Analysis of MAP/PH/1 Queueing model with Setup, Closedown, Multiple Vacations, Standby Server, Breakdown, Repair and Reneging

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Abstract

We have analyzed a single server queueing model in which the arrival of customers according to the Markovian arrival process, the service process according to phase type distributions and the standby server who is serving at a lower rate also follows the phase type distribution. If any of the customers present in the system when the server completes a vacation who starts the setup process to initiate service to the customers. After service completion, the main server begins the closedown process. The total number of customers are present in the system under the steady-state probability vector has been investigated by using the Matrix-Analytic method. We have examined the stability condition, the analysis of the busy period and derived some important performance measures of our model. Numerical results and graphical representation are discussed for the proposed model.

Keywords: Markovian Arrival Process, Phase type Distributions, Standby Server, Setup, Closedown, Multiple Vacation.

AMS Subject Classification (2010): **60K25, 68M20, 90B22.**

1. Introduction

On the basis of the study, the concept of the Markovian Arrival Process (MAP) has been introduced by Neuts (1981), the PH-representation is a Markov renewal process in service times and in general, MAP is the non-renewal process and it is commensurate to the Versatile modeling tool of Markovian Point Process (VMPP). This point process is fairly extensively described and well developed by the MAP is a specific type of semi-Markov process with a transition probability matrix (TPM). Later, it was realized that the VMPP and Batch Markovian Arrival Process (BMAP) are equivalent processes.

The Matrix-analytic methods(MAM) had been first introduced and examined by Neuts(1981). Qi-Ming He (2004), has analyzed the fundamentals of Matrix-Analytic methods such that the concept of arrival and the service process. Chakravarthy(2010) has described the various types of arrivals in which the customer's arrival follows the Markovian Arrival Process (MAP) with representation (D0, D1) of a square matrix whose order is m. The representation of the service times is (α, T) which follows phase type distributions whose matrix order is n. Let the generator Q is defined by Q = D0 + D1, is an irreducible stochastic matrix. The matrix D0 governs for transitions corresponding to no arrival, it has non-negative off-diagonal elements and non-singular with negative diagonal elements. The matrix D1 governs for transitions corresponding to arrival such that both diagonal and off-diagonal elements are non-negative.

If $\pi1$ is the unique probability vector of the Markov process described by the irreducible generator Q satisfying $\pi1Q = 0$ and $\pi1e = 1$. The constant $\lambda = \pi1D1e$ makes reference to the fundamental arrival rate, it will give us the expected number of customers arrive per unit time under the stationary version of the Markovian Arrival Process. The Marked Markovian Point Process (MMPP) is a special type of a doubly stochastic Poisson process whose arrival rate is modified by the states of an irreducible finite-state Continuous-Time Markov Chain (CTMC).

Attahiru Sule Alfa (1995) examined a discrete MAP/PH/1 in which the server offers service for a limited period of time and then the server goes on to another queue, in such a case it may consider server proceeds on a vacation. Jinbiao Wu et al. (2009) investigated the two types of arrivals such that positive and negative arrivals on the batch Markovian arrival process and the customer may go for G-queue with the second optional service. When the system empty, the server allows to take multiple vacations and they developed queue size distribution using the supplementary variable technique. Chesoong Kim et al. (2017) described unreliable BMAP/PH/N queueing type with breakdown occurrence moments are considered by Markovian arrival process and if the server fails immediately the repair period starts in which the duration of repair follows PH-distribution. Ayyappan and Shyamala (2014) have examined the concepts of setup time, breakdown and repair in a coherent way of batch arrival of customers with two heterogeneous servers.

Chakravarthy and Neuts (2014) have analyzed the queueing model of multi servers with two types of arrivals in which one type of customer is regular customers whose arrival follows the Markovian arrival process and another type is special customers whose arrival follows phase type renewal process. The regular customer requires only one server's attention but the special customer requires attention to all the servers. Furthermore, special customers possibly pre-empting the services of regular customers. Qi-Ming He and Attahiru Sule Alfa (2015) have studied the MAP/PH/K queueing model of the construction of Markov chains. Among these Markov chains, the first one is introduced through tracking service phases for the server which is construction of transition probability matrix in a straightforward manner and the second one is introduced through counting servers for phases which are using an algorithm for construction transition probability matrix. Zenios (1999) analyzed the queueing model with reneging such that the transplant waiting due to fear of organ transplantation that may lead to death. He has taken a survey at the end of 1996, the registered candidates for transplantation are 34,550 candidates but among these only 7,833 transplantations had performed and the remaining candidates were reneged due to impatience.

Arumuganathan and Jayakumar (2005) have analyzed the bulk queueing model with setup and closedown times. After completion of vacation if the queue size has reached N the server starts the setup process and the server starts the closedown process when the queue size less than N. Subsequently, they developed the cost model for their model. Wei Sun et al. (2012) developed Markovian queueing systems with three types of setup/closedown policies in which types are interruptible, skippable and insusceptible. Among these insusceptible explains if the customer comes during closedown times the service will start to the customer. The second type explains if a customer comes during closedown, they would be served after the closedown finishes and skipped the setup time and the third type tell the customers to arrive during the closedown time they could get service and they have to wait until the setup time finishes.

Tsung - Yin Wang (2012) has studied the Geo/G/1 queueing model with startup under N-policy. This N-policy is applicable for the server temporarily unavailable to the waiting customers. Service completed to all the customers, the server is being shut down in a closedown time. Zhisheng Niu (2003) examined the single server batch Markovian arrival processes of setup and closedown under single and multiple vacation queue in which they described the potential applications that the first one is switched virtual connection-based Internet protocols over Automated Teller Machine networks and the second one is multiple protocol label switched networks.

Many researchers incorporating their model with standby support. Sreekanth Kolledath et al. (2017) have analyzed a survey on standby support queueing models. They described different types of standby's are cold standby, warm standby, hot standby, mixed standby and standby switching. Among these the cold standby tells about the standby with zero failure rate, the warm standby tells about the standby with a lower failure rate than compared to the primary components, the hot standby tells about the standby has the same failure rate as the primary components, mixed standby is the new concept of the combination of cold and warm standby and the standby switching explains when switching the standby in place of the main one which may be unsuccessful that is standby switching failure has happened. In this situation, until switching is successful all the standby's are try to switch over one by one. Furthermore, we have referred to the concepts of standby in Subramanian and Sarm (1987) and Khalaf et al. (2011).

In real-life situations, pupils would like to do any of the work they have to do some of the preparatory action known as the setup process. After completion of that work, they do some shutdown action known as the closedown process. For example, the grocery store, supermarkets, Industries, computer systems, laptops, communication systems and hypermarkets. These examples are suitable for our model as setup time, closedown time, vacation, breakdown and standby server. Among these examples, the hypermarkets are an interesting concept of the combination of supermarkets and departmental stores. It has a wide range of shopping facilities such as including general merchandise and all kinds of grocery lines on one floor itself and it needs a large landscape to locate this one. In one trip itself, hypermarkets offer to the customers for buying whatever things they need in the routine shopping. These kinds of big-box stores need some amount of time to make the setup process and some amount of time to take for closing the hypermarkets. During the vacation times of hypermarkets, the reneging might happen due to impatience.

The remaining part of the article is organized as follows. We describe our mathematical model description in section 2. In section 3, we are generating our matrix formulation and notations of our model also included. In section 4 we discuss the stability condition and steady-state probability vector. In section 5, we have analyzed the busy period analysis and in section 6, measures of system performance are discussed. In section 7, presents some of the illustrated numerical and graphical representations. The main server and standby server service rates are compared in section 8. The conclusion of our model has been given in the last section 9.

2. The Mathematical model Description

In this model, we consider the arrival of customers follows the Markovian arrival process which represents (D0, D1), where D0 and D1 are square matrices of order is m′, the service process follows phase type distribution which represents (α, T) of order is n′ and the standby server service process also follows the phase type distribution which represents ($\alpha 1$, θT) of matrix order is n′ with T0 + Te = 1 such that T0 = -Te and we are taking α and $\alpha 1$ are the same for the purpose of differentiating the main server and standby server such that $\alpha = \alpha 1$. While the main server giving service to the customers, the server may breakdown at any time. When the breakdown has occurred, the main server goes for the repair process, immediately standby server switch over instead of the main server, then the standby server carry over the service process but at the lower rate compared to the main server service rate with representation θT where $0<\theta<1$. When the main server returns to the service station after rejuvenating from the repair, at that moment if the standby server is in the idle state. Obviously, the main server would be in the idle state until the customer's arrival to the system otherwise if the standby server is busy when the main server return to the system from the repair process then the main server would interrupt the standby server and carry over the service process. The breakdown times and repair times are exponentially distributed. After completing service to the customers, if there is no one in the system the main server starts the closedown process, afterward closedown the main server goes for vacation. When the main server return from vacation if there is no customer in the system then the server will go for vacation repeatedly until if the server finds at least one customer in the queue. The duration of vacation times follows exponential distribution at the rate η. After completion of vacation, if there is a customer in the queue then the main server does the setup process and then starts giving service to customers who are standing in the queue. Due to impatience, the customers who have an aspiration to get service may have reneged from the system that is they leave the system during the main server vacation period and the renege rate is ζ. The parameters of setup rate and closedown rate are σ and γ respectively.

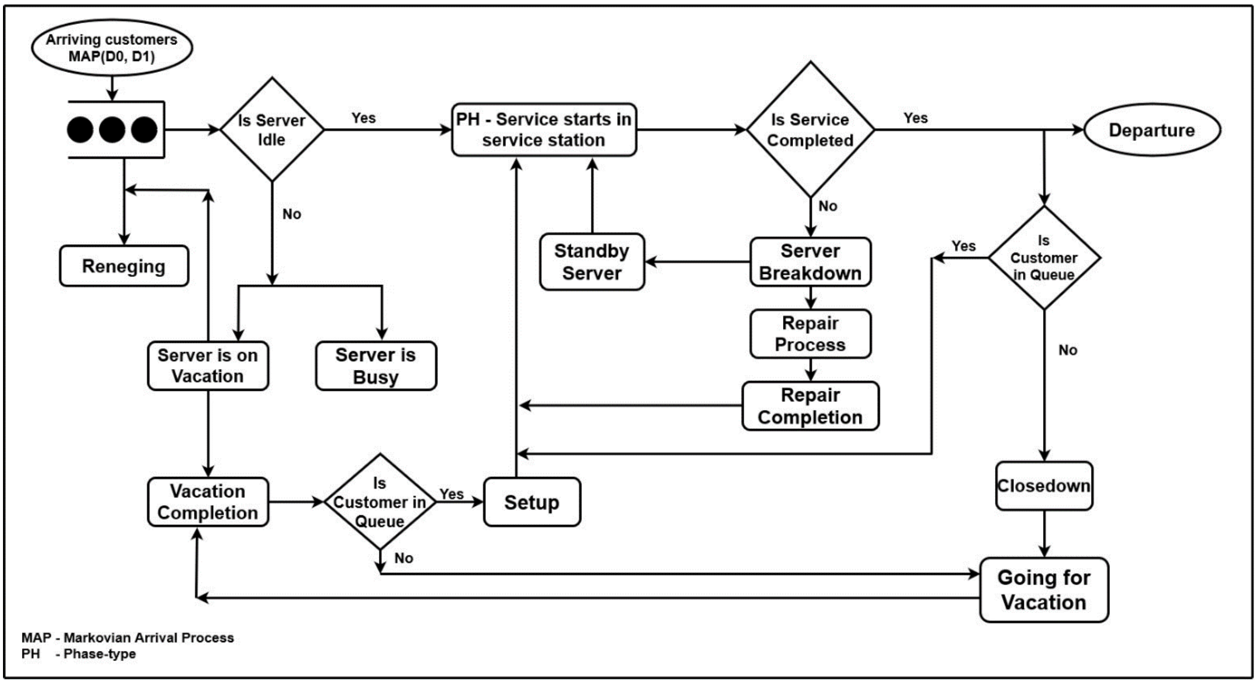

Figure 1: Pictorial Representation of our proposed model.

3. The Matrix Generation - QBD process

In this section, we have described the notation of our model as follows for the purpose of generating the QBD Process.

Notations for Matrix Generation

- $\otimes$ - Kronecker product of any two of the different order of matrices by using this symbol.
- $\oplus$ - Kronecker sum of any two of the different order of matrices by using this symbol.

- In - It denotes an n-dimensional Identity matrix.
- Im - It denotes an m-dimensional Identity matrix.
- Inm - It denotes an nm-dimensional Identity matrix.

- e - Column vector of suitable dimension each of its entry is 1.
- en - Column vector whose dimension is n and each of its entries are 1.
- enm - Column vector whose dimension is nm and each of its entries are 1.
- Let us denote λ be the arrival rate and is defined as λ = π1D1em, where π1 is the Probability vector of the generator matrix D = D0 + D1.
- The normal service rate of the main server and the standby server is denoted by δ and $\theta\delta$ where $\delta = [\alpha(-T)^{-1}en]^{-1}$.
- Define N(t) indicates the number of customers in the system.

- Define V(t) indicates the status of server at time t,

$$V(t) = \begin{cases} 1, & \text{ifthemainserverisinbusy,} \\ 2, & \text{ifthemainserverisinthebreakdown,} \\ 3, & \text{ifthemainserverisinthesetupprocess,} \\ 4, & \text{ifthemainserverisintheclosedown,} \\ 5, & \text{ifthemainserverisonvacation.} \end{cases}$$

- J(t) is the service process considered by phases.
- M(t) is the arrival process considered by phases.

Let {(N(t) , V(t) , J(t) , M(t) : t ... 0 } be the Continuous-Time Markov Chain (CTMC) with state level independent Quasi-Birth-and-Death process whose state space is as follows,

$$\Phi = l(0) \cup l(p).$$

where,
l(0) = {(0,2,s) : 1 ≤ s ≤ m} ∪ {(0,4,s) : 1 ≤ s ≤ m} ∪ (0,5,s) : 1 ≤ s ≤ m}.
for p ≥ 1,
l(p) = {(p,1,r,s):1 ≤ r ≤ n; 1 ≤ s ≤ m} ∪ {(p,2,r,s): 1 ≤ r ≤ n; 1 ≤ s ≤ m} ∪ {(p,3,s) : 1 ≤ s ≤ m} ∪ {(p,4,s) : 1 ≤ s ≤ m} ∪ {(p,5,s) : 1 ≤ s ≤ m}.
The infinitesimal matrix generation of the QBD Process is given by,

$$Q = \begin{bmatrix} B_{00} & B_{01} & 0 & 0 & 0 & 0 & \cdots & \cdots \\ B_{10} & A_1 & A_0 & 0 & 0 & 0 & \cdots & \cdots \\ 0 & A_2 & A_1 & A_0 & 0 & 0 & \cdots & \cdots \\ 0 & 0 & A_2 & A_1 & A_0 & 0 & \cdots & \cdots \\ 0 & 0 & 0 & A_2 & A_1 & A_0 & \cdots & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \\ \vdots & \vdots & \vdots & \vdots & & \ddots & \ddots & \ddots \end{bmatrix}$$

The entries in the block matrices of Q are defined as follows,

$$B_{00} = \begin{bmatrix} D0 - \Psi Im & \Psi Im & 0 \\ 0 & D0 - \gamma Im & \gamma Im \\ 0 & 0 & D0 \end{bmatrix},$$

$$B_{01} = \begin{bmatrix} 0 & \alpha 1 \otimes D1 & 0 & 0 & 0 \\ 0 & 0 & 0 & D1 & 0 \\ 0 & 0 & 0 & 0 & D1 \end{bmatrix},$$

$$B_{10} = \begin{bmatrix} 0 & T0 \otimes Im & 0 \\ \theta T0 \otimes Im & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \zeta Im & 0 \\ 0 & 0 & \zeta Im \end{bmatrix},$$

$$A_1 = \begin{bmatrix} T \oplus D0 - \tau Inm & \tau Inm & 0 & 0 & 0 \\ \Psi Inm & \theta T \oplus D0 - \Psi Inm & 0 & 0 & 0 \\ \alpha \otimes \sigma Im & 0 & D0 - \sigma Im & 0 & 0 \\ 0 & 0 & 0 & D0 - \gamma Im - \zeta Im & \gamma Im \\ 0 & 0 & \eta Im & 0 & D0 - \eta Im - \zeta Im \end{bmatrix},$$

$$A_0 = \begin{bmatrix} In \otimes D1 & 0 & 0 & 0 & 0 \\ 0 & In \otimes D1 & 0 & 0 & 0 \\ 0 & 0 & D1 & 0 & 0 \\ 0 & 0 & 0 & D1 & 0 \\ 0 & 0 & 0 & 0 & D1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} T0\alpha \otimes Im & 0 & 0 & 0 & 0 \\ 0 & \theta T0\alpha 1 \otimes Im & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \zeta Im & 0 \\ 0 & 0 & 0 & 0 & \zeta Im \end{bmatrix}$$

4. Stability Condition

We have analyzed our model under some condition that whether the system is stable.

4.1. Analysis of Stability condition

Let us define the matrix A as A = A0 + A1 + A2. It has clearly shown that the arrangement of the square matrix A is 2nm+3m and this matrix is an irreducible infinitesimal generator matrix.

The vector ξ is denoted by $\xi = (\xi 1, \xi 2, \xi 3, \xi 4, \xi 5)$. Let ξ be the steady-state probability vector of A satisfying $\xi A = 0$ and $\xi e = 1$, where $\xi 1$ and $\xi 2$ are of dimension nm and $\xi 3$, $\xi 4$, $\xi 5$ are of dimension m. The Markov process has the quasi-birth-and-death structure, there exits stability of our model should satisfy $\xi A0e < \xi A2e$, is the necessary and sufficient condition of a QBD process. The vector ξ is calculated by solving the following equations,

$$\xi 1[(T + T0\alpha) \oplus D - \tau Inm] + \xi 2[\Psi Inm] + \xi 3[\alpha \otimes \sigma Im] = 0.$$

$$\xi 1[\tau Inm] + \xi 2[(\theta T + \theta T0\alpha 1) \oplus D - \Psi Inm] = 0.$$

$$\xi 3[D - \sigma Im] + \xi 5[\eta Im] = 0.$$

$$\xi 4[D - \gamma Im] = 0.$$

$$\xi 4[\gamma Im] + \xi 5[D - \eta Im] = 0.$$

subject to the normalizing condition

$$\xi 1enm + \xi 2enm + \xi 3em + \xi 4em + \xi 5em = 1.$$

After some algebraical manipulation, the stability condition ξA0e < ξA2e which is turned to be as follows,

$$(\xi 1 + \xi 2)[en \otimes D1em] + (\xi 3 + \xi 4 + \xi 5)[D1em] < \xi 1[T0 \otimes em] + \xi 2[\theta T0 \otimes em] + (\xi 4 + \xi 5)[\zeta Im].$$

4.2. Analysis of Steady-state Probability vector

Let us take the variable x is the probability vector and is partitioned as $x = (x0, x1, x2, \ldots\ldots)$. Hence, x be the steady-state probability vector of Q. Here we are mentioning that $x0$ is of dimension 3m and $x1, x2, \ldots$ are of dimension 2nm+3m. Then x satisfies the condition $xQ = 0$ and $xe = 1$.

Moreover, when the stability condition has been satisfied with the subvectors of x except for $x0$ and $x1$, commensurate to the different level states are given by the equation as,

$$xj = x1\ R^{j-1}, \quad j \quad 2.$$

where the rate matrix R denotes the minimal non-negative solution of the matrix quadratic equation as $R^2A2 + RA1 + A0 = 0$, which is referred by the author Neuts(1981).

Since our system is stable, and if adding the square matrices of A0, A1 and A2 whose row sums are equal to zero, then the rate matrix R is a square matrix of order is 2nm+3m, it is obtained from the above quadratic equation and satisfies the relation RA2e = A0e.

The sub vectors of $x0$ and $x1$ have obtained by solving the following equations,

$$x0B00 + x1B10 = 0.$$
$$x0B01 + x1(A1 + RA2) = 0.$$

subject to the normalizing condition is

$$x0e3m + x1(I - R)^{-1}e2nm + 3m = 1.$$

Thus, the R matrix could be calculated mathematically using essential steps in the Logarithmic reduction algorithm of R are given below we have referred the author's Latouche and Ramaswami(1999).

Theorem: The structure of the rate matrix R is

$$R = \begin{bmatrix} R11 & R12 & 0 & 0 & 0 \\ R21 & R22 & 0 & 0 & 0 \\ R31 & R32 & R33 & 0 & 0 \\ R41 & R42 & R43 & R44 & R45 \\ R51 & R52 & R53 & 0 & R55 \end{bmatrix} \quad (1)$$

Proof: The computation of the matrix R, it is clearly shown that R must have the structure for our model as given in (1). The main server may be struck with breakdown while giving service leads to main server can go for repair, in this situation the standby server giving service at lower rate and the main server is being in service after return from the repair completion. Furthermore, here we will give proof of the construction of R. We can rewrite the matrix quadratic equation $R^2A2 + RA1 + A0 = 0$ is given by,

$$R = (R^2A2 + A0)(-A1)^{-1}$$

It can easily verify that the structure of the matrix $(-A1)^{-1}$ as follows,

$$(-A1)^{-1} = \frac{1}{V}\begin{bmatrix} f11 & f12 & 0 & 0 & 0 \\ f21 & f22 & 0 & 0 & 0 \\ f31 & f32 & f33 & 0 & 0 \\ f41 & f42 & f43 & f44 & f45 \\ f51 & f52 & f53 & 0 & f55 \end{bmatrix} \quad (2)$$

where the elements of $(-A1)^{-1}$ as follows,

$V = [[(T \oplus D0) - \tau Inm][(\theta T \oplus D0) - \Psi Inm] - [\Psi Inm][\tau Inm]] \times [[D0 - \sigma Im] \times [D0 - \gamma Im - \zeta Im][D0 - \eta Im - \zeta Im]],$

$f11 = [[(\theta T \oplus D0) - \Psi Inm][\sigma Im - D0][D0 - \gamma Im - \zeta Im][D0 - \eta Im - \zeta Im]],$

$f12 = [[\tau Inm][D0 - \sigma Im][D0 - \gamma Im - \zeta Im][D0 - \eta Im - \zeta Im]],$

$f21 = [[\Psi Inm][D0 - \sigma Im][D0 - \gamma Im - \zeta Im][D0 - \eta Im - \zeta Im]],$

$f22 = [[\tau Inm - (T \oplus D0)][D0 - \sigma Im][D0 - \gamma Im - \zeta Im][D0 - \eta Im - \zeta Im]],$

$f31 = [[\alpha \otimes \sigma Im][(\theta T \oplus D0) - \Psi Inm][D0 - \gamma Im - \zeta Im][D0 - \eta Im - \zeta Im]],$

$f32 = [[\alpha \otimes \sigma Im][\tau Inm][\gamma Im + \zeta Im - D0][D0 - \eta Im - \zeta Im]],$

$f33 = [[\Psi Inm][\tau Inm] - [(T \oplus D0) - \tau Inm][(\theta T \oplus D0) - \Psi Inm]] \times [[D0 - \gamma Im - \zeta Im] \times [D0 - \eta Im - \zeta Im]],$

$f41 = [[\alpha \otimes \sigma Im][(\theta T \oplus D0) - \Psi Inm][\gamma Im \times \eta Im]],$

$f42 = [[-(\alpha \otimes \sigma Im)][\tau Inm][\gamma Im \times \eta Im]],$

$f43 = [[\Psi Inm][\tau Inm] - [(T \oplus D0) - \tau Inm][(\theta T \oplus D0) - \Psi Inm]] \times [\gamma Im \times \eta Im],$

$f44 = [[\Psi Inm][\tau Inm] - [(T \oplus D0) - \tau Inm][(\theta T \oplus D0) - \Psi Inm]] \times [[D0 - \sigma Im] \times [D0 - \eta Im - \zeta Im]],$

$f45 = [[(T \oplus D0) - \tau Inm][(\theta T \oplus D0) - \Psi Inm] - [\Psi Inm][\tau Inm]] \times [[D0 - \sigma Im][\gamma Im]],$

$f51 = [[\alpha \otimes \sigma Im][\Psi Inm - (\theta T \oplus D0)][\eta Im][D0 - \gamma Im - \zeta Im]],$

$f52 = [[\alpha \otimes \sigma Im][\tau Inm][D0 - \gamma Im - \zeta Im][\eta Im]],$

$f53 = [[(T \oplus D0) - \tau Inm][(\theta T \oplus D0) - \Psi Inm] - [\Psi Inm][\tau Inm]] \times [[\eta Im][D0 - \gamma Im - \zeta Im]],$

$f55 = [[\Psi Inm][\tau Inm] - [(T \oplus D0) - \tau Inm][(\theta T \oplus D0) - \Psi Inm]] \times [[D0 - \sigma Im][D0 - \gamma Im - \zeta Im]].$

In the same way, pre-multiplying a diagonal block matrix with $(-A1)^{-1}$ matrix, it won't change the structure as seen in (2). Hence, the structure of matrix $A0(-A1)^{-1}$ is given by,

$$A0(-A1)^{-1} = \frac{1}{V}\begin{bmatrix} g11 & g12 & 0 & 0 & 0 \\ g21 & g22 & 0 & 0 & 0 \\ g31 & g32 & g33 & 0 & 0 \\ g41 & g42 & g43 & g44 & g45 \\ g51 & g52 & g53 & 0 & g55 \end{bmatrix} \quad (3)$$

where the elements of $A0(-A1)^{-1}$ as follows,

$g11 = [In \otimes D1]f11, \quad g12 = [In \otimes D1]f12, \quad g21 = [In \otimes D1]f21, \quad g22 = [In \otimes D1]f22$

$g31 = [D1]f31, \quad g32 = [D1]f32, \quad g33 = [D1]f33, \quad g41 = [D1]f41, \quad g42 = [D1]f42,$

$g43 = [D1]f43, \quad g44 = [D1]f44, \quad g45 = [D1]f45, \quad g51 = [D1]f51, \quad g52 = [D1]f52,$

$g53 = [D1]f53, \quad g55 = [D1]f55.$

Here, pre-multiplying a block matrix $A2$ with $(-A1)^{-1}$ matrix. Therefore, the structure of matrix $A2(-A1)^{-1}$ is given by,

$$A2(-A1)^{-1} = \frac{1}{V}\begin{bmatrix} h11 & h12 & 0 & 0 & 0 \\ h21 & h22 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ h41 & h42 & h43 & h44 & h45 \\ h51 & h52 & h53 & 0 & h55 \end{bmatrix} \quad (4)$$

where the elements of $A2(-A1)^{-1}$ as follows,

$h11 = [T0\alpha \otimes Im]f11, \quad h12 = [T0\alpha \otimes Im]f12, \quad h21 = [\theta T0\alpha \otimes Im]f21,$

$h22 = [\theta T0\alpha \otimes Im]f22, \quad h41 = [\zeta Im]f41, \quad h42 = [\zeta Im]f42, \quad h43 = [\zeta Im]f43,$

$h44 = [\zeta Im]f44, \quad h45 = [\zeta Im]f45, \quad h51 = [\zeta Im]f51, \quad h52 = [\zeta Im]f52,$

$h53 = [\zeta Im]f53, \quad h55 = [\zeta Im]f55.$

The sequence of $\{R^{(n)}, n = 0,1,2,3,\ldots.\}$ is defined by,

$$R^{(n+1)} = [(R^{(n)})^2 A2 + A0](-A1)^{-1}, \quad n = 0,1,2,3,\ldots.$$

The matrix quadratic equation $R^2A2 + RA1 + A0 = 0$ which has the minimal non negative solution as converges monotonically with $R^{(0)} = 0$. Hence, the structure of $\{A0(-A1)^{-1}\}$ and $\{(R^{(n)})^2 A2(-A1)^{-1}$, where $n = 1,2,3,\ldots.\}$ will remains the same as that of $(-A1)^{-1}$. Using $R^{(0)} = 0$, we can compute first iteration of R matrix i.e., $R^{(1)}$ then using first iteration of R matrix we can compute second iteration of R matrix i.e., $R^{(2)}$. Similarly, we can compute the further iterations of R matrix. Therefore, the each iteration of R matrix i.e., $R^{(n)}$ retains the same structure.

Logarithmic Reduction Algorithm of R

Step 0:
H ← $(-A_1)^{-1}A_0$, L ← $(-A_1)^{-1}A_2$, G = L, and T = H.
Step 1:
U = HL + LH
$M = H^2$
H ← $(I-U)^{-1}M$
M ← L^2
L ← $(I-U)^{-1}M$
G ← G + TL
T ← TH
Continue Step 1 Until $||e-Ge||_{\infty}\varepsilon$
Step 2:
$R = -A_0(A_1 + A_0G)^{-1}$.

5. Analysis of the Busy Period

• A Busy period is nothing but the interval between the customers arrives into the empty system and afterward the first interval once again the system becomes empty. So, it is the first passage from level 1 to 0. The busy cycle has described the first return time to level 0 with at least one visit to a state at any other level.

• Prior to examining the busy period, we have introduced an overview of the fundamental period. Under consideration of the QBD Process, it is the first passage time from level j to level $j-1, j$ 2.

• The cases $j = 0,1$ commensurate the boundary states have to be discussed individually. Note that for each and every level j, j   1 there corresponds (2nm+3m) states. Thus by the state (j,k) of level j we mention that the k^{th} state of level j when the states are arranged in alphabetical order.

• Let us denote $Gkk'(u,x)$ be the conditional probability that it started in the state (j,k) at time t = 0, the QBD process visits the level $j-1$ but not later than time x, we could make u transitions to the left and also entering the state (j,k').

Let us introduce the concept of the joint transform

$$\overline{G}kk'(z,s) = \sum_{u=1}^{\infty} z^u \int_0^{\infty} e^{-sx} dG\text{kk}'(u,x) \quad ; |z| \le 1, Re(s) \ge 0$$

and the matrix is denoted as follows

$$\overline{G}(z,s) = \overline{G}kk'(z,s)$$

then the above-defined matrix $\overline{G}(z,s)$ satisfies the equation

$$\overline{G}(z,s) = z(SI-A1)^{-1}A2 + (SI-A1)^{-1}A0\overline{G}^2(z,s)$$

• The matrix of $G = Gkk' = \overline{G}(1,0)$ would be taken for the first passage times, exclude for the boundary states. If we already know the matrix R then we could find the matrix G using the result

$$G = -(A1 + RA2)^{-1}A2$$

Otherwise, we may use the concept of a logarithmic reduction algorithm method to find the values of G matrix.

Notations of Boundary level states for Busy Period

- $Gkk'^{(1,0)}(u,x)$ denotes the conditional probability have been discussed for the first passage times from level 1 to the level 0 at time t = 0.
- $Gkk'^{(0,0)}(u,x)$ denotes the conditional probability have been discussed for the return time to the level 0.
- $\mathcal{F}1j$ denotes the mean first passage time from the level j to level $j-1$, given that the process is in the state (j,k) at time t = 0.
- $\overline{\mathcal{F}}1$ denotes the column vector with entries $\mathcal{F}1j$.
- $\mathcal{F}2j$ denotes the mean number of customers to be served during the first passage time from level j to level $j-1$, given that the first passage time has started in the state (j,k).
- $\overline{\mathcal{F}}2$ denotes the column vector with entries $\mathcal{F}2j$.
- $\overline{\mathcal{F}}1^{(1,0)}$ denotes the mean first passage time from level 1 to the level 0.
- $\overline{\mathcal{F}}2^{(1,0)}$ denotes the mean number of service completed during the first passage time from the level 1 to the level 0.
- $\overline{\mathcal{F}}1^{(0,0)}$ denotes the first return time to the level 0.
- $\overline{\mathcal{F}}2^{(0,0)}$ denotes the mean number of service completion in between first return time to the level 0.

For the boundary levels 1 and 0 we get,

$$\overline{G}^{(1,0)}(z,s) = z(SI - A1)^{-1}B10 + (SI - A1)^{-1}A0\overline{G}(z,s)\overline{G}^{(1,0)}(z,s)$$
$$0.3cm \quad \overline{G}^{(0,0)}(z,s) = (SI - B00)^{-1}B01\overline{G}^{(1,0)}(z,s)$$

Thus, the following instances are calculated using the matrices as $G(z,s)$, $\overline{G}^{(0,0)}(z,s)$ and $\overline{G}^{(1,0)}(z,s)$ are stochastic in nature. We can compute the instants as follows,

$$\overline{\mathcal{F}}1 = -\frac{\partial}{\partial s}\overline{G}(z,s)\Big|_{z=1,s=0} e = -[A1 + A0(I+G)]^{-1}e \tag{5}$$

$$\overline{\mathcal{F}}2 = \frac{\partial}{\partial z}\overline{G}(z,s)\Big|_{z=1,s=0} e = -[A1 + A0(I+G)]^{-1}A2e \tag{6}$$

$$\overline{\mathcal{F}}1^{(1,0)} = -\frac{\partial}{\partial s}\overline{G}^{(1,0)}(z,s)\Big|_{z=1,s=0} e = -[A1 + A0G]^{-1}(A0\overline{\mathcal{F}}1 + e) \tag{7}$$

$$\overline{\mathcal{F}}2^{(1,0)} = \frac{\partial}{\partial z}\overline{G}^{(1,0)}(z,s)\Big|_{z=1,s=0} e = -[A1 + A0G]^{-1}(A0\overline{\mathcal{F}}2 + B10e) \tag{8}$$

$$\overline{\mathcal{F}}1^{(0,0)} = -\frac{\partial}{\partial s}\overline{G}^{0,0}(z,s)\Big|_{z=1,s=0} e = -B00^{-1}[B01\overline{\mathcal{F}}1^{(1,0)} + e] \tag{9}$$

$$\overline{\mathcal{F}}2^{(0,0)} = \frac{\partial}{\partial z}\overline{G}^{(0,0)}(z,s)\Big|_{z=1,s=0} e = -B00^{-1}[B01\overline{\mathcal{F}}2^{(1,0)}]. \tag{10}$$

6. Measures of System Performance

The system performance measures are listed in this section for computation as follows,

1. Probability that the Main server is Busy in the system:
$P_{MSB} = \sum_{p=1}^{\infty} \sum_{r=1}^{n} \sum_{s=1}^{m} x_{p1rs}$

2. Probability that the Main server is the breakdown:
$P_{MSBD} = \sum_{s=1}^{m} x_{02s} + \sum_{p=1}^{\infty} \sum_{s=1}^{m} x_{p2rs}$

3. Probability of the Main server is in the setup process:
$P_{MSS} = \sum_{p=1}^{\infty} \sum_{s=1}^{m} x_{p3s}$

4. Probability of the Main server is in closedown period:
$P_{MSCD} = \sum_{p=0}^{\infty} \sum_{s=1}^{m} x_{p4s}$

5. Probability of the Main server is on vacation:
$P_{MSVAC} = \sum_{p=1}^{\infty} \sum_{s=1}^{m} x_{p5s}$

6. Expected system size:
$\mu_{NS} = \sum_{p=1}^{\infty} p x_p e_{2nm+3m} = x_1 (I - R)^{-2} e_{2nm+3m}$

7. Expected Queue size during the Main server is in Busy Period:
$\mu_{QMSB} = \sum_{p=1}^{\infty} \sum_{r=1}^{n} \sum_{s=1}^{m} (p-1) x_{p1rs} e_{nm}$

8. Expected Queue size during the Main server is in the breakdown:
$\mu_{MSBD} = \sum_{p=1}^{\infty} \sum_{r=1}^{n} \sum_{s=1}^{m} (p-1) x_{p2rs} e_{nm}$

9. Expected Queue size during the Main server is in the setup process:
$\mu_{MSS} = \sum_{p=1}^{\infty} \sum_{s=1}^{m} p x_{p3s} e_m$

10. Expected Queue size during the Main server is in the closedown period:
$\mu_{MSCD} = \sum_{p=0}^{\infty} \sum_{s=1}^{m} p x_{p4s} e_m$

11. Expected Queue size during the Main server is in Vacation:
$\mu_{MSV} = \sum_{p=0}^{\infty} \sum_{s=1}^{m} p x_{p5s} e_m$

12. Average Queue size:
$\mu_{QS} = \mu_{QMSB} + \mu_{MSBD} + \mu_{MSS} + \mu_{MSCD} + \mu_{MSV}$

7. Numerical Results

In this part, we are analyzing the model behavior in the form of numerical and graphical illustrations. The following five different MAP representations has mean value is same that is, 1 for all the different arrival process. These five sets of arrival values has taken as input data in published works in the literature, see Chakravarthy (2010).

Arrival in Erlang(ERLA) :

$$D0 = \begin{bmatrix} -3 & 3 & 0 \\ 0 & -3 & 3 \\ 0 & 0 & -3 \end{bmatrix}, D1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$$

Arrival in Exponential(EXPA) :

$$D0 = \begin{bmatrix} -1 \end{bmatrix}, D1 = \begin{bmatrix} 1 \end{bmatrix}$$

Arrival in Hyperexponential(HEXA) :

$$D0 = \begin{bmatrix} -1.90 & 0 \\ 0 & -0.19 \end{bmatrix}, D1 = \begin{bmatrix} 1.710 & 0.190 \\ 0.171 & 0.019 \end{bmatrix}$$

Arrival in MAP-Negative Correlation(MNCA) :

$$D0 = \begin{bmatrix} -1.00243 & 1.00243 & 0 \\ 0 & -1.00243 & 0 \\ 0 & 0 & -225.797 \end{bmatrix}, D1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.01002 & 0 & 0.99241 \\ 223.539 & 0 & 2.258 \end{bmatrix}$$

Arrival in MAP-Positive Correlation(MPCA) :

$$D0 = \begin{bmatrix} -1.00243 & 1.00243 & 0 \\ 0 & -1.00243 & 0 \\ 0 & 0 & -225.797 \end{bmatrix}, D1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.99241 & 0 & 0.01002 \\ 2.258 & 0 & 223.539 \end{bmatrix}$$

Let us consider three phase type distributions for the service process. Normalization of these three representations has done to get service rate δ. These sets of service values has taken as input data in published works in the literature, see Chakravarthy (2010).

Service in Erlang(ERLS) :

$$\alpha = \begin{pmatrix} 1,0 \end{pmatrix}, T = \begin{bmatrix} -2 & 2 \\ 0 & -2 \end{bmatrix}$$

Service in Exponential(EXPS) :

$$\alpha = \begin{pmatrix} 1 \end{pmatrix}, T = \begin{bmatrix} -1 \end{bmatrix}$$

Service in Hyperexponential(HEXS) :

$$\alpha = \begin{pmatrix} 0.8,0.2 \end{pmatrix}, T = \begin{bmatrix} -2.80 & 0 \\ 0 & -0.28 \end{bmatrix}$$

Illustrated Example 1:

We have examined the consequence of the renege rate ζ against the expected system size in the following table. We fix $\lambda = 1$; $\theta = 0.7$; $\Psi = 3$; $\gamma = 6$; $\sigma = 8$; $\delta = 4$; $\eta = 5$; $\tau = 2$.

Table 1: Expected System size ‘

Erlang service					
ζ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
8	0.2551100830	0.3191208819	0.4237650475	0.4033760196	19.5652579340
13	0.1876854481	0.2403341600	0.3314934478	0.3076333066	19.3768425434
18	0.1485579728	0.1928559259	0.2726619048	0.2483769578	19.2023395965
23	0.1229681989	0.1610892244	0.2317527947	0.2081541358	19.0357254576
28	0.1049145893	0.1383294179	0.2016045028	0.1790782187	18.8745536749
33	0.0914907791	0.1212157553	0.1784409219	0.1570846970	18.7176231039
38	0.0811162436	0.1078763507	0.1600754865	0.1398694196	18.5642607563
43	0.0728570202	0.0971852548	0.1451515493	0.1260294489	18.4140497046
48	0.0661255200	0.0884242695	0.1327815261	0.1146619073	18.2667099490

Table 2: Expected System size

Exponential service					
ζ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
8	0.2660071394	0.3337868611	0.4545621888	0.4165792561	19.5853519487
13	0.1959931546	0.2514044546	0.3554004824	0.3180855132	19.3960615220
18	0.1552672619	0.2017442209	0.2921818413	0.2570158696	19.2210068115
23	0.1285938228	0.1685137353	0.2482416632	0.2155124581	19.0539913023
28	0.1097574927	0.1447040698	0.2158754166	0.1854852167	18.8925006470
33	0.0957419592	0.1268006869	0.1910190187	0.1627575449	18.7353016465
38	0.0849044456	0.1128456776	0.1713193512	0.1449587238	18.5817042082
43	0.0762731206	0.1016612272	0.1553168399	0.1306438676	18.4312814390
48	0.0692360437	0.0924960096	0.1420568962	0.1188823783	18.2837471323

Table 3: Expected System size

Hyperexponential service					
ζ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
8	0.3475067705	0.4289873057	0.6229687643	0.5140562334	19.7220351977
13	0.2572426019	0.3236600000	0.4859931583	0.3946837704	19.5250809913
18	0.2043399957	0.2599631914	0.3987735746	0.3200301148	19.3452627410
23	0.1695337242	0.2172643308	0.3382751167	0.2690145569	19.1748608531
28	0.1448792919	0.1866366331	0.2938010054	0.2319627085	19.0107486316
33	0.1264946416	0.1635891620	0.2597065716	0.2038376098	18.8513991043
38	0.1122553475	0.1456147747	0.2327267733	0.1817628664	18.6959635987
43	0.1009001983	0.1312030431	0.2108393303	0.1639773995	18.5439220356
48	0.0916328750	0.1193894349	0.1927236849	0.1493429129	18.3949298110

From the above tables 1, 2 and 3, we conclude that while increasing the reneging rate, the expected system size decreases in case of the variety of arrangements of services and arrivals. Nevertheless, ERLA slowly decreases than the EXPA, HEXA rapidly. Among these, MPCA decreases much faster than the other arrivals.

Illustrated Example 2:

We have examined the consequence of the service rate δ of main server against the expected system size in the following table. We fix $\lambda = 1$; $\Psi = 5$; $\sigma = 8$; $\gamma = 6$; ζ=9; $\theta = 0.7$; $\eta = 5$; $\tau = 2$.

Table 4: Expected System size

Erlang service					
δ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
4	0.2379764747	0.2994591234	0.4013151438	0.3798100776	19.5258931834
5	0.2065039525	0.2487301273	0.3063421739	0.3207713481	14.4614145429
6	0.1880284636	0.2199680438	0.2582097489	0.2861007651	11.4819360299
7	0.1758546173	0.2015441054	0.2295481094	0.2632472972	9.5197772389
8	0.1672205738	0.1887802063	0.2106828814	0.2470346455	8.1299142452
9	0.1607759003	0.1794400127	0.1973928640	0.2349313955	7.0938468136
10	0.1557808065	0.1723229710	0.1875600607	0.2255502697	6.2917434540
11	0.1517955387	0.1667282839	0.1800107682	0.2180662304	5.6523974928
12	0.1485420686	0.1622200781	0.1740444599	0.2119573947	5.1308356413

Table 5: Expected System size

Exponential service					
δ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
4	0.2482351638	0.3132315037	0.4304393361	0.3923562659	19.5457697200
5	0.2120552776	0.2567327335	0.3227377870	0.3276776653	14.4728125522
6	0.1914637163	0.2252509825	0.2687014614	0.2904550358	11.4893149129
7	0.1781755609	0.2053235695	0.2368546405	0.2662474615	9.5249442035
8	0.1688892137	0.1916365013	0.2160806861	0.2492348851	8.1337368759
9	0.1620323328	0.1816856929	0.2015562395	0.2366207794	7.0967923028
10	0.1567611666	0.1741418179	0.1908778569	0.2268934813	6.2940852915
11	0.1525823509	0.1682358002	0.1827228795	0.2191636915	5.6543062914
12	0.1491880874	0.1634927441	0.1763070028	0.2128737519	5.1324233062

Table 6: Expected System size

Hyperexponential service					
δ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
4	0.3246765587	0.4027499695	0.5896485286	0.4848163361	19.6805443062
5	0.2546633015	0.3080706837	0.4156634604	0.3790445701	14.5503737715
6	0.2183704443	0.2586826177	0.3294321844	0.3228206429	11.5395557565
7	0.1965817028	0.2289316791	0.2796591165	0.2883845215	9.5600778968
8	0.1822049566	0.2092691192	0.2479171397	0.2652822023	8.1596645385
9	0.1720738738	0.1954065551	0.2262029037	0.2487705926	7.1167066591
10	0.1645808739	0.1851571755	0.2105570033	0.2364084960	6.3098612364
11	0.1588301261	0.1772982721	0.1988249493	0.2268191719	5.6671156423
12	0.1542858532	0.1710966489	0.1897458792	0.2191703336	5.1430353189

From the above tables 4, 5 and 6, we conclude that while increasing the main server service rate, the expected system size decreases in case of the variety of arrangements of services and arrivals. Eventhough, ERLA and EXPA decreases slowly, HEXA and MNCA decreases gradually and the MPCA decreases rapidly than compared to the other arrivals.

Illustrated Example 3:

We have examined the consequence of the repair rate Ψ against the expected system size in the following table. We fix $\lambda = 1$; $\theta = 0.7$; $\sigma = 8$; $\gamma = 6$; ζ=9; $\delta = 4$; $\eta = 5$; $\tau = 2$.

Table 7: Expected System size

Erlang service					
Ψ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
4	0.2351597114	0.2946564762	0.3905212078	0.3743593312	18.9274045812
6	0.2319256796	0.2890301295	0.3781560286	0.3679926754	18.2284868245
8	0.2300699236	0.2857783885	0.3712174710	0.3643031902	17.8330653966
10	0.2288467240	0.2836400981	0.3667572229	0.3618672995	17.5787028633
12	0.2279733379	0.2821209855	0.3636427225	0.3601306660	17.4013390100
14	0.2273159732	0.2809839024	0.3613424418	0.3588270709	17.2706052797
16	0.2268022053	0.2800998845	0.3595729794	0.3578113063	17.1702481848
18	0.2263890870	0.2793924588	0.3581691038	0.3569969784	17.0907820211
20	0.2260494077	0.2788132852	0.3570278602	0.3563293036	17.0262983354

Table 8: Expected System size

Exponential service					
Ψ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
4	0.2447096506	0.3075250014	0.4177427601	0.3860232879	18.9461545159
6	0.2406602054	0.3008823356	0.4032632476	0.3786534491	18.2459354054
8	0.2383541519	0.2970792602	0.3951810279	0.3744184322	17.8497897169
10	0.2368484292	0.2945978474	0.3900064894	0.3716437228	17.5949679582
12	0.2357825582	0.2928457732	0.3864044486	0.3696776380	17.4172877193
14	0.2349862035	0.2915406696	0.3837506133	0.3682089168	17.2863230878
16	0.2343676345	0.2905299637	0.3817132155	0.3670688570	17.1857901956
18	0.2338728056	0.2897237091	0.3800993764	0.3661576879	17.1061857821
20	0.2334677012	0.2890653405	0.3787892125	0.3654124806	17.0415905583

Table 9: Expected System size

Hyperexponential service					
Ψ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
4	0.3161898145	0.3913958479	0.5673779642	0.4725395327	19.0733802339
6	0.3066305618	0.3784862880	0.5422524487	0.4585297312	18.3645535023
8	0.3013461895	0.3712989978	0.5283974514	0.4506946122	17.9636497210
10	0.2979805242	0.3667051229	0.5196056510	0.4456686565	17.7058140995
12	0.2956447368	0.3635109574	0.5135264742	0.4421640141	17.5260552766
14	0.2939269314	0.3611595202	0.5090707869	0.4395779898	17.3935710289
16	0.2926095607	0.3593553104	0.5056641815	0.4375899624	17.2918789671
18	0.2915667102	0.3579267684	0.5029748399	0.4360133363	17.2113612827
20	0.2907203721	0.3567673791	0.5007976263	0.4347320116	17.1460279908

From the above tables 7, 8 and 9, we conclude that while increasing the repair rate, the expected system size decreases in case of the variety of arrangements of services and arrivals. Though, ERLA and EXPA decreases slowly, HEXA and MNCA decreases than ERLA, EXPA and the MPCA decreases fastly than compared to the other arrivals.

Illustrated Example 4:

We have examined the consequence of the vacation rate η against the expected system size in the following table. We fix $\lambda = 1$; $\Psi = 3$; $\sigma = 8$; $\theta = 0.7$; $\zeta=9$; $\delta = 4$; $\gamma = 6$; $\tau = 2$.

Table 10: Expected System size

Erlang service					
η	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
3	0.2048797964	0.2580248808	0.3496636085	0.3295547779	19.3974064241
5	0.2379764747	0.2994591234	0.4013151438	0.3798100776	19.5258931834
7	0.2609374022	0.3264662539	0.4321131261	0.4126041589	19.5849169838
9	0.2778495302	0.3454994764	0.4525827108	0.4356964425	19.6195291979
11	0.2908481194	0.3596511214	0.4671791623	0.4528335256	19.6425405482
13	0.3011626554	0.3705930064	0.4781152395	0.4660511754	19.6590576277
15	0.3095532427	0.3793097174	0.4866153454	0.4765521438	19.6715425632
17	0.3165160745	0.3864195048	0.4934122466	0.4850930542	19.6813385223
19	0.3223893119	0.3923304733	0.4989714475	0.4921737642	19.6892448290

Table 11: Expected System size

Exponential service					
η	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
3	0.2125347615	0.2684064944	0.3724672735	0.3392800892	19.4162589083
5	0.2482351638	0.3132315037	0.4304393361	0.3923562659	19.5457697200
7	0.2729429633	0.3424982484	0.4652043037	0.4269134345	19.6054778723
9	0.2911060532	0.3631501692	0.4884068653	0.4512048870	19.6405877132
11	0.3050432170	0.3785206714	0.5050054121	0.4692067998	19.6639788918
13	0.3160871004	0.3904145455	0.5174736353	0.4830756755	19.6807958537
15	0.3250602101	0.3998959660	0.5271852411	0.4940835793	19.6935236950
17	0.3324986711	0.4076338334	0.5349648003	0.5030296401	19.7035204476
19	0.3387673636	0.4140700670	0.5413374657	0.5104411633	19.7115955170

Table 12: Expected System size

Hyperexponential service					
η	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
3	0.2689766678	0.3360384794	0.4968181848	0.4105364262	19.5428638563
5	0.3246765587	0.4027499695	0.5896485286	0.4848163361	19.6805443062
7	0.3631292339	0.4465468202	0.6463852161	0.5328031754	19.7458368472
9	0.3913353241	0.4775803982	0.6847672480	0.5663216126	19.7850697502
11	0.4129378486	0.5007531426	0.7125078950	0.5910302936	19.8116452740
13	0.4300271643	0.5187319646	0.7335154646	0.6099806636	19.8310007263
15	0.4438915101	0.5330955374	0.7499870410	0.6249636134	19.8458016026
17	0.4553693955	0.5448394330	0.7632546739	0.6370991280	19.8575241033
19	0.4650307329	0.5546232529	0.7741738593	0.6471232193	19.8670586028

From the above tables 10, 11 and 12, we conclude that when we are increasing the vacation rate then the expected system size is also increases in the variety of arrangements of arrivals and services. Nonetheless, ERLA and EXPA increases slowly, HEXA and MNCA increases rapidly but in the case of MPCA increases gradually than compared to the other arrivals.

Illustrated Example 5:

We have examined the consequence of the setup rate σ against the expected system size in the following table. We fix $\lambda = 1$; $\theta = 0.7$; $\Psi = 3$; $\gamma = 6$; $\zeta=9$; $\delta = 4$; $\eta = 5$; $\tau = 2$.

Table 13: Expected System size

Erlang service					
σ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
7	0.2459777032	0.3089365521	0.4130365944	0.3898273658	19.5394272035
11	0.2230858302	0.2816829834	0.3792769140	0.3609173026	19.5001558027
15	0.2128018626	0.2693028343	0.3638927871	0.3476665782	19.4819360808
19	0.2069616016	0.2622371767	0.3551019328	0.3400646246	19.4714200228
23	0.2031965713	0.2576700983	0.3494163091	0.3351341033	19.4645746841
27	0.2005674189	0.2544758358	0.3454383758	0.3316773097	19.4597637942
31	0.1986274089	0.2521164492	0.3424995291	0.3291194092	19.4561977552
35	0.1971369360	0.2503025272	0.3402398004	0.3271501234	19.4534487560
39	0.1959559466	0.2488645462	0.3384482366	0.3255872405	19.4512648638

Table 14: Expected System size

Exponential service					
σ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
7	0.2563481281	0.3228302711	0.4424163840	0.4024462732	19.5593192999
11	0.2331297801	0.2952183454	0.4078875333	0.3733239025	19.5200048733
15	0.2226919076	0.2826652387	0.3921160745	0.3599734036	19.5017673293
19	0.2167619277	0.2754977271	0.3830919110	0.3523134799	19.4912413486
23	0.2129380461	0.2708635304	0.3772503625	0.3473450301	19.4843894587
27	0.2102672745	0.2676217056	0.3731608641	0.3438614526	19.4795737454
31	0.2082962544	0.2652268528	0.3701382388	0.3412836023	19.4760038850
35	0.2067817721	0.2633854671	0.3678132866	0.3392988602	19.4732517000
39	0.2055816379	0.2619255910	0.3659695047	0.3377236335	19.4710650544

Table 15: Expected System size

Hyperexponential service					
σ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
7	0.3333551730	0.4130708338	0.6030453013	0.4952896493	19.6941862430
11	0.3085394278	0.3833293074	0.5642591570	0.4650633254	19.6546155835
15	0.2974150057	0.3697515263	0.5463606703	0.4512114996	19.6362724604
19	0.2911082897	0.3619815009	0.5360605806	0.4432663296	19.6256896147
23	0.2870474039	0.3569506282	0.5293684884	0.4381139280	19.6188021143
27	0.2842140195	0.3534278749	0.5246714808	0.4345018467	19.6139618281
31	0.2821245619	0.3508236306	0.5211932306	0.4318290984	19.6103738246
35	0.2805199810	0.3488201520	0.5185139054	0.4297713517	19.6076075622
39	0.2792490027	0.3472310819	0.5163866252	0.4281381748	19.6054095731

From the above tables 13, 14 and 15, we examined that when we are increasing the setup rate, the expected system size decreases in the variety of arrangements of services and arrivals. Though, ERLA and EXPA decreases slowly, HEXA and MNCA decreases than ERLA, EXPA but in the case of MPCA decreases gradually than compared to the other arrivals.

Illustrated Example 6:

We have examined the consequence of the breakdown rate τ against the expected system size in the following table. We fix $\lambda = 1$; $\theta = 0.7$; $\Psi = 3$; $\gamma = 6$; $\zeta=9$; $\delta = 4$; $\eta = 5$; $\sigma = 8$.

Table 16: Expected System size

Erlang service					
τ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
1.2	0.2325503701	0.2902695261	0.3821097387	0.3693639979	18.5182290797
1.4	0.2339881967	0.2927164860	0.3872259225	0.3721512928	18.7936229079
1.6	0.2353697421	0.2950595706	0.3921235659	0.3748163882	19.0524217592
1.8	0.2366981857	0.2973051588	0.3968158982	0.3773669628	19.2960816647
2.0	0.2379764747	0.2994591234	0.4013151438	0.3798100776	19.5258931834
2.2	0.2392073439	0.3015268799	0.4056326106	0.3821522351	19.7430042545
2.4	0.2403933354	0.3035134296	0.4097787711	0.3843994331	19.9484393654
2.6	0.2415368142	0.3054233983	0.4137633345	0.3865572118	20.1431157099
2.8	0.2426399843	0.3072610704	0.4175953123	0.3886306959	20.3278568743

Table 17: Expected System size

Exponential service					
τ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
1.2	0.2416670895	0.3026331558	0.4081984191	0.3805408844	18.5362367193
1.4	0.2434137950	0.3054613010	0.4141292614	0.3837002241	18.8121367719
1.6	0.2450878289	0.3081652249	0.4198026751	0.3867164611	19.0714145056
1.8	0.2466935983	0.3107528650	0.4252344938	0.3895989516	19.3155280116
2.0	0.2482351638	0.3132315037	0.4304393361	0.3923562659	19.5457697200
2.2	0.2497162727	0.3156078338	0.4354307133	0.3949962680	19.7632892724
2.4	0.2511403881	0.3178880161	0.4402211261	0.3975261857	19.9691127063
2.6	0.2525107158	0.3200777309	0.4448221522	0.3999526729	20.1641586293
2.8	0.2538302269	0.3221822235	0.4492445268	0.4022818646	20.3492519179

Table 18: Expected System size

Hyperexponential service					
τ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
1.2	0.3102292500	0.3832114374	0.5520675036	0.4635680590	18.6585235243
1.4	0.3140934994	0.3884433265	0.5621199692	0.4692715155	18.9377955214
1.6	0.3177817606	0.3934329312	0.5717146896	0.4747016151	19.2002701798
1.8	0.3213058493	0.3981967858	0.5808816904	0.4798774333	19.4474184089
2.0	0.3246765587	0.4027499695	0.5896485286	0.4848163361	19.6805443062
2.2	0.3279037649	0.4071062588	0.5980405239	0.4895341627	19.9008081433
2.4	0.3309965207	0.4112782628	0.6060809685	0.4940453865	20.1092456548
2.6	0.3339631373	0.4152775420	0.6137913152	0.4983632567	20.3067843045
2.8	0.3368112576	0.4191147133	0.6211913473	0.5024999226	20.4942570689

From the above tables 16 ,17 and 18, we conclude that maximizing the breakdown rate then the expected system size is also maximizes in different arrangements of services and arrivals of ERLA, EXPA, HEXA, MNCA and MPCA. Nevertheless, Erlang arrival and exponential arrival increases slowly, hyperexponential arrival and negative correlation arrival increases rapidly but in the case of positive correlation arrival increases gradually than compared to the other arrivals.

Illustrated Example 7:

We have examined the consequence of the standby server service rate $\theta\delta$ against the expected system size in the following table. We fix $\lambda = 1$; $\Psi = 3$; $\sigma = 8$; $\gamma = 6$; ζ=9; $\delta = 4$; $\eta = 5$; $\tau = 2$.

Table 19: Expected System size

Erlang service					
$\theta\delta$	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
2.2	0.2493345337	0.3189650962	0.4442998622	0.4016843196	21.5840044974
2.4	0.2451019008	0.3117401217	0.4282485811	0.3936201806	20.8508674817
2.6	0.2413398016	0.3052737730	0.4140070993	0.3863659406	20.1663913544
2.8	0.2379764747	0.2994591234	0.4013151438	0.3798100776	19.5258931834
3.0	0.2349534663	0.2942073951	0.3899556725	0.3738598100	18.9252717370
3.2	0.2322228221	0.2894444617	0.3797470180	0.3684374863	18.3609199326
3.4	0.2297449241	0.2851080872	0.3705365387	0.3634777414	17.8296526306
3.6	0.2274868176	0.2811457369	0.3621954802	0.3589252447	17.3286467252
3.8	0.2254209087	0.2775128340	0.3546148072	0.3547329083	16.8553911568

Table 20: Expected System size

Exponential service					
$\theta\delta$	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
2.2	0.2620613084	0.3355792516	0.4791805431	0.4171768202	21.6079764679
2.4	0.2569279619	0.3273268675	0.4610597903	0.4080428441	20.8733500039
2.6	0.2523461596	0.3199161494	0.4449036704	0.3998098925	20.1875135419
2.8	0.2482351638	0.3132315037	0.4304393361	0.3923562659	19.5457697200
3.0	0.2445286090	0.3071765721	0.4174381012	0.3855803680	18.9440047937
3.2	0.2411715731	0.3016705653	0.4057075188	0.3793968794	18.3786007973
3.4	0.2381183070	0.2966453703	0.3950849869	0.3737337400	17.8463630966
3.6	0.2353304623	0.2920432536	0.3854325696	0.3685297555	17.3444602783
3.8	0.2327757010	0.2878150282	0.3766327917	0.3637326873	16.8703739928

Table 21: Expected System size

Hyperexponential service					
$\theta\delta$	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
2.2	0.3547810935	0.4431870322	0.6674822570	0.5295597332	21.7696904127
2.4	0.3437154609	0.4283603500	0.6389033334	0.5131722013	21.0252665947
2.6	0.3337276417	0.4149413304	0.6130742909	0.4983227530	20.3304822178
2.8	0.3246765587	0.4027499695	0.5896485286	0.4848163361	19.6805443062
3.0	0.3164438165	0.3916345369	0.5683321841	0.4724883079	19.0712565623
3.2	0.3089294302	0.3814663829	0.5488748637	0.4611988887	18.4989294453
3.4	0.3020484686	0.3721358261	0.5310622021	0.4508287634	17.9603060193
3.6	0.2957283944	0.3635488745	0.5147098562	0.4412755675	17.4525004402
3.8	0.2899069400	0.3556245899	0.4996586252	0.4324510585	16.9729466378

From the above tables 19, 20, and 21, we conclude that increasing the standby service rate, the expected system size decreases in case of the variety of arrangements of services and arrivals. Eventhough, ERLA and EXPA decreses slowly, HEXA and MNCA decreases their values than ERLA and the MPCA decreases more rapidly than compared to the other arrivals.

Illustrated Example 8:

We have examined the consequence of the closedown rate γ against the expected system size in the following table. We fix $\lambda = 1$; $\Psi = 3$; $\sigma = 8$; $\theta = 0.7$; $\zeta=9$; $\delta = 4$; $\eta = 5$; $\tau = 2$.

Table 22: Expected System size

Erlang service					
γ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
6	0.2379764747	0.2994591234	0.4013151438	0.3798100776	19.5258931834
9	0.2395334980	0.3014656569	0.4047552133	0.3816022397	19.5279632231
12	0.2402164837	0.3023627617	0.4062973564	0.3823623790	19.5286980831
15	0.2405789359	0.3028461514	0.4071288880	0.3827566641	19.5290420827
18	0.2407949971	0.3031378197	0.4076307286	0.3829876942	19.5292306987
21	0.2409343973	0.3033278535	0.4079577337	0.3831347288	19.5293453188
24	0.2410296700	0.3034587825	0.4081830537	0.3832340971	19.5294202030
27	0.2410977093	0.3035529176	0.4083450702	0.3833043942	19.5294718322
30	0.2411480148	0.3036229156	0.4084655580	0.3833559509	19.5295089408

Table 23: Expected System size

Exponential service					
γ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
6	0.2482351638	0.3132315037	0.4304393361	0.3923562659	19.5457697200
9	0.2498484226	0.3154037298	0.4343315782	0.3941987613	19.5478091696
12	0.2505494261	0.3163754300	0.4360797056	0.3949768093	19.5485318940
15	0.2509192056	0.3168992191	0.4370235337	0.3953792754	19.5488698925
18	0.2511387310	0.3172153589	0.4375937005	0.3956146569	19.5490551173
21	0.2512799490	0.3174213865	0.4379655094	0.3957642577	19.5491676409
24	0.2513762514	0.3175633626	0.4382218574	0.3958652568	19.5492411422
27	0.2514449089	0.3176654569	0.4384062772	0.3959366509	19.5492918133
30	0.2514956029	0.3177413838	0.4385434836	0.3959889793	19.5493282322

Table 24: Expected System size

Hyperexponential service					
γ	**ERLA**	**EXPA**	**HEXA**	**MNCA**	**MPCA**
6	0.3246765587	0.4027499695	0.5896485286	0.4848163361	19.6805443062
9	0.3269565720	0.4059890577	0.5960709245	0.4871727892	19.6825830695
12	0.3279220141	0.4074410391	0.5989721822	0.4881523092	19.6833068224
15	0.3284224851	0.4082249413	0.6005447623	0.4886538794	19.6836459575
18	0.3287159437	0.4086986463	0.6014975096	0.4889451390	19.6838321413
21	0.3289030001	0.4090076556	0.6021201914	0.4891292753	19.6839454308
24	0.3290296693	0.4092207649	0.6025502744	0.4892530810	19.6840195391
27	0.3291194782	0.4093741107	0.6028601346	0.4893403108	19.6840706941
30	0.3291854945	0.4094882160	0.6030909498	0.4894040754	19.6841075030

From the above tables 22, 23 and 24, we conclude that increasing the closedown rate then the expected system size is also increases in the variety of arrangements of services and arrivals. Eventhough, ERLA and EXPA increases slowly, HEXA and MNCA increases their values than EXPA but in the case of MPCA increases gradually than compared to the other arrivals.

Illustrated Example 9:

We fix $\lambda = 1$; $\Psi = 3$; $\theta = 0.7$; $\zeta = 9$; $\eta = 5$; $\tau = 2$; $\gamma = 6$.

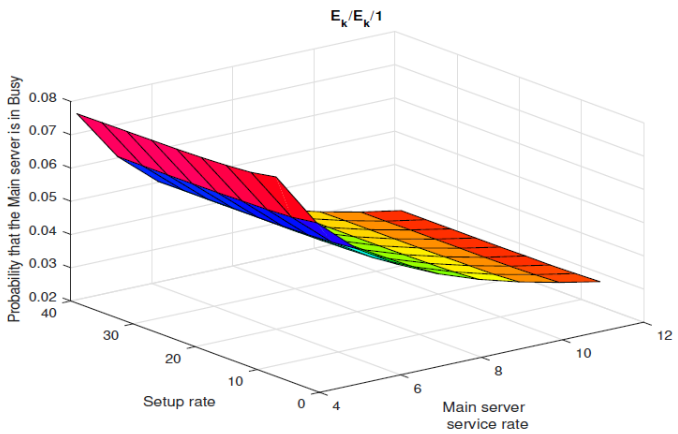

Figure 2: The graph of $E_k/E_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

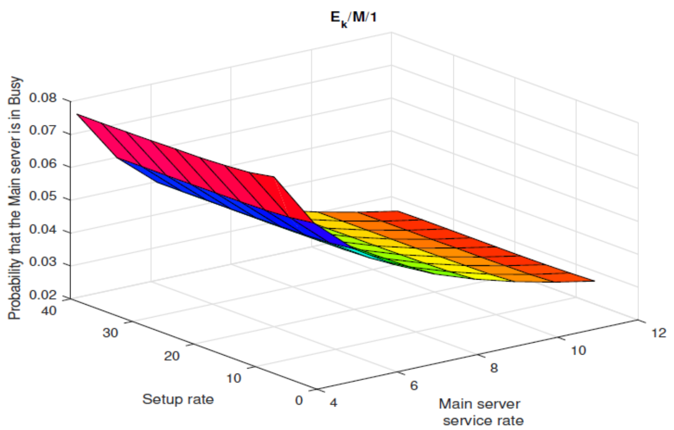

Figure 3: The graph of $E_k/M/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

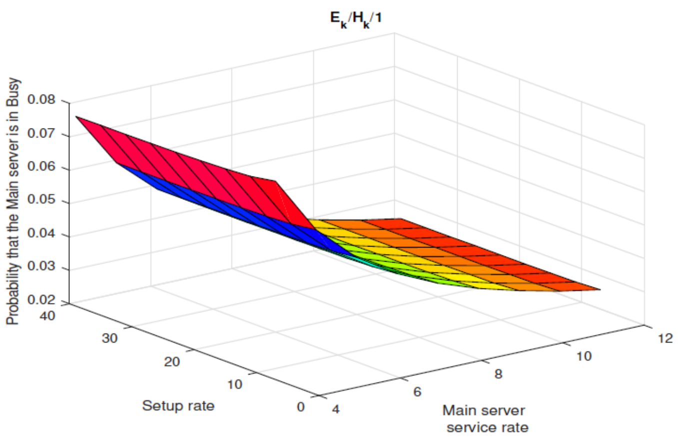

Figure 4: The graph of $E_k/H_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

We observe from figures 2, 3 and 4 that the impact of setup rate and main server service rate on the probability of the main server service is in the busy mode. We have examined the probability of the main server is in the busy mode decreases while we are increasing both the setup rate and main server service rate for the arrangement of Erlang arrival with ERLS, EXPS and HEXS. However, the Erlang arrival decreases slowly in hyperexponential service.

We fix $\lambda = 1$; $\Psi = 3$; $\theta = 0.7$; $\zeta = 9$; $\eta = 5$; $\tau = 2$; $\gamma = 6$.

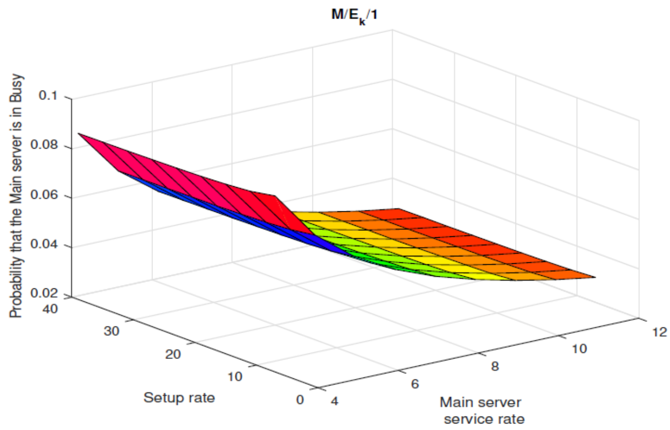

Figure 5: The graph of $M/E_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

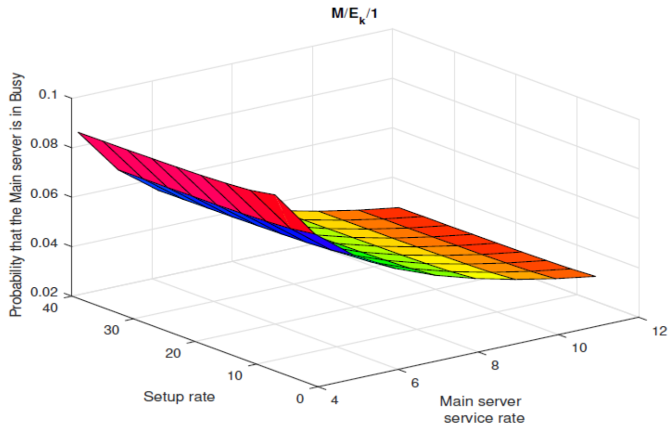

Figure 6: The graph of $M/M/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

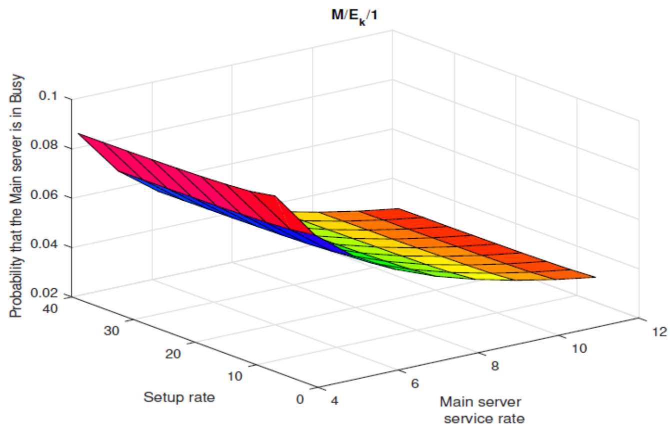

Figure 7: The graph of $M/H_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

We observe from figures 5, 6 and 7 that the impact of setup rate and main server service rate on the probability of the main server service is in the busy mode. We have examined the probability of the main server is in the busy mode decreases while we are increasing both the setup rate and main server service rate for the arrangement of exponential arrival with ERLS, EXPS and HEXS. Nevertheless, the Erlang service times decreases than the hyperexponential service times.

We fix $\lambda = 1$; $\Psi = 3$; $\theta = 0.7$; $\zeta = 9$; $\eta = 5$; $\tau = 2$; $\gamma = 6$.

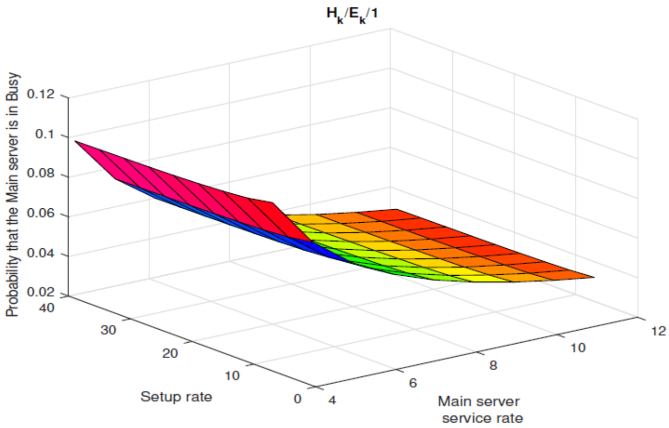

Figure 8: The graph of $H_k/E_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

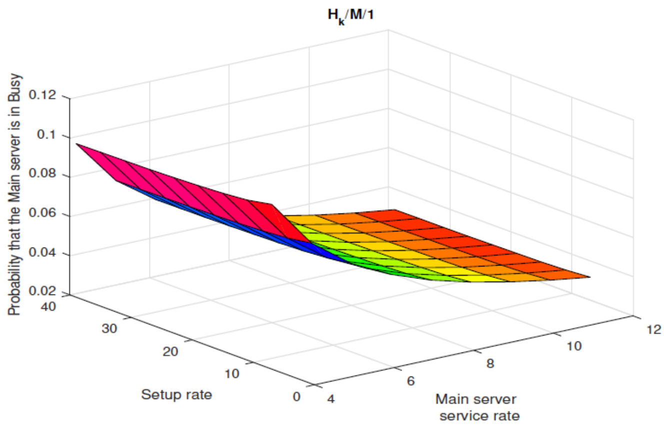

Figure 9: The graph of $H_k/M/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

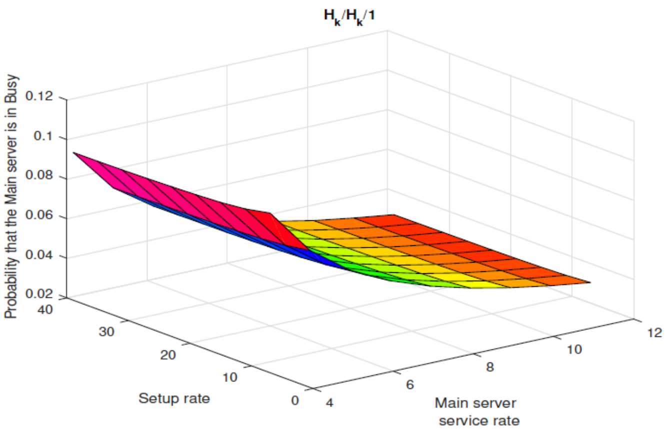

Figure 10: The graph of $H_k/H_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

We observe from figures 8, 9 and 10 that the consequence of setup rate and main server service rate on the probability of the main server is in the busy mode. We have examined the probability of the main server is in the busy mode decreases while we are increasing both the setup rate and main server service rate for the arrangement of hyperexponential arrival with ERLS, EXPS and HEXS. Meanwhile, the hyperexponential arrival decreases slowly in hyperexponential service times.

We fix $\lambda = 1$; $\Psi = 3$; $\theta = 0.7$; $\zeta = 9$; $\eta = 5$; $\tau = 2$; $\gamma = 6$.

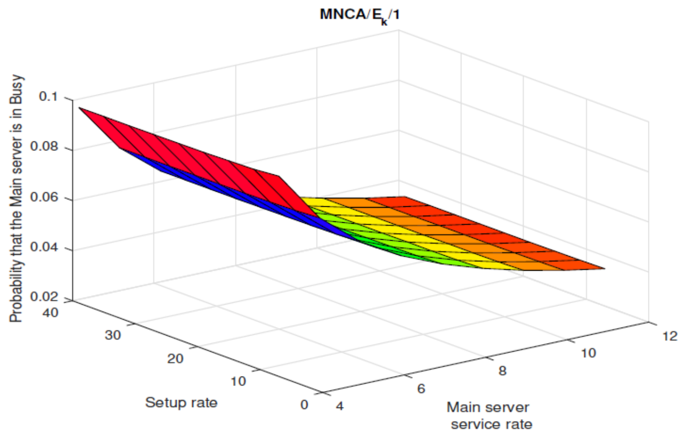

Figure 11: The graph of the $MAP - NC/E_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

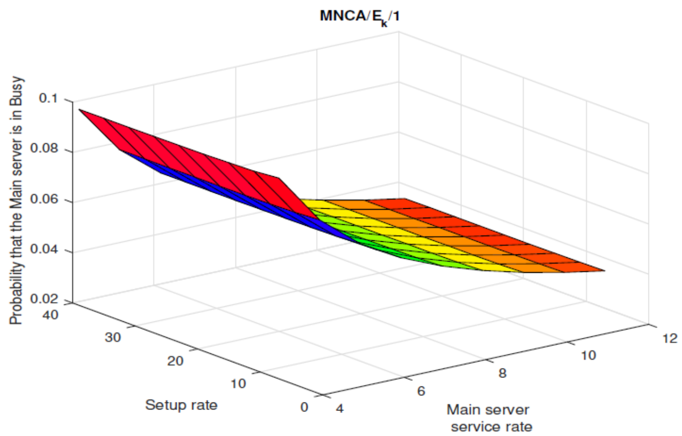

Figure 12: The graph of the $MAP - NC/M/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

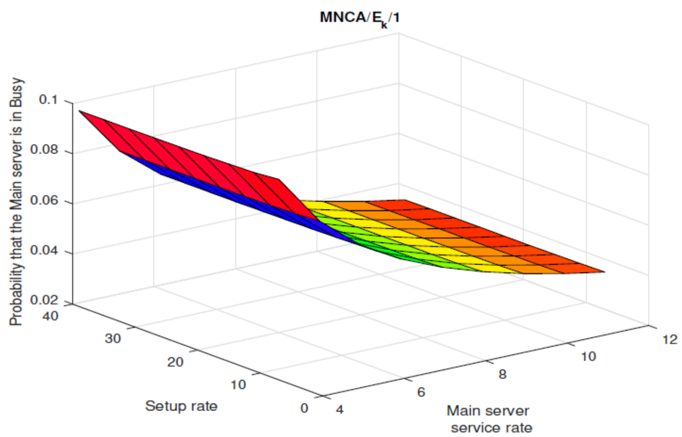

Figure 13: The graph of the $MAP - NC/H_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

We observe from figures 11, 12 and 13 that the impact of setup rate and main server service rate on the probability of the main server service is in the busy mode. We have examined the probability of the main server is in the busy mode decreases while we are increasing both the setup rate and main server service rate for the arrangement of MAP-Negative correlation arrival (MNCA) with ERLS, EXPS and HEXS. Nevertheless, MAP-Negative correlation arrival decreases slowly in HEXS than ERLS service times.

We fix $\lambda = 1$; $\Psi = 3$; $\theta = 0.7$; $\zeta = 9$; $\eta = 5$; $\tau = 2$; $\gamma = 6$.

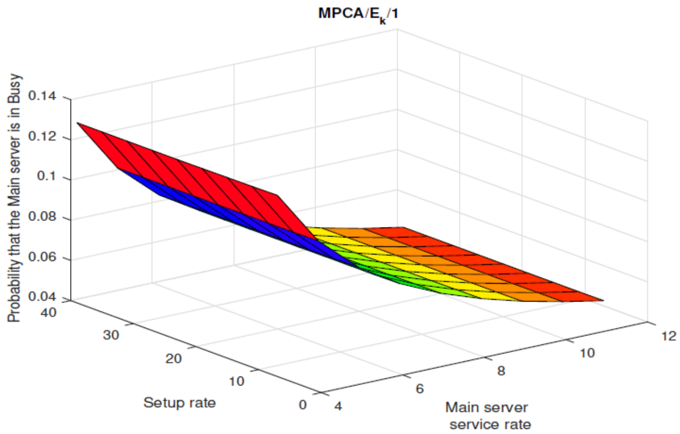

Figure 14: The graph of the $MAP - PC/E_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

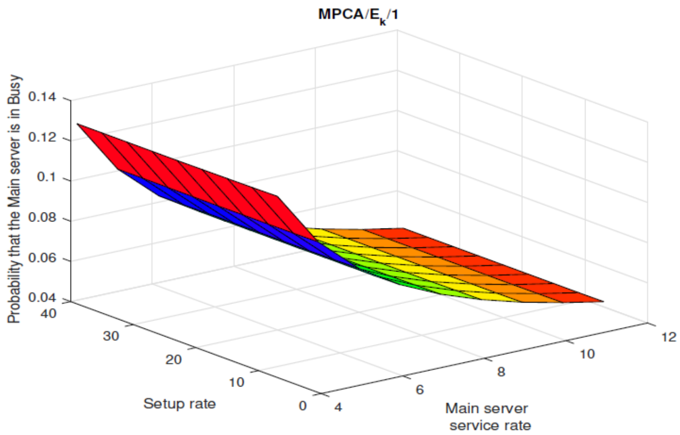

Figure 15: The graph of the $MAP - PC/M/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

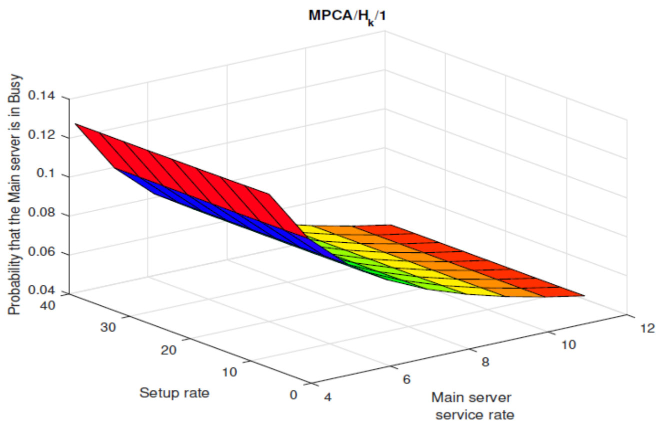

Figure 16: The graph of the $MAP - PC/H_k/1$ - setup rate(σ) and main server service rate(δ) versus probability that the main server is in busy

We observe from figures 14, 15 and 16 that the impact of setup rate and main server service rate on the probability of the main server service is in the busy mode. We have examined the probability of the main server is in the busy mode decreases while we are increasing both the setup rate and main server service rate for the arrangement of MAP-Positive correlation arrival(MPCA) with ERLS, EXPS and HEXS. Nevertheless, MAP-Positive correlation arrival decreases slowly in the hyperexponential service times.

Illustrated Example 10:

We fix $\lambda = 1$; $\delta = 4$; $\Psi = 3$; $\theta = 0.7$; $\eta = 5$; $\tau = 2$; $\sigma = 8$.

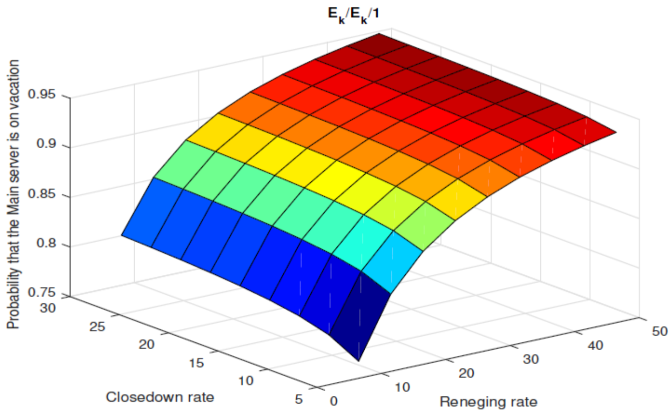

Figure 17: The graph of the $E_k/E_k/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

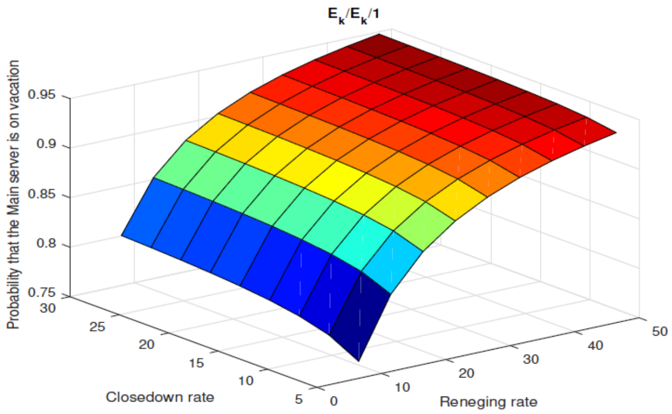

Figure 18: The graph of the $E_k/M/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

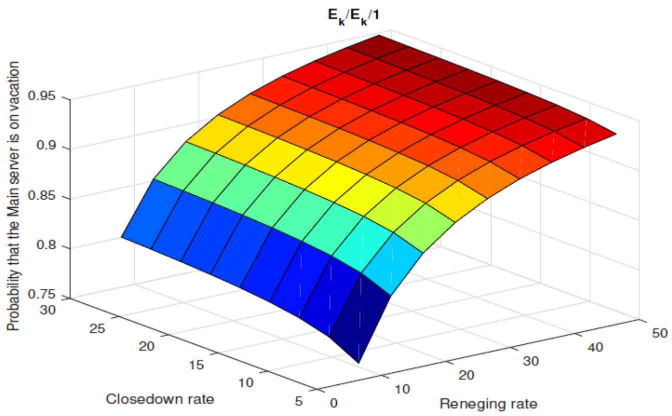

Figure 19: The graph of the $E_k/H_k/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

We observe from figures 17,18 and 19 that the impact of closedown rate and reneging rate on the probability of the main server service is on vacation. We have examined the probability of the main server is in the vacation increases while we are increasing both the closedown rate and reneging rate for the arrangement of Erlang arrival with ERLS, EXPS and HEXS. However, the Erlang arrival increases fastly in hyperexponential service times.

We fix $\lambda = 1$; $\delta = 4$; $\Psi = 3$; $\theta = 0.7$; $\eta = 5$; $\tau = 2$; $\sigma = 8$.

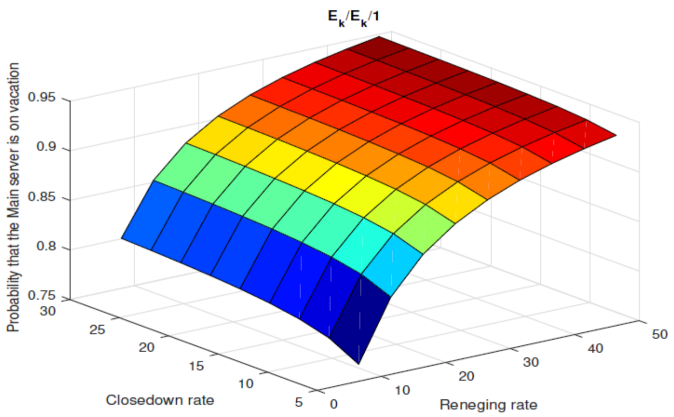

Figure 20: The graph of the $M/E_k/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

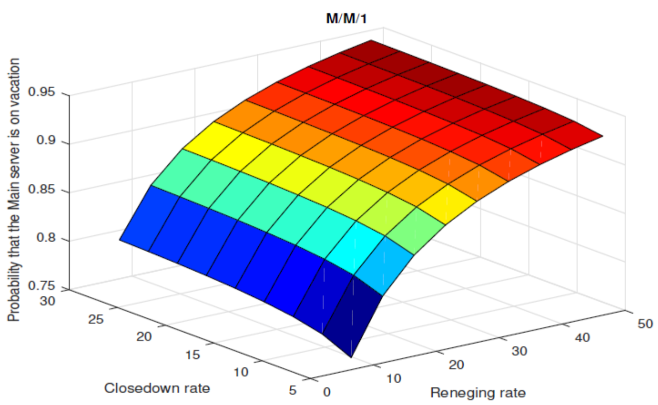

Figure 21: The graph of the $M/M/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

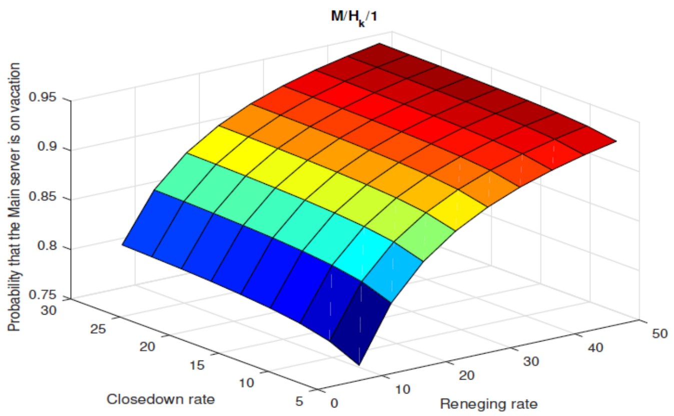

Figure 22: The graph of the $M/H_k/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

We observe from the figures 20, 21 and 22 that it shows the consequence of closedown rate and reneging rate on the probability of the main server service is on vacation. We have examined that the probability of the main server is in the vacation increases while we are increasing both the closedown rate and reneging rate for the arrangement of exponential arrival with ERLS, EXPS and HEXS. Therefore, the exponential arrival highly increases in hyperexponential service times.

We fix $\lambda = 1$; $\delta = 4$; $\Psi = 3$; $\theta = 0.7$; $\eta = 5$; $\tau = 2$; $\sigma = 8$.

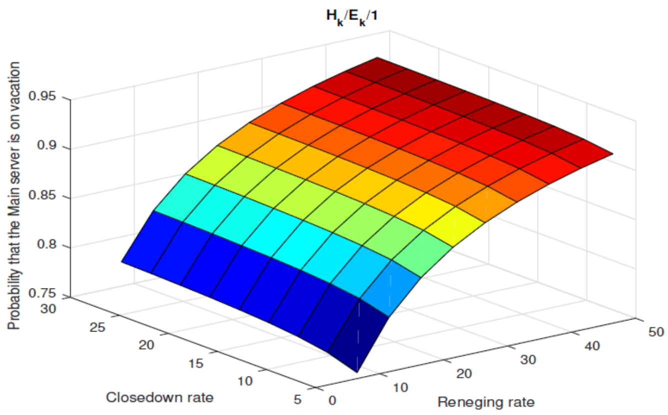

Figure 23: The graph of the $H_k/E_k/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

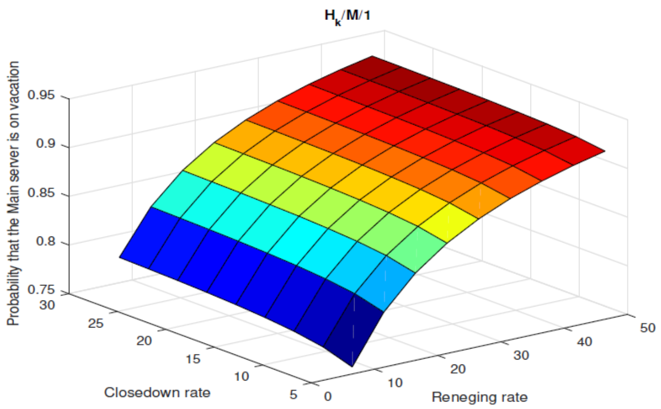

Figure 24: The graph of the $H_k/M/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

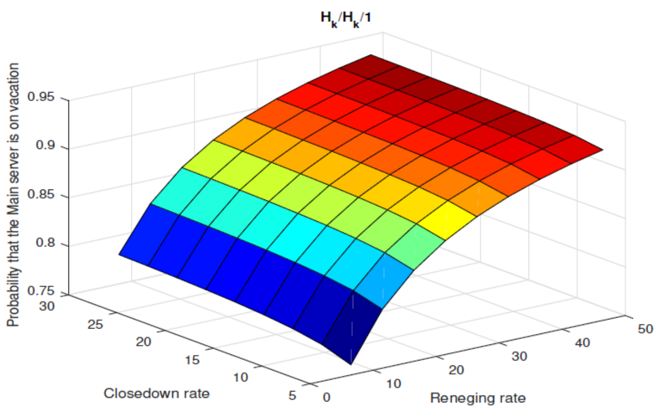

Figure 25: The graph of the $H_k/H_k/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

We observe from figures 23, 24 and 25 that it shows the consequence of closedown rate and reneging rate on the probability of the main server is on vacation. We have examined the probability of the main server is in the vacation increases while we are increasing both the closedown rate and reneging rate for the arrangement of hyperexponential arrival with ERLS, EXPS and HEXS. Nonetheless, the hyperexponential arrival times increases slowly in the case of Erlang service times.

We fix $\lambda = 1$; $\delta = 4$; $\Psi = 3$; $\theta = 0.7$; $\eta = 5$; $\tau = 2$; $\sigma = 8$.

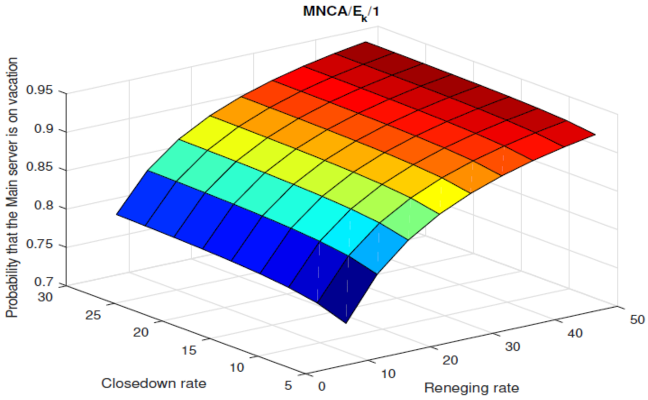

Figure 26: The graph of the $MAP - NC/E_k/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

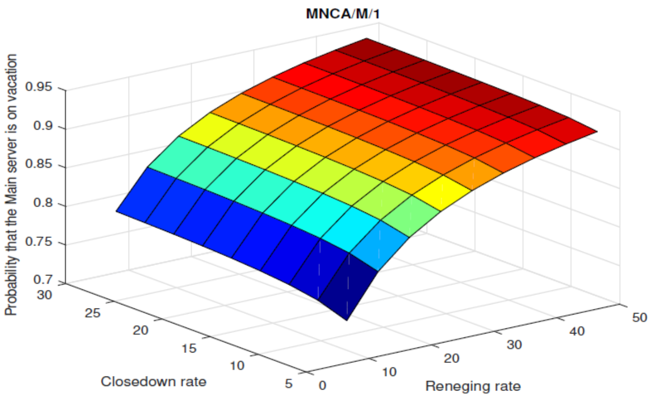

Figure 27: The graph of the $MAP - NC/M/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

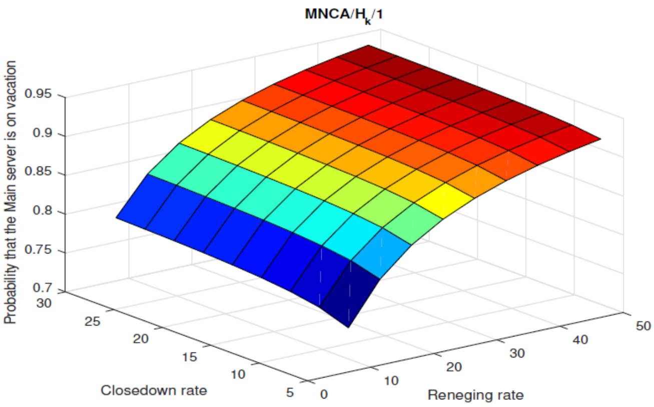

Figure 28: The graph of the $MAP - NC/H_k/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

We observe from figures 26, 27 and 28 that it shows the consequence of closedown rate and reneging rate on the probability of the main server service is on vacation. We have examined the probability of the main server is in the vacation increases while we are increasing both the closedown rate and reneging rate for the arrangement of MAP-Negative correlation arrival(MNCA) with ERLS, EXPS and HEXS. Nevertheless, MAP-Negative correlation arrival increases fastly in case of hyperexponential service times.

We fix $\lambda = 1$; $\delta = 4$; $\Psi = 3$; $\theta = 0.7$; $\eta = 5$; $\tau = 2$; $\sigma = 8$.

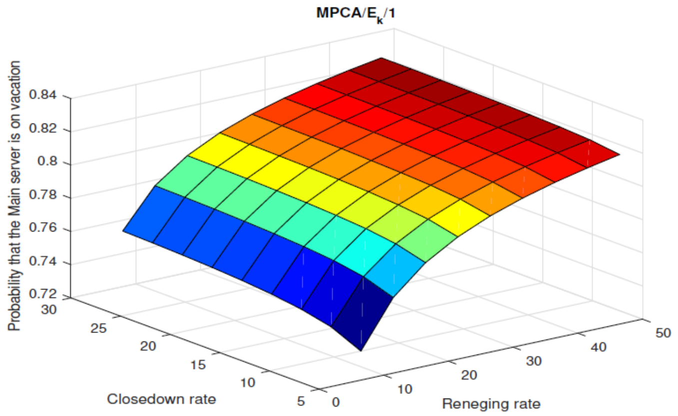

Figure 29: The graph of the $MAP - PC/E_k/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

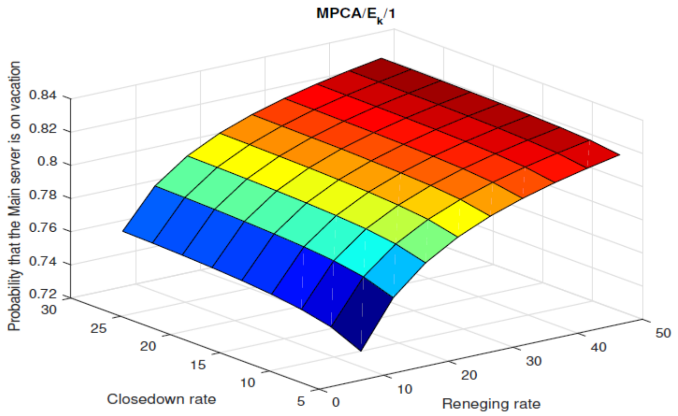

Figure 30: The graph of the $MAP - PC/M/1$ - closedown rate(γ) and reneging rate(ζ) versus probability that the main server is on vacation

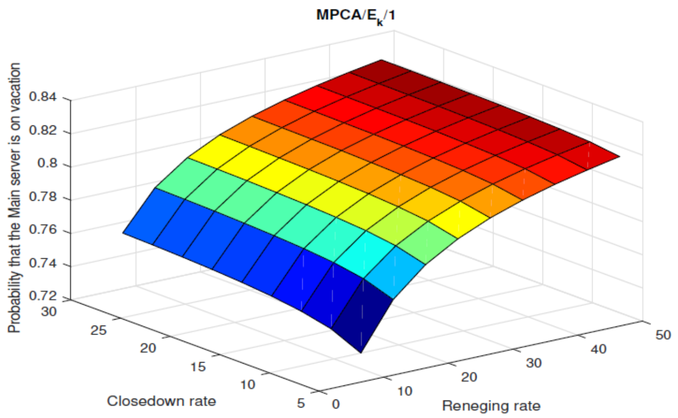

Figure 31: The graph of the $MAP - PC/H_k/1$ - closedown rate(ζ) and reneging rate(γ) versus probability that the main server is on vacation

We observe from figures 29, 30 and 31 that it shows the consequence of closedown rate and reneging rate on the probability of the main server service is on vacation. We have examined the probability of the main server is in the vacation increases while we are increasing both the closedown rate and reneging rate for the arrangement of MAP-Positive correlation arrival(MPCA) with ERLS, EXPS and HEXS. Moreover, the hyperexponential service fastly increases in MAP-Positive correlation arrival.

Illustrated Example 11:

We fix $\lambda = 1$; $\delta = 4$; $\eta = 5$; $\tau = 2$; $\sigma = 8$; $\gamma = 6$; $\zeta = 9$.

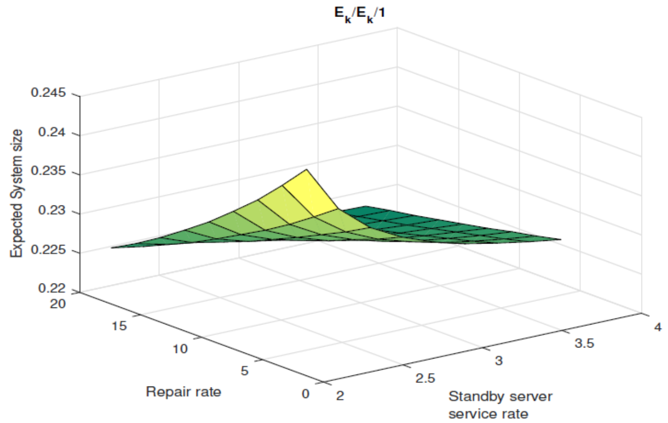

Figure 32: The graph of $E_k/E_k/1$ - repair rate(Ψ) and standby server service rate($\theta\delta$) versus the expected system size

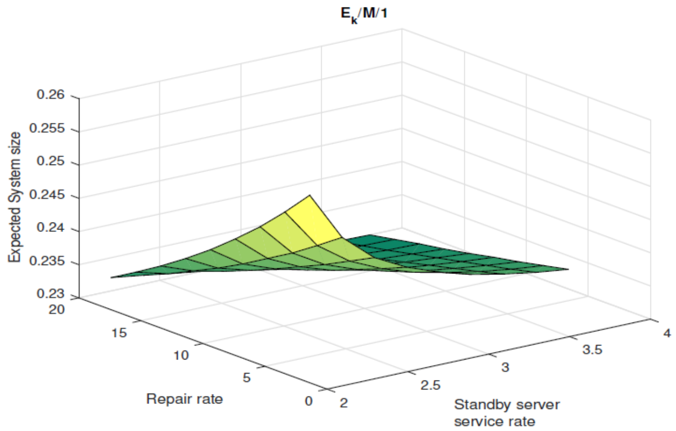

Figure 33: The graph of $E_k/M/1$ - repair rate(Ψ) and standby server service rate($\theta\delta$) versus the expected system size

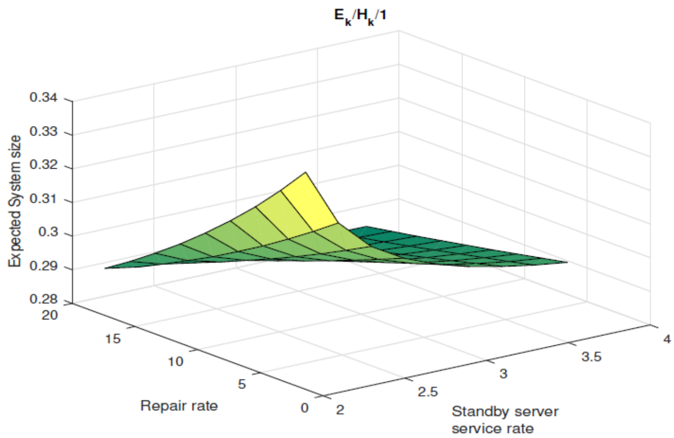

Figure 34: The graph of $E_k/H_k/1$ - repair rate(Ψ) and standby server service rate($\theta\delta$) versus the expected system size

We observe from the figures 32, 33 and 34 that it shows the consequence of standby server service rate and repair rate on the expected system size. We have examined that the expected system size decreases while we are increasing both the repair rate and standby server service rate for the arrangement of Erlang arrival with ERLS, EXPS and HEXS. However, the Erlang arrival fastly decreases with hyperexponential service.

We fix $\lambda = 1$; $\delta = 4$; $\eta = 5$; $\tau = 2$; $\sigma = 8$; $\gamma = 6$; $\zeta = 9$.

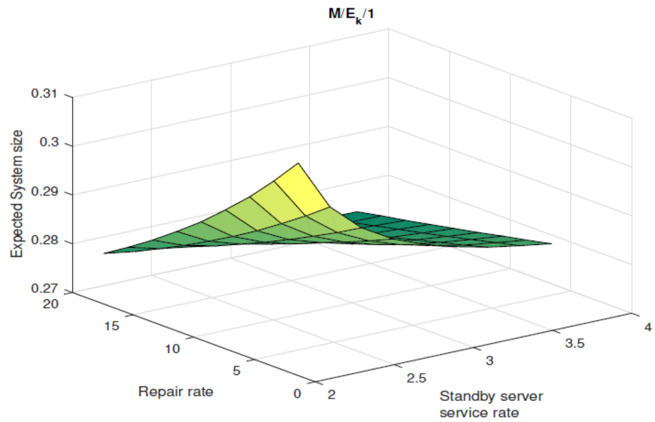

Figure 35: The graph of $M/E_k/1$ - repair rate(Ψ) and standby server service rate($\theta\delta$) versus the expected system size

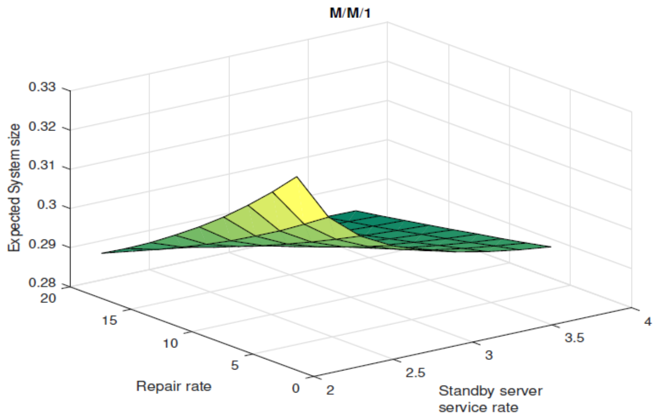

Figure 36: The graph of $M/M/1$ - repair rate(Ψ) and standby server service rate($\theta\delta$) versus the expected system size

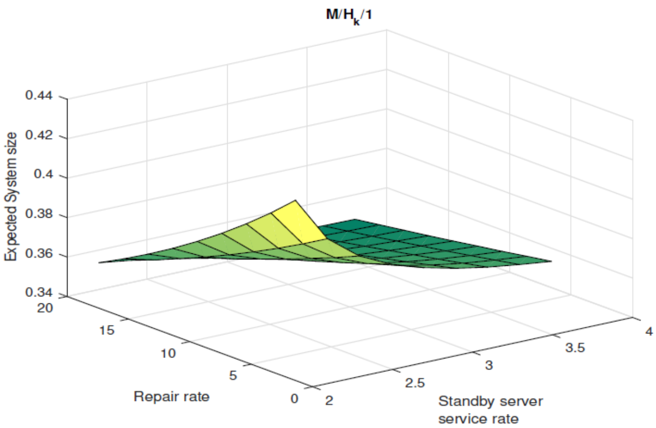

Figure 37: The graph of $M/H_k/1$ - repair rate(Ψ) and standby server service rate($\theta\delta$) versus the expected system size

We observe from the figures 35, 36 and 37 that it shows the consequence of standby server service rate and repair rate on the expected system size. We have examined that the expected system size decreases while we are increasing both the repair rate and standby server service rate for the arrangement of exponential arrival with services of ERLS, EXPS and HEXS. Nevertheless, exponential arrival decreases fastly in hyperexponential service, gradually in exponential service and slowly in Erlang service times.

We fix $\lambda = 1$; $\delta = 4$; $\eta = 5$; $\tau = 2$; $\sigma = 8$; $\gamma = 6$; $\zeta = 9$.

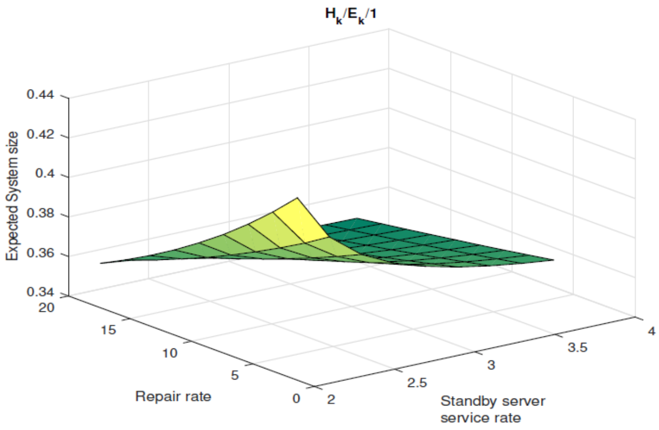

Figure 38: The graph of $H_k/E_k/1$ - repair rate(Ψ) and standby server service rate($\theta\delta$) versus the expected system size

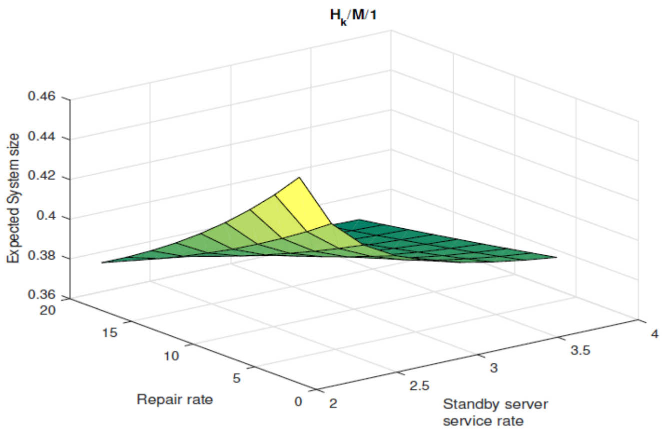

Figure 39: The graph of $H_k/M/1$ - repair rate(Ψ) and standby server service rate($\theta\delta$) versus the expected system size

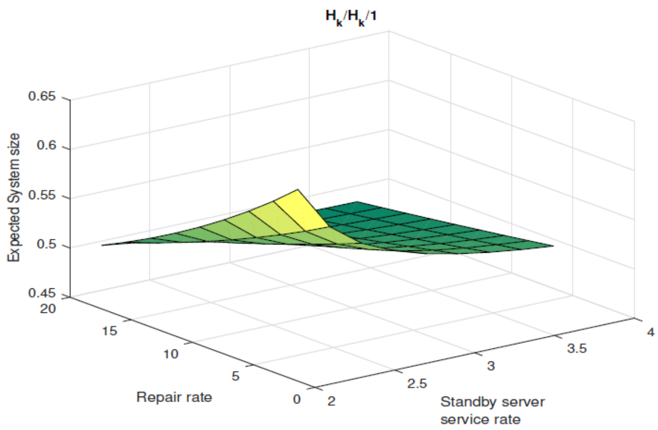

Figure 40: The graph of $H_k/H_k/1$ - repair rate(Ψ) and standby server service rate($\theta\delta$) versus the expected system size

We observe from the figures 38, 39 and 40 that it shows the consequence of standby server service rate and repair rate on the expected system size. We have examined that the expected system size decreases slowly while we are increasing both the repair rate and standby server service rate for the arrangement of hyperexponential arrival with services of ERLS, EXPS and HEXS. Moreover, the hyperexponential service times decreases than the Erlang service times with the hyperexponential arrival times.

We fix $\lambda = 1$; $\delta = 4$; $\eta = 5$; $\tau = 2$; $\sigma = 8$; $\gamma = 6$; $\zeta = 9$.

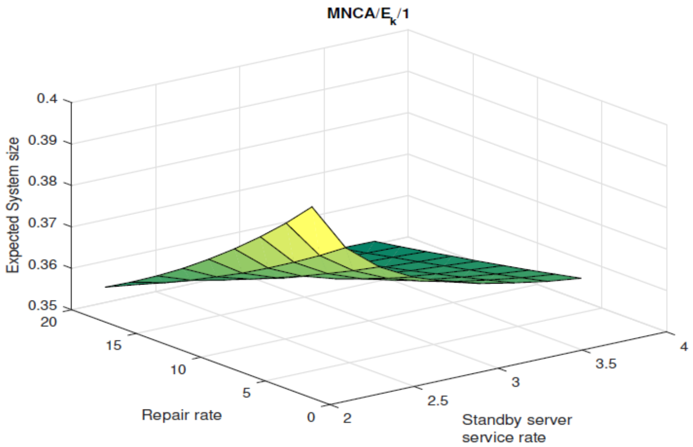

Figure 41: The graph of the $MAP - NC/E_k/1$ - standby server service rate($\theta\delta$) and repair rate(Ψ) versus the expected system size

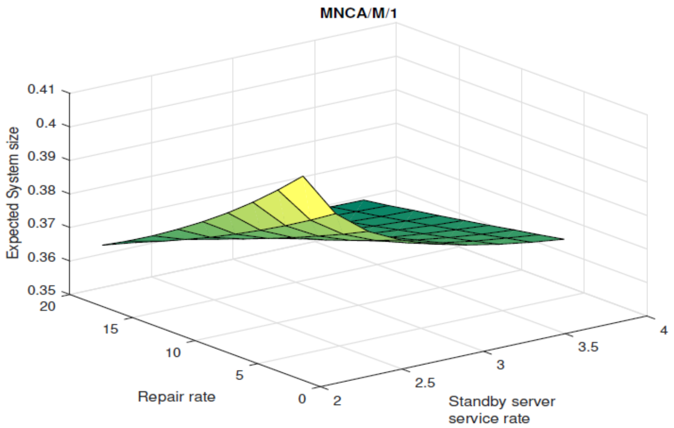

Figure 42: The graph of the $MAP - NC/M/1$ - standby server service rate($\theta\delta$) and repair rate(Ψ) versus the expected system size

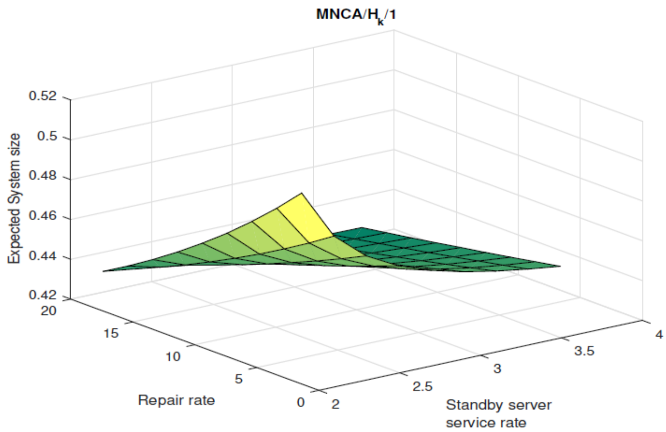

Figure 43: The graph of the $MAP - NC/H_k/1$ - standby server service rate($\theta\delta$) and repair rate(Ψ) versus the expected system size

We observe from the figures 41, 42 and 43 that it shows the consequence of standby server service rate and repair rate on the expected system size. We have examined that the expected system size decreases randomly while we are increasing both the repair rate and standby server service rate for the arrangement of MAP-Negative correlation arrival(MNCA) with services of ERLS, EXPS and HEXS. Nonetheless, the MAP-Negative correlation arrival decreases slowly in Erlang service times and fastly in hyperexponential service times.

We fix $\lambda = 1$; $\delta = 4$; $\eta = 5$; $\tau = 2$; $\sigma = 8$; $\gamma = 6$; $\zeta = 9$.

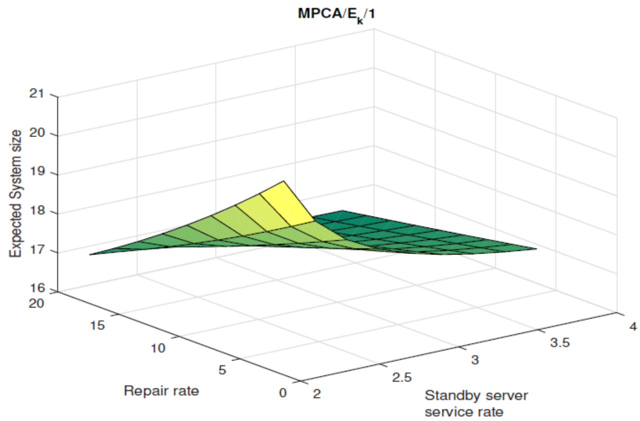

Figure 44: The graph of the $MAP-PC/E_k/1$ - standby server service rate($\theta\delta$) and repair rate(Ψ) versus the expected system size

Figure 45: The graph of the $MAP-PC/M/1$ - standby server service rate($\theta\delta$) and repair rate(Ψ) versus the expected system size

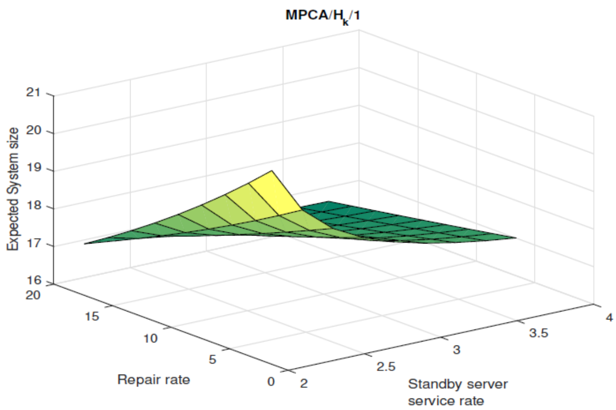

Figure 46: The graph of the $MAP-PC/H_k/1$ - standby server service rate($\theta\delta$) and repair rate(Ψ) versus the expected system size

We observe from the figures 44, 45 and 46 that it shows the consequence of standby server service rate and repair rate on the expected system size. We have examined that the expected system size decreases rapidly while we are increasing both the repair rate and standby server service rate for the arrangement of MAP-Positive correlation arrival(MPCA) with services of ERLS, EXPS and HEXS. Furthermore, the Erlang service times slowly decreases than the exponential and hyperexponential service times in case of MAP-Positive correlation arrival.

8. Comparing the service rate of the Main server and Standby server

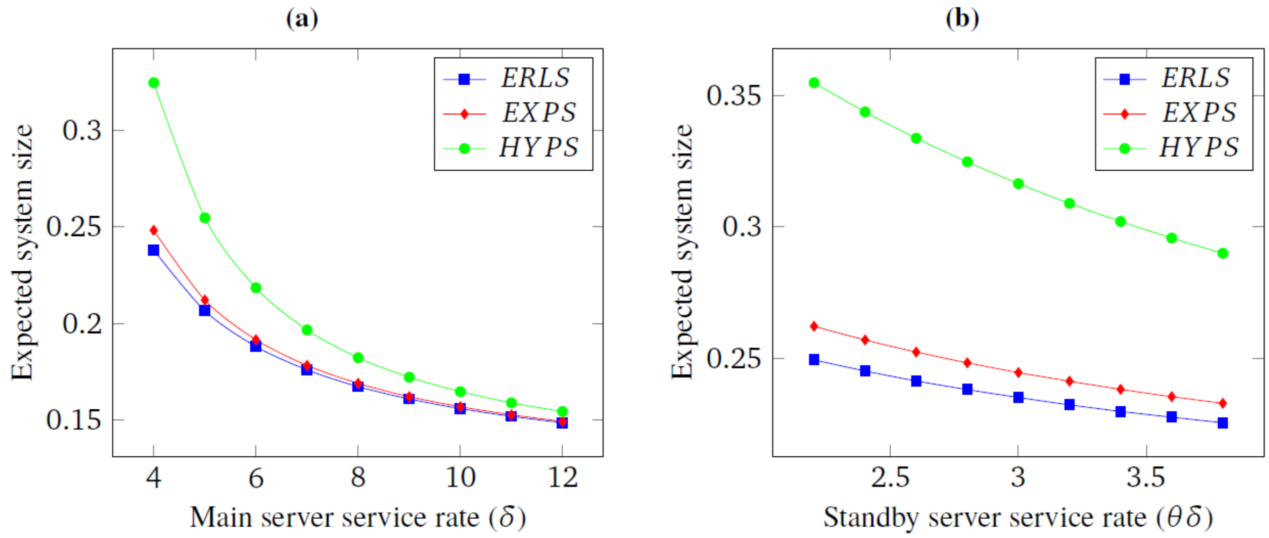

Figure 47: Expected system sizes Vs Main server and Standby server service rate of Erlang arrival

From figure 47, by comparing both the service rate of the main server and standby server contrast to the expected system size, it decreases rapidly in main server service rate and in the case of the standby server service rate decreases slowly.

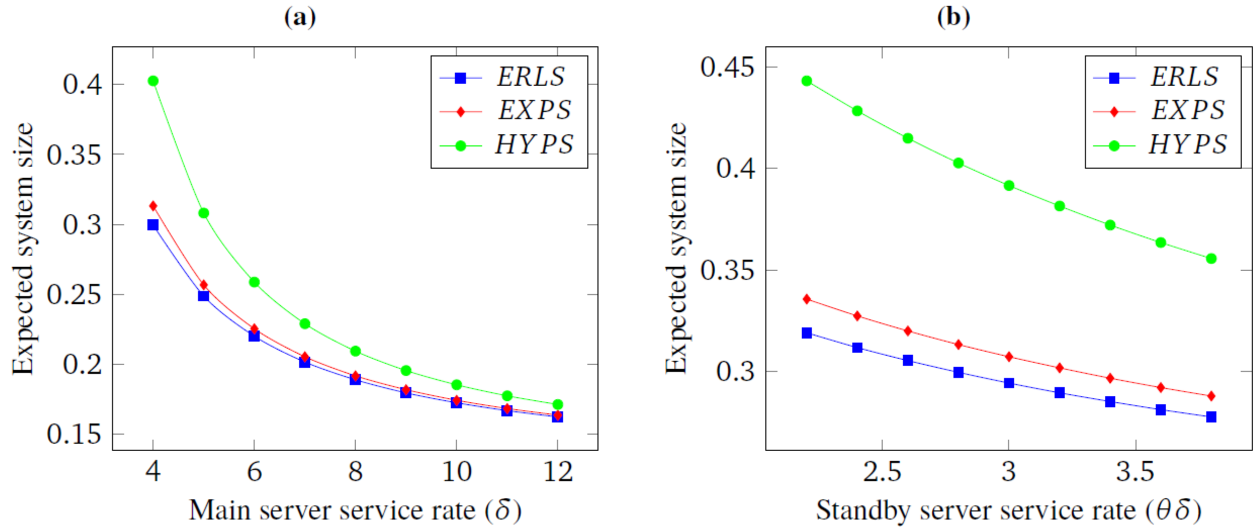

Figure 48: Expected system sizes Vs Main server and Standby server service rate of exponential arrival

From figure 48, by comparing both the service rate of the main server and standby server against to the expected system size, it decreases fastly in main server service rate and in the case of the standby server service rate decreases gradually.

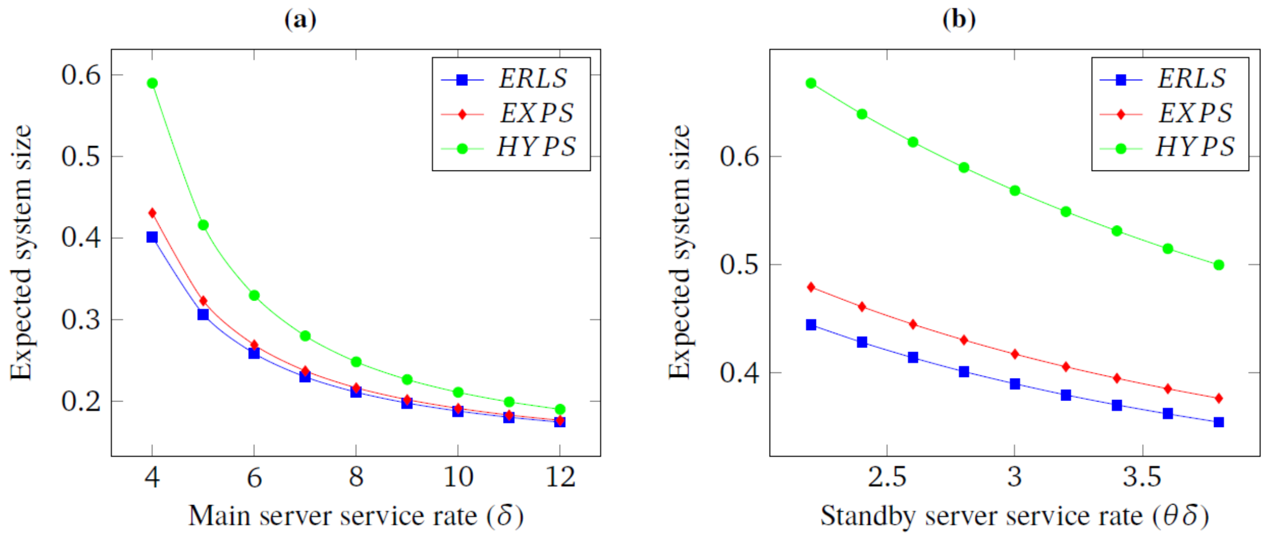

Figure 49: Expected system sizes Vs Main server and Standby server service rate of hyperexponential arrival

From figure 49, by comparing both the service rate of the main server and standby server contrast to the expected system size, it decreases more rapidly in main server service rate and in the case of the standby server service rate decreases gradually.

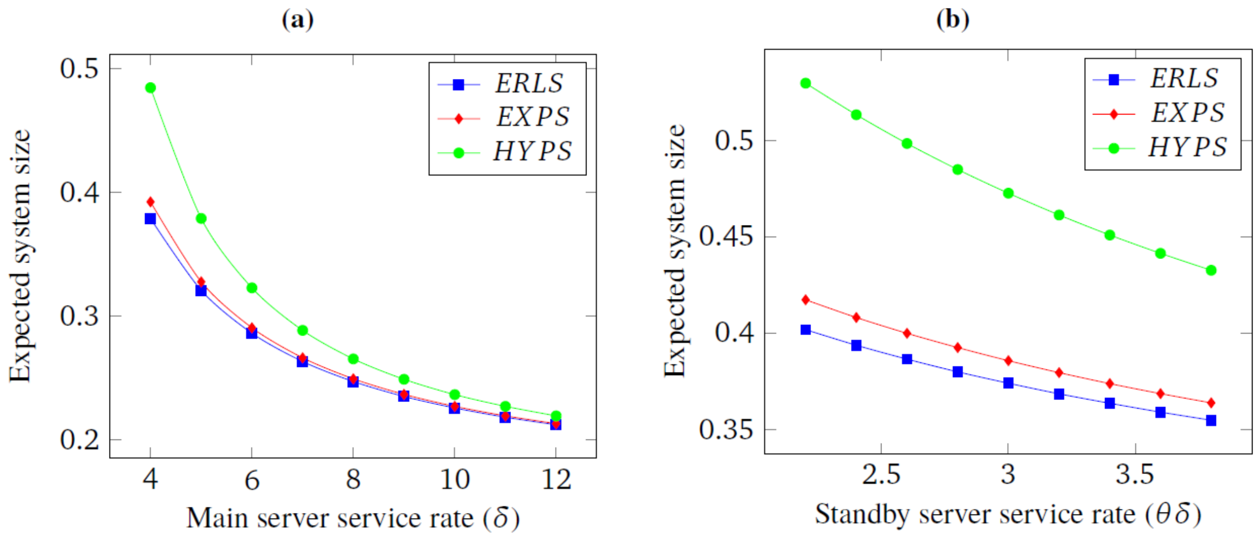

Figure 50: Expected system sizes Vs Main server and Standby server service rate of MAP-Negative correlation arrival

From figure 50, by comparing the service rate of the main server and standby server contrast to the expected system size, it decreases rapidly in main server service rate and in the case of the standby server service rate decreases slowly.

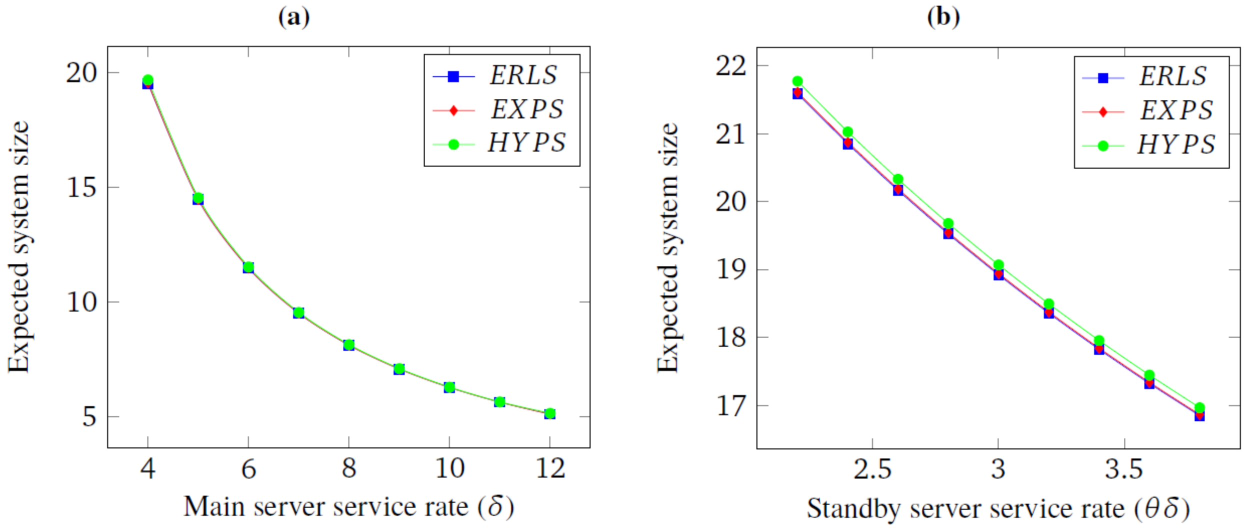

Figure 51: Expected system sizes Vs Main server and Standby server service rate of MAP-Positive correlation arrival

In figure 51, by comparing the service rate of the main server and standby server on the expected system size, main server service rate decreases such that all types of services converge and in the case of the standby server service rate decreases rapidly.

9. Conclusion

In our model, customers arrive in Markovian Arrival Process and the process of service in phase type distribution with server breakdown, multiple vacations, reneging, standby server, setup, closedown and repair. In our work, we also compute the busy period analysis. Using numerical arrivals and services we tabulated the expected system size values for the breakdown rate, repair rate, standby server service rate, setup rate, closedown rate, service rate and vacation rate. We have compared the both the setup rate and main server service rate contrast to the probability that the main server is in the busy mode, both the closedown rate and reneging rate contrast to the probability that the main server is on vacation and both the standby server service rate and repair rate contrast to expected system size showed through the graphical demonstrations. Furthermore, We have compared the service rate of the main server and standby server graphically.

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