_2=x_1+(f_1-1)h_f$, $x_3=x_2+h_d=t_e(1)$, $x_4=t_a(2)=t_b(2)$,
$x_5=x_4+(f_2-1)h_f$, $x_6=x_5+h_d=t_e(2)$, $x_7=t_a(3)=t_b(3)$, $x_8=x_7+(f_3-1)h_f$,
$x_9=x_8+h_d=t_e(3)$, $x_{10}=x_7+(f_2-1)h_f$.

5.1 Loading regimes for system $L_2F_nC_{DL}$

At a customer's call, only one lift goes, i.e. the nearest lift. Then, starting from the third customer, at the preceding of a customer's arrival instant, based on the same probability, one lift occupies f_1-th ($f_1=2,3,\ldots n;$) floor, another f_2 -th ($f_2=2,3,..n;$) floor (see Fig. 5.2).

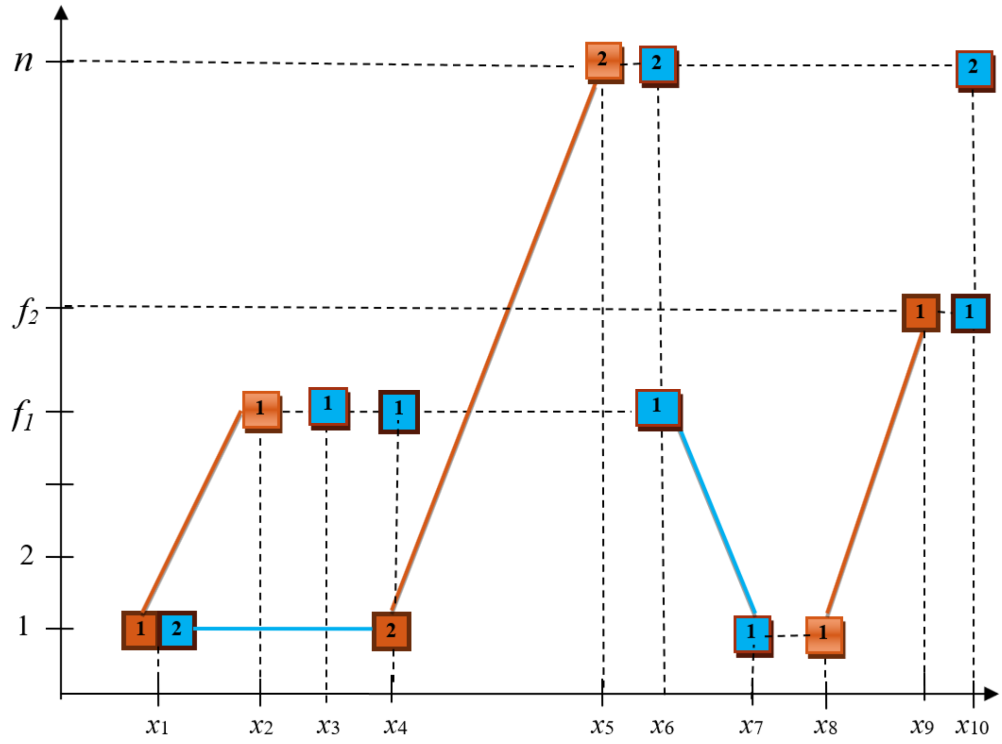

Fig.5.2

$x_1=t_a(1)=t_b(1)$, $x_2=x_1+(f_1-1)h_f$, $x_3=x_2+h_d=t_e(1)$, $x_4=t_a(2)=t_b(2)$, $x_5=x_4+(f_2-1)h_f$, $x_6=x_5+h_d=t_e(2)$, $x_7=t_a(3)$, $x_8=x_7+(f_3-1)h_f$, $x_9=x_8+h_d=t_b(3)$, $x_{10}=x_9+(f_1-1)h_f$.

Remark 5.1. In the loading regime the behavior of the system $L_2F_n\,C_{DL}$ coincides with the behavior of the system $L_1F_nC_{IL}$, because as soon as one lift reaches the last (n-th) floor, afterward in fact, only one lift (which occupies the first floor) is operating, because it will always be a nearest lift for the arrived customer.

6. Calculating various characteristics of the lifts' system $L_2F_nC_{IL}$ (two lifts, n floors, independent lifts). Loading regime

As we consider $L_2F_nC_{IL}$ system, then, at customers' call, both lifts go independently from each other. Taking into consideration that in the loading regime, at the preceding customer's arrival instant, one lift occupies the first floor, another one j-th (j=2, 3,..., n) floor (see Fig. 5.2). Then the calculations yield to:

$CTT(L_2F_nC_{IL})=(n(n-1)(2n-1)/8+(n-1)^2(n-2)/2)h_f/(n-1)^2+h_d=(3n/4-1/8-1/8(n-1))h_f+h_d$;

$SRT(L_2F_nC_{IL})=(n(n-1)(2n-1)/24+(n-1)(n-2)(2n-3)/6)h_f/(n-1)^2+h_d=$

$=[5n/12-19/24+5/24(n-1)]h_f+h_d$.

Denote

$a=1/8-1/8\,(n-1) = (1/8)\,(1-1/(n-1))=(1/8)[(n-2)/(n-1)]<0.125$;

$b=-19/24+5/24(n-1) =(1/24)[5/(n-1)-19]/<0.8$;

As CTT and SRT are linear functions of n, with coefficients 3n/4 and 5n/12 and /a/<0.125 and /b/<0.8, then, for n>10 we can neglect a and b and hence, put:

$CTT(L_2F_nC_{IL})=(3n/4)h_f+h_d$; $SRT(L_2F_nC_{IL})=(5n/12)h_f+h_d$. (6.1)

If we put $h_f=1$, $h_d=0$, then $CTT(L_2F_nC_{IL}) = 3n/4$, $SRT(L_2F_nC_{IL}) = 5n/12$.

6.1 System $L_2F_nC_{IL}$ (two lifts, n floors, independent lifts). Mixed regime

$\lambda_{12} = \lambda_{13} = \ldots = \lambda_{1n}$, $\lambda_1 = \Sigma_{k=2}^n \lambda_{1k}$, $\lambda_{21} = \lambda_{31} = \ldots = \lambda_{n1}$, $\lambda_2 = \Sigma_{k=2}^n \lambda_{k1}$ are the intensity of the transformed flows of customers. In this case, at the preceding customer's arrival instant, one lift occupies the first floor and another f-th floor (f=2, 3,…,n). The probability to have a customer at the first floor is $\lambda_1/(\lambda_1 + \lambda_2)$ and at the other (upper) floors, it equals $\lambda_2/(\lambda_1 + \lambda_2)$. Thus, for a Customer Total Time and Single Race Time after the routine calculations we have

$$CTT(L_2F_nC_{IL})=(\lambda_1/((\lambda_1+\lambda_2))((n-1)/2)+\lambda_1/(\lambda_1+\lambda_2))(1/(n-1)^2)((n(n-1)(2n-1)/8)+$$
$$+(n-1)^2(n-2)/2)h_f+h_d$$
$$SRT(L_2F_nC_{IL})=(\lambda_1/(\lambda_1+\lambda_2))(1/(n-1)^2)+\lambda_1/(\lambda_1+\lambda_2)((n(n-1)(2n-1)/48)+$$
$$+(n-1)(n-2)(2n-3)/12)h_f+h_d.$$

If n>10 then we have the following formulas

$$CTT(L_2F_nC_{IL}) = ((n/4)(2\lambda_1+3\lambda_2)/(\lambda_1+\lambda_1))h_f+h_d;$$
$$SRT(L_2F_nC_{IL}) = ((5n/24)/\lambda_2/(\lambda_1+\lambda_2))h_f+h_d. \tag{6.2}$$

Corollary 6.1. If $\lambda_1 = \lambda_2$, then, it follows from formula (6.2):

$$CTT(L_2F_nC_{IL})=(5n/8)nh_f+h_d;\ SRT(L_2F_nC_{IL})=(5n/48)h_f+h_d \tag{6.3}$$

Introduce the control policy, which means that at the end of a customer's service instant, the lift should go to the f1-th floor (if there is no lift), otherwise, the lift must go to the f2-th floor. Our aim is to find f1 and f2, which minimizes the value of CTT(L2FnCxx). Then, at the preceding of a customer's arrival instant, one lift occupies f1-th floor, another f2-th floor. Similarly, to (6.3), we have:

$$CTT(L_2F_nC_{f1f2})=(\lambda_1/(\lambda_1+\lambda_2))(f_1-1+(n-1)/2)+(\lambda_2/(\lambda_1+\lambda_2))((f_1-1)^2/(n-1))+f_1+$$
$$+(f_2-f_1)/2(n-1)((f_2-f_1)/2+f_1-1)+((f_2-f_1)(f_2-1)/2(n-1)+(n-f_2+f_1-1))h_f+h_d;$$
$$SRT(L_2F_nC_{f1f2})=(\lambda_1/(\lambda_1+\lambda_2))(f_1-1)h_1+(\lambda_2/(\lambda_1+\lambda_2))(f_1-1)^2/2n+((f_2-f_1)^2 4(n-1)+$$
$$+(n-f_2)/2(n-1))h_f+h_d. \tag{6.4}$$

If h_f=1 and h_d=0, then for n>10 we have

$$f_1=\max[1,(n/4)(1-3\lambda_1/\lambda_2)];\ f_2=(f_1+2n)/3 \tag{6.5}$$

Using (6.4) and (6.5) we have

$$CTT(L_2F_nC_{f1f2})=((\lambda_1/(\lambda_1+\lambda_2))(f_1-1+(n-1)/2)+\ +(\lambda_2/(\lambda_1+\lambda_2))(3(f_1-1)^2+(n-f_1)(f_1+2n-4))3(n-1)$$
$$SRT((L_2F_nC_{f1f2})=(\lambda_1/(\lambda_1+\lambda_2))(f_1-1)+(\lambda_2/(\lambda_1+\lambda_2))(3(f_1-1)^2+(n-f_1)^2)/6(n-1) \tag{6.6}$$

Remark 6.1. For a rare flow, if $\lambda_1>0$, $\lambda_2=0$, then, it follows from (6.6), that f1=1. In this case, in fact, only one lift operates, because in the end of the service, it comes to the first flow and there are no customers in another floor. Therefore, it does not matter the location of the second lift.

Such systems can be used in the residence buildings, where there are two lifts with different capacities. The lift with a larger capacity is used for the delivery of the furniture and other big goods. For n>10, it follows from (6.6)

$$CTT(L_2FnC_{1,1}) = (n-1)/2,\ SRT(L_2FnC_{1,1}) = 0.$$

Remark 6.2. For a rare flow, if ◎◎◎= 0, ◎◎◎> 0, then, for n>10, it follows from (6.5) and (6.6)
$f_1=n/4$, $f_2=3n/4$,
$CTT(L_2F_nC_{f1,f2})=5n/8$, $SRT(L_2F_nC_{f1,f2}) = n/8$ (6.7)
i.e. at the preceding customer's arrival moment, one lift should occupy (n/4) –th floor, another (3n/4)–th floor.

The comparison (6.5) and (6.6) shows that introduced control leads to an advantage in a customer's average service time 16% and regarding the single race time 10%.

Remark 6.3. If ◎◎◎◎◎ then for n>10 it follows from (6.6) and (6.7)

$f_1=1, f_2=2n/3, CTT(L_2F_nC_{f1,f2})=7n/12, SRT(L_2F_nC_{f1,f2})=(n/12),$ (6.8)

i.e. at the preceding customer's arrival moment, one lift must occupy the first floor and another, 2n/3 floor (see, Fig.6.2). We assume that 2n/3 is an integer number

$x_1=t_a(1)=t_b(1), x_2=x_1+(f_1-1)h_f, x_3=x_2+h_d=t_e(1), x_4=t_a(2)=t_b(2), x_5=x_4+(f_2-1)h_f,$ $x_6=x_5+ +h_d=t_e(2), x_7=t_a(3), x_8=x_7+(f_3-1)h_f, x_9=x_8+h_d=t_b(3), x_{10}=x_9+(f_1-1)h_f, x_{11}=x_{10}+h_d= t_e(3).$

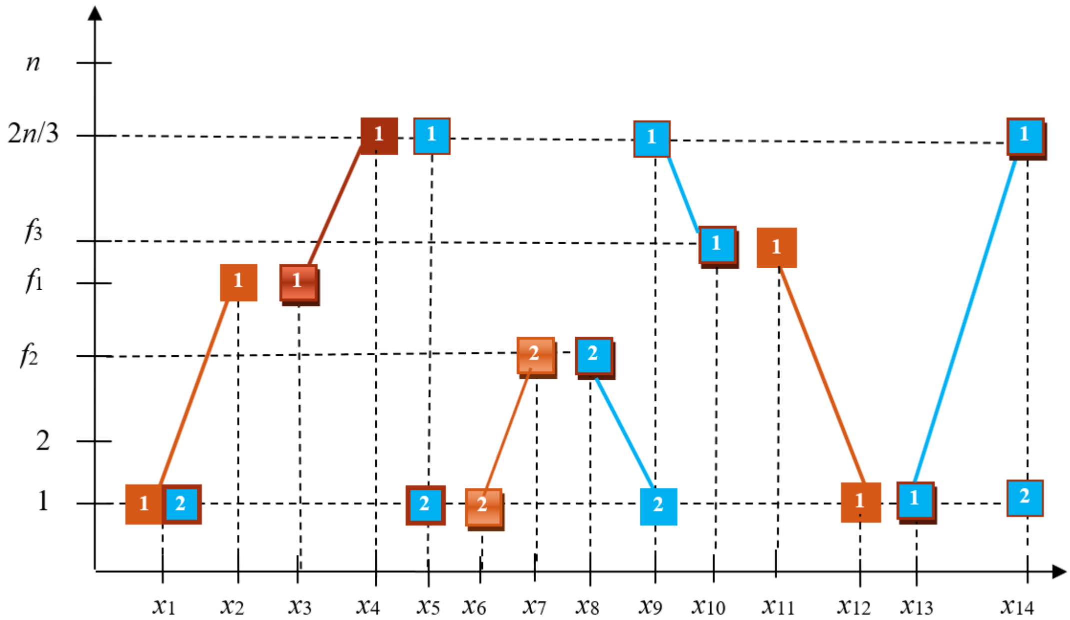

Fig.6.2

$x_1=t_a(1)=t_b(1)$, $x_2=x_1+(f_1-1)h_f$, $x_3=x_2+h_d=t_e(1)$, $x_4=x_3+(2n/3-f_1)h_d$,
$x_5 =t_a(2)$, $x_6= x_5+h_d$, $x_7=x_6+(f_2 -1)h_f$, $x_8=x_7+h_d$, $x_9= x_8+(f_2-1)h_f$,
$x_{10}= x_9+(2n/3-f_3)h_f$, $x_{11}= x_{10}+h_d$, $x_{12}= x_{11}+(f_3-1)h_f$, $x_{13}= x_{12}+h_d$, $x_{14}= x_{13}+(2n/3-1)h_f$.

In Fig.6.2, there are shown the lifts' positions at different instants, for the system $L_2F_nC_{1,2n/3}$. At the preceding of a customer's arrival instant, one lift occupies the first floor, another floor 2n/3 (see, Fig.6.2; (x_5, x_9, x_{14})). The comparison (6.8) with (6.9) shows that the control gives an advantage of the customer's average service time of 4% and regarding the single race time, of 2%.

Consider the system $L_l F_n C_{f1,f2,..,fl}$, where l – is a number of lifts in the system. Denote f_j (j=1,2,..,l) the floor, which must be occupied by j-th lift in an optimal regime.

We will show that for the system $L_l F_n C_{f1,f2,..,fl}$ in the mixed regime $f_1,f_2,\ldots,f_l$, the following recurrent formulas are applied:

$$\begin{aligned} &2\lambda_1(n-1)+ \lambda_1(3f_1-f_2-2) = 0 \\ &f_{i-1} -2f_j+f_{j+1}=0, \ j=2,3,\ldots,l-1 \\ &2n -3f_l+f_{l-1}=0 \end{aligned} \tag{6.9}$$

Below, in Fig.6.3, there are shown the lifts' positions at the different instants, for the system $L_k F_n C_{f1,f2,\ldots,fl}$. We remind that t_i is a customer's arrival instant into the system. If we put l=3 then f_1, f_2, f_3 satisfies the equation (6.9). According to an optimal control at each preceding customer's arrival instant, one lift must occupy f_1-th floor, another f_2-th floor and the third lift - f_3-th floor. Suppose that n>10. Then,

$$f_1=\max[1, n\lambda_2+(\lambda_2- \lambda_1(\nu-1)(2\lambda-1)/2l\lambda_2] =\max\{1, n/2l(1-(2l-1)\lambda_1/ \lambda_2)\},$$

$$\begin{aligned} &f_j=f_1+2(j-1)(n-f_1)/(2l-1)= f_1[1+2(j-1)/(2l-1)]+[2(j-1)n/(2l-1)]= \\ &= f_1[2(l-j)+1]/(2l-1)+2n(j-1)/(2l-1), \end{aligned} \tag{6.10}$$

where (6.10) is the unique solution of (6.9).

Let's put l=3, $\lambda_1 =\lambda_2$, then, for n>10, we have f1=1, f2=2n/5, f3=4n/5, (see Fig. 6.3)

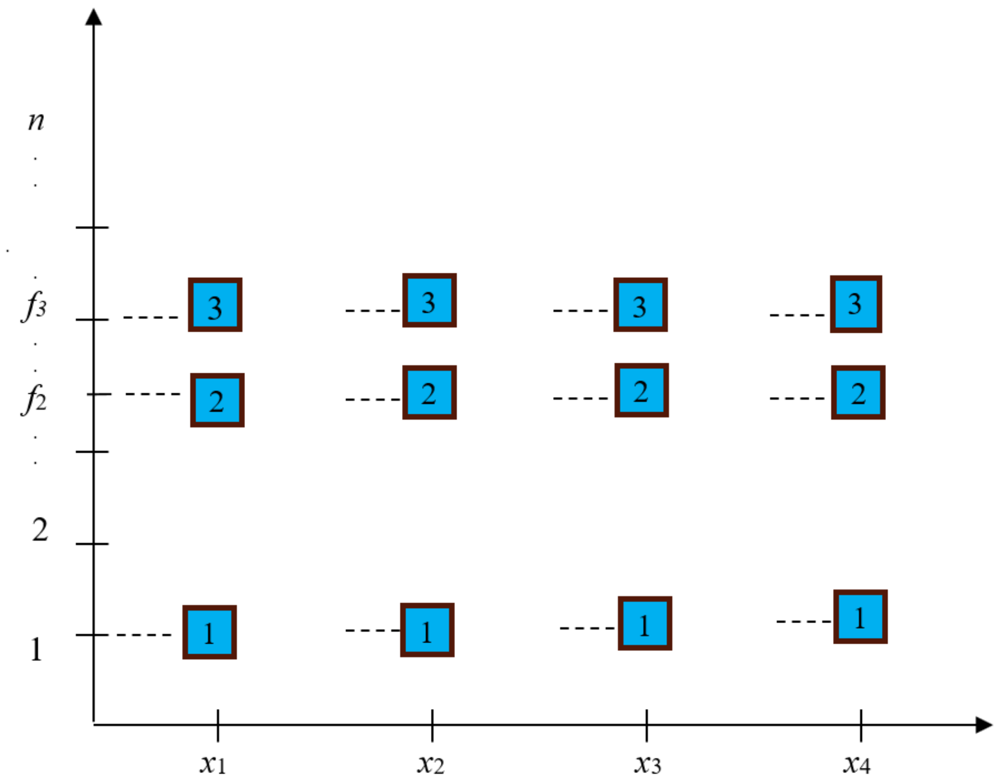

Fig. 6.3

In the Fig. 6.3, there are shown the lifts' positions at the preceding customer's arrival instant, for the system $L_3 F_n C_{f1,f2,f3}$.

Remark 6.4. For the system $L_3F_nC_{f1,f2,f3}$ starting from some moment at the preceding of a customer arrival instant lifts have the position shown in the Fig.6.3. Customers arriving to the floors 1,2,…f_1/2 will be served by the first lift, customers at the f_1/2, f_1/2+1,… f_1, f_1+1,…,f_2-f_1 by second lift and so on. After finishing service i-th lift must come to the f_i floor.

7. Simulation of the systems with several lifts in loading and unloading regimes

In [6], based on Wolfram Mathematica, several programs had been developed for the simulation of the input data and imitation systems with one lift. Here we consider two lifts systems, using related developed programs. The numerical comparison between such parameters as the waiting lift times CWT and the total time CTT show the advantage of the two lifts systems. The programs for the simulation systems with two lifts had been developed and they will be applied below. Systems with more than two lifts have more complicated structures and a special approach will be realized later.

Let us consider an office building where offices are placed on floors from 2 up to n, e.g. n=10. On each of the even floor 2, 4, 6, 8, 10 there are 20 working places for customers. On each odd floor 3, 5, 7, 9, there are 30 working places for customers. Altogether there are n_c=220 working places for customers in the building. Every morning, during a time interval, e.g. half hour (t_m=1800 Sec), all the customers should be on the floors which have their offices and should start to work. In large towns, it is difficult to reach a certain building at scheduled time. Therefore, we use approximate the time when the customers reach their offices at independent times, uniformly distributed over the time interval t_m=1800 Sec.

We also suppose that for any customer who had come at the 1st floor lift hall, at uniformly distributed time ta, in the time interval tm, and his destination floor f_d was randomly selected, without the repetition of the list of all the working places in this building, We created the program simulated for each i-th customer at his initial four-plot history {i, 1, f_d, t_a}. The i-th customer waits for the lift, during a time t_b, when the customer can come into the lift cabin. The lift is going up and reaches the destination floor f_d , at time t_e. Then, the customer goes out from the cabin. His initial history extended to {i, 1, f_d, t_a, t_b, t_e}. We have also created a program which produces these extensions of histories for all the customers. The efficiency of a lift's system can be evaluated by estimating CWT and CTT, by using the obtained extended histories. In the following table, these parameters were estimated using the data of the loading work by 1- lift system with roominess r=10.

Floor	2	3	4	5	6	7	8	9	10
CWT	54.458	63.217	40.180	56.152	56.936	65.222	66.083	47.991	55.474
CTT	64.458	79.217	62.680	82.819	90.436	102.389	108.334	95.325	109.974

Table 7.1. Estimates of mean values CWT-waiting cabin time and CTT total time destination all floors by 1-lift system with r=10 in loading process are given below.

This table shows the weak dependency parameters CWT and CTT from roominess. Therefore, we can accept r=10 as an appropriate value of the cabin's roominess. But in the case of unloading the parameters CWT and CTT, it will be too large, if r=10. This problem with 1-Lift system can be lost by increasing the cabin roominess, say to take r=16. But the most natural way to solve this problem will be the use of a 2-Lift system with r=10, for both lifts, in loading and unloading cases. Consider the system with one lift in an unloading regime with a line decreasing probability density in a time interval with a length t_m=1800 Sec. The line probability density is a hypotenuse in triangle with two orthogonal sides with lengths t_m and 1/ (2 t_m). Then, S[x] =1-2 x/t_m + (x /t_mT)^2 is a probability in the randomly taking initial history t_a >x. The graphic of S[x] is shown below:

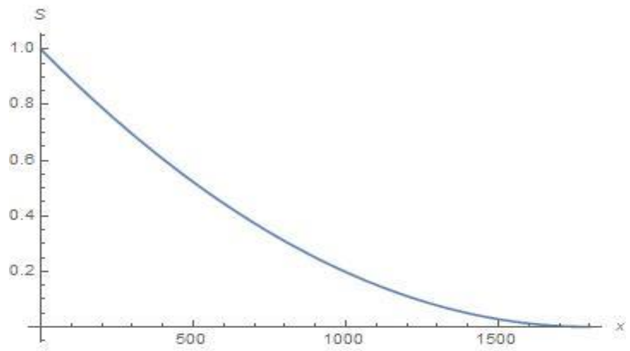

Fig.7.1. Probability function S[x]

Let the lift goes from f_1 floor down and there are some customers at the lower floors f_2 ($f_2 < f_1$), who will go down to the 1-st floor. Then the lift must stop at the f_2- floor, only if, due to the lift's roominess, there is a free space in the cabin. Otherwise, the lift passes f_2 without stopping and the customers at the f_2-floor must wait for another appearance of the lift. We are interested in the problem of finding the dependence of CWT and CTT on lift's roominess. Our programs regarding the simulation of such systems are based on a conception of the lift's cycles. The lift's cycle is defined as the time interval between two sequential instants when the lift arrives to the first floor and all the customers (if there are) leave the lift cabin. At the end of the lift's cycle, all characteristics of the system can be calculated by using of the extended six-plot histories {i, f_a, f_d, t_a, t_s, t_e}. Note that in this paper only one-day data are used, but our programs can work with many independent days' data. Then, the accuracy of the table's numbers will be exact. Below, Table 7.2, demonstrates the dependency of CWT and CTT on the value of the lift's roominess.

R	8	10	12	14	16	18	20	22
CWT	382.3	236.0	146.0	90.1	64.0	51.4	43.8	43.0
CTT	400.7	255.7	167.6	112.7	88.6	77.9	70.5	69.8

Table 7.2. The estimated mean values of CWT and CTT, over all floors in an unloading process, are given using the customers' one-day data, for one lift with different values of roominess.

From Table 7.2 we can see that by increasing the lift's roominess we obtain a reduction of CWT and CTT. Here we do not consider the problem of the accuracy s numbers in the Tables. However, note that the large values of roominess imply undesirable big sizes of the lift's cabin. Here we face the necessity to find a solution to the problem. We consider that instead of one lift with a large roominess it is worth trying to use two lifts with a smaller roominess. We suppose that these two lifts are independent. We consider systems with 2 lifts which are serving different floors. Let the 1st lift stop at odd floors 3, 5, 7, 9 and the 2nd lift stop on even floors 2, 4, 6, 8, 10. Suppose that both lifts have the same roominess r=10. For each lift, we can use our programs with initial simulation and extended histories in loading and unloading processes. Using these two lifts' similar parameters, e.g. h_f=2.5; h_d = 5; n_c = 220; t_m=1800 Sec.

Floor	3	5	7	9
CWT	38.339	33.811	33.986	37.515
CTT	43.339	47.478	56.486	68.349

Table 7.3. Customers' estimates of mean values CWT - waiting times and CTT - total serves customers' times during unloading for Lift-1 used on odd floors 3, 5, 7, 9.

Floor	2	4	6	8	10
CWT	44.425	28.631	41.062	31.321	29.299
CTT	46.925	39.381	59.812	56.821	63.049

Table 7.4. Customers' estimates of mean values CWT - waiting times and CTT - total serves customers' times during unloading for Lift-2 used on even floors 2, 4, 6, 8, 10.

In both Table 7.3 and Table 7.4, CWT and CTT take fewer values than in the system with one lift and r=10.Thus, we have found the solution to the problem. We use only one lift with a large roominess or instead of one lift with a large roominess, we use two or more lifts with less roominess. The simulation of the systems with 2 lifts shows that the cycle time is less and moreover, both CWT and CTT take smaller values than in the systems with one lift with the same roominess.

8. Conclusions

In the paper, for various lifts' systems, the universal mathematical models have been constructed and the main characteristics (CWT – a customer's waiting time before service, CTT – a customer's total time, roominess of the lift's cabin and others), are introduced. It is also introduced the new type of customer flows, called transforming flows (transforming point processes). Such processes can be transformed from any point process into processes with a rare event. For such flows of customers, by analytical approaches, the formulas for calculating the main characteristics of efficiency lift systems (CWT, CTT and others) are suggested. The various control rules, reducing the different characteristics of the lifts' systems are introduced. The several graphs, reflecting the behaviour of the different lift systems, are shown. The introduced characteristics allow find the optimal number of lifts in buildings and skyscrapers. The paper offers an important solution for investigating the lifts' systems, which are natural in modern metropolises. Simulations works of lifts' models is the base for such investigations which adequately describe the lifts' systems behaviour.

Collecting and analysing the data of the simulations allow obtain the suitable recommendations for planning the future lifts' systems. The programs for the simulation of the lifts' systems have been prepared. Numerical examples show how the optimal roominess of a lift's cabin can be ensured.

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