Estimation of Stress-Strength Reliability Model Using Finite Mixture of M-Transformed Exponential Distributions

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Abstract

In this paper Stress –Strength reliability is studied where various cases have been considered for stress (Y) and strength (X) variables viz., the strength follows finite mixture of M-Transformed Exponential distributions and stress follows exponential, Lindley and M-Transformed Exponential distributions. The reliability of a system and the parameters are obtained by the method of maximum likelihood method. At the end results are illustrated with the help of numerical evaluations and real life data.

Key words: Reliability, M-Transformed Exponential Distribution, Stress, Strength.

1. Introduction

The term "stress-strength reliability" specify the quantity R=P(Y < X), where X is a random strength and Y random stress such that the system fails if the stress Y exceeds the strength Y. Church and Harris imported the term stress-strength for the first time. The stress-strength reliability and its problems for considerable distributions have been discussed by Church and Harris [1970], Woodward and Kelley [1977], Beg and Singh [1979], Awad and Gharraf [1986], Surles and Padgett [1998, 2001], Raqab and Kundu [2005], Mokhlis [2005], and Saraçoğlu et al. [2011]. Kotz et al. [2003] have presented an inspection of all approaches and results on the stress-strength reliability in the last four decades. Adil H. khan and T.R Jan [2014] have studied the stress-strength reliability for two parameter Lindley distribution, Exponential distribution and Gamma distribution. And have considered different conditions for stress and strength variables.

In the present paper, we discuss stress strength reliability and have considered that the strength variable follows finite mixture of M-Transformed Exponential distribution and stress variables follows finite mixture of exponential or Lindley or M-Transformed Exponential distributions. The structural properties and uses of M-Transformed Exponential distribution as a lifetime distribution are studied by Dinesh Kumar et al. [2017]. We debate the estimation method for finite mixture of M-Transformed Exponential distribution by the method of maximum likelihood estimation. The M-Transformed Exponential distribution with parameters α is defined by its probability density function (p.d.f.)

$$f(x;\alpha) = \frac{2e^{-\frac{x}{\alpha}}}{\alpha\left(2-e^{-\frac{x}{\alpha}}\right)^2} \quad ; \quad \alpha, x > 0 \tag{1}$$

In the present paper, we consider three cases

1) Stress follows exponential distribution and strength follows finite mixture of M-Transformed Exponential distribution.
2) Stress follows Lindley distribution and strength follows finite mixture of M-Transformed Exponential distribution.
3) Stress follows finite mixture of Lindley distributions and strength follows finite mixture of M-Transformed Exponential distributions.
4) Stress and strength both follows finite mixture of M-Transformed Exponential distribution.

2. Statistical Model

In this model we assume that random variables X (strength) and Y (stress) are independent and the values of X and Y are non-negative. The reliability of a component with strength X and stress Y imposed on it is given by

$$R = P(X > Y) = \int_o^\infty \left[\int_0^x g(y)dy\right] f(x)dx \tag{2.1}$$

where $f(x)$ and $g(y)$ are pdf of strength and stress respectively.
A finite mixture of M-Transformed Exponential distributions with v components can be expressed as

$$\mathrm{f(x) = p_1f_1(x) + p_2f_2(x) + \cdots + p_vf_v(x)}\ ; \mathrm{p_i} > 0, i = 1,2, \dots , v \quad \sum_{\mathrm{i=1}}^{\mathrm{v}} \mathrm{p_i} = 1 \tag{2.2}$$

3. Reliability Computations

Let X be the strength of the v-components with probability density functions $\mathrm{f_i(x); i = 1,2, \dots, v}$. The pdf of X which follows finite mixture of M-Transformed Exponential distributions is

$$\mathrm{f_i(x) = p_i}\frac{2e^{-\frac{x}{\alpha_i}}}{\alpha_i\left(2-e^{-\frac{x}{\alpha_i}}\right)^2}\ ;\ \mathrm{x > 0,\ \alpha_i > 0, p_i > 0, i = 1,2, \dots}, v, \textstyle\sum_{\mathrm{i=1}}^{\mathrm{v}} \mathrm{p_i} = 1$$

Case I: The stress Y follows exponential distribution

As Y follows exponential distribution, pdf of Y is given by

$$\mathrm{g(y) = \lambda e^{-\lambda y}, \lambda > 0}. y > 0$$

For two components $v = 2$, and

$$\mathrm{f(x) = p_1}\frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + \mathrm{p_2}\frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2}\ ;\ \mathrm{p_1 + p_2 = 1, \alpha_1, \alpha_2, x > 0}$$

As X and Y are independent then from (2), Reliability function $\mathrm{R_2}$ is

$$\mathrm{R_2} = \int_0^\infty\int_0^{\mathrm{x}} \mathrm{\lambda e^{-\lambda y}}\left[\mathrm{p_1}\frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + \mathrm{p_2}\frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2}\right]\mathrm{dxdy}$$

$$= \int_0^\infty\int_0^{\mathrm{x}} \mathrm{\lambda e^{-\lambda y}}\left[\mathrm{p_1}\sum_{m_1=1}^{\infty}\frac{m_1}{2^{m_1}\alpha_1}e^{-\frac{m_1x}{\alpha_1}} + \mathrm{p_2}\sum_{m_2=1}^{\infty}\frac{m_2}{2^{m_2}\alpha_2}e^{-\frac{m_2x}{\alpha_2}}\right]\mathrm{dxdy}$$

$$\mathrm{R_2} = \int_0^\infty \left(1 - \mathrm{e^{-\lambda x}}\right)\sum_{i=1}^{2}\sum_{m_i=1}^{\infty}\frac{\mathrm{p_i}m_i}{2^{m_i}\alpha_i}e^{-\frac{m_ix}{\alpha_i}}\,\mathrm{dx}$$

$$= 1 - \sum_{i=1}^{2}\sum_{m_1=1}^{\infty}\frac{p_im_i}{2^{m_i}(\lambda\alpha_i + m_i)}\ ; \qquad \sum_{\mathrm{i=1}}^{2}\mathrm{p_i} = 1 \tag{3.1}$$

For three components $v = 3$, we have

$$\mathrm{f(x)} = \mathrm{p}_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + \mathrm{p}_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} + \mathrm{p}_3 \frac{2e^{-\frac{x}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{x}{\alpha_3}}\right)^2}$$

$$;\ \mathrm{p}_1+\mathrm{p}_2+\mathrm{p}_3 = 1, \alpha_1, \alpha_2, \alpha_3, \mathrm{x} > 0$$

$$\mathrm{R}_3 = \int_0^\infty \int_0^x \lambda \mathrm{e}^{-\lambda y}\left[\mathrm{p}_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + \mathrm{p}_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} + \mathrm{p}_3 \frac{2e^{-\frac{x}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{x}{\alpha_3}}\right)^2}\right]\mathrm{dxdy}$$

$$\mathrm{R}_3 = \int_0^\infty \int_0^x \lambda \mathrm{e}^{-\lambda y}\left[\mathrm{p}_1 \sum_{m_1=1}^{\infty} \frac{m_1}{2^{m_1}\alpha_1} e^{-\frac{m_1 x}{\alpha_1}} + \mathrm{p}_2 \sum_{m_2=1}^{\infty} \frac{m_2}{2^{m_2}\alpha_2} e^{-\frac{m_2 x}{\alpha_2}} + \mathrm{p}_3 \sum_{m_3=1}^{\infty} \frac{m_3}{2^{m_3}\alpha_3} e^{-\frac{m_3 x}{\alpha_3}}\right]\mathrm{dxdy}$$

$$\mathrm{R}_3 = \int_0^\infty \left(1-\mathrm{e}^{-\lambda x}\right) \sum_{i=1}^{3} \sum_{m_i=1}^{\infty} \frac{\mathrm{p}_i m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i x}{\alpha_i}}\,\mathrm{dx}$$

$$\mathrm{R}_3 = 1 - \sum_{i=1}^{3} \sum_{m_i=1}^{\infty} \frac{p_i m_i}{2^{m_i}(\lambda\alpha_i + m_i)} \quad ; \quad \sum_{i=1}^{3} \mathrm{p_i} = 1 \tag{3.2}$$

In general for v-components, $\mathrm{f(x)} = \mathrm{p}_1\mathrm{f}_1(\mathrm{x}) + \mathrm{p}_2\mathrm{f}_2(\mathrm{x}) + \cdots + \mathrm{p_v}\mathrm{f_v}(\mathrm{x})$; $\sum_{i=1}^{v} \mathrm{p_i} = 1$, we hav

$$\mathrm{R_v} = 1 - \sum_{i=1}^{v} \sum_{m_i=1}^{\infty} \frac{p_i m_i}{2^{m_i}(\lambda\alpha_i + m_i)} \quad ; \quad \sum_{i=1}^{v} \mathrm{p_i} = 1 \tag{3.3}$$

Case II: The stress Y follows two parameter Lindley distribution

As Y follows Lindley distribution, pdf of Y is given by

$$\mathrm{g(y)} = \frac{\theta^2}{\theta+\lambda}(1+\lambda \mathrm{y})\mathrm{e}^{-\theta \mathrm{y}} \quad ; \quad \mathrm{y} > 0, \theta > 0, \alpha > -\theta$$

For two components $v = 2$

$$\mathrm{f(x)} = \mathrm{p}_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + \mathrm{p}_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} \ ;\ \mathrm{p}_1+\mathrm{p}_2 = 1, \alpha_1, \alpha_2, \mathrm{x} > 0$$

As X and Y are independent then from (2), Reliability function R_2 is

$$\mathrm{R}_2 = \int_0^\infty \int_0^x \frac{\theta^2}{\theta+\lambda}(1+\lambda \mathrm{y})\mathrm{e}^{-\theta \mathrm{y}}\left(\mathrm{p}_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + \mathrm{p}_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2}\right)\mathrm{dxdy}$$

$$\mathrm{R}_2 = \int_0^\infty \int_0^x \frac{\theta^2}{\theta+\lambda}(1+\lambda \mathrm{y})\mathrm{e}^{-\theta \mathrm{y}}\left(\mathrm{p}_1 \sum_{m_1=1}^{\infty} \frac{m_1}{2^{m_1}\alpha_1} e^{-\frac{m_1 x}{\alpha_1}} + \mathrm{p}_2 \sum_{m_2=1}^{\infty} \frac{m_2}{2^{m_2}\alpha_2} e^{-\frac{m_2 x}{\alpha_2}}\right)\mathrm{dxdy}$$

$$\mathrm{R}_2 = \int_0^\infty \left(1 - \frac{\theta+\lambda+\lambda\theta \mathrm{x}}{\theta+\lambda} e^{-\theta x}\right)\left(\sum_{i=1}^{2} \sum_{m_i=1}^{\infty} \mathrm{p_i} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i x}{\alpha_i}}\right)\mathrm{dx}$$

$$\mathrm{R}_2 = 1 - \sum_{i=1}^{2} \sum_{m_i=1}^{\infty} \mathrm{p_i} \frac{m_i}{2^{m_i}\alpha_i}\left(\int_0^\infty e^{-\left(\theta+\frac{m_i}{\alpha_i}\right)x} dx + \frac{\lambda\theta}{\theta+\lambda}\int_0^\infty x e^{-\left(\theta+\frac{m_i}{\alpha_i}\right)x} dx\right)$$

$$\mathrm{R}_2 = 1 - \sum_{i=1}^{2} \sum_{m_i=1}^{\infty} \mathrm{p_i} \frac{m_i}{2^{m_i}}\left(\frac{1}{m_i+\theta\alpha_i} + \frac{\lambda\theta\alpha_i}{(\theta+\lambda)(m_i+\theta\alpha_i)^2}\right) \tag{3.4}$$

For three components $v = 3$, we have

$$\mathrm{f(x)} = \mathrm{f(x)} = \mathrm{p}_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + \mathrm{p}_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} + \mathrm{p}_3 \frac{2e^{-\frac{x}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{x}{\alpha_3}}\right)^2}$$

$$;\ \mathrm{p}_1+\mathrm{p}_2+\mathrm{p}_3 = 1, \alpha_1, \alpha_2, \alpha_3, \mathrm{x} > 0$$

$$R_3 = \int_0^\infty \int_0^x \frac{\theta^2}{\theta+\lambda}(1+\lambda y)e^{-\theta y}\left[p_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} + p_3 \frac{2e^{-\frac{x}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{x}{\alpha_3}}\right)^2}\right] dxdy$$

$$R_3 = \int_0^\infty \left(1 - \frac{\theta+\lambda+\lambda\theta x}{\theta+\lambda}e^{-\theta x}\right)\left(\sum_{i=1}^{3}\sum_{m_1}^{\infty} p_i \frac{m_i}{2^i \alpha_i} e^{-\frac{m_i x}{\alpha_i}}\right) dx$$

$$R_3 = 1 - \sum_{i=1}^{3}\sum_{m_i=1}^{\infty} p_i \frac{m_i}{2^{m_i}\alpha_i}\int_0^\infty \left(e^{-\left(\frac{m_i}{\alpha_i}+\theta\right)x} + \frac{\lambda\theta}{\theta+\lambda} x e^{-\left(\frac{m_i}{\alpha_i}+\theta\right)x}\right) dx$$

$$R_3 = 1 - \sum_{i=1}^{3}\sum_{m_i=1}^{\infty} p_i \frac{m_i}{2^{m_i}}\left(\frac{1}{m_i+\theta\alpha_i} + \frac{\lambda\theta\alpha_i}{(\theta+\lambda)(m_i+\theta\alpha_i)^2}\right) \tag{3.5}$$

In general for v-components, $f(x) = p_1 f_1(x) + p_2 f_2(x) + \cdots + p_v f_v(x)$; $\sum_{i=1}^{v} p_i = 1$, we have

$$R_v = 1 - \sum_{i=1}^{v}\sum_{m_i=1}^{\infty} p_i \frac{m_i}{2^{m_i}}\left(\frac{1}{m_i+\theta\alpha_i} + \frac{\lambda\theta\alpha_i}{(\theta+\lambda)(m_i+\theta\alpha_i)^2}\right) \tag{3.6}$$

Special case

When $\lambda = 1$, two parameter Lindley distribution reduces to one parameter Lindley distribution and then the reliability function is given as

$$R_v = 1 - \sum_{i=1}^{v}\sum_{m_i=1}^{\infty} p_i \frac{m_i}{2^{m_i}}\left(\frac{1}{m_i+\theta\alpha_i} + \frac{\theta\alpha_i}{(\theta+1)(m_i+\theta\alpha_i)^2}\right) \tag{3.7}$$

Case III: The stress Y follows mixture of two parameter Lindley distributions

As Y follows mixture of M-Transformed Exponential distributions, pdf of X and Y is given by

For two components, $v = 2$

$$f(x) = p_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} \;;\; p_1+p_2 = 1, \alpha_1, \alpha_2, x > 0$$

$$g(y) = p_3 \frac{\theta_3^2}{\theta_3+\lambda_3}(1+\lambda_3 y)e^{-\lambda_3 y} + p_4 \frac{\theta_4^2}{\theta_4+\lambda_4}(1+\lambda_4 y)e^{-\theta_4 y} \;;\; p_3+p_4 = 1,$$

As X and Y are independent then from (2), Reliability function R_2 is

$$R_2 = \int_0^\infty \int_0^x \left(p_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2}\right)\left(p_3 \frac{\theta_3^2}{\theta_3+\lambda_3}(1+\lambda_3 y)e^{-\lambda_3 y} + p_4 \frac{\theta_4^2}{\theta_4+\lambda_4}(1+\lambda_4 y)e^{-\theta_4 y}\right) dxdy$$

$$R_2 = \sum_{j=i+2}^{4}\sum_{i=1}^{2}\int_0^\infty \int_0^x p_i \sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x} p_j \frac{\theta_j^2}{\theta_j+\lambda_j}\left(1+\lambda_j y\right)e^{-\lambda_j y} dxdy$$

$$R_2 = \sum_{j=i+2}^{4}\sum_{i=1}^{2}\int_0^\infty p_i p_j \sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x}\left(1 - \frac{\theta_j+\lambda_j+\lambda_j\theta_j x}{\theta_j+\lambda_j}e^{-\theta_j x}\right) dx$$

$$R_2 = 1 - \sum_{j=i+2}^{4}\sum_{i=1}^{2}\sum_{m_i=1}^{\infty} \frac{p_i p_j m_i}{2^{m_i}\alpha_i}\int_0^\infty \left(e^{-\left(\frac{m_i}{\alpha_i}+\theta_j\right)x} + \frac{\lambda_j\theta_j}{\theta_j+\lambda_j} x e^{-\left(\frac{m_i}{\alpha_i}+\theta_j\right)x}\right) dx$$

$$R_2 = 1 - \sum_{j=i+2}^{4}\sum_{i=1}^{2}\sum_{m_i=1}^{\infty} \frac{p_i p_j m_i}{2^{m_i}}\left[\frac{1}{m_i+\theta_j\alpha_i} + \frac{\lambda_j\theta_j\alpha_i}{\left(\theta_j+\lambda_j\right)\left(m_i+\theta_j\alpha_i\right)^2}\right] \tag{3.8}$$

For three components $v = 3$, we have

$$f(x) = p_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} + p_3 \frac{2e^{-\frac{x}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{x}{\alpha_3}}\right)^2} \;;\; p_1+p_2+p_3 = 1, \alpha_1, \alpha_2, \alpha_3, x > 0$$

$$g(y) = p_4 \frac{\lambda_4^2}{\lambda_4+\alpha_4}(1+\alpha_4 y)e^{-\lambda_4 y} + p_5 \frac{\lambda_5^2}{\lambda_5+\alpha_5}(1+\alpha_5 y)e^{-\lambda_5 y} + p_3 \frac{\lambda_6^2}{\lambda_6+\alpha_6}(1+\alpha_6 y)e^{-\lambda_6 y}$$
$$;\ p_4+p_5+p_6=1, \lambda_4, \lambda_5, \lambda_6, y>0$$

X and Y are independent then from (I), Reliability function R is

$$R_3 = \int_0^\infty \int_0^x \left(p_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} + p_3 \frac{2e^{-\frac{x}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{x}{\alpha_3}}\right)^2} \right)$$
$$\times \left(\frac{p_4\lambda_4^2}{\lambda_4+\alpha_4}(1+\alpha_4 y)e^{-\lambda_4 y} + \frac{p_5\lambda_5^2}{\lambda_5+\alpha_5}(1+\alpha_5 y)e^{-\lambda_5 y} + \frac{p_3\lambda_6^2}{\lambda_6+\alpha_6}(1+\alpha_6 y)e^{-\lambda_6 y} \right) dxdy$$

$$R_3 = \sum_{j=i+3}^{6}\sum_{i=1}^{3}\int_0^\infty\int_0^x p_i \sum_{m_i=1}^{\infty}\frac{m_i}{2^{m_i}\alpha_i}e^{-\frac{m_i}{\alpha_i}x}\, p_j \frac{\theta_j^2}{\theta_j+\lambda_j}(1+\lambda_j y)e^{-\lambda_j y} dxdy$$

$$R_3 = \sum_{j=i+3}^{4}\sum_{i=1}^{2}\int_0^\infty p_i p_j \sum_{m_i=1}^{\infty}\frac{m_i}{2^{m_i}\alpha_i}\, e^{-\frac{m_i}{\alpha_i}x}\left(1-\frac{\theta_j+\lambda_j+\lambda_j\theta_j x}{\theta_j+\lambda_j}e^{-\theta_j x}\right)dx$$

$$R_3 = 1-\sum_{j=i+3}^{6}\sum_{i=1}^{3}\sum_{m_i=1}^{\infty}\frac{p_i p_j m_i}{2^{m_i}\alpha_i}\int_0^\infty \left(e^{-\left(\frac{m_i}{\alpha_i}+\theta_j\right)x} + \frac{\lambda_j\theta_j}{\theta_j+\lambda_j}xe^{-\left(\frac{m_i}{\alpha_i}+\theta_j\right)x}\right)dx$$

$$R_3 = 1-\sum_{j=i+3}^{6}\sum_{i=1}^{3}\sum_{m_i=1}^{\infty}\frac{p_i p_j m_i}{2^{m_i}}\left[\frac{1}{m_i+\theta_j\alpha_i}+\frac{\lambda_j\theta_j\alpha_i}{(\theta_j+\lambda_j)(m_i+\theta_j\alpha_i)^2}\right] \tag{3.9}$$

In general for v-components,

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + \cdots + p_v f_v(x)\ ;\ \sum_{i=1}^{v} p_i = 1$$

$$g(y) = p_{v+1}f_{v+1}(y) + p_{v+2}f_{v+2}(y) + \cdots + p_{2v}f_{2v}(y)\ ;\ \sum_{i=v+1}^{2v} p_i = 1$$

$$R_v = \int_0^\infty\int_0^x \left(p_1 f_1(x) + p_2 f_2(x) + \cdots + p_k f_k(x)\right)\left(p_{v+1}f_{v+1}(y) + p_{v+2}f_{v+2}(y) + \cdots + p_{2v}f_{2v}(y)\right)dxdy$$

$$R_v = \sum_{j=i+v}^{2v}\sum_{i=1}^{v}\int_0^\infty\int_0^x p_i \sum_{m_i=1}^{\infty}\frac{m_i}{2^{m_i}\alpha_i}e^{-\frac{m_i}{\alpha_i}x}\, p_j \frac{\theta_j^2}{\theta_j+\lambda_j}(1+\lambda_j y)e^{-\lambda_j y} dxdy$$

$$R_v = \sum_{j=i+v}^{2v}\sum_{i=1}^{v}\int_0^\infty p_i p_j \sum_{m_i=1}^{\infty}\frac{m_i}{2^{m_i}\alpha_i}\, e^{-\frac{m_i}{\alpha_i}x}\left(1-\frac{\theta_j+\lambda_j+\lambda_j\theta_j x}{\theta_j+\lambda_j}e^{-\theta_j x}\right)dx$$

$$R_v = 1-\sum_{j=i+v}^{2v}\sum_{i=1}^{v}\sum_{m_i=1}^{\infty}\frac{p_i p_j m_i}{2^{m_i}\alpha_i}\int_0^\infty \left(e^{-\left(\frac{m_i}{\alpha_i}+\theta_j\right)x} + \frac{\lambda_j\theta_j}{\theta_j+\lambda_j}xe^{-\left(\frac{m_i}{\alpha_i}+\theta_j\right)x}\right)dx$$

$$R_v = 1-\sum_{j=i+v}^{2v}\sum_{i=1}^{v}\sum_{m_i=1}^{\infty}\frac{p_i p_j m_i}{2^{m_i}}\left[\frac{1}{m_i+\theta_j\alpha_i}+\frac{\lambda_j\theta_j\alpha_i}{(\theta_j+\lambda_j)(m_i+\theta_j\alpha_i)^2}\right] \tag{3.10}$$

Special case

When $\lambda_j = 1$, two parameter Lindley distribution reduces to one parameter Lindley distribution and then R_k is the reliability function when X follows mixture of one parameter Lindley distribution and Y follows mixture of one M-Transformed Exponential distribution and is given as

$$R_v = 1-\sum_{j=i+v}^{2v}\sum_{i=1}^{v}\sum_{m_i}^{\infty}\frac{p_i p_j m_i}{2^{m_i}}\left[\frac{1}{m_i+\theta_j\alpha_i}+\frac{\theta_j\alpha_i}{(\theta_j+1)(m_i+\theta_j\alpha_i)^2}\right] \tag{3.11}$$

Case IV: The stress Y follows mixture of M-Transformed Exponential distributions

As Y follows mixture of M-Transformed Exponential distributions, pdf of X and Y is given by

For two components, $v = 2$

$$f(x) = p_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} \ ; \ p_1+p_2=1, \alpha_1, \alpha_2, x>0$$

$$g(y) = p_3 \frac{2e^{-\frac{y}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{y}{\alpha_3}}\right)^2} + p_4 \frac{2e^{-\frac{y}{\alpha_4}}}{\alpha_4\left(2-e^{-\frac{y}{\alpha_4}}\right)^2} \ ; \ p_3+p_4=1,$$

As X and Y are independent then from (2), Reliability function R_2 is

$$R_2 = \int_0^\infty \int_0^x \left(p_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} \right) \left(p_3 \frac{2e^{-\frac{y}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{y}{\alpha_3}}\right)^2} + p_4 \frac{2e^{-\frac{y}{\alpha_4}}}{\alpha_4\left(2-e^{-\frac{y}{\alpha_4}}\right)^2} \right) dxdy$$

$$R_2 = \sum_{j=i+2}^{4} \sum_{i=1}^{2} \int_0^\infty \int_0^x p_i \sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x} p_j \sum_{m_j=1}^{\infty} \frac{m_j}{2^{m_j}\alpha_j} e^{-\frac{m_j}{\alpha_j}x} dxdy$$

$$R_2 = \sum_{j=i+2}^{4} \sum_{i=1}^{2} \int_0^\infty p_i p_j \sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x} \left(1 - \sum_{m_j=1}^{\infty} \frac{1}{2^{m_j}} e^{-\frac{m_j}{\alpha_j}x}\right) dx$$

$$R_2 = 1 - \sum_{j=i+2}^{4} \sum_{i=1}^{2} p_i p_j \int_0^\infty \left(\sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x} \right) \left(\sum_{m_j=1}^{\infty} \frac{1}{2^{m_j}} e^{-\frac{m_j}{\alpha_j}x} \right) dx$$

$$R_2 = 1 - \sum_{j=i+2}^{4} \sum_{i=1}^{2} p_i p_j \left(\frac{\alpha_j}{\alpha_i + \alpha_j} \right) \tag{3.12}$$

For three components, $v = 3$

$$f(x) = p_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} + p_3 \frac{2e^{-\frac{x}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{x}{\alpha_3}}\right)^2} \ ; \ p_1+p_2+p_3=1$$

$$g(y) = p_4 \frac{2e^{-\frac{y}{\alpha_4}}}{\alpha_4\left(2-e^{-\frac{y}{\alpha_4}}\right)^2} + p_5 \frac{2e^{-\frac{y}{\alpha_5}}}{\alpha_5\left(2-e^{-\frac{y}{\alpha_5}}\right)^2} + p_6 \frac{2e^{-\frac{x}{\alpha_6}}}{\alpha_6\left(2-e^{-\frac{x}{\alpha_6}}\right)^2} \ ; \ p_4+p_5+p_6=1,$$

As X and Y are independent then from (2), Reliability function R_2 is

$$R_3 = \int_0^\infty \int_0^x \left(p_1 \frac{2e^{-\frac{x}{\alpha_1}}}{\alpha_1\left(2-e^{-\frac{x}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x}{\alpha_2}}}{\alpha_2\left(2-e^{-\frac{x}{\alpha_2}}\right)^2} + p_3 \frac{2e^{-\frac{x}{\alpha_3}}}{\alpha_3\left(2-e^{-\frac{x}{\alpha_3}}\right)^2} \right) \left(p_4 \frac{2e^{-\frac{y}{\alpha_4}}}{\alpha_4\left(2-e^{-\frac{y}{\alpha_4}}\right)^2} + p_5 \frac{2e^{-\frac{y}{\alpha_5}}}{\alpha_5\left(2-e^{-\frac{y}{\alpha_5}}\right)^2} + p_6 \frac{2e^{-\frac{x}{\alpha_6}}}{\alpha_6\left(2-e^{-\frac{x}{\alpha_6}}\right)^2} \right) dxdy$$

$$R_3 = \sum_{j=i+3}^{6} \sum_{i=1}^{3} \int_0^\infty \int_0^x p_i \sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x} p_j \sum_{m_j=1}^{\infty} \frac{m_j}{2^{m_j}\alpha_j} e^{-\frac{m_j}{\alpha_j}x} dxdy$$

$$R_3 = \sum_{j=i+3}^{6} \sum_{i=1}^{3} \int_0^\infty p_i p_j \sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x} \left(1 - \sum_{m_j=1}^{\infty} \frac{1}{2^{m_j}} e^{-\frac{m_j}{\alpha_j}x}\right) dx$$

$$R_3 = 1 - \sum_{j=i+3}^{6} \sum_{i=1}^{3} p_i p_j \int_0^\infty \left(\sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x} \right) \left(\sum_{m_j=1}^{\infty} \frac{1}{2^{m_j}} e^{-\frac{m_j}{\alpha_j}x} \right) dx$$

$$R_3 = 1 - \sum_{j=i+3}^{6} \sum_{i=1}^{3} p_i p_j \left(\frac{\alpha_j}{\alpha_i + \alpha_j} \right) \tag{3.13}$$

In general for v-components,

$$f(x) = p_1 f_1(x) + p_2 f_2(x) + \cdots + p_v f_v(x)\ ; \quad \sum_{i=1}^{v} p_i = 1$$

$$g(y) = p_{v+1} f_{v+1}(y) + p_{v+2} f_{v+2}(y) + \cdots + p_{2v} f_{2v}(y)\ ; \quad \sum_{i=v+1}^{2v} p_i = 1$$

$$R_v = \int_0^{\infty}\int_0^{x} \left(p_1 f_1(x) + p_2 f_2(x) + \cdots + p_v f_v(x)\right)\left(p_{v+1} f_{v+1}(y) + p_{v+2} f_{v+2}(y) + \cdots + p_{2v} f_{2v}(y)\right) dxdy$$

$$R_v = \sum_{j=i+v}^{2v}\sum_{i=1}^{v}\int_0^{\infty}\int_0^{x} p_i \sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x}\, p_j \sum_{m_j=1}^{\infty} \frac{m_j}{2^{m_j}\alpha_j} e^{-\frac{m_j}{\alpha_j}x}\, dxdy$$

$$R_v = \sum_{j=i+v}^{2v}\sum_{i=1}^{v}\int_0^{\infty} p_i p_j \sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x}\left(1 - \sum_{m_j=1}^{\infty} \frac{1}{2^{m_j}} e^{-\frac{m_j}{\alpha_j}x}\right) dx$$

$$R_v = 1 - \sum_{j=i+2v}^{2v}\sum_{i=1}^{v} p_i p_j \int_0^{\infty}\left(\sum_{m_i=1}^{\infty} \frac{m_i}{2^{m_i}\alpha_i} e^{-\frac{m_i}{\alpha_i}x}\right)\left(\sum_{m_j=1}^{\infty} \frac{1}{2^{m_j}} e^{-\frac{m_j}{\alpha_j}x}\right) dx$$

$$R_v = 1 - \sum_{j=i+v}^{2v}\sum_{i=1}^{v} p_i p_j \left(\frac{\alpha_j}{\alpha_i + \alpha_j}\right) \tag{3.14}$$

4. Maximum Likelihood Estimation

$$L(\alpha_1, \alpha_2 p_1/\hat{y}) = \prod_{j=1}^{n}\left[p_1 \frac{2e^{-\frac{x_i}{\alpha_1}}}{\alpha_1\left(2 - e^{-\frac{x_i}{\alpha_1}}\right)^2} + p_2 \frac{2e^{-\frac{x_i}{\alpha_2}}}{\alpha_2\left(2 - e^{-\frac{x_i}{\alpha_2}}\right)^2}\right]$$

$$L(\alpha_1, \alpha_2 p_1/\hat{y}) = \frac{2^n n!}{n_1!\, n_2!}\left(\frac{p_1}{\alpha_1}\right)^{n_1}\left(\frac{p_2}{\alpha_2}\right)^{n_2}\prod_{i=1}^{n_1}\frac{e^{-\frac{x_i}{\alpha_1}}}{\left(2 - e^{-\frac{x_i}{\alpha_1}}\right)^2}\prod_{i=1}^{n_2}\frac{e^{-\frac{x_i}{\alpha_2}}}{\left(2 - e^{-\frac{x_i}{\alpha_2}}\right)^2}$$

$$\log L(\alpha_1, \alpha_2 p_1/\hat{y}) = log\left(\frac{2^n n!}{n_1!\, n_2!}\right) + n_1 log p_1 - n_1 log \alpha_1 + n_2 \log(1 - p_1) - n_2 log \alpha_2 + \sum_{i=1}^{n_1}\left[-\frac{x_i}{\alpha_1} - 2log\left(2 - e^{-\frac{x_i}{\alpha_1}}\right)\right] + \sum_{i=1}^{n_2}\left[-\frac{x_i}{\alpha_2} - 2log\left(2 - e^{-\frac{x_i}{\alpha_2}}\right)\right]$$

Now,

$$\frac{\partial \log L}{\partial \alpha_1} = 0$$

$$-\frac{n_1}{\alpha_1} + \sum_{i=1}^{n_1}\left[\frac{x_i}{{\alpha_1}^2} + \frac{2x_i e^{-\frac{x_i}{\alpha_1}}}{{\alpha_1}^2\left(2 - e^{-\frac{x_i}{\alpha_1}}\right)}\right] = 0$$

$$\alpha_1 = \frac{1}{n_1}\sum_{i=1}^{n_1}\left[x_i + \frac{2x_i e^{-\frac{x_i}{\alpha_1}}}{\left(2 - e^{-\frac{x_i}{\alpha_1}}\right)}\right] \tag{4.1}$$

$$\frac{\partial \log L}{\partial \alpha_2} = 0$$

$$-\frac{n_2}{\alpha_2} + \sum_{i=1}^{n_2}\left[\frac{x_i}{{\alpha_2}^2} + \frac{2x_i e^{-\frac{x_i}{\alpha_2}}}{{\alpha_2}^2\left(2 - e^{-\frac{x_i}{\alpha_2}}\right)}\right] = 0$$

$$\alpha_2 = \frac{1}{n_2}\sum_{i=1}^{n_2}\left[x_i + \frac{2x_i e^{-\frac{x_i}{\alpha_2}}}{\left(2 - e^{-\frac{x_i}{\alpha_2}}\right)}\right] \tag{4.2}$$

$\frac{\partial \log L}{\partial p_1} = 0$

$$\frac{n_1}{p_1} - \frac{n_2}{1-p_1} = 0$$

$$\hat{p}_1 = \frac{n_1}{n}\ ; n_1 + n_2 + 1 \tag{4.3}$$

Generalizing the above results for k-components we get

$$\alpha_i = \frac{1}{n_i}\sum_{j=1}^{n_1}\left[x_j + \frac{2x_i e^{-\frac{x_j}{\alpha_i}}}{\left(2 - e^{-\frac{x_j}{\alpha_i}}\right)}\right] \quad and \quad \hat{p}_i = \frac{n_i}{n}\ ; \quad n = \sum_{i=1}^{v} n_i \tag{4.4}$$

It is oblivious that first equation is nonlinear in α_j and is not easily solvable, therefore for obtaining their estimates we propose to use numerical iteration method. Therefore α_j can be obtained by using the nonlinear equation

$$h(\alpha_i) = \alpha_i \tag{4.5}$$

where

$$h(\alpha_i) = \frac{1}{n_i}\sum_{j=1}^{n_1}\left[x_j + \frac{2x_i e^{-\frac{x_j}{\alpha_i}}}{\left(2 - e^{-\frac{x_j}{\alpha_i}}\right)}\right] \tag{4.6}$$

since $\hat{\alpha}_i$ is a fixed point solution of $h(\alpha_i) = \alpha_i$, therefore it can be obtained by using simple iteration procedure as $h\left(\alpha_{i(k)}\right) = \alpha_{i(k+1)}$, where $\alpha_{i(k)}$ and $\alpha_{i(k+1)}$ are kth and (k+1)th iterations of $\hat{\alpha}_i$. When the difference between $\alpha_{i(k)}$ and $\alpha_{i(k+1)}$ is very small we stop the iteration and once we obtain the estimates $\hat{\alpha}_i$ *and* $\hat{p}_i$ the M.L.E of reliabilities for different cases are given as

1. M.L.E of reliability function when stress follows exponential distribution with known parameter λ and strength follows finite mixture of M-Transformed Exponential distribution with parameter α_i is

$$\hat{R}_v = 1 - \sum_{i=1}^{v}\sum_{m_i=1}^{\infty}\frac{\hat{p}_i m_i}{2^{m_i}(\lambda\hat{\alpha}_i + m_i)}$$

2. M.L.E of reliability function when stress follows Lindley distribution with known parameters λ and θ and strength follows finite mixture of M-Transformed Exponential distributions with parameter α_i is

$$\hat{R}_v = 1 - \sum_{i=1}^{v}\sum_{m_i=1}^{\infty}\hat{p}_i\frac{m_i}{2^{m_i}}\left(\frac{1}{m_i + \theta\hat{\alpha}_i} + \frac{\lambda\theta\hat{\alpha}_i}{(\theta+\lambda)(m_i + \theta\hat{\alpha}_i)^2}\right)$$

3. M.L.E of reliability function when stress follows finite mixture of Lindley distributions with known parameters λ_j *and* θ_j and strength follows finite mixture of M-Transformed Exponential distributions with parameter α_i is

$$\hat{R}_v = 1 - \sum_{j=i+v}^{2v}\sum_{i=1}^{v}\sum_{m_i=1}^{\infty}\frac{\hat{p}_i\hat{p}_j m_i}{2^{m_i}}\left[\frac{1}{m_i + \theta_j\hat{\alpha}_i} + \frac{\lambda_j\theta_j\hat{\alpha}_i}{(\theta_j + \lambda_j)\left(m_i + \theta_j\hat{\alpha}_i\right)^2}\right]$$

4. M.L.E of reliability function when stress and strength both follows finite mixture of M-Transformed Exponential distributions with parameters α_i and α_j is

$$\hat{R}_v = 1 - \sum_{j=i+v}^{2v}\sum_{i=1}^{v}\hat{p}_i\hat{p}_j\left(\frac{\hat{\alpha}_j}{\hat{\alpha}_i + \hat{\alpha}_j}\right)$$

5. Numerical Evaluation

In this section, we have evaluated reliability of a system with two components for specific values of a parameters involved in the reliability expressions in section 3 using graphical approach. Here we will discuss how reliability of two component system behaves when stress, strength or probability parameters are changed.

Case I: Stress follows exponential distribution and strength follows finite mixture of M-Transformed Exponential distribution

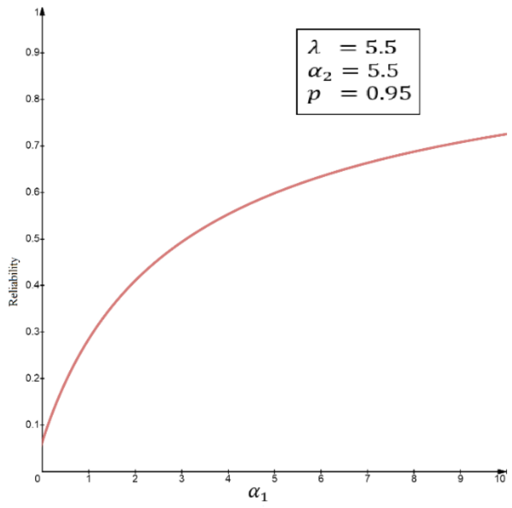

Figure 1

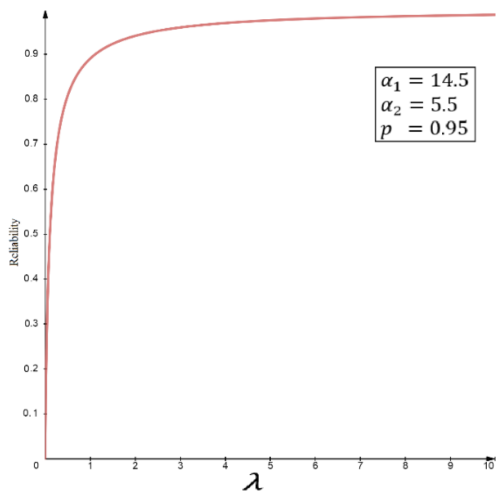

Figure 2

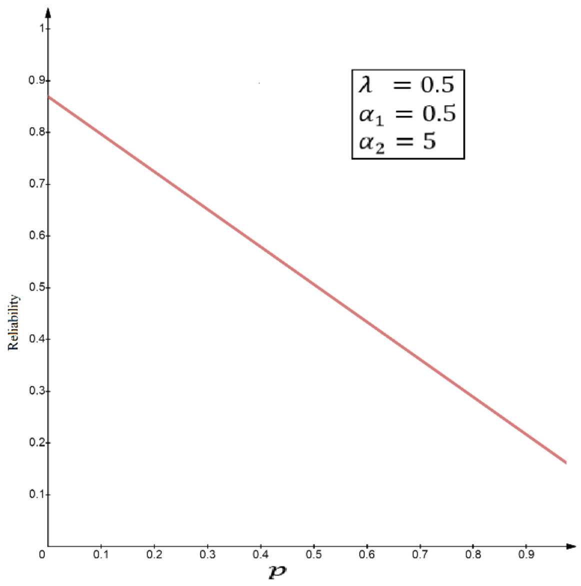

Figure 3

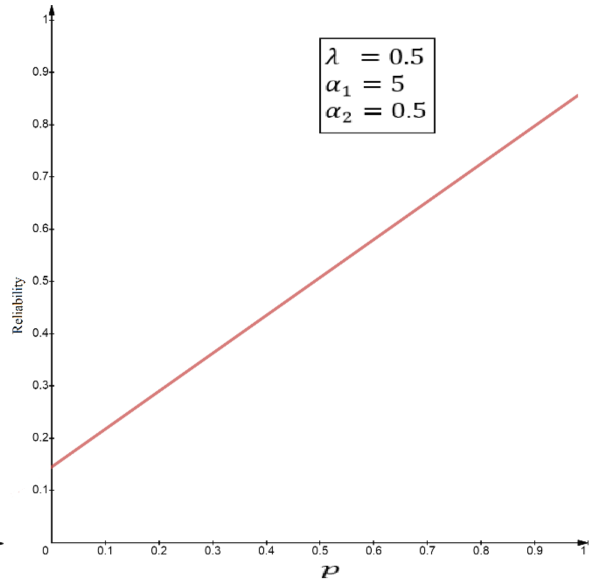

Figure 4

Case II: Stress follows Lindley distribution and strength follows finite mixture of M-Transformed Exponential distribution

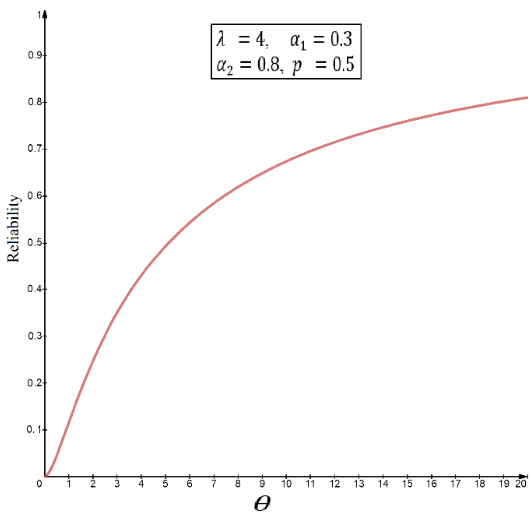

Figure 5

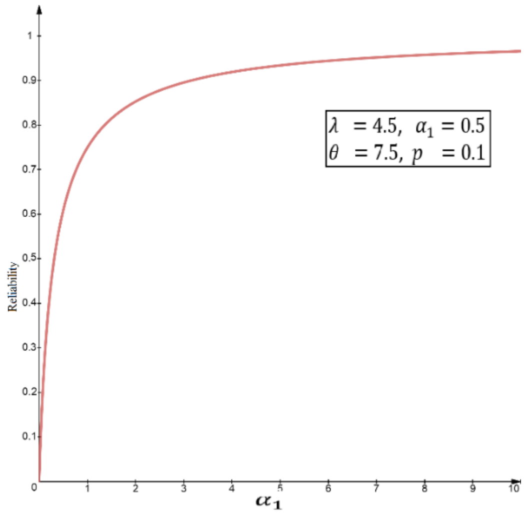

Figure 6

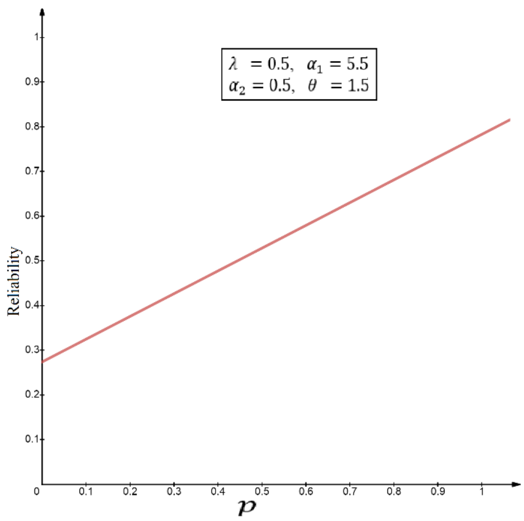

Figure 7

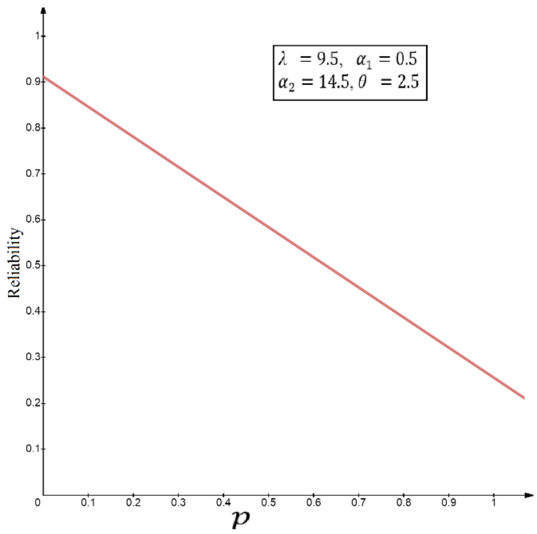

Figure 8

Case III: Stress follows finite mixture of Lindley distributions and strength follows finite mixture of M-Transformed Exponential distributions

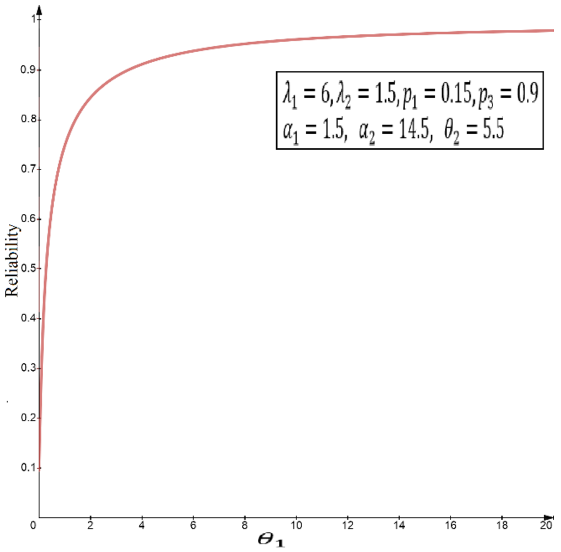

Figure 9

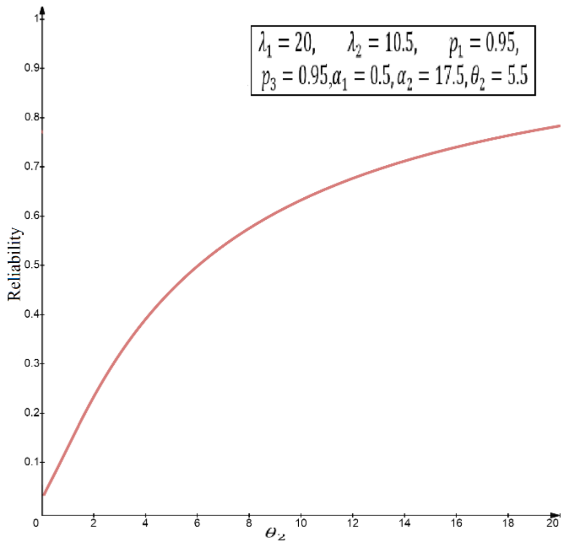

Figure 10

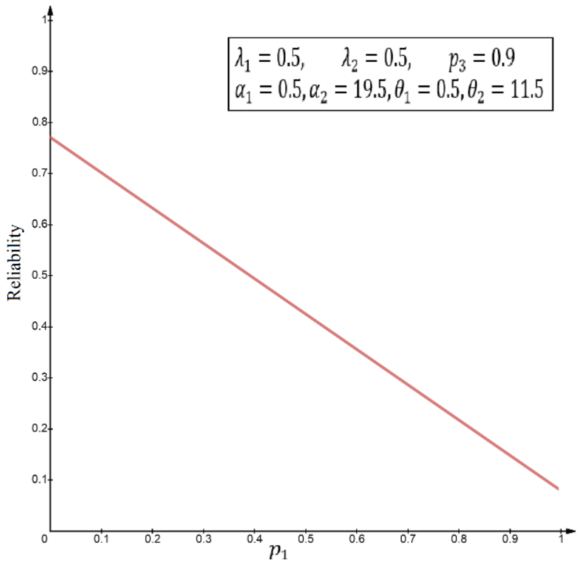

Figure 11

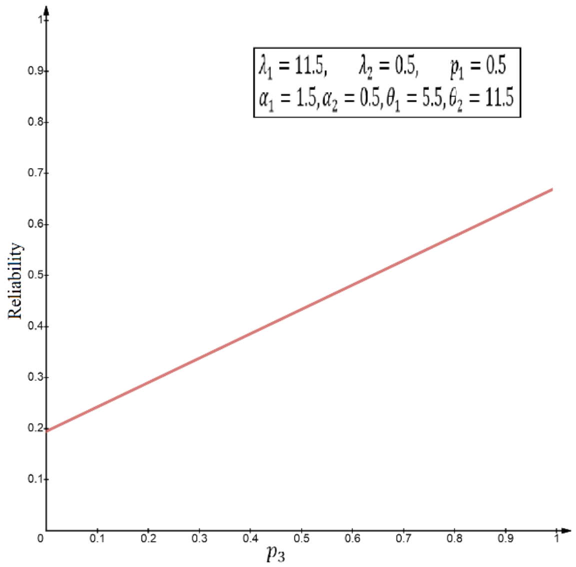

Figure 12

Case IV: Stress and strength both follows finite mixture of M-Transformed Exponential distributions

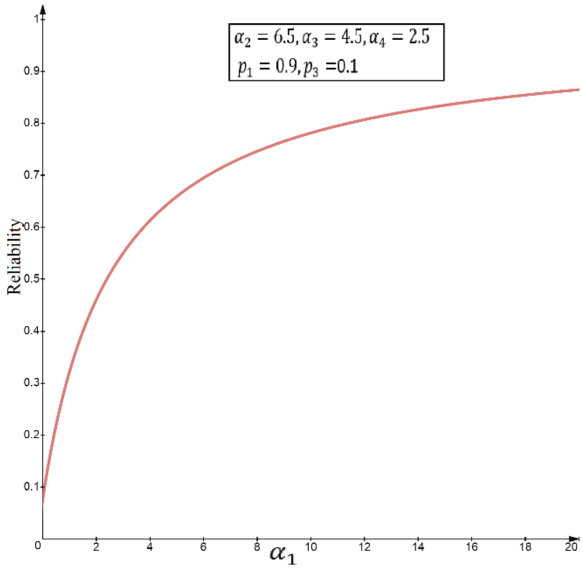

Figure 13

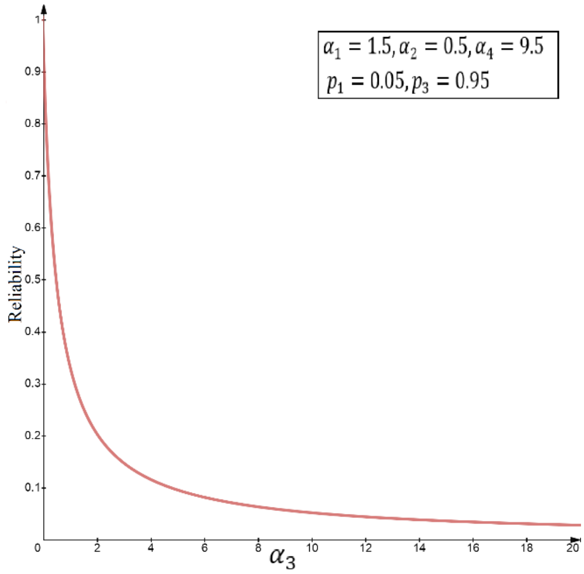

Figure 14

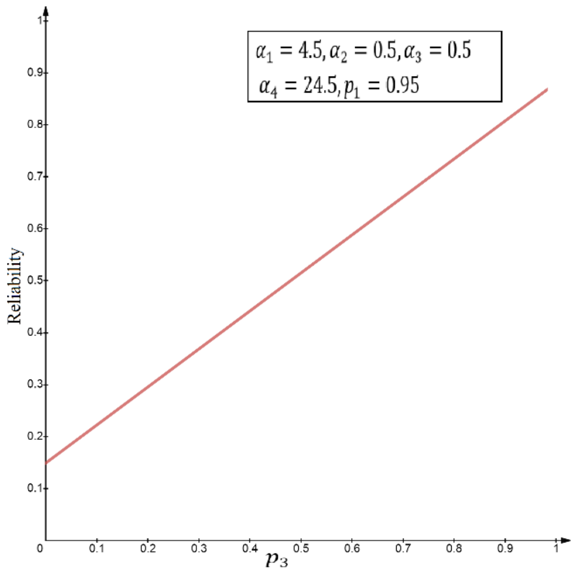

Figure 15

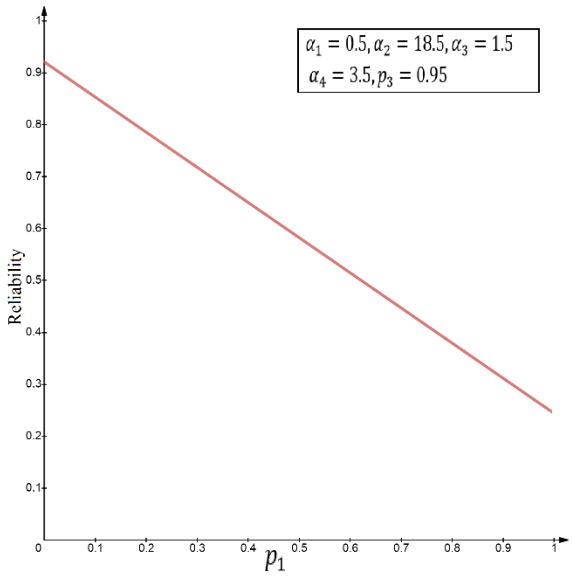

Figure 16

6. Real Data Analysis

In this sub section we analyze 33 leukaemia patients with two causes of death AG positive (presence of Auer rods and/or significant granulature of the leukaemic cells) or AG negative (both Auer rods and granulature are absent). The survival times in weeks are given

AG positive patients: 65, 156, 100, 134, 16, 108, 121, 4, 39, 143, 56, 26, 22, 1, 1, 5, 65

AG negative patients: 56, 65, 17, 7, 16, 22, 3, 4, 2, 3, 8, 4, 3, 30, 4, 43

We use the iterative procedure to obtain MLE of the parameters α_1 and α_2 using equation (4.5) and the final value of estimates are $\hat{\alpha}_1 = 90.65061, \hat{\alpha}_2 = 25.44067$ *and* $\hat{p}_1$ =0.5151.Based on these estimates the MLE of the reliability are given below

Stress and strength both follows finite mixture of M-Transformed Exponential distributions $\widehat{\mathrm{R}}_2 = 0.68$

Table 1: Stress follows exponential distribution and strength follows finite mixture of M-Transformed Exponential distribution

R	0.9161	0.9547	0.9689	0.9763	0.9809	0.9840	0.9862	0.9880	0.9892	0.9903
λ	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5

Table 2: Stress follows Lindley distribution and strength follows finite mixture of M-Transformed Exponential distribution

θ \ λ	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0.5	0.9317	0.9264	0.9238	0.9223	0.9212	0.9205	0.9200	0.9195	0.9191	0.9189
1	0.9409	0.9340	0.9298	0.9270	0.9250	0.9235	0.9224	0.9215	0.9207	0.9200
1.5	0.9450	0.9375	0.9327	0.9292	0.9265	0.9245	0.9228	0.9215	0.9204	0.9195
2	0.9473	0.9399	0.9346	0.9306	0.9275	0.9250	0.9230	0.9214	0.9200	0.9187
2.5	0.9490	0.9415	0.9360	0.9317	0.9283	0.9255	0.9232	0.9212	0.9195	0.9181
3	0.9500	0.9427	0.9371	0.9326	0.9289	0.9258	0.9233	0.9211	0.9191	0.9174
3.5	0.9507	0.9436	0.9380	0.9333	0.9294	0.9262	0.9234	0.9209	0.9188	0.9169
4	0.9514	0.9444	0.9387	0.9340	0.9299	0.9264	0.9234	0.9208	0.9185	0.9164
4.5	0.9520	0.9450	0.9393	0.9344	0.9303	0.9266	0.9235	0.9207	0.9182	0.9160
5	0.9523	0.9455	0.9398	0.9348	0.9305	0.9268	0.9235	0.9206	0.9180	0.9156

Table 3: Stress follows finite mixture of Lindley distributions and strength follows finite mixture of M-Transformed Exponential distributions, $\mathrm{p}_3 = 0.35, \lambda_2 = 6.5, \theta_2 = 5.5$

θ_1 \ λ_1	1	2	3	4	5	6	7	8	9	10
1	0.9680	0.9656	0.9644	0.9637	0.9632	0.9629	0.9626	0.9624	0.9622	0.9621
2	0.9784	0.9762	0.9748	0.9739	0.9733	0.9728	0.9725	0.9722	0.9719	0.9717
3	0.9818	0.9794	0.9780	0.9768	0.9760	0.9754	0.9749	0.9744	0.9741	0.9738
4	0.9833	0.9810	0.9792	0.9779	0.9770	0.9761	0.9754	0.9750	0.9744	0.9740
5	0.9842	0.9817	0.9798	0.9784	0.9771	0.9762	0.9754	0.9747	0.9741	0.9736
6	0.9848	0.9822	0.9801	0.9785	0.9772	0.9761	0.9751	0.9743	0.9736	0.9730
7	0.9852	0.9825	0.9803	0.9786	0.9771	0.9758	0.9747	0.9738	0.9730	0.9723
8	0.9855	0.9827	0.9804	0.9786	0.9769	0.9755	0.9743	0.9733	0.9723	0.9715
9	0.9858	0.9829	0.9805	0.9785	0.9768	0.9753	0.9740	0.9728	0.9717	0.9708
10	0.9860	0.9830	0.9806	0.9784	0.9766	0.9750	0.9735	0.9723	0.9711	0.9701

7. Conclusion

In this paper we have obtained stress strength reliability of a system with v-components using different distributions for stress variables viz. exponential distribution, M-Transformed exponential distribution and for strength variable we have used M-Transformed exponential distribution. In the last section we showed that reliability of 2-component system can be monotonically increasing and monotonically decreasing for specific values of parameter. Thus by proper choice of parameters leads to high reliability. The maximum likelihood estimates of parameters involved in the reliability function of v-components system are also obtained using iteration method.

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