Reliability Test Plan for the Marshall Olkin Length Biased Lomax Distribution

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Abstract

In this article, a generalization of the length biased Lomax distribution called the Marshall Olkin length biased Lomax distribution is introduced. A few statistical and reliability properties of the new distribution is discussed. The parameters of the introduced model are estimated by the method of maximum likelihood estimation. The suitability of the model is verified empirically by applying a real life data set. Also, establish a reliability test plan for acceptance or rejection of a lot of products submitted for inspection with lifetimes directed by this distribution.

Keywords: Marshall Olkin family, Length biased Lomax, Maximum Likelihood Estimates, Reliability Test Plan, Operating Characteristic Function.

I. Introduction

The applications in different areas such as lifetime analysis, financial modelling, business, economics, insurance, clinical trials shows that the data sets arising from various areas may require more flexible distributions. So there is an increasing trend in developing generalized families of distributions by adding one or more additional parameters to the baseline distributions.

Marshall and Olkin (1997) introduced a method of adding a new shape parameter into a family of distributions. Considering a baseline distribution with cumulative density function, G(x), of a random variable X, then the cumulative density function of the Marshall and Olkin - G (MO-G) family of distributions is

$$F(x) = \frac{G(x)}{1-(1-\gamma)(1-G(x))} \tag{1}$$

The corresponding probability density function of (1) is given by,

$$f(x) = \frac{\gamma g(x)}{[1-(1-\gamma)(1-G(x))]^2} \tag{2}$$

where $\gamma > 0$ is a shape parameter. Clearly, for $\gamma = 1$, we obtain the baseline distribution, i.e., F(x) = G(x). Many authors have proposed various distributions belonging to the Marshall Olkin family of distributions such as Marshall Olkin Weibull by Ghitany et al. (2005), Marshall Olkin Pareto by Alice et al. (2003), Marshall Olkin semi-Weibull by Alice et al. (2005), Marshall Olkin Lomax by Ghitany et al. (2007), Marshall Olkin semi-Burr and Marshall Olkin Burr by Jayakumar et al. (2008) and Marshall Olkin q-Weibull by Jose et al. (2010), Marshall Olkin Frechet distribution by Krishna et al. (2013), Marshall Olkin gamma by Ristic et al. (2007), Marshall Olkin Lomax by Ghitany et al. (2007), Marshall Olkin linear failure-rate by Ghitany et al. (2007), Marshall Olkin log-logistic by Gui (2013), and Marshall Olkin exponential Weibull by Pogny et al. (2015).

The Lomax distribution has so many applications in life testing, reliability modeling and is also used to fit business failure data. This distribution was proposed by Lomax (1954). Using generalized probability weighted moment, Abdullah and Abdullah (2010) estimated the parameters of Lomax distribution.

A continuous random variable X is said to have a Lomax distribution with parameters θ and β if its probability density function (pdf) is,

$$f(x\mid\theta,\beta)=\frac{\theta\left(\frac{\beta+x}{\beta}\right)^{-\theta}}{\beta+x}. \tag{3}$$

for $0<x<\infty,\theta>0$ and $\beta>0.$

In the rest of the paper, we define the new model and the statistical properties of the proposed distribution are discussed in Section 2. The failure rate function is given in section 3. Estimation of parameters by using maximum likelihood estimation method is presented in Section 4. An application of proposed model on real life data set are provided in Section 5. In Section 6, we develop reliability test plans to decide whether to accept or reject a submitted lot of products whose lifetime is assumed to be a Marshall Olkin length biased lomax distribution. Section 7 gives the description of the tables and applications. In Section 8, concluding remarks are presented.

II Marshall Olkin Length Biased Lomax distribution

The concept of length-biased distributions originated from the study of weighted distributions. The weighted or moment distributions arise in the context of unequal probability sampling; have a great importance in reliability, biomedicine, and ecology, among others. The weighted distributions are first considered by Fisher (1934) and they have been found applications in several areas such as ecology, fisheries etc. In this regard, Gupta and Akman(1995) considered certain applications of the weighted distributions in biomedical.

To introduce the concept of a weighted distribution[Patil (2002)], suppose that X is a random variable(r.v.) with its natural probability density function (pdf) f(x|c), where the natural parameter $\theta\in\Theta$ (Θ is the parameter space). A weighted distribution with kernel f(x|θ) and weight function w(x, β) is defined as g(x|θ, β) = w(x, β)f(x|θ)/C(θ, β).
where, w(x, β)> 0 and C(θ, β) = E_f [w(X, β)]. When X is a non-negative random variable and w(x,β) = x, the resultant weighted distribution is known as length-biased distribution

$$f_{X_w}(x)=\frac{w(x)f_X(x)}{E[w(x)]} \tag{4}$$

Assuming that E[w(x)] $<\infty$.

The length biased distribution is obtained by taking weight as w(x) = x which can be denoted by a random variable X_L which has pdf expressed as

$$f_{X_L}(x) = \frac{x f_X(x)}{\mu} \tag{5}$$

Where μ = E(x) $< \infty$.

Afaq Ahmad et al. (2016) proposed the length biased weighted lomax distribution (here we will call it Length Biased Lomax (LBL) distribution) through assigning weight to the lomax distribution by following the idea of Fisher (1934). Afaq Ahmad et al. (2016) proved that the LBL is more flexible than the lomax distribution.

The Probability Density Function (PDF) and Cumulative Distribution Function (CDF) of LBL distribution are;

$$g(x) = \frac{(\theta-1)\theta x\left(\frac{x}{\beta}+1\right)^{-\theta-1}}{\beta^2} \tag{6}$$

for $x > 0, \theta, \beta > 0$

and

$$G(x) = 1-\beta^{\theta-1}(\beta+x)^{-\theta}(\beta+\theta x) \tag{7}$$

In this article, we proposed a new extension of the LBL distribution called the Marshall Olkin Length Biased Lomax (MOLBL) distribution. The new model is obtained using the Marshall Olkin family of distributions. Some of its statistical properties are constructed with the hope that it will bring wider applications in reliability and other areas of research. The new distribution can be defined by inserting (7) in (1). Then, the CDF of the MOLBL model, say F(x) = $F(x;\alpha,\beta,\gamma)$ is;

$$F(x) = \frac{1-\beta^{\theta-1}(\beta+\mathrm{x}\theta)(\beta+x)^{-\theta}}{1-(1-\gamma)\left(\beta^{\theta-1}(\beta+\mathrm{x}\theta)(\beta+x)^{-\theta}\right)} \tag{8}$$

where $\theta > 0$ and $\gamma > 0$ are shape parameters and $\beta >$ 0 is the scale parameter.

By putting Eqs. (6) and (7) in Eq.(2) , we obtain the PDF of the MOLBL distribution

$$f(x) = \frac{\gamma(\theta-1)\theta x\left(\frac{x}{\beta}+1\right)^{-\theta-1}}{\beta^2\left(1-(1-\gamma)\beta^{\theta-1}(\beta+x)^{-\theta}(\beta+\theta x)\right)^2} \tag{9}$$

For γ = 1, the MOLBL reduces to the LBL distribution. The Survival Function (SF) of X is;

$$S(x) = \frac{\gamma\beta^{\theta}(\beta+\mathrm{x}\theta)}{\beta(\beta+x)^{\theta}+(\gamma-1)\beta^{\theta}(\beta+\mathrm{x}\theta)}$$

Figure 1 shows some possible shapes of the PDF of the MOLBL distribution for selected values of parameters.

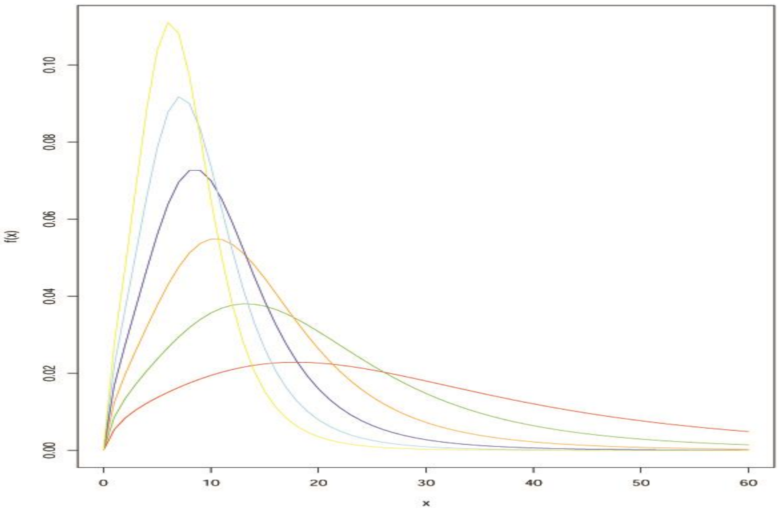

Figure 1: Graph for MOLBL density function at different parameter values. (θ,β,γ) = red (4, 10,15), Green (5,10, 15), Orange (6, 10 , 15), Blue (7, 10, 15), Skyblue(8,10,15), Yellow(9,10,15).

III. Failure Rate Function

The failure rate or hazard rate function of the MOLBL distribution is given by,

$$h(x)=\frac{\gamma(\theta-1)\theta x\left(\frac{x}{\beta}+1\right)^{-\theta-1}}{\beta^{2}\left(1-(1-\gamma)\beta^{\theta-1}(\beta+x)^{-\theta}(\beta+\theta x)\right)^{2}\left(1-\frac{1-\beta^{\theta-1}(\beta+x\theta)(\beta+x)^{-\theta}}{1-(1-\gamma)\beta^{\theta-1}(\beta+x\theta)(\beta+x)^{-\theta}}\right)}$$

where $\theta>0$, $\gamma>0$ and $\beta>0$.

Figs. 2 and 3 shows some possible shapes of the hazard rate function of the MOLBL distribution for selected values of parameters.

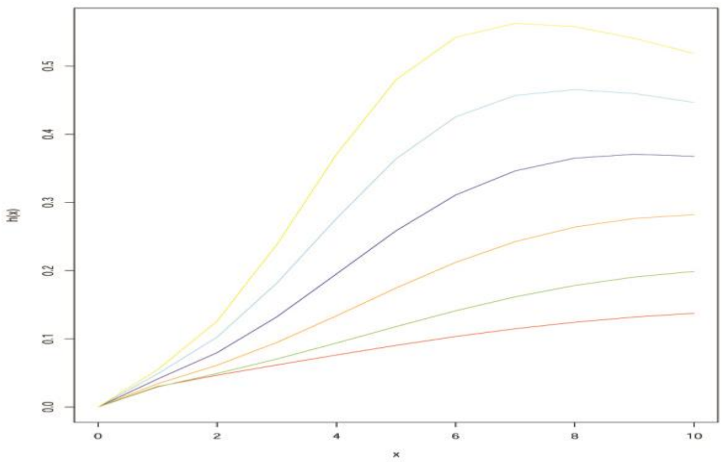

Figure 2 : Graph for hazard rate function at different parameter values. (θ,β,γ) = red (5, 6,10), Green (6,6, 15), Orange (7, 6 , 18), Blue (8, 6, 20), Skyblue(9,6,22), Yellow(10,6,25).

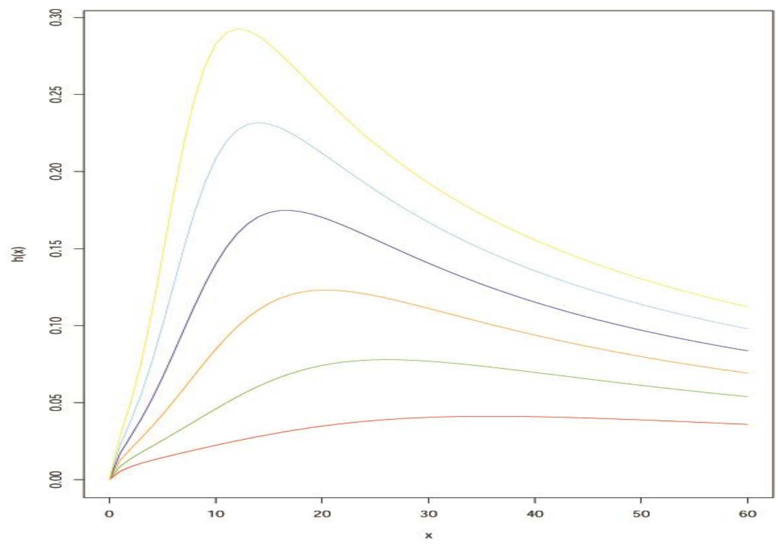

Figure 3 **: Graph for hazard rate function at different parameter values.** , (θ, β, γ) = red (4, 10,15), Green (5,10, 15), Orange (6, 10 , 15), Blue (7, 10, 15), Skyblue(8,10,15), Yellow(9,10,15).

IV Maximum Likelihood Estimation

In this section, the unknown parameters of the MOLBL distribution are estimated by using the maximum likelihood method. Let $x_1; x_2; \ldots x_n$ be a random sample of size n from the MOLBL distribution. The likelihood function for the MOLBL distribution is;

$$L = \prod_{i=1}^{n} \frac{\gamma(\theta-1)\theta x_i \left(\frac{x_i}{\beta}+1\right)^{-\theta-1}}{\beta^2 \left(1-(1-\gamma)\beta^{\theta-1}(\beta+x_i)^{-\theta}(\beta+\theta x_i)\right)^2}$$

The log-likelihood function reduces to

$$LogL = -2\sum_{i=1}^{n} \log\left(1-(1-\gamma)\beta^{\theta-1}(\beta+x_i)^{-\theta}(\beta+\theta x_i)\right) - (\theta+1)\sum_{i=1}^{n}\log\left(\frac{x_i}{\beta}+1\right)$$
$$+\sum_{i=1}^{n}\log(x_i) + n\log(\gamma) + n\log(\theta-1) + n\log(\theta) - 2n\log(\beta).$$

Now, differentiate the above equation with respect to parameters,

$$\frac{\partial \text{logL}}{\partial \theta} = -\sum_{i=1}^{n} \log\left(\frac{x_i}{\beta}+1\right) + \frac{n}{\theta-1} + \frac{n}{\theta}$$
$$-(1-\gamma)\beta^{\theta-1}x_i(\beta+x_i)^{-\theta} - (1-\gamma)\beta^{\theta-1}\log(\beta)(\beta+x_i)^{-\theta}(\beta+\theta x_i) +$$
$$-2\sum_{i=1}^{n} \frac{(1-\gamma)\beta^{\theta-1}(\beta+x_i)^{-\theta}(\beta+\theta x_i)\log(\beta+x_i)}{1-(1-\gamma)\beta^{\theta-1}(\beta+x_i)^{-\theta}(\beta+\theta x_i)}$$

$$\frac{\partial \mathrm{logL}}{\partial \beta} = -(\theta+1)\sum_{i=1}^{n} - \frac{x_i}{\beta^2\left(\frac{x_i}{\beta}+1\right)} - \frac{2n}{\beta} - 2\sum_{i=1}^{n} \frac{\begin{array}{l}(1-\gamma)\theta\beta^{\theta-1}(\beta+\theta x_i)(\beta+x_i)^{-\theta-1} - \\ (1-\gamma)\beta^{\theta-1}(\beta+x_i)^{-\theta} - \\ (1-\gamma)(\theta-1)\beta^{\theta-2}(\beta+\theta x_i)(\beta+x_i)^{-\theta}\end{array}}{1-(1-\gamma)\beta^{\theta-1}(\beta+x_i)^{-\theta}(\beta+\theta x_i)}$$

And

$$\frac{\partial \mathrm{logL}}{\partial \gamma} = \frac{n}{\gamma} - 2\sum_{i=1}^{n} \frac{\beta^{\theta-1}(\beta+x_i)^{-\theta}(\beta+\theta x_i)}{1-(1-\gamma)\beta^{\theta-1}(\beta+x_i)^{-\theta}(\beta+\theta x_i)}$$

The maximum likelihood estimator of the parameters can be obtained by solving the above equations. This solution can also be obtained by using R software with *nlm* or *maxLik* package.

V Data Analysis

In this section, we provide an application to real data to illustrate the importance of the MOLBL distribution. The MLEs of the model parameters are computed and goodness-of-fit statistics for this model are compared with the Length biased Lomax model introduced by Afaq Ahmad et al. (2016). For this, we consider the data set from Lee and Wang (2003). This data set consists of 128 observations on remission times (in months) of a random sample of bladder cancer patients.

Data set: 0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.20, 2.23, 0.26, 0.31, 0.73, 0.52, 4.98, 6.97, 9.02, 13.29, 0.40, 2.26, 3.57, 5.06, 7.09, 11.98, 4.51, 2.07, 0.22, 13.8, 25.74, 0.50,2.46, 3.64, 5.09, 7.26, 9.47, 14.24, 19.13, 6.54, 3.36, 0.82, 0.51, 2.54, 3.70, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 1.76, 8.53, 6.93, 0.62, 3.82, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 3.25, 12.03, 8.65, 0.39, 10.34, 14.83, 34.26, 0.90,2.69, 4.18, 5.34, 7.59, 10.66, 4.50, 20.28, 12.63, 0.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 6.25, 2.02, 22.69, 0.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 8.37, 3.36, 5.49, 0.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 12.02, 6.76, 0.40, 3.02, 4.34, 5.71, 7.93, 11.79, 18.1, 1.46, 4.40, 5.85, 2.02, 12.07.

In order to compare the two models, we consider the criteria like AIC (Akaike information criterion) and BIC (Bayesian information criterion). The better distribution corresponds to lesser AIC, and BIC values.

AIC = 2k-2logL and BIC = klogn-2logL

where k is the number of parameters in the statistical model, n is the sample size and -2logL is the maximized value of the log-likelihood function under the considered model.

We have fitted Length biased Lomax and Marshal Olkin Length biased lomax models to this data. These two distributions are fitted to the subject data using maximum likelihood estimation. The MLEs of the parameters with standard errors(SE) and the corresponding loglikelihood values, AIC, BIC are displayed in Table 1.

Table 1: MLEs and some goodness of fit measures for Comparison.

Distribution	Parameter	Estimates	SE	-2logL	AIC	BIC
LBL	$\hat{\alpha}$	3.1142	0.5056	814.94	818.95	824.65
	$\hat{\beta}$	5.2372	1.6951			
MOLBL	$\hat{\alpha}$	2.6495	0.2230	808.02	814.01	822.57
	$\hat{\beta}$	0.6115	0.5824			
	$\hat{\gamma}$	15.2889	17.6980			

From Table 1, it has been observed that the Marshal Olkin Length biased Lomax distribution has the lesser AIC and BIC values as compared to Length biased Lomax Distribution. Hence we can conclude that the Marshal Olkin Length biased Lomax distribution leads to a better fit than the Length biased Lomax distribution.

VI Reliability Test Plan

The acceptance sampling plan is affected with the inspection of a sample of products taken from a lot and the decision whether to accept or to reject the lot based on the quality of the product in statistical quality control . Here we discuss the reliability test plan for accepting or not to accepting a lot when the lifetime of the product follows the Marshal olkin length biased Lomax distribution. In a life testing experiment, the process is to restrict the test by a predetermined time t and notice the number of failures. If the number of failures at the end of time t does not exceed a given number c (acceptance number), then we accept the lot with a given probability of at least p*. But if the number of failures exceeds c before time t, then the test is terminated, and the lot is rejected. For such an event like a truncated life test and the associated decision rule, we are interested in finding the smallest sample size to make at a decision. Assume that the lifetime of a product follows the Marshal olkin length biased Lomax distribution with cumulative distribution function (cdf)

$$F(t) = \frac{1-\beta^{\theta-1}(\beta+t\theta)(\beta+t)^{-\theta}}{1-(1-\gamma)\left(\beta^{\theta-1}(\beta+t\theta)(\beta+t)^{-\theta}\right)}; \quad 0 < t < \infty \tag{10}$$

where $\theta > 0$ and $\gamma > 0$ are shape parameters and $\beta >$ 0 is the scale parameter. If θ and γ are known, then the average lifetime depends only on β. Let β_0 be the required minimum average lifetime. Then

$$F(t,\theta,\gamma,\beta) \le F(t,\theta,\gamma,\beta_0) \Leftrightarrow \beta \ge \beta_0.$$

A sampling plan is specified by the following quantities:

- the number of units (n) on test
- the acceptance number (c)
- the maximum test duration (t)
- the minimum average lifetime (β_0).

The consumers risk, which is the probability of accepting a bad lot should not exceed the value 1-

p*,where p* is a lower bound for the probability that a lot of true value β below β_0 is rejected by the sampling plan. For fixed p* the sampling plan is characterized by (n, c, $\frac{t}{\beta_0}$). By considering the large lots, the binomial distribution may be used to determine the acceptance probability. The problem is to determine the smallest positive integer n for given values of c and $\frac{t}{\beta_0}$ such that

$$L(p_0) = \sum_{i=0}^{c} \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq 1-p^* \tag{11}$$

where $p_0 = F(t,\theta,\gamma,\beta_0)$ given by (10) indicates the failure probability before time t which depends only on the ratio $\frac{t}{\beta_0}$. The function L(p) is known as the operating characteristic(OC) function of the sampling plan, i.e. the probability of acceptance of the lot as a function of the probability of failure $p(\beta) = F(t;\theta,\gamma,\beta)$. The average lifetime of the product is increasing with β, and therefore the failure probability p(β) decreases, implying that the OC function is increasing in β. The minimum values of n satisfying eq (11) are obtained for θ =3 , γ = 2, p* = 0.75,0.90,0.95,0.99 and $\frac{t}{\beta_0}$ = 0.75, 1, 1.25, 1.75, 2, 3 and 3.5. The results are displayed in Table 2. If $p_0 = F(t,\theta,\gamma,\beta_0)$ is very small and n is large, the binomial probability may be approximated by the Poisson probability with parameter $\lambda = np_0$, so eq (11) becomes

$$L_1(p_0) = \sum_{i=0}^{c} \frac{\lambda^i}{i!} e^{-\lambda} \leq 1-p^* \tag{12}$$

The minimum values of n satisfying eq (12) are obtained for the same combination of values of θ,γ and $\frac{t}{\beta_0}$ for various values of p* and is presented in Table 3. The operating characteristic function of the sampling plan $(n,c,\frac{t}{\beta_0})$ gives the probability of accepting the lot and is given by

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p_0^i (1-p_0)^{n-i}$$

where p = F(t, β) is considered as a function of β. The values of n and c are obtained by means of operating characteristics function for given value of p* and $\frac{t}{\beta_0}$ are displayed in Table 4 by considering the fact that p = F($\frac{t}{\beta_0} / \frac{\beta}{\beta_0}$)

Table 2: Minimum sample size for the specified ratio t/β_0, confidence level p^*, acceptance number c, θ =3 and γ = 2 using binomial approximation.

p^*	c	t/β_0						
		0.75	1	1.25	1.75	2	3	3.5
.75	0	5	4	3	2	2	2	1
	1	11	8	6	5	4	3	3
	2	15	11	9	7	6	5	4
	3	20	15	12	9	8	6	6
	4	25	19	15	11	10	8	8
	5	30	23	18	13	12	9	9
	6	35	27	21	15	14	11	10
	7	40	29	24	17	16	13	11
	8	45	31	27	19	18	14	13
	9	50	35	28	21	20	15	14
	10	52	38	30	23	21	16	15
.90	0	9	6	5	3	3	2	2
	1	15	11	8	6	6	4	4
	2	20	15	12	9	8	6	6
	3	28	19	16	12	10	8	7
	4	33	22	20	15	12	10	8
	5	38	26	22	17	14	11	10
	6	43	30	24	18	16	12	11
	7	48	34	26	20	18	14	13
	8	53	37	29	22	20	16	15
	9	58	41	32	24	22	18	16
	10	61	44	35	26	24	20	17
.95	0	11	8	6	4	4	3	3
	1	18	14	10	7	6	5	4
	2	24	17	14	10	9	6	6
	3	32	21	18	13	11	8	8
	4	35	25	22	15	13	10	9
	5	41	29	26	17	15	12	11
	6	45	33	29	19	17	14	13
	7	51	37	31	22	19	15	14
	8	56	41	32	24	21	16	15
	9	60	45	35	26	23	18	16
	10	66	48	38	28	25	19	18
.99	0	18	12	9	6	6	4	4
	1	27	19	14	10	9	6	6
	2	31	22	17	12	11	8	7
	3	44	27	21	16	13	10	10
	4	49	31	25	18	16	12	12
	5	50	35	29	20	19	14	13
	6	55	40	32	23	21	15	14
	7	61	44	35	26	24	17	15
	8	67	49	38	28	25	19	17
	9	74	52	41	30	27	21	18
	10	77	55	44	32	29	22	20

Table 3: Minimum sample size for the specified ratio t/β_0, confidence level p^*, acceptance number c, θ =3 and γ = 2 using poisson approximation.

p^*	c	t/β_0						
		0.75	1	1.25	1.75	2	3	3.5
.75	0	6	5	4	3	3	2	2
	1	11	9	7	6	5	4	4
	2	17	13	10	8	7	6	6
	3	22	17	13	10	9	8	7
	4	26	21	17	12	11	9	9
	5	31	25	20	14	13	11	11
	6	35	29	22	16	15	13	13
	7	40	32	24	19	17	15	14
	8	45	35	27	21	19	16	15
	9	49	37	30	23	21	17	16
	10	54	40	32	25	23	18	17
.90	0	10	7	6	5	4	4	3
	1	16	13	10	8	7	6	6
	2	22	20	14	10	10	8	7
	3	28	23	18	13	12	10	9
	4	33	25	20	15	14	11	11
	5	39	28	23	18	16	13	13
	6	44	32	26	20	18	15	15
	7	49	36	29	22	21	17	16
	8	55	39	32	25	24	19	17
	9	59	43	36	27	26	21	19
	10	63	47	38	29	27	22	20
.95	0	13	9	8	6	6	5	4
	1	20	15	12	9	9	7	7
	2	26	20	16	12	11	9	10
	3	32	24	20	15	15	11	11
	4	39	28	23	18	17	13	12
	5	43	32	27	20	18	15	14
	6	49	36	30	23	21	17	16
	7	54	40	32	25	24	19	18
	8	59	44	36	27	26	20	19
	9	65	48	40	30	28	22	21
	10	70	52	42	32	29	24	22
.99	0	19	15	12	9	8	7	6
	1	28	20	17	13	12	10	9
	2	36	26	21	16	16	13	12
	3	42	32	25	19	18	15	14
	4	49	37	29	22	20	17	15
	5	54	41	33	26	23	18	17
	6	60	44	36	28	25	20	19
	7	66	49	39	30	29	23	21
	8	72	53	43	33	31	26	23
	9	77	58	47	36	34	27	25
	10	83	61	49	38	35	28	26

Table 4: Values of the operating characteristic function of the sampling plan $(n, c, \frac{t}{\beta_0})$

				$\frac{\beta}{\beta_0}$						
p^*	n	c	$\frac{t}{\beta_0}$	2	4	6	8	10	12	14
.75	15	2	0.75	0.8142	0.9862	0.9979	0.9995	0.9998	0.9999	0.9999
	11	2	1	0.7820	0.9804	0.9967	0.9992	0.9997	0.9999	0.9999
	9	2	1.25	0.7445	0.9730	0.9951	0.9987	0.9995	0.9998	0.9999
	7	2	1.75	0.6687	0.9541	0.9905	0.9973	0.9990	0.9996	0.9998
	6	2	2	0.6803	0.9535	0.9899	0.997	0.9989	0.9995	0.9997
	5	2	3	0.5387	0.9016	0.9739	0.9913	0.9965	0.9984	0.9992
	4	2	3.5	0.6287	0.9233	0.9793	0.9929	0.9971	0.9987	0.9993
.90	20	2	0.75	0.6743	0.9695	0.9952	0.9988	0.9996	0.9998	0.9999
	15	2	1	0.6088	0.9542	0.9918	0.9979	0.9993	0.9997	0.9998
	12	2	1.25	0.5668	0.9405	0.9884	0.9969	0.9989	0.9995	0.9998
	9	2	1.75	0.4887	0.9089	0.9794	0.9939	0.9978	0.9991	0.9995
	8	2	2	0.4682	0.8965	0.9753	0.9925	0.9972	0.9988	0.9994
	6	2	3	0.3833	0.8383	0.9535	0.9839	0.9935	0.9970	0.9985
	6	2	3.5	0.2757	0.7619	0.9225	0.9711	0.9878	0.9942	0.9970
.95	24	2	0.75	0.5612	0.9512	0.9918	0.998	0.9993	0.9997	0.9998
	17	2	1	0.5247	0.9369	0.9883	0.9969	0.9990	0.9996	0.9998
	14	2	1.25	0.4570	0.9122	0.9820	0.9950	0.9983	0.9993	0.9997
	10	2	1.75	0.4088	0.8815	0.9722	0.9916	0.9969	0.9987	0.9994
	9	2	2	0.3771	0.8614	0.9653	0.9892	0.9959	0.9982	0.9991
	6	2	3	0.3833	0.8383	0.9535	0.9839	0.9935	0.9970	0.9985
	6	2	3.5	0.2757	0.76199	0.9225	0.9711	0.9878	0.9942	0.9970
.99	31	2	0.75	0.3854	0.909	0.9834	0.9958	0.9986	0.9994	0.9997
	22	2	1	0.3432	0.883	0.9762	0.9936	0.9978	0.9991	0.9996
	17	2	1.25	0.3183	0.8615	0.9693	0.9913	0.9969	0.9987	0.9994
	12	2	1.75	0.2765	0.8194	0.9540	0.9857	0.9947	0.9977	0.9989
	11	2	2	0.2341	0.7820	0.9401	0.9804	0.9925	0.9967	0.9984
	8	2	3	0.1734	0.6909	0.8965	0.9614	0.9838	0.9925	0.9962
	7	2	3.5	0.1692	0.6687	0.8823	0.9541	0.9801	0.9905	0.9951

VII Illustration of Table and application of sampling plan

Assume that the life time distribution is Marshall-Olkin length biased lomax distribution with θ =3 and γ = 2 . Suppose that the experimenter is interested in constituting that the correct unknown average lifetime is at least 1000 hours. Suppose that it is desired to stop the the experiment at t=1000 hours. So if consumers risk is is set to be $1-p^* = .25$ then from Table 2 sampling plan is $(n = 11, c = 2, t/\beta_0 = 1)$. ie; if during 1000 hours,not more than 2 failures out of 11 are observed then the experimenter can say with confidence limit 0.75 that the average life is at least 1000 hours. If we use Poisson approximation to binomial the corresponding value is n=13.

For the sampling plan $(n = 11, c = 2, t/\beta_0 = 1)$ with the consumer risk 0.25 under the Marshall-olkin length biased lomax distribution the operating characteristic values from Table 4 are,

$\frac{\beta}{\beta_0}$	2	4	6	8	10	12	14
L(p)	0.7820	0.9804	0.9967	0.9992	0.9997	0.9999	0.9999

This shows that when $\frac{\beta}{\beta_0} = 4$ producers risk is 0.02 and when $\frac{\beta}{\beta_0} = 12$ it is negligible.

Application: Consider the following ordered failure times of the product, gathered from a software development project. The data set regarding software reliability was presented by Wood (1996), analyzed via the acceptance sampling viewpoint by Rosaiah and Kantam(2005), Rao, Ghitany and Kantam (2008,2009), Lio, Tsai and Wu (2010) ,Rao and Kantam (2010) and Jose et al. (2018). This data consists of a sample of 18 observations.

Data Set : 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, 5218, 5823, 6539, 7083, 7487,7846,8205,8564,8923,9282.

Let the specified average lifetime be 1000 hours, and let the testing time be 750 hours. This leads to the ratio t/β_0 = 0.75 with corresponding n = 18 and c = 1 for p* = 0.95. Therefore, the sampling plan for the above sample data is (n = 18, c = 1, t/β_0 = 0.75). We accept the lot only if the number of failures not after 750 hours is less than or equal to one. However, the confidence level is assured by the sampling plan only if the given lifetimes follow the MOLBL distribution.

In the above sample, there is only one failure at 519 hours before termination t = 750 hours. Hence, we accept the product. Here, we notice that termination time t is smaller when using this sampling plan compaired with the sampling plan suggested by Krishna et al. (2013) and Rosaiah et al.(2005). So, when we consider this present sampling plan, we can reduce the cost and the experimental time.

VIII Conclusion

In this paper, We consider the Marshal Olkin length biased Lomax distribution and discuss some properties. We fit the distribution to a real-life data set. Also, a reliability test plan is developed when the lifetimes of the items follow the Marshal Olkin length biased Lomax distribution. The results are illustrated by a numerical example. Table 2 provides the minimum sample size for the specified ratio t/β_0 , confidence level p*, acceptance number c for θ=3 and γ = 2 using binomial approximation. Table 3 provides the minimum sample size using Poisson approximation. Table 4 provides values of the operating characteristic function of the sampling plan (n, c, t/β_0) for given confidence level p* with θ =3 , γ = 2 and acceptance number c = 2. The new test plans deliver positive results in comparison with the existing works and can be activated for making optimal decisions.

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