A Truncated Two Parameter Pranav Distribution and its Applications

Kamlesh Kumar Shukla
•
Department of Statistics, College of Science, Mainefhi,
State of Eritrea (N.E. Africa)
Email: kkshukla22@gmail.com

Abstract

In this paper, two parameter truncated Pranav distribution has been proposed. Some statistical properties including moments, coefficient of variation, skewness and index of dispersion have been derived and presented graphically. Survival and Hazard functions are derived and its behaviors are presented graphically. Maximum likelihood method of estimation has been used to estimate the parameters of proposed model. Simulation study has also been carried out. A proposed distribution has been applied on two data sets and compares its superiority over two parameter and one parameter classical distributions.

Keywords: Truncated, Pranav distribution, Lindley distribution, Skewness, Kurtosis

I. Introduction

In the recent past decades, life time modeling has been becoming popular in distribution theory, where many statisticians are involved in introducing new models. Some of the life time models are very popular and applied in biological, engineering and agricultural areas, such as Lindley distribution; Lindley (1958), weighted Lindley distribution; Gitany et al. (2008), Akash distribution; Shanker (2015), Ishita distribution; Shanker and Shukla (2017), Pranav distribution; Shukla (2018) etc and extension of above mentioned distribution has also been becoming popular in different areas of statistics.

Shukla (2018) has introduced a Pranav distribution which is mixture of exponential distribution having scale parameter θ and gamma distribution having shape parameter 4 and scale parameter θ, is defined by its pdf and cdf :

$$f_1(y;\theta)=\frac{\theta^4}{\theta^4+6}\left(\theta+y^3\right)e^{-\theta y} \quad ;y>0,\,\theta>0 \tag{1.1}$$

$$F_2(y;\theta)=1-\left[1+\frac{\theta y\left(\theta^2y^2+3\theta y+6\right)}{\theta^4+6}\right]e^{-\theta y} \quad ;y>0,\theta>0 \tag{1.2}$$

Shukla (2018) has discussed in details about its mathematical and statistical properties. Estimation of parameter using both the method of moment and the maximum likelihood estimation have mentioned in his paper and its application to model lifetime data from engineering and biomedical sciences.
Recently, Umeh and Ibenegbu (2019) have introduced extension of Pranav distribution and named as two parameter Pranav (TPPD) distribution and its pdf and cdf are defined as follows:

$$f_2(y;\theta)=\frac{\theta^4}{\alpha\theta^4+6}\left(\alpha\theta+y^3\right)e^{-\theta y}\quad ;y>0,\ \theta>0,\alpha\geq 0 \tag{1.3}$$

$$F_2(y;\theta,\alpha)=1-\left[1+\frac{\theta y\left(\theta^2y^2+3\theta y+6\right)}{(\alpha\theta^4+6)}\right]e^{-\theta y}\quad ;y>0,\theta>0,\alpha\geq 0 \tag{1.4}$$

Umeh and Ibenegbu (2019) have discussed in details about its mathematical and statistical properties. Estimation of parameter using both the method of moment and the maximum likelihood estimation including its application have mentioned in their paper.

Truncated type of distribution are more effective in application to modeling life time data because its limits used as bound either upper or lower or both according to given data. Truncated normal distribution is proposed by Johnson et al (1994). It has wide application in economics and statistics. Many researchers have been proposed truncated type of distribution and applied in different areas of statistics, especially in censor data such as truncated Wiebull distribution; Zange and Xie (2010) , truncated Lomax distribution; Aryuyuen and Bodhisuwan (2018), truncated Pareto distribution; Janinetti and Ferraro (2008), truncated Lindley distribution; Singh et al (2014). Truncated version of distribution can be defined as:

Definition1. Let X be a random variable that is distributed according to some pdf $g(x;\Theta..)$ and cdf $G(x;\Theta..)$, where Θ is a parameter vector of X.

Let X lies within the interval $[a,b]$ where $-\infty<a\leq x\leq b<\infty$ then the conditional on $a\leq x\leq b$ is distributed. We have the pdf of truncated distribution as reported by Singh et al (2014) defined by:

$$f(x;\Theta..)=g(x/a\leq x\leq b;\Theta..)=\frac{g(x;\Theta..)}{G(b;\Theta..)-G(a;\Theta..)}\frac{-b\pm\sqrt{b^2-4ac}}{2a} \tag{1.5}$$

where $f(x;\Theta..)=g(x;\Theta..)$ for all $a\leq x\leq b$ and $f(x;\Theta)=0$ elsewhere.
Notice that $f(x;\theta)$ in fact $f(x;\theta)$ is a pdf of X on interval $[a,b]$.

$$f(x;\Theta..)=\int_a^b f(x;\Theta..)dx=\frac{1}{G(b;\Theta..)-G(a;\Theta..)}\int_a^b g(x;\Theta..)dx$$

$$=\frac{1}{G(b;\Theta..)-G(a;\Theta..)}G(b;\Theta..)-G(a;\Theta..)=1 \tag{1.6}$$

The cdf of truncated distribution is given by

$$F(x;\Theta..)=\int_a^x f(x;\Theta..)dx=\frac{G(x;\Theta..)-G(a;\Theta..)}{G(b;\Theta..)-G(a;\Theta..)} \tag{1.7}$$

The main objectives of this paper are (i) to propose new truncated distribution using two parameter Pranav distribution, which is called as Truncated Two Parameter Pranav distribution (TTPPD) (ii) to know statistical mathematical properties and its suitability, it has been compared with classical distributions of two parameter as well as one parameter using two lifetime datasets. The study has been divided in eight sections. Introduction about the paper is described in the first section. In the second section, TTPPD has been derived. Mathematical and statistical properties including its moment have been discussed in third section. Behavior of hazards rate has been presented mathematically as well as graphically in fourth section. Moments and its related expression have been discussed in fifth section. Simulation study of the presented distribution has been discussed to check estimation parameters using Bias and Mean square error in sixth section.

Estimation of parameter of proposed distribution has been discussed in seven section where its applications and comparative study of other classical two parameter life time distributions as well as one parameter distributions have been illustrated using life time data. In the last, conclusions have been drawn according to studied of behavior and properties of TTPPD.

II. Truncated Two Parameter Pranav distribution

In this section, pdf and cdf of new truncated distribution is proposed and named Truncated two parameter Pranav distribution(TTPPD) , using (1.5) & (1.7) of definition1 and from (1.3) & (1.4) , which is defined as :

Definition 2: Let X be random variable which is distributed as TTPPD with scale parameter θ, location parameter a & b ,and shape parameter α , will be denoted by TTPPD (a,b,θ,α). The pdf and cdf of X are respectively:

$$f(x;\theta,\alpha)=\frac{\theta^4(x^3+\theta\alpha)e^{-\theta x}}{\left((\theta^3a^3+\alpha\theta^4+3a^2\theta^2+6a\theta+6)e^{-\theta a}-(\theta^3b^3+\alpha\theta^4+3b^2\theta^2+6b\theta+6)e^{-\theta b}\right)}$$

(1.8)

$$F(x;\theta,\alpha)=\frac{\left((\theta^3a^3+\alpha\theta^4+3a^2\theta^2+6a\theta+6)e^{-\theta a}-(\theta^3x^3+\alpha\theta^4+3x^2\theta^2+6x\theta+6)e^{-\theta x}\right)}{\left((\theta^3a^3+\alpha\theta^4+3a^2\theta^2+6a\theta+6)e^{-\theta a}-(\theta^3b^3+\alpha\theta^4+3b^2\theta^2+6b\theta+6)e^{-\theta b}\right)}$$

(1.9)

Where $-\infty<a\le x\le b<\infty$, and $\theta>0,\alpha>0$

Following conditions can be categories:

(i) When $a=0$ and $b=\infty$ it reduces to two parameter Pranav distribution.
(i) When $a=0$ it is known as right truncated two parameter Pranav distribution.
(ii) When $b=\infty$ it is known as left truncated two parameter Pranav distribution.
(iii) When and it reduces to two parameter Pranav distribution.
(iv) When $a=0$, $b=\infty$ and $\alpha=1$ it reduces to Pranav distribution.

Performance of pdf of TTPPD for varying values of parameter has been illustrated in the fig.1. From the figure1, it is clearly indicates that parameters a & b are the location parameter, θ scale parameter and α is the shape parameter, and value of pdf is decreasing as increased value of θ when ($\theta<1$) at fixed values of a, b and α , whereas pdf is increasing as increased value of θ when ($\theta>1$) at fixed values of a, b and α .

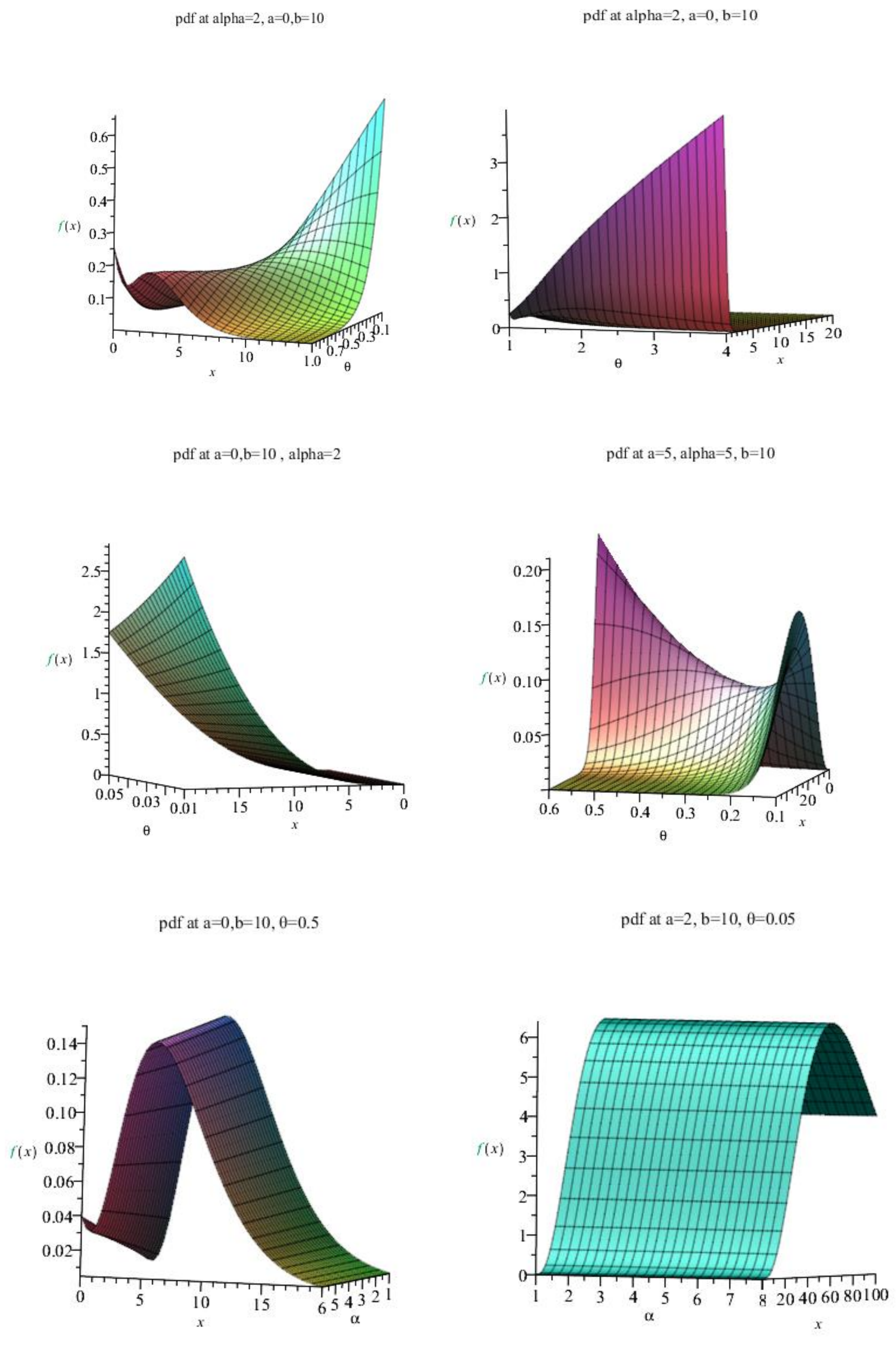

Figure 1. *pdf plots of TTPPD for varying values of parameters*

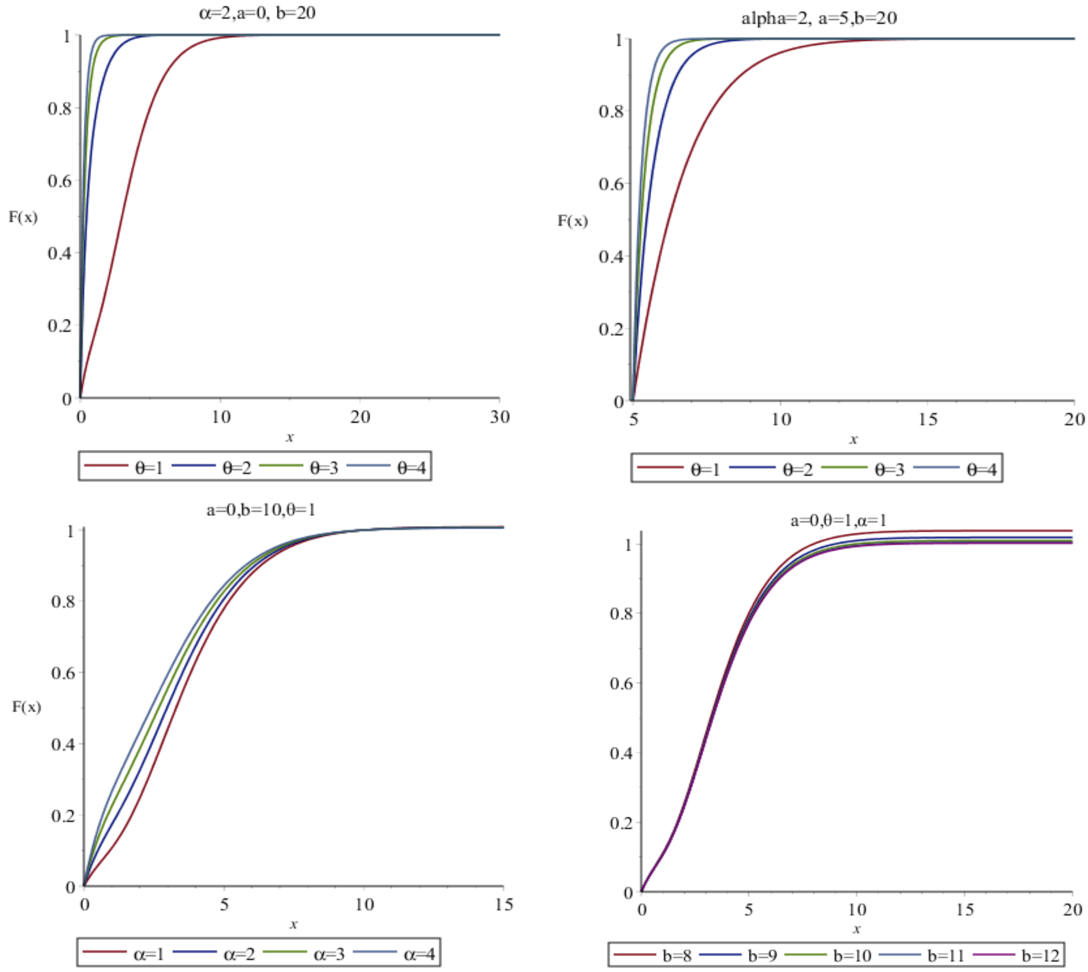

Figure 2. *cdf plots of TTPPD for varying values of parameter*

III. Survival and Hazard function

$S(x)$ and $h(x)$ are the survival function and hazard function respectively, which are defined as:

$$S(x) = 1 - F(x)$$

$$S(x;\theta,\alpha) = \frac{\left((\theta^3 x^3 + \alpha\theta^4 + 3x^2\theta^2 + 6x\theta + 6)e^{-\theta x} - (\theta^3 b^3 + \alpha\theta^4 + 3b^2\theta^2 + 6b\theta + 6)e^{-\theta b}\right)}{\left((\theta^3 a^3 + \alpha\theta^4 + 3a^2\theta^2 + 6a\theta + 6)e^{-\theta a} - (\theta^3 b^3 + \alpha\theta^4 + 3b^2\theta^2 + 6b\theta + 6)e^{-\theta b}\right)}$$

$$h(x) = \frac{f(x)}{S(x)}$$

$$h(x) = \frac{\theta^4 (x^3 + \theta\alpha)e^{-\theta x}}{\left((\theta^3 x^3 + \alpha\theta^4 + 3x^2\theta^2 + 6x\theta + 6)e^{-\theta x} - (\theta^3 b^3 + \alpha\theta^4 + 3b^2\theta^2 + 6b\theta + 6)e^{-\theta b}\right)}$$

From the equation, It is notice that $h(x)$ is independent of parameter 'a'

Behavior of hazard function of TTPPD for varying values of parameter is presented in figure3:

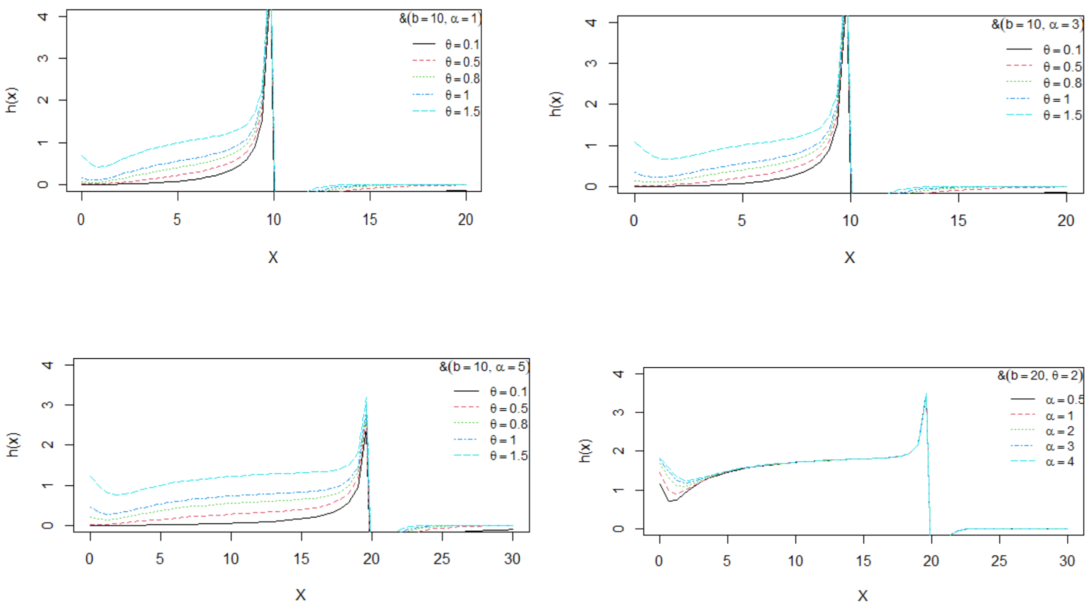

Figure 3. *h(x) plots of TTPPD for varying values of parameter*

IV. Moments and Mathematical Properties

Theorem: Suppose X follows doubly TTPPD(θ,α,a,b). Then the r th moment about origin μ_r' of TTPPD is

$$\mu_r' = \frac{\theta^4\alpha\{\gamma(r+1,\theta b)-\gamma(r+1,\theta a)\}+\{\gamma(r+4,\theta b)-\gamma(r+4,\theta a)\}}{\theta^r\begin{pmatrix}(a^3\theta^3+3a^2\theta^2+6a\theta+\alpha\theta^4+6)e^{-\theta a}-\\(b^3\theta^3+3b^2\theta^2+6b\theta+\alpha\theta^4+6)e^{-\theta b}\end{pmatrix}};r=1,2,3,...$$

Proof: Considering $K=\begin{Bmatrix}(a^3\theta^3+3a^2\theta^2+6a\theta+\alpha\theta^4+6)e^{-\theta a}-\\(b^3\theta^3+3b^2\theta^2+6b\theta+\alpha\theta^4+6)e^{-\theta b}\end{Bmatrix}$

in (2.1), we have

$$\mu_r' = \frac{\theta^4}{K}\int_a^b x^r\left(\theta\alpha+x^3\right)e^{-\theta x}dx$$

$$=\frac{\theta^4}{K}\left[\int_a^b \alpha\theta e^{-\theta x}x^r dx+\int_a^b e^{-\theta x}x^{r+3}dx\right]$$

Taking $u=\theta x, x=\dfrac{u}{\theta}$

$$= \frac{\theta^4}{K}\left[\begin{array}{l}\frac{\theta\alpha}{\theta^{r+1}}\left\{\int_0^{\theta b} e^{-u}x^r du - \int_0^{\theta a} e^{-u}x^r du\right\} + \\ \frac{1}{\theta^{r+4}}\left\{\int_0^{\theta b} e^{-u}u^{r+3}du - \int_0^{\theta a} e^{-u}x^{r+3}du\right\}\end{array}\right]$$

Where $\gamma(\alpha, z) = \int_0^z e^{-x}x^{\alpha-1}dx, \alpha > 0, x > 0$ is the lower incomplete gamma function.

$$= \frac{\theta^4}{K}\left[\frac{\alpha\{\gamma(r+1,\theta b)-\gamma(r+1,\theta a)\}}{\theta^r} + \frac{\gamma(r+4,\theta b)-\gamma(r+4,\theta a)}{\theta^{r+4}}\right]$$

$$= \frac{1}{K}\left[\frac{\theta^4\alpha\{\gamma(r+1,\theta b)-\gamma(r+1,\theta a)\}+\{\gamma(r+4,\theta b)-\gamma(r+4,\theta a)\}}{\theta^r}\right]$$

$$= \frac{\theta^4\alpha\{\gamma(r+1,\theta b)-\gamma(r+1,\theta a)\}+\{\gamma(r+4,\theta b)-\gamma(r+4,\theta a)\}}{\theta^r\left(\begin{array}{l}(a^3\theta^3+3a^2\theta^2+6a\theta+\alpha\theta^4+6)e^{-\theta a} - \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\alpha\theta^4+6)e^{-\theta b}\end{array}\right)}$$

(4.1)

Now taking $r = 1, 2$, mean and variance can be obtained as

$$\mu_1' = \frac{\theta^4\alpha\{\gamma(2,\theta b)-\gamma(2,\theta a)\}+\{\gamma(5,\theta b)-\gamma(5,\theta a)\}}{\theta\left(\begin{array}{l}(a^3\theta^3+3a^2\theta^2+6a\theta+\alpha\theta^4+6)e^{-\theta a} - \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\alpha\theta^4+6)e^{-\theta b}\end{array}\right)}$$

$$\mu_2' = \frac{\theta^4\alpha\{\gamma(3,\theta b)-\gamma(3,\theta a)\}+\{\gamma(6,\theta b)-\gamma(6,\theta a)\}}{\theta^2\left(\begin{array}{l}(a^3\theta^3+3a^2\theta^2+6a\theta+\alpha\theta^4+6)e^{-\theta a} - \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\alpha\theta^4+6)e^{-\theta b}\end{array}\right)}$$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

Similarly rest two moment of origin as well as coefficient of variation, coefficient of skewness, coefficient of kurtosis and Index of dispersion can be obtained, substituting $r = 3, 4$ in the equation (4.1), which are as follows:

$$\mu_3' = \frac{\theta^4\alpha\{\gamma(4,\theta b)-\gamma(4,\theta a)\}+\{\gamma(7,\theta b)-\gamma(7,\theta a)\}}{\theta^3\left(\begin{array}{l}(a^3\theta^3+3a^2\theta^2+6a\theta+\alpha\theta^4+6)e^{-\theta a} - \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\alpha\theta^4+6)e^{-\theta b}\end{array}\right)}$$

$$\mu_4' = \frac{\theta^4\alpha\{\gamma(5,\theta b)-\gamma(5,\theta a)\}+\{\gamma(8,\theta b)-\gamma(8,\theta a)\}}{\theta^4\left(\begin{array}{l}(a^3\theta^3+3a^2\theta^2+6a\theta+\alpha\theta^4+6)e^{-\theta a} - \\ (b^3\theta^3+3b^2\theta^2+6b\theta+\alpha\theta^4+6)e^{-\theta b}\end{array}\right)}$$

Coefficient of Variation= $\frac{\left(\mu_2' - (\mu_1')^2\right)^{1/2}}{\mu_1'}$, Coefficient of Skweness= $\frac{\left(\mu_3' + 3\mu_2'\mu_1' - (\mu_1')^2\right)}{(\mu_2' - (\mu_1')^2)^{3/2}}$, Coefficient of Kurtosis= $\dfrac{\left(\mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4\right)}{(\mu_2' - (\mu_1')^2)^2}$,

Index of dispersion= $\dfrac{\left(\mu_2' - (\mu_1')^2\right)}{\mu_1'}$, graph of above measures are presented in figure 4,5,6,7,8 &9.

Expressions of other central moments are not being given here because they have lengthy expressions. However, they can be easily obtained.

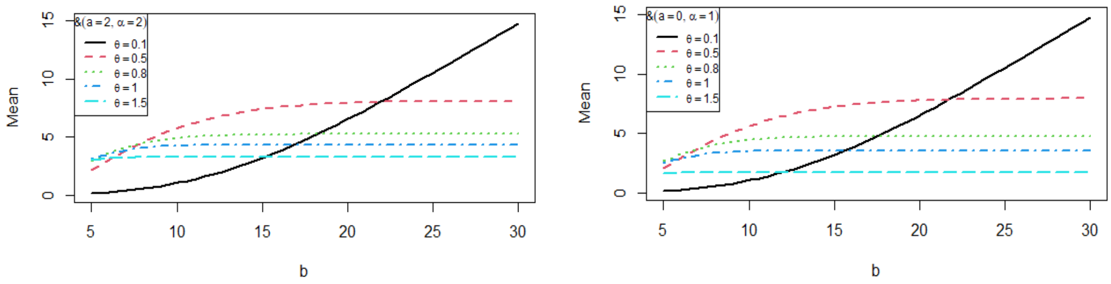

Figure4. *Mean of TTPPD on varying value of parameters*

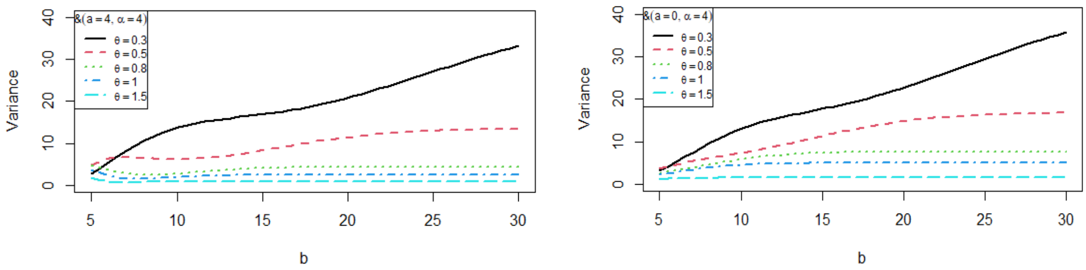

Figure5.*Variance of TTPPD on varying value of parameters*

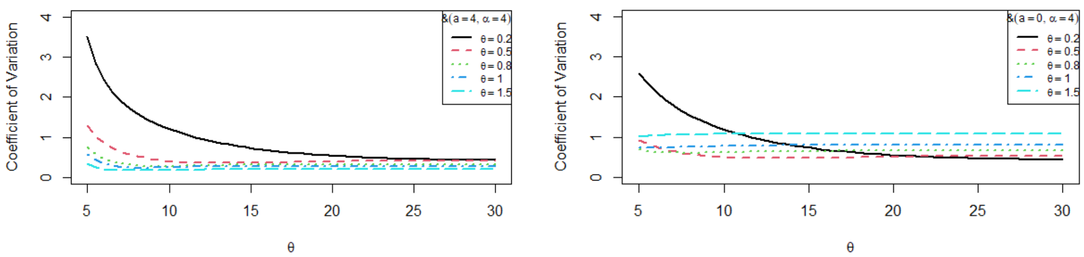

Figure 6.*Variance of TTPPD on varying value of parameters*

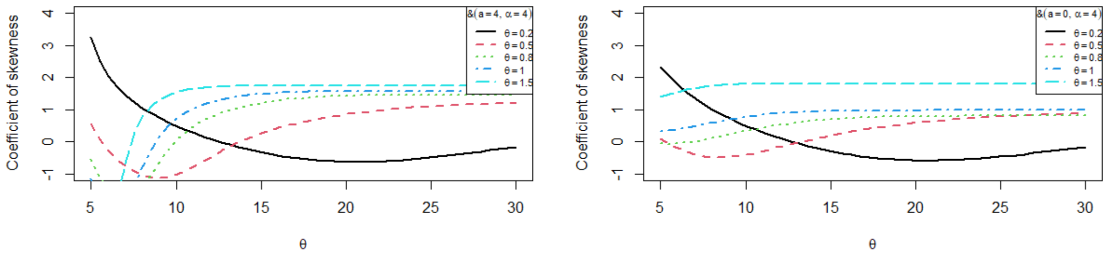

Figure7.*Variance of TTPPD on varying value of parameters*

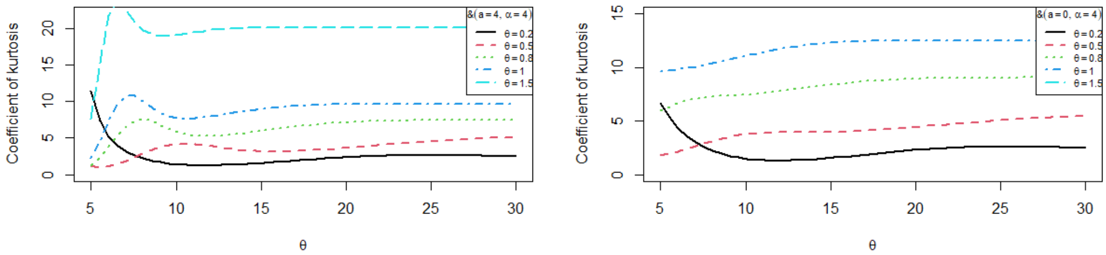

Figure8.*Variance of TTPPD on varying value of parameters*

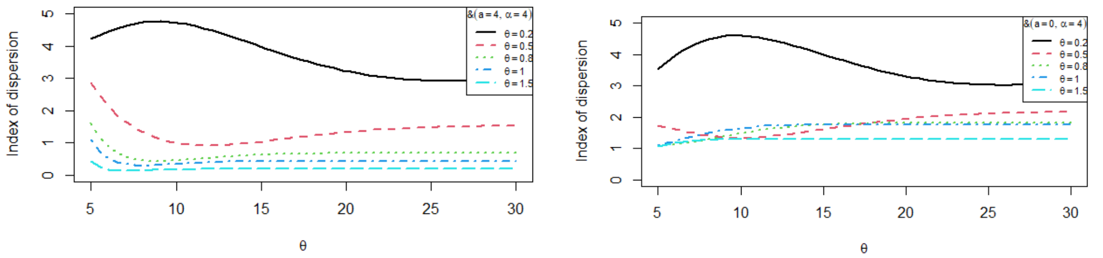

Figure9.*Variance of TTPPD on varying value of parameters*

V. Maximum likelihood Method of Estimation

Let $\left(x_1, x_2, x_3, \ldots, x_n\right)$ be a random sample of size n from (1.1). The likelihood function, L of Pranav distribution is given by

$$L=\left(\frac{\theta^4}{\left((\theta^3a^3+\alpha\theta^4+3a^2\theta^2+6a\theta+6)e^{-\theta a}-(\theta^3b^3+\alpha\theta^4+3b^2\theta^2+6b\theta+6)e^{-\theta b}\right)}\right)^n\prod_{i=1}^{n}\left(\alpha\theta+x_i^3\right)e^{-n\theta\bar{x}}$$

and its log likelihood function is thus obtained as

$$\ln L=n\ln\left(\frac{\theta^4}{\left((\theta^3a^3+\alpha\theta^4+3a^2\theta^2+6a\theta+6)e^{-\theta a}-(\theta^3b^3+\alpha\theta^4+3b^2\theta^2+6b\theta+6)e^{-\theta b}\right)}\right)+$$

$$\sum_{i=1}^{n}\ln\left(\alpha\theta+x_i^3\right)-n\theta\bar{x}$$

Taking $\hat{a}=\min\left(x_1,x_2,x_3,\ldots,x_n\right)$, $\hat{b}=\max\left(x_1,x_2,x_3,\ldots,x_n\right)$, the maximum likelihood estimate $\hat{\theta}$ of parameter θ is the solution of the log-likelihood equation $\frac{\partial \log L}{\partial \theta} = 0$. It is obvious that $\frac{\partial \log L}{\partial \theta} = 0$ will not be in closed form and hence some numerical optimization technique can be used e the equation for θ. In this paper the nonlinear method available in R software has been used to find the MLE of the parameter θ.

VI. Simulation Study

In this section, simulation of study of (2.1) has been carried out. Acceptance and Rejection method has been used to generate random number. Bias Error and Mean square Error have been calculated for varying values parameters θ and α whereas parameter a and b kept constant.

Table 1. *Simulation of TSD at a=10, b=100,* $\theta = 0.1$ *and* $\alpha = 1$

Sample Size (n)	θ	α	Bias Error(θ)	MSE(θ)	Bias Error(α)	MSE(α)
20	0.1	1.0	0.035225	0.024817	1.121145	25.13931
	0.5	2.0	0.015225	0.004636	1.071145	22.94702
	1.0	3.0	-0.00977	0.001911	1.021145	20.85473
	1.5	4.0	-0.03477	0.024186	0.971145	18.86244
40	0.1	1.0	0.016613	0.01104	1.228047	60.32398
	0.5	2.0	0.006613	0.001749	1.203047	57.89288
	1.0	3.0	-0.00589	0.001386	1.178047	55.51179
	1.5	4.0	-0.01839	0.013523	1.153047	53.18069
60	0.1	1.0	0.011075	0.00736	0.818698	40.21598
	0.5	2.0	0.004409	0.001166	0.802031	38.59525
	1.0	3.0	-0.00392	0.000924	0.785365	37.00786
	1.5	4.0	-0.01226	0.009016	0.768698	35.4538
80	0.1	1.0	0.013449	0.014469	0.010876	0.009463
	0.5	2.0	0.008449	0.00571	-0.00162	0.000211
	1.0	3.0	0.002199	0.000387	-0.01412	0.01596
	1.5	4.0	-0.00405	0.001313	-0.02662	0.056708

Table 2. *Simulation of TSD at a=10, b=100,* $\theta = 0.5$ *and* $\alpha = 2$

Sample Size (n)	θ	α	Bias Error(θ)	MSE(θ)	Bias Error(α)	MSE(α)
20	0.1	1.0	0.031624	0.020001	1.096027	24.0255
	0.5	2.0	0.011624	0.002702	1.046027	21.88345
	1.0	3.0	-0.01338	0.003578	0.996027	19.84139
	1.5	4.0	-0.03838	0.029454	0.946027	17.89934
40	0.1	1.0	0.016796	0.011285	0.764069	23.35205
	0.5	2.0	0.006796	0.001848	0.739069	21.84891
	1.0	3.0	-0.0057	0.001301	0.714069	20.39577
	1.5	4.0	-0.0182	0.013255	0.689069	18.99264
60	0.1	1.0	0.015861	0.015095	0.059787	0.214469
	0.5	2.0	0.009195	0.005073	0.04312	0.111562
	1.0	3.0	0.000861	4.45E-05	0.026454	0.041988
	1.5	4.0	-0.00747	0.00335	0.009787	0.005747
80	0.1	1.0	0.012453	0.012405	0.027456	0.060306
	0.5	2.0	0.007453	0.004443	0.014956	0.017894
	1.0	3.0	0.001203	0.000116	0.002456	0.000483
	1.5	4.0	-0.00505	0.002038	-0.01004	0.008071

Table 3. *Simulation of TSD at a=10, b=100,* $\theta=1$ *and* $\alpha=3$

Sample Size (n)	θ	α	Bias Error(θ)	MSE(θ)	Bias Error(α)	MSE(α)
20	0.1	1.0	0.03261	0.021269	0.989088	19.56588
	0.5	2.0	0.01261	0.00318	0.939088	17.63771
	1.0	3.0	-0.01239	0.00307	0.889088	15.80953
	1.5	4.0	-0.03739	0.02796	0.839088	14.08136
40	0.1	1.0	0.01753	0.012292	0.631985	15.97622
	0.5	2.0	0.00753	0.002268	0.606985	14.73725
	1.0	3.0	-0.00497	0.000988	0.581985	13.54828
	1.5	4.0	-0.01747	0.012208	0.556985	12.40931
60	0.1	1.0	0.015636	0.014669	0.071319	0.305184
	0.5	2.0	0.008969	0.004827	0.054652	0.179212
	1.0	3.0	0.000636	2.43E-05	0.037986	0.086574
	1.5	4.0	-0.0077	0.003555	0.021319	0.02727
80	0.1	1.0	0.012021	0.011561	0.044151	0.155947
	0.5	2.0	0.007021	0.003944	0.031651	0.080144
	1.0	3.0	0.000771	4.76E-05	0.019151	0.029342
	1.5	4.0	-0.00548	0.002401	0.006651	0.003539

Table 4. *Simulation of TSD at a=10, b=100 ,* $\theta=1.5$ *and* $\alpha=4$

Sample Size (n)	θ	α	Bias Error(θ)	MSE(θ)	Bias Error(α)	MSE(α)
20	0.1	1.0	0.052018	0.054117	0.279555	1.563022
	0.5	2.0	0.032018	0.020503	0.229555	1.053912
	1.0	3.0	0.007018	0.000985	0.179555	0.644801
	1.5	4.0	-0.01798	0.006467	0.129555	0.335691
40	0.1	1.0	0.035047	0.04913	0.055135	0.121594
	0.5	2.0	0.025047	0.025093	0.030135	0.036325
	1.0	3.0	0.012547	0.006297	0.005135	0.001055
	1.5	4.0	4.66E-05	8.68E-08	-0.01987	0.015785
60	0.1	1.0	0.02594	0.040373	0.017838	0.019092
	0.5	2.0	0.019273	0.022288	0.001172	8.24E-05
	1.0	3.0	0.01094	0.007181	-0.01549	0.014406
	1.5	4.0	0.002607	0.000408	-0.03216	0.062062
80	0.1	1.0	0.016822	0.022637	0.059504	0.283254
	0.5	2.0	0.011822	0.01118	0.047004	0.176747
	1.0	3.0	0.005572	0.002483	0.034504	0.09524
	1.5	4.0	-0.00068	3.68E-05	0.022004	0.038733

VII. Applications on Life time data

In this section, TTPPD has been applied on following three data sets, where maximum likelihood method of estimation has been used for estimation of its parameter. Parameter θ is estimated whereas another parameters a, and b are considered as lowest and highest values of data. i. e. $a=\min(x)$ and $b=\max(x)$, where x is data set. Goodness of fit has been decided using Akaike information criteria (AIC), Bayesian Information criteria (BIC) and Kolmogorov Simonov test (KS) values respectively, which are calculated for each distribution and also compared with p-value. As we know that best goodness of fit of the distribution can be decide on the basis minimum value of

KS, AIC and BIC and maximum p-value for K.S.

Data Set 1: The data is given by Birnbaum and Saunders (1969) on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 100 observations with maximum stress per cycle 31,000 psi. The data ($\times 10^{-3}$) are presented below (after subtracting 65**).**

5 25 31 32 34 35 38 39 39 40 42 43
43 43 44 44 47 47 48 49 49 49 51 54
55 55 55 56 56 56 58 59 59 59 59 59
63 63 64 64 65 65 65 66 66 66 66 66
67 67 67 68 69 69 69 69 71 71 72 73
73 73 74 74 76 76 77 77 77 77 77 77
79 79 80 81 83 83 84 86 86 87 90 91
92 92 92 92 93 94 97 98 98 99 101 103
105 109 136 147

Data Set 2: This data set is the strength data of glass of the aircraft window reported by Fuller *et al* (1994):

18.83 20.8 21.657 23.03 23.23 24.05 24.321 25.5 25.52 25.8 26.69 26.77
26.78 27.05 27.67 29.9 31.11 33.2 33.73 33.76 33.89 34.76 35.75 35.91
36.98 37.08 37.09 39.58 44.045 45.29 45.381

Table 5: *MLE's, - 2ln L, AIC, K-S and p-values of the fitted distributions for data set-1*

Distributions	ML Estimates	$-2\ln L$	AIC	BIC	K-S	p-value
TTPPD	$\theta = 0.05527$ $\alpha = 2.02011$	**927.37**	**931.37**	**930.14**	**0.136**	0.056
TTPLD	$\theta = 0.02238$ $\alpha = 15.80197$	957.94	961.94	967.15	0.191	**0.001**
TPPD	$\theta = 0.05853$ $\alpha = 2.13206$	934.06	938.06	940.93	0.173	**0.005**
TPWD	$\theta = 0.00272$ $\theta = 1.39558$	989.35	993.35	998.56	0.294	**0.000**
TAD	$\theta = 0.03917$	939.13	941.13	942.05	0.153	**0.017**
TLD	$\theta = 0.02199$	958.88	960.88	962.31	0.186	**0.001**
Pranav	$\theta = 0.05853$	934.06	936.06	937.49	0.167	**0.007**
Lindley	$\theta = 0.02886$	983.10	985.10	986.54	0.252	**0.000**
Exponential	$\theta = 0.01463$	**1044.87**	**1046.87**	**1048.30**	**0.336**	**0.000**

Table 6: *MLE's, - 2ln L, AIC, K-S and p-values of the fitted distributions for data set-2*

Distributions	ML Estimates	$-2\ln L$	AIC	BIC	K-S	p-value
TTPPD	$\theta = 0.12066$ $\alpha = 0.5999$	**201.80**	**205.80**	**204.57**	**0.107**	0.829
TTPLD	$\theta = 0.12981$ $\alpha = 2.17254$	232.77	236.77	239.64	0.282	**0.011**
TPPD	$\theta = 0.12981$ $\alpha = 1.9946$	232.77	236.77	239.64	0.282	**0.011**
TPWD	$\theta = 0.00203$ $\theta = 1.80566$	241.61	245.61	247.61	0.353	**0.000**
TAD	$\theta = 0.08776$	201.96	203.96	205.58	0.112	**0.786**
TLD	$\theta = 0.05392$	202.18	204.18	205.61	0.117	**0.738**
Pranav	$\theta = 0.12981$	232.77	234.77	236.20	0.267	**0.019**
Lindley	$\theta = 0.06299$	253.98	255.98	256.98	0.365	**0.000**
Exponential	$\theta = 0.032452$	**274.52**	**276.52**	**277.52**	**0.458**	**0.000**

VIII. Conclusions

In this paper, Truncated Two Parameter Distribution (TTPPD) has been proposed. Its mathematical and statistical properties including coefficient of variation, skewness, kurtosis and Index of dispersion have been derived and presented graphically. Maximum likelihood method has been used for estimation of its parameters. Goodness of fit of TTPPD has been discussed with two life time data sets and compared with truncated two parameter Lindley distribution (TTPLD), two parameter Weibull distribution (TPWD), two parameter Pranav distribution (TPPD), truncated Akash distribution (TAD), truncated Lindley distribution (TLD), Pranav, Lindley and Exponential distribution. It has been observed from above results, TTPPD gives good fit over above mentioned distributions on both the data sets.

References

[1] Aryuyuen, S.(2018), Truncated Two parameter Lindley distribution and application, *The Journal of Applied Science,* 17(1),19-32

[2] Aryuyuen,S. and Bodhisuwan, W. (2018). The truncated power Lomax distribution: Properties and applications, *Walailak Journal of Science and Technology*, 16 , 655-668.

[3] Birnbaum, Z.W., Saunders, S.C. (1969), Estimation for a family of life distribution with applications to fatigue. *Journal of applied probability*, 6(2), 328-347.

[4] Fuller, E.J., Frieman, S., Quinn, J., Quinn, G., and Carter, W.(1994): Fracture mechanics approach to the design of glass aircraft windows: A case study, SPIE Proc 2286, 419-430.

[5] Ghitany ME, Atieh B, Nadarajah S. Lindley distribution and its Applications. *Mathematics Computing and Simulation.* 2008;78:493– 506.

[6] Johnson, N. L., Kotz,S. and Balakrishnan,N. (1994), Continuous Univariate Distributions, Wiley, New York (1994).

[7] Lindley, D.V. (1958): Fiducial distributions and Bayes' Theorem, *Journal of the Royal Statistical Society*, Series B, 20, 102 – 107.

[8] Shukla, K. K. and Shanker, R., The Truncated Akash distribution: properties and applications communicated.

[9] Shukla, K. K (2018), Pranav distribution with Properties and Applications, Biometrics and Biostatistics International Journal,7(3),244-254.

[10] Singh,S. K., Singh,U. and Sharma, V.K. (2014), The truncated Lindley distribution: Inference and application, *Journal of Statistics Applications & Probability*, 3, 219-228.

[11] Umeh,E. and Ibenegbu, A. (2019): A two parameter Pranav distribution with properties and its application, *J. Biostat. Epidemiol*. 5(1),74-90

[12] Zange,Z., and Xie, M., (2011),On the upper truncated Weibull distribution and its reliability implications, *Reliability Engineering & System Safety*, 96 , 194-200.

[13] Zaninetti,L. and Ferraro,M. (2008), On the truncated Pareto distribution with applications, *Central European Journal of Physics*, 6 , 1-6.

Received: June 21, 2020
Accepted: September 01, 2020