Application of Non-Parametric Bootstrap Technique for evaluating MTTF and Reliability of a Complex Network with Non-Identical Component Failure Laws

Aditya Tiwary

•

Dept. of Fire Technology & Safety Engineering, IPS Academy,
Institute of Engineering and science, Indore (M.P), India
raditya2002@gmail.com

Abstract

This paper describes a non-parametric bootstrap technique and Monte Carlo simulation technique to evaluate MTTF and reliability for a complex network, where components have different failure density functions. Algorithm for complete reliability analysis with boot strapping and Monte Carlo simulation (MCS) technique for the complex network has been developed. The algorithm has been implemented on a bridge network. The result obtained has been compared with those obtained using MCS.

Keywords: Monte Carlo simulation, bootstrap, failure density function, mean time to failure, reliability.

I. Introduction

Reliability evaluation of a system is an important issue. Various analytical methods for reliability evaluation of network have been presented in the literature [1, 2, 3]. Average reliability indices are evaluated using analytical technique where as simulation techniques are used to generate probability distribution of these indices. Billinton et al. [4] developed a technique for reliability evaluation of distribution system. Volkanavski et al. [5] used fault tree analysis for power system reliability evaluation. Various MCS based methodologies have been developed for reliability evaluation [6, 7]. Several variations of MCS methods have been developed to probabilistically evaluate long term reliability of power system [8]. Billinton et al. [9] have developed a new MCS technique which is based on a system state transition sampling approach for composite power system reliability. Evaluation of Tsai and Lu [10] utilized the non-parametric bootstrap to estimate available transfer capacity (ATC). Othman et al. [11] developed a novel approach to determine transmission reliability margin using bootstrap technique. Reliability indices accounting omission of random repair time for distribution systems usingMonte Carlo simulation [14]. Jirutitijaroen et al. [15] developed a comparison of simulation methods for power system reliability indexes and their distribution. Determination of reliability indices for distribution system using a state transition sampling technique accounting random down time omission was discussed by Tiwary et al. [16]. Bootstrapping based technique for evaluating reliability indices of RBTS distribution system neglecting random down time was evaluated [17]. Volkanavski et al. [18] proposed application of fault tree analysis for assessment of the power system reliability. Li et al. [19] studies the impact of covered overhead conductors on distribution reliability and safety.

Reliability enhancement of distribution system using Teaching Learning based optimization considering customer and energy based indices was obtained in Tiwary et al. [20]. A smooth bootstrapping based technique for evaluating distribution system reliability indices neglecting random interruption duration is developed [21]. Sarantakos et al. [22] introduced a method to include component condition and substation reliability into distribution system reconfiguration. Battu et al. [23] discussed a method for reliability compliant distribution system planning using Monte Carlo simulation. Tiwary et al. [24] has discussed a methodology for reliability evaluation of an electrical power distribution system, which is radial in nature.Uspensky et al. [25] has developed a method for reliability assessment of the digital relay protection system. A probabilistic method to assess the reliability impact of EVs penetration is discussed [26]. A BN-based unified modeling method for performance and reliability is proposed [27]. A framework for dynamic prediction of reliability weaknesses in power transmission systems based on imbalanced data is discussed [28]. Shrestha et al. [29] proposed the development of an operational adequacy evaluation framework for operational planning of bulk electric power systems. Tiwary et al. [30] has discussed a methodology for evaluation of customer orientated indices and reliability of a meshed power distribution system.

Evaluation of reliability and MTTF of a network become tedious analytically if the failure density functions of different components are non-exponential and different. In such situation simulation approach provides a simple way to obtain these indices. MCS procedure requires large number of samples to obtain desired accuracy results. Hence re-sampling techniques (boot-strap) have been used in various studies. Boot strapping requires lesser number of samples and thus considerable reduction in CPU time may be achieved. In view of this in this paper boot strapping has been employed along with MCS to evaluate reliability and MTTF of a complex reliability network.

II. Monte Carlo Simulation

The reliability indices of a system can be evaluated using one of two basic approaches, direct analytical technique or stochastic simulation. Simulation is a very valuable method which is widely used in the solution of real engineering problems. Analytical technique represents the system by a mathematical model, which is often simplified and evaluates the reliability indices from this model using direct mathematical solutions. Simulation technique, on the other hand, estimates the reliability indices by simulating the actual process and random behavior of the system [1]. Computational algorithm for evaluating mean time to failure (MTTF) and reliability by using MCS technique is as follows:

Step-1 Obtain minimal cut-set for the given network.

Step-2 Obtain random variates for time to failure for each of the components by considering their respective failure density functions. Based on the cut sets, obtain time to failure of the system.

Step-3 Repeat step-2 for large number of times. e.g. NS=10000.

Step-4 Estimate mean time to failure $(M\hat{T}TF)$ by using following relation:

$$M\hat{T}TF = \frac{1}{NS}\sum_{i=1}^{NS} t_i \tag{1}$$

t_i is time to failure for ith sample

Step-5 Obtain coefficient of variations for convergence

$$\beta = s / M\hat{T}TF \tag{2}$$

β is coefficient of variation for MTTF

Where $s = \hat{\sigma} / \sqrt{NS}$

(3)

$$\hat{\sigma}^2 = \frac{1}{NS} \sum_{i=1}^{NS} (t_i - M\hat{T}TF)^2 \tag{4}$$

Step-6 If

$\beta < \xi$

Then solution has converged.

ξ is tolerance specified e.g. 0.0010.

If not converged, increase sample "NS" and repeat from step-2.

Step-7 Obtain the function I_i as follows:

$I_i = 0$ if $t_i \le t_r$

$= 1$ if $t_i > t_r$

t_r is the duration for which reliability is required.

Step-8 Estimate reliability by using following relation:

$$\hat{R}(t_r) = \frac{1}{NS} \sum_{i=1}^{NS} I_i$$

Thus reliability may be calculated for various values of time t_r.

III. Non-parametric Bootstrap technique

The non-parametric bootstrap technique, randomly replace the position ofactual data yielding to several new samples of data [10, 11, 12, 13]. Theprocedure of non-parametric bootstrap technique commences with anindependent random sample. A smaller size of time to failure is obtainedusing MCS and assume that it is given as

$$[t_{f1}, t_{f2}, \ldots, t_{fNS'}] \tag{5}$$

Re-sampling with replacement (boot strapping) is adopted from (5) and 'NB' samples are obtained as follows.

$$\{t_{fB1}^{(i)}, t_{fB2}^{(i)}, \ldots, t_{fB,NS'}^{(i)}\} \tag{6}$$

$i = 1, \ldots\ldots\ldots\ldots NB$

(6) is set of time to failure as obtained from (5) after re-sampling and may have repeated time to failure from (5).Now for each re-sampled set of data (6) sample mean is calculated

$$\bar{t}_{fB}^{(i)} = \frac{1}{NS} \sum_{k=1}^{NS'} t_{fBK}^{(i)} \tag{7}$$

$i = 1, \ldots\ldots\ldots\ldots NB$

Over all estimated mean time to failure is calculated as follows

$$M\hat{T}TF_B = \frac{1}{NB}\sum_{i=1}^{NB}\bar{t}_{fB}^{(i)} \tag{8}$$

$M\hat{T}TF_B$ is the estimated mean time to failure obtained using boot strapping. Convergence may be checked using coefficient of variation.

Computational algorithm for evaluating mean time to failure (MTTF) using non-parametric bootstrap technique is given as follows:

Step-1 Obtain data set of time to failures using MCS as explained inprevious section. This is given by relation (5). Note that $NS' << NS$.

Step-2 Obtain re-sampled data set as given by relation (6).

Step-3 Calculate mean time to failure using relation (7) and (8).

Step-4 Obtain coefficient of variations for convergence

$$\beta = s / M\hat{T}TF_B$$

Where $s = \hat{\sigma} / \sqrt{NB}$

$$\hat{\sigma}^2 = \frac{1}{NB-1}\sum_{i=1}^{NB}(\bar{t}_{fBi}^{(i)} - M\hat{T}TF_B)^2$$

Step-5 If

$$\beta < \xi$$

Then solution has converged.

ξ is tolerance specified e.g. 0.0010

Step-6 In each resample calculate reliability for time

$$r_B^{(i)}(t_r) = \frac{n^{(i)}(t_r)}{NS}$$

$n^{(i)}(t_r)$ number of data is for which time to failure is greater than t_r

$i = 1, \ldots\ldots NB$

Step-7 Calculate average reliability

$$\bar{r}(t_r) = \frac{1}{NB}\sum_{i=1}^{NB} r_B^{(i)}(t_r)$$

Thus reliability $\bar{r}(t_r)$ is obtained for various time values.

IV. Result and discussion

The two techniques non-parametric bootstrap technique and MCS technique are used to evaluate MTTF and reliability for bridge network as shown in Fig.-1.

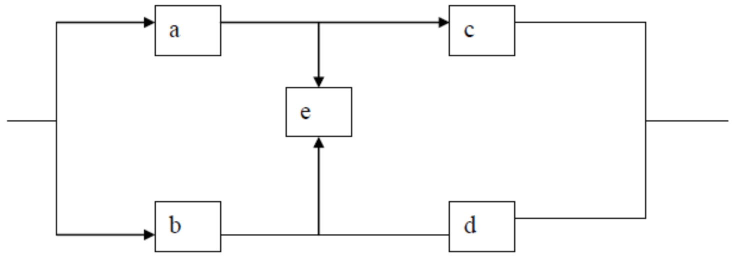

Fig-1 Complex network with five components a, b, c, d and e respectively.

Developed algorithms have been implemented by considering failure density function of components 'a' and 'b' exponentially, component 'c' and 'd' uniformly distributed and component 'e' Normally distributed. The failure rate for components 'a' and 'b' have been taken as 0.1 and 0.2 per year respectively. The range for the uniformly distributed components c and d are $0 \le t \le 5$ and $0 \le t \le 8$ respectively. For the component 'e' which is Normally distributed the mean value is 5 and the standard deviation is 1. Table-1 and Table-2 shows the values of MTTF obtained by MCS and nonparametric bootstrap technique respectively.

Table 1 MTTF as obtained using MCS and CPU time

Number of Samples, NS	MTTF, yrs.	CPU Time in seconds
101000	3.7955	0.117450

Table 2 MTTF as obtained using Boot strap technique and CPU time

Number of Samples, NS'	$MTTF_B$, yrs.	CPU Time in seconds
100	3.8390	0.047849

The error is 1.133 % using nonparametric bootstrap technique as compared to MCS. The CPU time required for convergence on Intel core 2 duo processor, 2.10 GHZ by using bootstrap technique have reduced by 59.26 %. The plot for coefficient of variation with respect to number of samples by using MCS and nonparametric bootstrap technique is shown in Fig.-2 and Fig.-3 respectively.

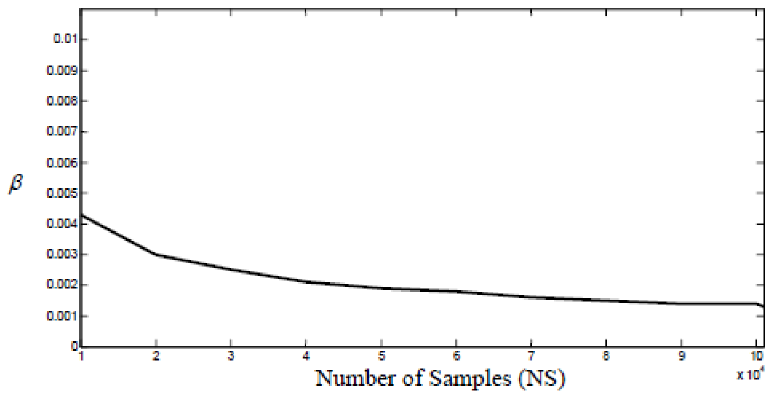

Fig-2 Variation of coefficient of variation with respect to number of samples in MCS

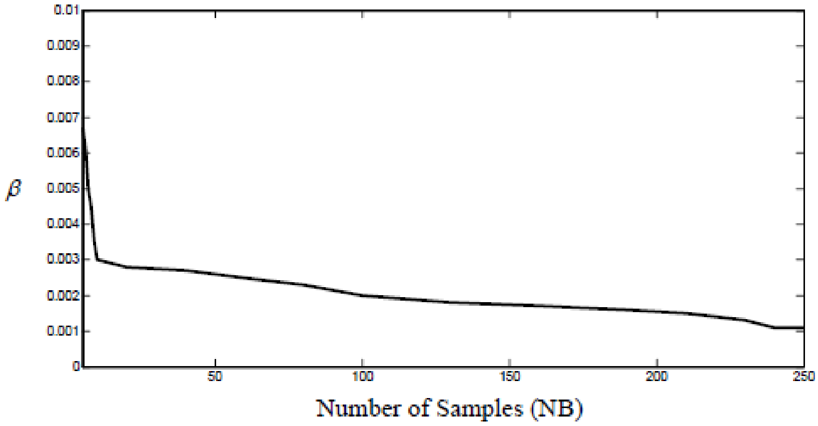

Fig-3 Variation of coefficient of variation with boot strap samples

The plot of reliability with respect to time by using MCS and bootstrap technique is shown in Fig.-4. Fig.-5 gives relative frequency distribution of mean time to failure $(\bar{t}_{fB}^{(i)})$ as obtained using boot strapping. It is observed that the shape is that of Normal distribution as is the case with boot strapping.

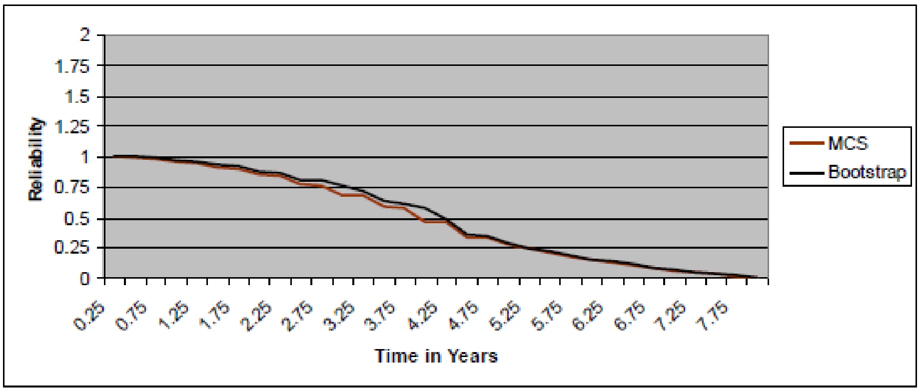

Fig-4 Plots showing variation of reliability with respect to time of operation using MCS and boot strap technique

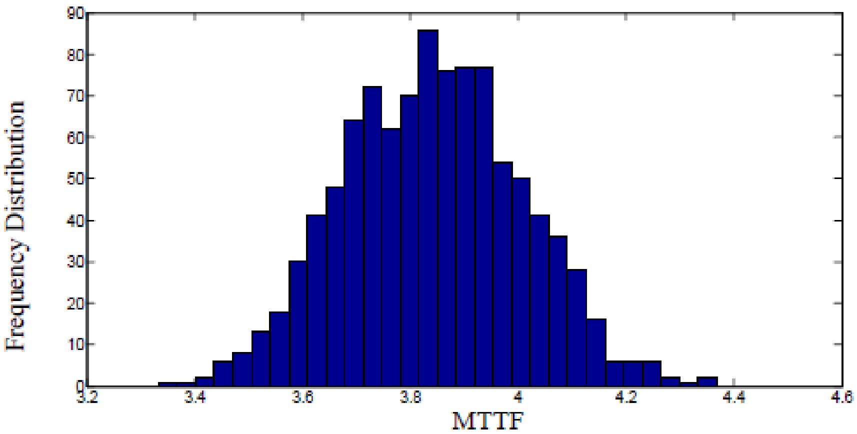

Fig-5 Frequency distribution of MTTF as obtained from non-parametric bootstrap technique

V. Conclusion

The paper has provided a comparison between MCS and enhanced sampling using bootstrap technique for calculating MTTF and reliability for the complex network by considering different distributions. The result obtained by two methods is in close agreement. The error introduced in MTTF using boot strapping is 1.133 % with respect to MTTF obtained using general MCS. Considerable reduction in CPU time is observed using bootstrap technique. The convergence has been visualized by the coefficient of variation.

References

1. R. Billinton, R. N. Allan. (2007). Reliability Evaluation of Engineering systems: Concepts and techniques. Springer International edition.
2. R. Billinton, R. N. Allan. (1996). Reliability evaluation of power system. Springer International Edition.
3 R. N. Allan, R. Billinton. (2000). Probabilistic assessment of power systems. Proc. IEEE, 88(2):140-162.
4 R. Billinton, M. S. Grover. (1975). Reliability evaluation in distribution and transmission system. Proc. IEEE, 122(5):517-524.
5 Volkanavski, M. Cepin, B. Mavko. (2009). Application of fault tree analysis for assessment of the power system reliability. Reliability engineering and system safety, 94(6):1116-1127.
6 E. Zio, L. Podofillini, V. Zille. (2006). A combination of Monte Carlo simulation and cellular automata for computing the availability of complex network system. Reliability Engineering and system safety, 91: 181-190.
7 P. Jirutitijaroen, C. singh. (2008). Comparison of simulation methods for power system reliability indexes and their distribution. IEEE Trans. power systems, 23(2):486-492.
8 R. Billinton, W. Wangdee. (2006). Delivery point reliability indices of a bulk electric system using sequential Monte Carlo simulation. IEEE Trans. on power Delivery, 21(1): 345-352.
9 R. Billinton, W. Li. (1993). A system state transition sampling method for composite system reliability evaluation. IEEE Trans. on power system, 8(3):761-770.
10 C. Y. Tsai, C. N. Lu. (2001). Bootstrap application in ATC estimation. IEEE Power Eng Rev, 21: 40-2.
11 M. M. Othaman, I. Musirin. (2001). A novel approach to determine transmission reliability margin using parametric bootstrap technique. Int. J. of Electrical Power and Energy System, 33:1666-1674.
12 H. C. Frey, D. E. Burmaster. (1999). Methods for characterizing variability and uncertainty: comparision of bootstrap simulation and likelihood based approaches. Int J Risk Anal, 19:109-30.
13 R. Wehrens, H. Putter, L. M. C. Buydens. (2000). The bootstrap: a tutorial. Chemom Intell Lab Syst, 54:35-52.
14 L. D. Arya, S. C. Choube, R. Arya, Aditya Tiwary. (2012). Evaluation of Reliability indices accounting omission of random repair time for distribution systems using Monte Carlo simulation. Int. J. of Electrical Power and Energy System, 42:533-541.
15 Jirutitijaroen P, singh C. (2008).Comparison of simulation methods for power system reliability indexes and their distribution. IEEE Trans Power Syst, 23:486–92.

16 Aditya Tiwary, R. Arya, S. C. Choube and L. D. Arya. (2013).Determination of reliability indices for distribution system using a state transition sampling technique accounting random down time omission. Journal of The Institution of Engineers (India): series B (Springer), 94:71-83.

17 Aditya Tiwary, R. Arya, L. D. Arya and S. C. Choube. (2017). Bootstrapping based technique for evaluating reliability indices of RBTS distribution system neglecting random down time. The IUP Journal of Electrical and Electronics Engineering, X: 48-57.

18 Volkanavski, Cepin M, Mavko B. (2009).Application of fault tree analysis for assessment of the power system reliability. Reliab Eng Syst Safety, 94:1116–27.

19 Li BM, Su CT, Shen CL. (2010).The impact of covered overhead conductors on distribution reliability and safety. Int J Electr Power Energy Syst, 32:281–9.

20 Aditya Tiwary. (2017). Reliability enhancement of distribution system using Teaching Learning based optimization considering customer and energy based indices. International Journal on Future Revolution in Computer Science & Communication Engineering, 3:58-62.

21 R. Arya, Aditya Tiwary, S. C. Choube and L. D. Arya. (2013). A smooth bootstrapping based technique for evaluating distribution system reliability indices neglecting random interruption duration. Int. J. of Electrical Power and Energy System, 51:307-310.

22 I. Sarantakos, D. M. Greenwood, J. Yi, S. R. Blake, P. C. Taylor. (2019). A method to include component condition and substation reliability into distribution system reconfiguration. Int. J. of Electrical Power and Energy System, 109:122-138.

23 N. R. Battu, A. R. Abhyankar, N. Senroy. (2019). Reliability Compliant Distribution System Planning Using Monte Carlo Simulation. Electric power components and systems, 47:985-997.

24 Aditya Tiwary. (2019). Reliability evaluation of radial distribution system – A case study. Int. J. of Reliability: Theory and Applications, 14, 4(55):9-13.

25 Michael Uspensky. (2019). Reliability assessment of the digital relay protection system. Int. J. of Reliability: Theory and Applications, 14:10-17.

26 M. P. Anand, B. Bagen, A. Rajapakse. (2020). Probabilistic reliability evaluation of distribution systems considering the spatial and temporal distribution of electric vehicles. Int. J. of Electrical Power & Energy Systems.117.

27 Yi Ren, B. Cui, Q. A. Feng, D. Yang, D. Fan, Bo Sun, M. Li. (2020).A reliability evaluation method for radial multi-microgrid systems considering distribution network transmission capacity. Computers & Industrial Engineering.139.

28 C. Sun, X. Wang, Y. Zheng, F. Zhang. (2020). A framework for dynamic prediction of reliability weaknesses in power transmission systems based on imbalanced data. Int. J. of Electrical Power & Energy Systems. 117.

29 T. K. Shrestha, R. Karki, P. Piya. (2020). Development of an operational adequacy evaluation framework for operational planning of bulk electric power systems. International Journal of Reliability, Quality and Safety Engineering.

30 Aditya Tiwary. (2020).Customer orientated indices and reliability evaluation of meshed power distribution system. Int. J. of Reliability: Theory and Applications, 15, 1(56):10-19.

Received: July 15, 2020
Accepted: September 21, 2020