Optimal order quantity and pricing policies for EPL model with rework and quality inspection

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Abstract

This article develops a three-echelon supply chain coordination policy composing of a supplier, a manufacturer, and a wholesaler. The economic production lot-size (EPL) model comprises of perfect and defective items, quality inspection, return policy and reworking of defective items by the manufacturer. The defective items produced are reworked at a cost just after the regular production time. Here, return policy is considered between the outside supplier and the supplier, and the manufacturer and the wholesaler. Also, we considered the production cost as a function of production rate. We have formulated the profit functions of each member of the supply chain and optimized the total profit function of the whole supply chain system. Next, we have shown that the profit function of each member is concave. A numerical example with graphical representation presented to illustrate the proposed model.

Keywords: Supply chain, Rework, Inspection error, Return policy, quality inspection
AMS Subject Classification: 90B05, 90B30, 90B50

1 Introduction

Ordering and pricing decisions play an important role in optimizing the costs and profits of supply chain members. Recently, many researchers were executed in the area of the supply chain by applying ordering and pricing decisions. Also, the demand of customers is usually price-sensitive and they are looking for the lower price therefore firm attract customers by utilizing optimal pricing policies. In the study of supply chain network, the members of the network like supplier, manufacturer, wholesaler and retailer play an equal role and are interconnected for the proper functioning of the supply chain. To highlight such type of phenomenon, Sana (2011) developed an integrated production-inventory model for supplier, manufacturer and retailer consisting of perfect and imperfect quality items. He maximized total profit of supply chain in coordination with production and inventory decisions. Banerjee (1986) shows that the joint optimal ordering policy, together with an appropriate price adjustment, can be economically beneficial for both the purchaser and the vendor. Ben-Daya (1999) discussed a multi-stage lot sizing models for imperfect production processes.

Khan *et al.* (2014) presented a simple but integrated mathematical model that incorporates quality inspection errors at the buyer’s end and learning in production at the vendor’s end for determining an optimal vendor-buyer inventory policy. Khanna *et al.* (2017) developed a finite production model to inquire the optimal production quantity for maximizing the total expected profit per unit time. They constructed model in an imperfect production, imperfect inspection and imperfect rework environment. An inventory model with a money back agreement between the retailer and the supplier means defective products are returned to the supplier via the vehicle that brings new order, was considered by Vishkaei *et al.* (2014) instead of selling defective items at a lower price at the end of the period. Eroglu and Ozdemir (2007) addressed an EOQ model for defective items which are classified as scraps and imperfects, they are sold on a discounted selling price as a single lot. Authors assumed defective rate is a random variable with uniformly distributed.

Further, Priyan and Manivannan (2016); Noori-daryan and Taleizadeh (2018); Wang Chiu (2006) investigated the concept of inspection, random defective rate, return policy and rework process for different situations. In the classical economic production lot size (EPL) model, it is often assumed that the production cost is constant and predetermined, and items produced are of perfect quality. In reality, unit production cost depend on the production rate. The cost of the production process decreases with increased production rate. During production, both perfect and imperfect quality items are produced, and imperfect quality items are reworked or send back. Khouja and Mehrez (1994) developed an EPL model with unit production costs depending on production rate. They considered production rate as a decision variable. In this paper, result shows that optimal production rate may be different from the production rate that minimizes unit production cost.

Taleizadeh and Noori-daryan (2014) proposed a mathematical model which shows that by increasing the ordering cost, the number of shipments received by the supplier and the producer is decreased and the total cost of the supply chain network is increased. Khan *et al.* (2011) used the similar approach of Salameh and Jabeh (2000) to produce an optimal order production quantity with imperfect items. They incorporated the screening costs more accurately in the economic order sizing decisions. Huang *et al.* (2011) developed a model as a three-level dynamic non-cooperative game composed of multiple suppliers, a single manufacturer and multiple retailers. They used both analytical and computational methods for the derivative of the optimal decisions of all members of the supply chain.

In manufacturing industries, researchers and practitioners gives importance to develop and pricing policies and inventory control models in supply chain management. In this direction, some notable researches were addressed by Khedlekar and Singh (2019); Liu et al. (2009); Cárdenas-Barrón (2008,2009) and others. Pal *et al.* (2012) showed that the integrated profit function is more profitable compared to the profit of the whole chain. In this paper, production system may follows a probability density function. Mukherjee *et al.* (2019) investigated learning effect in production and investment process for quality improvement at the vendor's end, and lot-size dependent lead-time at the buyer's end. They observed that the expected annual total cost and investment required to improve the production process quality decrease, as the value of learning exponent increases. Barzegar Astanjin and Sajadieh (2017) shows that a coordination in supply chains is more beneficial if the defective rate is high. The investment in quality does not essentially leads to reduced costs, while the coordination of decision making strengthen such policies. In this paper, we extend EPL model by considering production cost as a function of production rate with reworkable items.

2 Model Assumptions and Notations

2.1 Notations

1. The level of positive inventory at time t of the supplier
2. The level of positive inventory at time t of the manufacturer
3. The level of positive inventory at time t of the wholesaler
4. The supplier's selling price per unit
5. The inventory holding cost per unit per unit time at supplier
6. The inventory holding cost per unit per unit time at manufacturer

7. The inventory holding cost per unit per unit time at wholesaler
8. The ordering cost of the supplier per order
9. The ordering cost of the manufacturer per order
10. The ordering cost of the wholesaler per order
11. The inspection cost per item of the supplier
12. The inspection cost per item of the manufacturer
13. The inspection cost per item of the wholesaler
14. The production rate of the manufacturer per unit time
15. The reworking rate of the defective items by the manufacturer per unit time, $P' = zP$, $z = 1$ (say)
16. The purchasing cost of raw material per item of the supplier
17. Buyback/repurchase price per returned items of the supplier per item
18. Reduced price of the defective items per unit, $p'_m = xp_m$
19. Buyback price of the returned items of the wholesaler per item, $p'_w = yp_w$
20. The proportion of defective items at the supplier, $0 \leq \alpha < 1$
21. The proportion of defective items at the manufacturer, $0 \leq \beta < 1$
22. The proportion of defective items at the wholesaler, $0 \leq \gamma < 1$
23. The cycle length of the supplier
24. The cycle length of the manufacturer
25. The cycle length of the wholesaler
26. The cycle length of the manufacturer for production and selling both
27. The cycle length of the manufacturer for reworking and selling both
28. The cycle length of the manufacturer for selling
29. The cycle length of the wholesaler for collecting and selling both
30. The cycle length of the wholesaler for selling
31. The manufacturer's suggested retail price (MSRP)
32. The manufacturer's demand rate per unit time, where $D_m = a - bp_s$
33. The wholesaler's demand rate per unit time, where $D_w = a - bp_m + \theta(M_p - p_m)$
34. The customer's demand rate per unit time, where $D_c = a - bp_w$
35. The market potential
36. The price sensitivity factor of demand
37. The sensitivity which determines the effect on demand when a wholesaler sells product below or above MSRP
38. The production cost per unit
39. The fixed cost like labour, energy and technology cost
40. The variation constant of tool/die costs
41. The total profit of the supplier per unit time, ($)
42. The total profit of the manufacturer per unit time, ($)
43. The total profit of the wholesaler per unit time, ($)
44. The total profit of the supply chain per unit time, ($)

Decision variables:

1. The ordering quantity(lot size) of the supplier per cycle
2. The manufacturer's selling price per unit
3. The wholesaler's selling price per unit

2.2 Assumptions

A single type of item is developed in the model. Shortage is not allowed. The model consists of a supplier, a manufacturer and a wholesaler. The demand is constant and price-sensitive. We considered a return policy between a supplier and a manufacturer; a manufacturer and a wholesaler. The manufacturer does rework after manufacturing with the same rate.

3 The Mathematical Model

Fig. 1 represents the outline of the proposed model. We considered a supplier, a manufacturer and a wholesaler. In this section we developed the profit function of each member of the supply chain. The total inventory system is depicted in Fig. 2.

3.1 The Supplier's Model

Firstly, supplier inspects all the raw material received from the outside supplier. Then, the supplier returns the defective items to the outside supplier and supplied good quality items to the manufacturer at a demand rate D_m. Suppose $I_s(t)$ is on hand inventory of the supplier at time t.

Initially, the inventory cycle begins with maximum stock-level $(1-\alpha)Q$ at $t=0$. During the time interval $[0, T_s]$, the stock-level decreases due to price-dependent demand. Finally, inventory level becomes zero at $t = T_s$. The differential equation satisfies in time interval $[0, T_s]$ is as follows

$$\frac{dI_s(t)}{dt} = -D_m, \ 0 \le t \le T_s \tag{1}$$

with boundary condition $I_s(0) = (1-\alpha)Q$ and $I_s(T_s) = 0$.

Condition: $(1-\alpha)Q \ge D_m T_s$

Solving Eq. (1), we obtain inventory level $I_s(t)$ as

$$I_s(t) = (1-\alpha)Q - D_m t; \ \ 0 \le t \le T_s \tag{2}$$

At $t = T_s$, above equation becomes

$$(1-\alpha)Q - D_m T_s = 0$$

$$T_s = \frac{(1-\alpha)Q}{D_m} \tag{3}$$

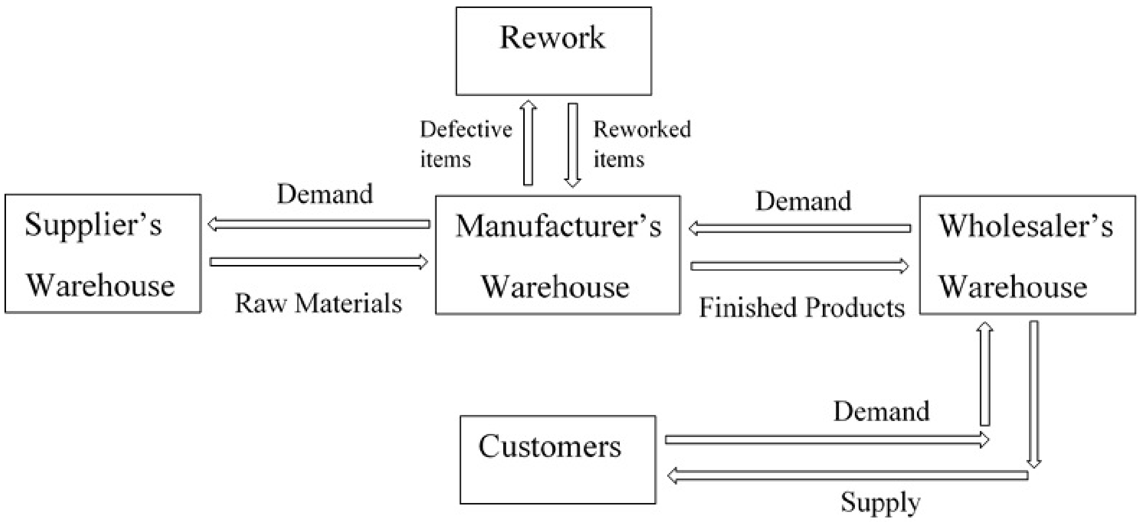

Figure 1: A three-level supply chain

Now, the total profit consists the following values:

Holding cost of the supplier

$$HC = \frac{1}{T_s} C_h^s \int_0^{T_s} I_s(t)\ dt = \frac{C_h^s(1-\alpha)Q}{2}$$

Ordering cost

$$OC = \frac{C_o^s}{T_s} = \frac{C_o^s D_m}{(1-\alpha)Q}$$

$$IC = \frac{C_i^s Q}{T_s} = \frac{C_i^s D_m}{(1-\alpha)}$$

Purchasing cost

$$PC = \frac{C_p Q}{T_s} = \frac{C_p D_m}{(1-\alpha)}$$

Hence, The total profit of the supplier is given by

TP_s = sales revenue - purchasing cost - inspection cost - holding cost - annual ordering cost

$$TP_s = \left(p_s + \frac{\alpha}{1-\alpha} C_p'\right) D_m - (C_p + C_i^s)\frac{D_m}{1-\alpha} - \frac{C_h^s(1-\alpha)Q}{2} - \frac{C_o^s D_m}{(1-\alpha)Q} \tag{4}$$

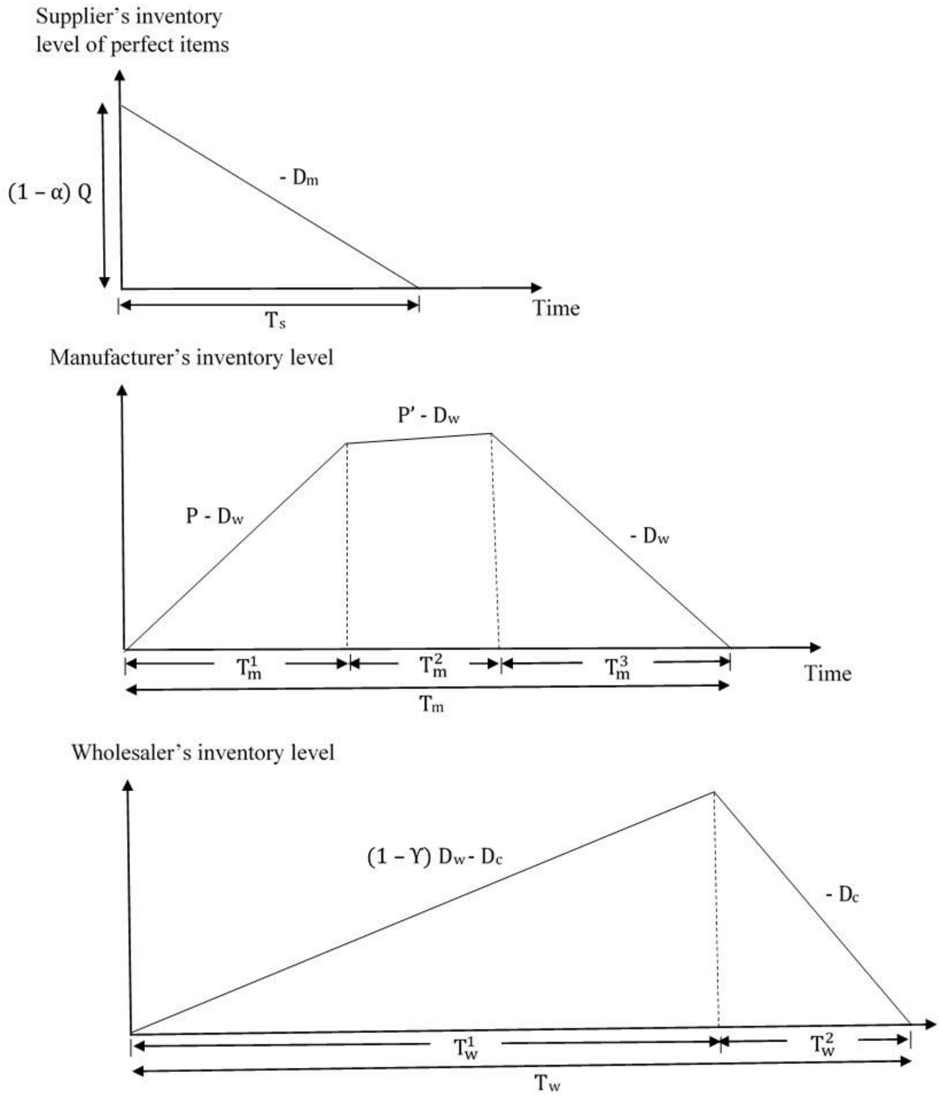

Figure 2: The inventory levels of the supply chain members

3.2 The Manufacturer's Model

The manufacturer produces the finished products from raw materials at a rate P during the time interval $[0, T_m^1]$with satisfying the wholesaler demand at the rate D_w. in the same interval, some defective products are manufactured which are reworked by manufacturer with the same rate P in the interval $[T_m^1, T_m^1 + T_m^2]$. In the interval $[T_m^1 + T_m^2, T_m]$, the inventory level decreases due to wholesaler's demand rate D_w and reaches zero at $t = T_m$. To avoid shortage, we consider $P \geq D_w$. Also, the unit production cost as $C(P) = p_s + \frac{L}{P} + \Gamma P$. Suppose $I_m(t)$ is on hand inventory of the manufacturer at time t. Thus, to represent the manufacturer's inventory system, the governing differential equations are

$$\frac{dI_m(t)}{dt} = (1-\beta)P - D_w, \; 0 \leq t \leq T_m^1 \tag{5}$$

$$\frac{dI_m(t)}{dt} = P - D_w, \; T_m^1 \leq t \leq T_m^1 + T_m^2 \tag{6}$$

$$\frac{dI_m(t)}{dt} = -D_w, \; T_m^1 + T_m^2 \leq t \leq T_m \tag{7}$$

with boundary conditions $I_m(0) = 0$ and $I_m(T_m) = 0$.

Solving Eqs. (5) - (7), we obtain the inventory level $I_m(t)$ as

$$I_m(t) = \{(1-\beta)P - D_w\}t; \quad 0 \leq t \leq T_m^1$$

$$I_m(t) = \{(1-\beta)P - D_w\}T_m^1 + (P - D_w)(t - T_m^1); \quad T_m^1 \leq t \leq T_m^1 + T_m^2$$

$$I_m(t) = D_w(T_m - t); \quad T_m^1 + T_m^2 \leq t \leq T_m$$

Also,

$$T_m^1 = \frac{(1-\alpha)Q}{P} \tag{8}$$

$$T_m^2 = \frac{\beta(1-\alpha)Q}{P} \tag{9}$$

$$T_m^3 = \frac{(1-\alpha)Q}{D_w}\left[1 - \frac{(1+\beta)D_w}{P}\right] \tag{10}$$

and

$$T_m = \frac{(1-\alpha)Q}{D_w} \tag{11}$$

Now, the total profit of the manufacturer consists the following terms:

Holding cost of the manufacturer

$$HC = \frac{1}{T_m} C_h^m \int_0^{T_m} I_m(t)\, dt = \frac{C_h^m (1-\alpha)Q}{2}\left[1 - (1 + \beta + \beta^2)\frac{D_w}{P}\right]$$

Ordering cost

$$OC = \frac{C_o^m}{T_m} = \frac{C_o^m D_w}{(1-\alpha)Q}$$

Inspection cost

$$IC = \frac{C_i^m (P T_m^1 + P' T_m^2)}{T_m} = C_i^m D_w (1 + \beta z)$$

Production cost

$$PC = \frac{1}{T_m}\left[P \int_0^{T_m^1} C(P)\, dt + P' \int_{T_m^1}^{T_m^1 + T_m^2} C(P')\, dt\right]$$

$$= [C(P) + z\beta C(P')]D_w$$

Supplier cost

$$SC = \frac{p_s(1-\alpha)Q}{T_m} = p_s D_w$$

Hence, the total profit of the manufacturer is given by

TP_m = sales revenue - supplier cost - inspection cost - holding cost - annual ordering cost - production cost

$$\begin{aligned} TP_m &= (p_m - p_s - \gamma p_m')D_w - C_i^m D_w(1 + \beta z) - \frac{C_h^m(1-\alpha)Q}{2}\left[1 - (1 + \beta + \beta^2)\frac{D_w}{P}\right] \\ &- \frac{C_o^m D_w}{(1-\alpha)Q} - [C(P) + z\beta C(P')]D_w \end{aligned} \tag{12}$$

3.3 The Wholesaler's Model

The wholesaler inspects all the products received from the manufacturer and return the defective products back to the manufacturer. In the interval $[0, T_w^1]$, the stock-level at the wholesaler is accumulates at the rate $(1-\gamma)D_w - D_c$. In the interval $[T_w^1, T_w]$, the accumulated inventory depletes with the rate D_c and reaches zero at $t = T_w$. Suppose I_w is on hand inventory at time t. The differential equations in time interval $[0, T_w]$

$$\frac{dI_w(t)}{dt} = (1-\gamma)D_w - D_c,\ \ 0 \le t \le T_w^1 \tag{13}$$

$$\frac{dI_w(t)}{dt} = -D_c,\ T_w^1 \le t \le T_w \tag{14}$$

with the boundary conditions $I_w(0) = 0$ and $I_w(T_w) = 0$.

From Eqs. (13) and (14), we have the inventory level $I_w(t)$ as

$$I_w(t) = \{(1-\gamma)D_w - D_c\}t;\quad 0 \le t \le T_w^1 \tag{15}$$

$$I_w(t) = D_c(T_w - t);\quad T_w^1 \le t \le T_w \tag{16}$$

We have the following equations

$$(1-\gamma)D_w T_w^1 \ = (1-\beta)r_p T_m^1 + \beta r_p T_m^2$$

$$(1-\gamma)D_w T_w^1 \ = (1-\alpha)Q$$

This implies

$$T_w^1 = \frac{(1-\alpha)Q}{(1-\gamma)D_w}$$

$$T_w^2 = \frac{(1-\alpha)Q}{D_c}\left[1 - \frac{D_c}{(1-\gamma)D_w}\right]$$

At $t = T_w^1$ in Equations (15) and (16)

$$\{(1-\gamma)D_w - D_c\}T_w^1 = D_c(T_w - T_w^1)$$

$$T_w = \frac{(1-\alpha)Q}{D_c}$$

Now, the total profit consists the following values:

Holding cost of the wholesaler

$$HC = \frac{1}{T_m} C_h^m \int_0^{T_w} I_w(t)\ dt = \frac{C_h^W(1-\alpha)Q}{2}\left[1 - \frac{D_c}{(1-\gamma)D_w}\right]$$

Ordering cost

$$OC = \frac{C_o^W}{T_w} = \frac{C_o^W D_c}{(1-\alpha)Q}$$

Inspection cost

$$IC = \frac{C_i^W D_w T_w^1}{T_w} = \frac{C_i^W D_c}{(1-\gamma)}$$

Hence, the total profit of the wholesaler is given by

TP_w= sales revenue - inspection cost - holding cost - ordering cost

$$TP_w = (p_w - p_m + \gamma p_w')D_c - \frac{C_i^W D_c}{(1-\gamma)} - \frac{C_h^W(1-\alpha)Q}{2}\left[1 - \frac{D_c}{(1-\gamma)D_w}\right] - \frac{C_o^W D_c}{(1-\alpha)Q} \tag{17}$$

Finally, the total profit of chain is the summation of the total profits of the supplier, the manufacturer and the wholesaler shown in Equation 18,

$$TP = TP_s + TP_m + TP_w \tag{18}$$

so we have

$$\begin{aligned} TP &= \left(p_s + \frac{\alpha}{1-\alpha} C'_p\right)(a - bp_s) - (C_p + C_i^s)\frac{(a-bp_s)}{(1-\alpha)} - \frac{C_h^s(1-\alpha)Q}{2} - \frac{C_o^s(a-bp_s)}{(1-\alpha)Q} \\ &+\{(1-\gamma x)p_m - p_s\}\{a - bp_m + \theta(M_p - p_m)\} - C_i^m\{a - bp_m + \theta(M_p - p_m)\}(1+\beta) \\ &- \frac{C_h^m(1-\alpha)Q}{2}\left[1 - (1+\beta+\beta^2)\frac{\{a-bp_m+\theta(M_p-p_m)\}}{P}\right] \\ &- \frac{C_o^m\{a-bp_m+\theta(M_p-p_m)\}}{(1-\alpha)Q} - \left(p_s + \frac{L}{P} + \gamma P\right)\{a - bp_m + \theta(M_p - p_m)\}(1+\beta) \\ &+\{(1+\gamma y)p_w - p_m\}(a - bp_w) - \frac{C_i^w(a-bp_w)}{(1-\gamma)} - \frac{C_o^w(a-bp_w)}{(1-\alpha)Q} \\ &- \frac{C_h^w(1-\alpha)Q}{2}\left[1 - \frac{(a-bp_w)}{(1-\gamma)\{a-bp_m+\theta(M_p-p_m)\}}\right] \end{aligned} \tag{19}$$

Theorem 4.1 *The supplier's total profit function $TP_s(Q)$ is concave.*

Proof. Concavity of the suppler's total profit can be proved by taking the second-order derivative of $TP_s(Q)$ respect to Q which is strictly negative,

$$\frac{d^2TP_s}{dQ^2} = -\frac{2C_o^s}{(1-\alpha)Q^3}(a - bp_s) < 0.$$

The root of the first derivative of objective function Eq. (4) with respect to Q is global maximum of total profit function (Fig. 3). That means the optimal ordering quantity of supplier is

$$\frac{dTP_s}{dQ} = -\frac{(1-\alpha)C_h^s}{2} + \frac{C_o^s}{(1-\alpha)Q^2}(a - bp_s)$$

this implies,

$$Q^* = \frac{1}{(1-\alpha)}\sqrt{\frac{2C_o^s(a-bp_s)}{C_h^s}}. \tag{20}$$

Theorem 4.2 *The manufacturer's total profit function $TP_m(p_m)$ is concave.*

Proof. Differentiating Eq. (12) with respect to p_m

$$\begin{aligned} \frac{dTP_m}{dp_m} &= -\frac{(1-\alpha)(b+\theta)Q(1+\beta+\beta^2)C_h^m}{2P} + (1+z\beta)(b+\theta)C_i^m + \frac{(b+\theta)C_o^m}{(1-\alpha)Q} \\ &+(1-x\gamma)\{a - bp_m + \theta(M_p - p_m)\} - (b+\theta)\{(1-\gamma x)p_m - p_s\} \\ &+(b+\theta)\left\{\left(p_s + \frac{L}{P} + \gamma P\right) + z\beta\left(p_s + \frac{L}{zP} + \gamma z P\right)\right\} \end{aligned}$$

The necessary condition $\frac{dTP_m}{dp_m} = 0$, for existence of optimal solution, yields

$$\begin{aligned} p_m^* &= \frac{1}{2(x\gamma-1)(b+\theta)}\Big[a(x\gamma - 1) - (1+z\beta)(b+\theta)C_i^m + (x\gamma-1)\theta M_p \\ &-(b+\theta)p_s + \frac{(b+\theta)C_o^m}{Q(\alpha-1)} - \frac{Q(\alpha-1)(1+\beta+\beta^2)(b+\theta)C_h^m}{2P} \\ &- \frac{(b+\theta)}{P}\{L(1+\beta) + P^2(1+z^2\beta)\gamma + P(1+z\beta)p_s\}\Big]. \end{aligned} \tag{21}$$

Also,

$$\frac{d^2TP_m}{dp_m^2} = -2(b+\theta)(1-x\gamma) < 0.$$

Therefore, TP_s is a concave function of p_m and thus (12) provides global maximum profit of the manufacturer.

Theorem 4.3 *The wholesaler's total profit function $TP_w(p_w)$ is concave.*

Proof. Differentiate Eq. (17) with respect to p_w

$$\begin{aligned} \frac{dTP_w}{dp_w} &= \frac{bC_i^w}{(1-\gamma)} + \frac{bC_o^w}{Q(1-\alpha)} - \frac{bQ(1-\alpha)C_h^w}{2(1-\gamma)\{a-bp_m+\theta(M_p-p_m)\}} \\ &+(1+y\gamma)(a - bp_w) - b\{(1+\gamma y)p_w - p_m\} \end{aligned}$$

The necessary condition $\frac{dTP_w}{dp_w} = 0$, for existence of optimal solution, yields

$$p_w^* = \frac{1}{2b(1+y\gamma)}\left[a(1+y\gamma) + bp_m + \frac{bC_i^w}{(1-\gamma)} + \frac{bC_o^w}{Q(1-\alpha)} - \frac{bQ(1-\alpha)C_h^w}{2(1-\gamma)\{a-bp_m+\theta(M_p-p_m)\}}\right]. \tag{22}$$

Further, $\frac{d^2TP_w}{dp_w^2} = -2b(1+y\gamma) < 0$ ensures concavity condition. Substituting the optimal value of the wholesaler's selling price in (17), we have the global maximum profit of the wholesaler.

Theorem 4.4 *The unit production cost function $C(P)$ is convex.*

Proof Since

$$C(P) = p_s + \frac{L}{P} + \gamma P \tag{23}$$

$$\frac{dC(P)}{dP} = -\frac{L}{P^2} + \gamma \tag{24}$$

$$\frac{d^2C(P)}{dP^2} = \frac{2L}{P^3} > 0 \tag{25}$$

$$P = \sqrt{\frac{L}{\gamma}}$$

so, this production rate leads to minimum production cost.

4 Numerical Examples

Example 1. Suppose the input data is as follows: $a = 250$, $b = 0.6$, $p_s = 25$, $C_h^s = 3$, $C_h^m = 4$, $C_h^w = 5$, $C_o^s = 100$, $C_o^m = 250$, $C_o^w = 200$, $C_i^s = 3$, $C_i^m = 2$, $C_i^w = 3$, $P = 100$, $C_p = 10$, $C_p' = 6$, $\alpha = 0.2$, $\beta = 0.5$, $\gamma = 0.1$, $x = 0.5$, $y = 0.4$, $L = 1$, $\Gamma = 0.8$, $M_p = 50$ and $\theta = 0.5$. We check the condition

$$\frac{d^2TP_s}{dQ^2} = -\frac{2C_o^s}{(1-\alpha)Q^3}(a - bp_s) = -0.015339 < 0.$$

Hence the optimal solution is $Q^* = 156.46$, $TP_s^* = 2033.25$ (Fig. 3).

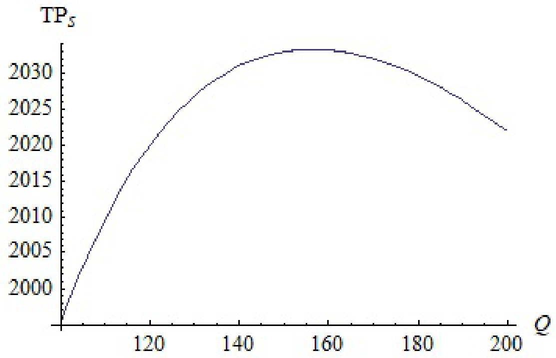

Figure 3: Supplier's total profit versus Q

Example 2. Let us take the values of Example 1 in appropriate units with $Q^* = 156.46$ and apply the solution procedure, to find the optimal solutions $p_m^* = 221.385$, $TP_m^* = 605.331$ (Fig. 4).

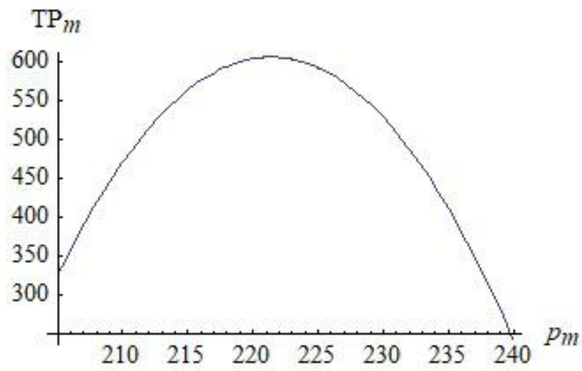

Figure 4: Manufacturer's total profit versus p_m

Example 3. By using above parameters $Q^* = 156.46$, $p_m^* = 221.385$. the optimal values of solution are calculated as: $p_w^* = 311.829$, $TP_w^* = 6545.47$. Fig. 5 illustrate the optimal price p_w^* and the optimal wholesaler's profit TP_w^*.

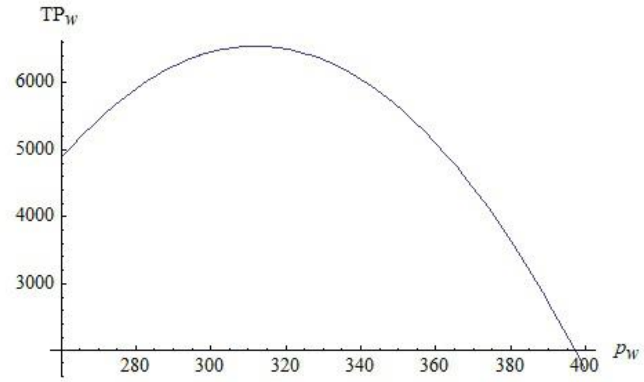

Figure 5: Wholesaler's total profit versus p_w

5 Conclusions

We developed a three-echelon supply chain including a supplier, a manufacturer and a wholesaler. The wholesaler order his demands to the manufacturer and the manufacturer order his demands to the supplier. The shortage is not permitted. The goal of the paper is to optimise the total profit of the supply chain members. The decision variables of the proposed model are the manufacturer's selling price, the wholesaler's selling price and the number of orders received by the supplier, respectively.

Here, demand is price-sensitive and rework at the manufacturer is allowed. However, a Nash-equilibrium method is considered among the members of the supply chain. A numerical example is presented to show the practicality of the proposed model. We considered the unit production cost as a function of production rate. We found that the profits of the chain members are concave with respect to their decision variables.

For future research, the present model can be extended under uncertain demand, the stock out situation, profit sharing, multiple competing wholesalers, delivery lead time may be incorporated.

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