Performance Analysis of Network System Configured as Series-Parallel Subject to Different Repair Policies: Copula Approach to Joint Probability Distribution

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Abstract

A network system is designed in this paper. It has two labs connected in parallel with four units each; working under 3-out-of-4: G; policy, two servers connected in parallel working under 1-out-of-2: G; policy and a router. The labs, servers and the router are connected in series all together. By making use of a supplementary variable and Laplace transforms to varies measures of system reliability, the network system has been studied and evaluation of Sensitivity, Availability, Reliability, (Mean time to failure) MTTF, and cost analysis for particular values of the failure and repair rates is made. Conclusion has been done with computed results demonstrated by tables, figures and graphs.

Keywords: Network, System, Availability; Reliability; Gumbel Hougard, series-parallel.

1. Introduction

It is very vital that a network system or any industrial system should be defendable and consistent; such system is more desirable than any other. The availability and reliability of that system plays a big role in its production output, performance, industrial growth and expected profit. In trying to enhance the performance of such desirable network/industrial systems many researchers evaluate different systems. Kabiru and Singh [1] made a reliability assessment of complex system consisting two subsystems connected in series configuration using Gumbel-Hougaard family copula distribution. Yusuf and Ismail [6] focused on reliability analysis of communication network system with redundant Relay Station under Partial and Complete Failure.

Singh and Poonia [14] study the performance analysis of a complex repairable system with two subsystems in series configuration with an imperfect switch. Using transition diagrams and systems of first order differential difference equations Yusuf and Sani in [2] developed and solved a system recursively to obtain the steady-state availability, busy period of repair men and profit function. Kakkar and Chitkara [15] consider finding the reliability of two unit parallel industrial system. Lado and Singh [4] develop a model of a complex repairable system having two subsystems A and B which is connected in a series configuration. Kakkar and Chitkara [9] present reliability of two dissimilar parallel units in the present of preventive maintenance. Yusuf and Mahmud [3] use Markov model of a system derived through the system state transition table and differential difference equations which are further used to evaluate the system availability and mean time to system failure and profit.

Using Copula Singh and Gulati [13] studied the performance analysis of complex system in series configuration under different failure and repair discipline. Garg [18] use fuzzy Kolmogorov's differential equations to make an approach for analyzing the reliability of industrial system. Yusuf and Bala [21] made an analysis of reliability characteristics of a parallel system with external supporting devices for operation.

2. State Description, Notation, and Assumptions

2.1 State description

S_0: This state represents a fully functional system with three working units and one standby unit in both labs, one operational server and one standby server, and working router.
S_1: This represents operational state with one unit down in lab 1.
S_2: This represents failure state due to failure of two units in lab 1.
S_3: This represents operational state with one unit down in lab 2.
S_4: This represents failure state due to failure of two units in lab 2.
S_5: This represents operational state with one server down.
S_6: This represents failure state due to failure of both servers.
S_7: This represents failure state due to failure of the main router.
S_8: This represents operational state with one unit in lab 1 down and one server under repair.
S_9: This represents operational state with one unit in lab 2 down and one server under repair.
S_{10}: This represents operational with one server down and a unit in lab 2 under repair.
S_{11}: This represents operational with one server down and a unit in lab 1 under repair.

2.2 Notations

- t Stands for Time variable on a time scale.
- s Stands for Laplace transform variable for all expressions.
- β_1 Stands for Failure rate of any unit in lab 1.
- β_2 Stands for Failure rate of any unit in lab 2.
- β_3 Stands for Failure rate of any server.
- β_4 Stands for Failure rate of the router.
- $\phi(a)$ Stands for all Repair rates.
- $P_i(t)$ Stands for the probability that the system is in S_i state at instants for i = 0 to 9.
- $\bar{P}(t)$ Stands for Laplace transformation of the state transition probability P(t).
- $P_i(a,t)$ Stands for the probability that a system is in state S_i for $i = 1,2,\ldots,11$, the system is running under repair and elapse repair time is (a, t) with repair variable a and time variable t.
- $E_p(t)$ Stands for Expected profit during the time interval [0, t).
- K_1, K_2 Stands for Revenue and service cost per unit time respectively.

2.3 Assumptions

It's assumed that at the beginning all the units in labs, servers and the router are working perfectly. At least three units from the two labs, one server and a router are needed for the system to operate. Failure rates are regarded as the same and may follow exponential distribution. Likewise repairs follow general distribution.

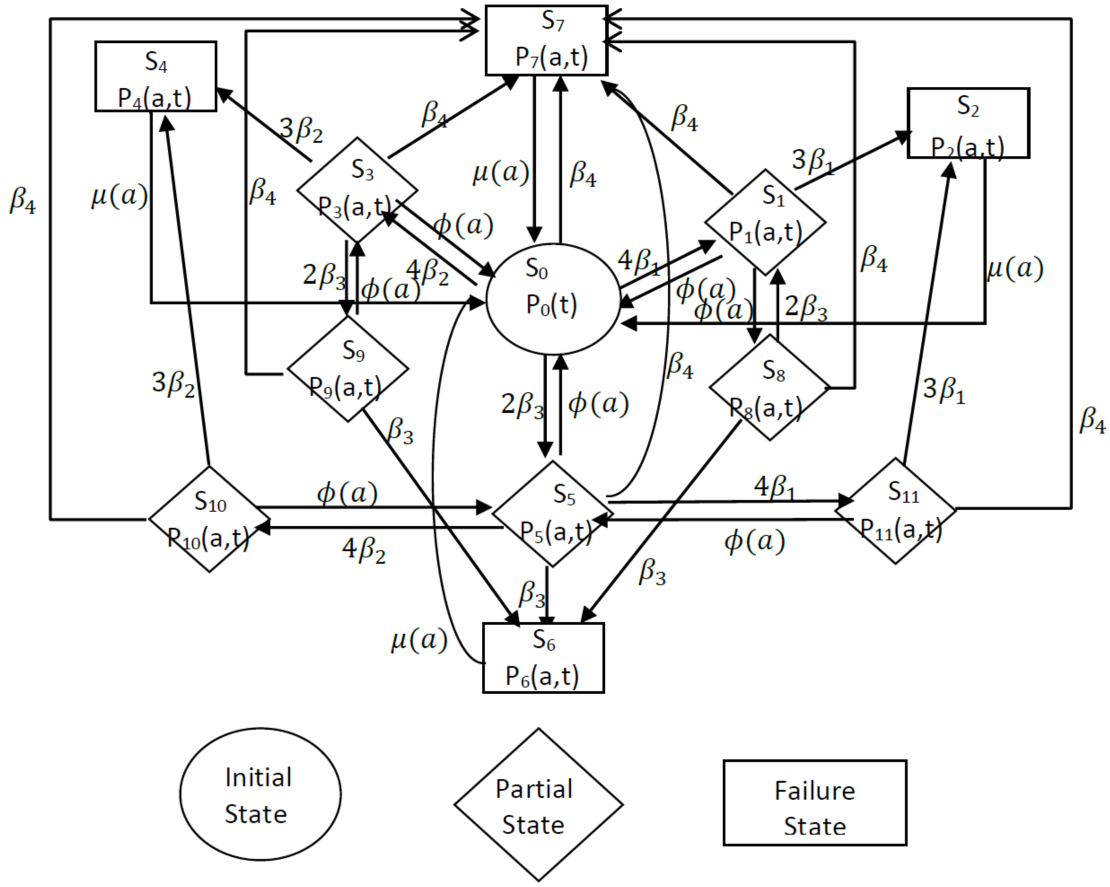

Figure 1: Transition Diagram

3. Mathematical Model Formulation

Bearing in mind the probability and continuity arguments, train set of difference differential equations and the present mathematical model are lump together as:

$$\left(\frac{\partial}{\partial t}+4\beta_1+4\beta_2+2\beta_3+\beta_4\right)p_0(t)=\int_0^\infty \emptyset(a)p_1(a,t)da+\int_0^\infty \mu(a)p_2(a,t)da+\int_0^\infty \emptyset(a)p_3(a,t)da+\int_0^\infty \mu(a)p_4(a,t)da+\int_0^\infty \emptyset(a)p_5(a,t)da+\int_0^\infty \mu(a)p_6(a,t)da+\int_0^\infty \mu(a)p_7(a,t)da \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+3\beta_1+2\beta_3+\beta_4+\emptyset(a)\right)p_1(a,t)=0 \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+\mu(a)\right)p_2(a,t)=0 \tag{3}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+3\beta_2+2\beta_3+\beta_4+\emptyset(a)\right)p_3(a,t)=0 \tag{4}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+\mu(a)\right)p_4(a,t)=0 \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+4\beta_1+4\beta_2+\beta_3+\beta_4+\emptyset(a)\right)p_5(a,t)=0 \tag{6}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+\mu(a)\right)p_6(a,t)=0 \tag{7}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+\mu(a)\right)p_7(a,t)=0 \tag{8}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+\beta_3+\beta_4+\emptyset(a)\right)p_8(a,t)=0 \tag{9}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+\beta_3+\beta_4+\emptyset(a)\right)p_9(a,t)=0 \tag{10}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+3\beta_2+\beta_4+\emptyset(a)\right)p_{10}(a,t)=0 \tag{11}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+3\beta_1+\beta_4+\emptyset(a)\right)p_{11}(a,t)=0 \tag{12}$$

Boundary condition

$$p_1(a,t)=4\beta_1 p_0(t) \tag{13}$$
$$p_2(a,t)=3\beta_1 p_1(a,t) \tag{14}$$
$$p_3(a,t)=4\beta_2 p_0(t) \tag{15}$$
$$p_4(a,t)=3\beta_2 p_3(a,t) \tag{16}$$
$$p_5(a,t)=2\beta_3 p_0(t) \tag{17}$$
$$p_6(a,t)=\beta_3 p_5(a,t) \tag{18}$$
$$p_7(a,t)=\beta_4\big(p_0(t)+p_1(a,t)+p_3(a,t)+p_5(a,t)+p_8(a,t)+p_9(a,t)+p_{10}(a,t)+p_{11}(a,t)\big) \tag{19}$$
$$p_8(a,t)=2\beta_3 p_1(a,t) \tag{20}$$
$$p_9(a,t)=2\beta_3 p_3(a,t) \tag{21}$$
$$p_{10}(a,t)=4\beta_2 p_5(a,t) \tag{22}$$
$$p_{11}(a,t)=4\beta_1 p_5(a,t) \tag{23}$$

Solution of the Model:

Using initial condition, P_0 (0) =1and Laplace transformation of equations (1) to (23) we have the following:

$$(s+4\beta_1+4\beta_2+2\beta_3+\beta_4)\overline{p_0}(s)=\int_0^\infty \emptyset(a)\overline{p_1}(a,s)da+\int_0^\infty \mu(a)\overline{p_2}(a,s)da+\int_0^\infty \emptyset(a)\overline{p_3}(a,s)da+\int_0^\infty \mu(a)\overline{p_4}(a,s)da+\int_0^\infty \emptyset(a)\overline{p_5}(a,s)da+\int_0^\infty \mu(a)\overline{p_6}(a,s)da+\int_0^\infty \mu(a)\overline{p_7}(a,s)da \tag{24}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial a}+3\beta_1+2\beta_3+\beta_4+\emptyset(a)\right)p_1(a,s)=0 \tag{25}$$
$$\left(s+\frac{\partial}{\partial a}+\mu(a)\right)\overline{p_2}(a,s)=0 \tag{26}$$
$$\left(s+\frac{\partial}{\partial a}+3\beta_2+2\beta_3+\beta_4+\emptyset(a)\right)\overline{p_3}(a,s)=0 \tag{27}$$
$$\left(s+\frac{\partial}{\partial a}+\mu(a)\right)\overline{p_4}(a,s)=0 \tag{28}$$
$$\left(s+\frac{\partial}{\partial a}+4\beta_1+4\beta_2+\beta_3+\beta_4+\emptyset(a)\right)\overline{p_5}(a,s)=0 \tag{29}$$
$$\left(s+\frac{\partial}{\partial a}+\mu(a)\right)\overline{p_6}(a,s)=0 \tag{30}$$
$$\left(s+\frac{\partial}{\partial a}+\mu(a)\right)\overline{p_7}(a,s)=0 \tag{31}$$
$$\left(s+\frac{\partial}{\partial a}+\beta_3+\beta_4+\emptyset(a)\right)\overline{p_8}(a,s)=0 \tag{32}$$
$$\left(s+\frac{\partial}{\partial a}+\beta_3+\beta_4+\emptyset(a)\right)\overline{p_9}(a,s)=0 \tag{33}$$
$$\left(s+\frac{\partial}{\partial a}+3\beta_2+\beta_4+\emptyset(a)\right)\overline{p_{10}}(a,s)=0 \tag{34}$$
$$\left(s+\frac{\partial}{\partial a}+3\beta_1+\beta_4+\emptyset(a)\right)\overline{p_{11}}(a,s)=0 \tag{35}$$

Laplace transform of boundary conditions

$$\overline{p_1}(0,s)=4\beta_1\overline{p_0}(s) \tag{36}$$
$$\overline{p_2}(0,s)=3\beta_1\overline{p_1}(0,s) \tag{37}$$
$$\overline{p_3}(0,s)=4\beta_2\overline{p_0}(s) \tag{38}$$
$$\overline{p_4}(0,s)=3\beta_2\overline{p_3}(0,s) \tag{39}$$
$$\overline{p_5}(0,s)=2\beta_3\overline{p_0}(s) \tag{40}$$
$$\overline{p_6}(0,s)=\beta_3\overline{p_5}(0,s) \tag{41}$$
$$\overline{p_7}(0,s)=\beta_4\big(\overline{p_0}(s)+\overline{p_1}(0,s)+\overline{p_3}(0,s)+\overline{p_5}(0,s)+\overline{p_8}(0,s)+\overline{p_9}(0,s)+\overline{p_{10}}(0,s)+\overline{p_{11}}(0,s)\big) \tag{42}$$
$$\overline{p_8}(0,s)=2\beta_3\overline{p_1}(0,s) \tag{43}$$

$$\overline{p_9}(0,s) = 2\beta_3\overline{p_3}(0,s) \tag{44}$$
$$\overline{p_{10}}(0,s) = 4\beta_2\overline{p_5}(0,s) \tag{45}$$
$$\overline{p_{11}}(0,s) = 4\beta_1\overline{p_5}(0,s) \tag{46}$$

The solution of (24) to (35) with (36) to (46) results as;

$$\overline{P_0}(s) = \frac{1}{D(s)} \tag{47}$$
$$\overline{P_1}(s) = \frac{4\beta_1}{D(s)}\left\{\frac{1-\bar{S}_\phi(s+3\beta_1+2\beta_3+\beta_4)}{s+3\beta_1+2\beta_3+\beta_4}\right\} \tag{48}$$
$$\overline{P_2}(s) = \frac{12{\beta_1}^2}{D(s)}\left\{\frac{1-\bar{S}_\mu(s)}{s}\right\} \tag{49}$$
$$\overline{P_3}(s) = \frac{4\beta_2}{D(s)}\left\{\frac{1-\bar{S}_\phi(s+3\beta_2+2\beta_3+\beta_4)}{s+3\beta_2+2\beta_3+\beta_4}\right\} \tag{50}$$
$$\overline{P_4}(s) = \frac{12{\beta_2}^2}{D(s)}\left\{\frac{1-\bar{S}_\mu(s)}{s}\right\} \tag{51}$$
$$\overline{P_5}(s) = \frac{2\beta_3}{D(s)}\left\{\frac{1-\bar{S}_\phi(s+4\beta_1+4\beta_2+\beta_3+\beta_4)}{s+4\beta_1+4\beta_2+\beta_3+\beta_4}\right\} \tag{52}$$
$$\overline{P_6}(s) = \frac{2{\beta_3}^2}{D(s)}\left\{\frac{1-\bar{S}_\mu(s)}{s}\right\} \tag{53}$$
$$\overline{P_7}(s) = \frac{\beta_4(1+4\beta_1+4\beta_2+2\beta_3+16\beta_1\beta_3+16\beta_2\beta_3)}{D(s)}\left\{\frac{1-\bar{S}_\mu(s)}{s}\right\} \tag{54}$$
$$\overline{P_8}(s) = \frac{8\beta_1\beta_3}{D(s)}\left\{\frac{1-\bar{S}_\phi(s+\beta_3+\beta_4)}{s+\beta_3+\beta_4}\right\} \tag{55}$$
$$\overline{P_9}(s) = \frac{8\beta_2\beta_3}{D(s)}\left\{\frac{1-\bar{S}_\phi(s+\beta_3+\beta_4)}{s+\beta_3+\beta_4}\right\} \tag{56}$$
$$\overline{P_{10}}(s) = \frac{8\beta_2\beta_3}{D(s)}\left\{\frac{1-\bar{S}_\phi(s+3\beta_2+\beta_4)}{s+\beta_3+\beta_4}\right\} \tag{57}$$
$$\overline{P_{11}}(s) = \frac{8\beta_1\beta_3}{D(s)}\left\{\frac{1-\bar{S}_\phi(s+3\beta_1+\beta_4)}{s+\beta_3+\beta_4}\right\} \tag{58}$$

Where $D(s) = s + 4\beta_1 + 4\beta_2 + 2\beta_3 + \beta_4 - \Big\{4\beta_1\{\bar{S}_\phi(s + 3\beta_1 + 2\beta_3 + \beta_4)\} + 12{\beta_1}^2\{\bar{S}_\mu(s)\} + 4\beta_2\{\bar{S}_\phi(s + 3\beta_2 + 2\beta_3 + \beta_4)\} + 12{\beta_2}^2\{\bar{S}_\mu(s)\} + 2\beta_3\{\bar{S}_\phi(s + 4\beta_1 + 4\beta_2 + \beta_3 + \beta_4)\} + 2{\beta_3}^2\{\bar{S}_\mu(s)\} + \beta_4(1 + 4\beta_1 + 4\beta_2 + 2\beta_3 + 16\beta_1\beta_3 + 16\beta_2\beta_3)\{\bar{S}_\mu(s)\}\Big\}$ (59)

The followings are Laplace transformations of the state transition probabilities when the system is in initial, partial failure and failed condition at any time:

$$\overline{P_{up}}(s) = \overline{P_0}(s) + \overline{P_1}(s) + \overline{P_3}(s) + \overline{P_5}(s) + \overline{P_8}(s) + \overline{P_9}(s) + \overline{P_{10}}(s) + \overline{P_{11}}(s) \tag{60}$$

$$\overline{P_{up}}(s) = \overline{P_0}(s)\Big(1 + 4\beta_1\left\{\frac{1-\bar{S}_\phi(s+3\beta_1+2\beta_3+\beta_4)}{s+3\beta_1+2\beta_3+\beta_4}\right\} + 4\beta_2\left\{\frac{1-\bar{S}_\phi(s+3\beta_2+2\beta_3+\beta_4)}{s+3\beta_2+2\beta_3+\beta_4}\right\} + 2\beta_3\left\{\frac{1-\bar{S}_\phi(s+4\beta_1+4\beta_2+\beta_3+\beta_4)}{s+4\beta_1+4\beta_2+\beta_3+\beta_4}\right\} + 8\beta_1\beta_3\left\{\frac{1-\bar{S}_\phi(s+\beta_3+\beta_4)}{s+\beta_3+\beta_4}\right\} + 8\beta_2\beta_3\left\{\frac{1-\bar{S}_\phi(s+\beta_3+\beta_4)}{s+\beta_3+\beta_4}\right\} + 8\beta_2\beta_3\left\{\frac{1-\bar{S}_\phi(s+3\beta_2+\beta_4)}{s+\beta_3+\beta_4}\right\} + 8\beta_1\beta_3\left\{\frac{1-\bar{S}_\phi(s+3\beta_1+\beta_4)}{s+\beta_3+\beta_4}\right\}\Big) \tag{61}$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s)$$

4. Analytical Study of the Model for Particular Case

Availability Analysis

With $\bar{S}_\phi(s) = \frac{\phi}{s+\phi}$, $\bar{S}_\mu(s) = \frac{\mu}{s+\mu}$, $\frac{1-\bar{S}_\phi(s)}{s} = \frac{1}{s+\phi}$, $\frac{1-\bar{S}_\mu(s)}{s} = \frac{1}{s+\mu}$ and considering the values of different parameters as $\beta_1 = 0.03, \beta_2 = 0.02, \beta_3 = 0.05, \beta_4 = 0.06$, $\phi = \mu = 1$ in (61), the expression for availability by taking the inverse Laplace transform is obtained as:

$$\bar{P}_{up}(t) = 0.03892291366e^{-2.838301077t} - 0.06425625912e^{-1.489776405t} - 0.00241062060e^{-1.283327761t} + 0.0002345183193e^{-1.229731692t} + 1.036198863e^{-0.01716306460t} - 0.00204112976e^{-1.150000000t} - 0.006655302231e^{-1.110000000t} \tag{63}$$

Table 1 and Figure 2 below shows different values of $P_{up}(t)$ using (63) and time variable t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9,

Table 1: Variation of Availability with respect to time

Time(t)	Availability
0	1
2	0.9970
4	0.9672
6	0.9348
8	0.9033
10	0.8728
12	0.8433
14	0.8149
16	0.7874
18	0.7608

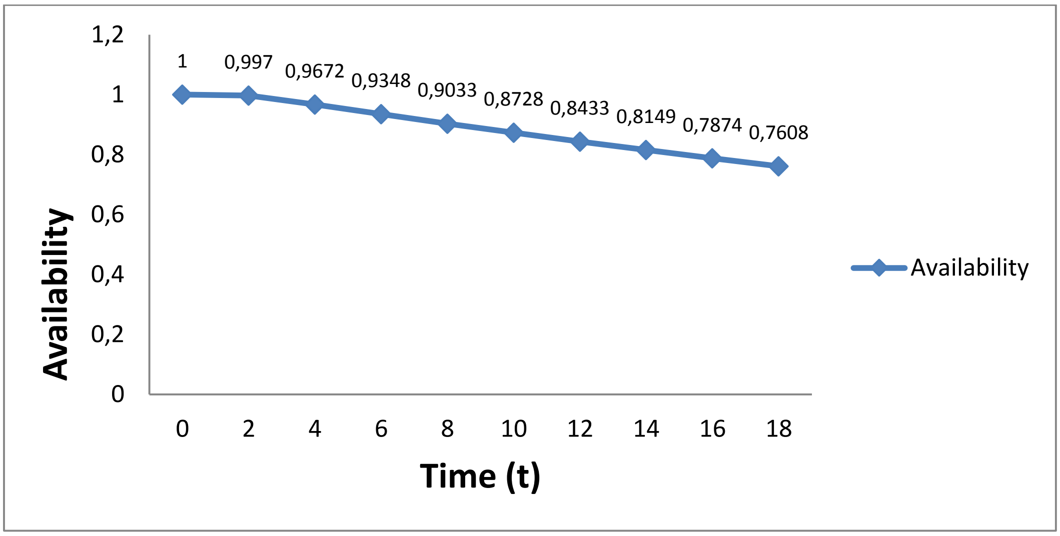

Figure 2: Availability as function of time

Reliability Analysis

All repair rates, ϕ, μ, set to zero in equation (61), the values of failure rates as $\beta_1 = 0.03, \beta_2 = 0.02, \beta_3 = 0.05, \beta_4 = 0.06$ and then computing inverse Laplace transform, express the reliability for the system as;

$$R(t) = -2.889523810e^{-0.3600000000t} + 0.03809523810e^{-0.1500000000t} + 0.08000000000e^{-0.1100000000t} + 0.5714285714e^{-0.2200000000t} + 1.2000000000e^{-0.2500000000t} + 2.\,e^{-0.3100000000t} \quad (64)$$

Table 2 and Figure 3 below shows different values of $P_{up}(t)$ using (63) and time variable t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Table 2: Computation of reliability for different values of time

Time(t)	Reliability
0	1
2	0.8577
4	0.6451
6	0.4553
8	0.3106
10	0.2081
12	0.1382
14	0.0917
16	0.0610
18	0.0409

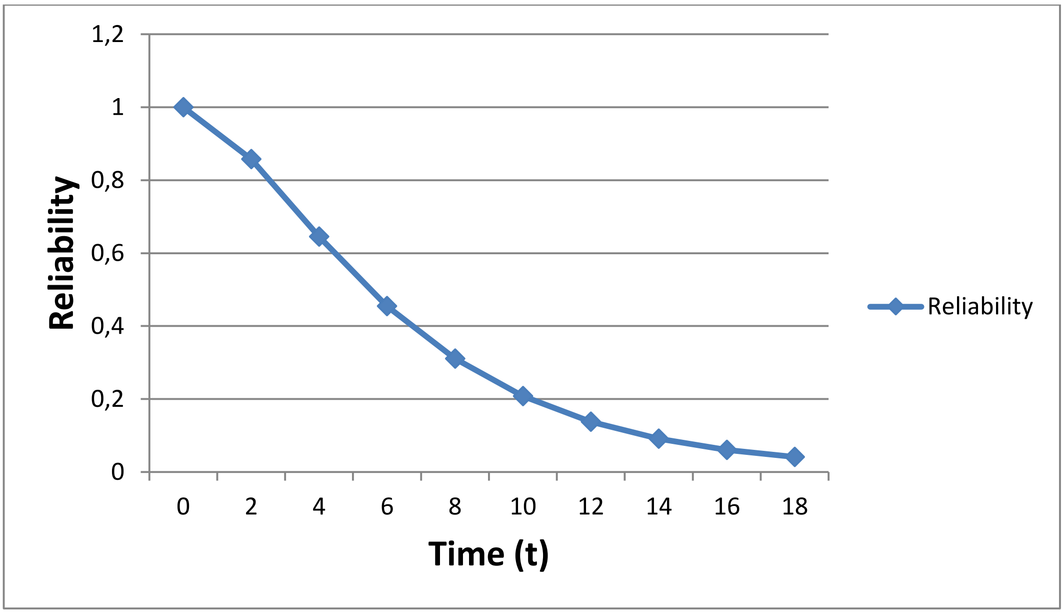

Figure 3: Reliability as function of time

Mean Time to Failure (MTTF) Analysis

All repairs are set to zero in equation (49), and then taking limit, as s tends to zero express MTTF as:

$$MTTF = \lim_{s\to 0} \overline{P_{up}}(s) = \frac{1}{4\beta_1+4\beta_2+2\beta_3+\beta_4}\left(1+\frac{4\beta_1}{3\beta_1+2\beta_3+\beta_4}+\frac{4\beta_2}{3\beta_2+2\beta_3+\beta_4}+\frac{2\beta_3}{4\beta_1+4\beta_2+\beta_3+\beta_4}+\frac{8\beta_1\beta_3}{\beta_3+\beta_4}+\frac{8\beta_2\beta_3}{\beta_3+\beta_4}+\frac{8\beta_2\beta_3}{3\beta_1+\beta_4}+\frac{8\beta_1\beta_3}{3\beta_1+\beta_4}\right) \tag{65}$$

Table 3 and figure 4 below express the variation of MTTF with respect to failure rates, by fixing$\beta_2 = 0.02, \beta_3 = 0.05, \beta_4 = 0.06$ and varying β_1 as $0.01, 0.02, 0.03, 0.04, 005, 0.06, 0.07, 0.08, 0.09$, fixing $\beta_1 = 0.03, \beta_3 = 0.05, \beta_4 = 0.06$ and varying β_2 as $0.01, 0.02, 0.03, 0.04, 005, 0.06, 0.07, 0.08, 0.09$, fixing$\beta_1 = 0.03, \beta_2 = 0.02, \beta_4 = 0.06$ and varying β_3 as $0.01, 0.02, 0.03, 0.04, 005, 0.06, 0.07, 0.08, 0.09$, fixing $\beta_1 = 0.03, \beta_2 = 0.02, \beta_3 = 0.05$, and varying β_4 as$0.01, 0.02, 0.03, 0.04, 005, 0.06, 0.07, 0.08, 0.09$, in (61).

Table 3: Computation of MTTF corresponding to the various values of failure rates

Failure rate	MTTF β_1	MTTF β_2	MTTF β_3	MTTF β_4
0.01	8.0406	7.2282	8.6971	9.6404
0.02	7.4263	6.8927	8.0926	8.9180
0.03	6.8927	6.5597	7.6128	8.3040
0.04	6.4307	6.2478	7.2208	7.7725
0.05	6.0297	5.9624	6.8927	7.3061
0.06	5.6798	5.7036	6.6126	6.8927
0.07	5.3726	5.4699	6.3694	6.5231
0.08	5.1012	5.2586	6.1555	6.1903
0.09	4.8600	5.0673	5.9650	5.8889

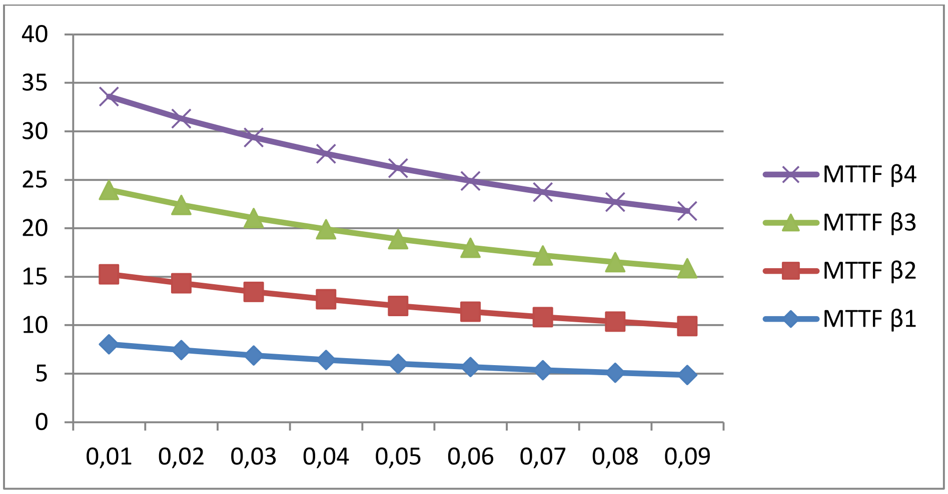

Figure 4: MTTF corresponding to the various values of failure rates

Sensitivity Analysis corresponding to (MTTF)

Table 4 and figure 5 below, express the sensitivity in MTTF of the system, using the partial differentiation of MTTF with respect to the failure rates of the system and setting parameters as $\beta_1 = 0.03, \beta_2 = 0.02, \beta_3 = 0.05, \beta_4 = 0.06$.

Table 4: MTTF sensitivity as function of time

Failure rate	∂(MTTF)/ β_1	∂(MTTF)/ β_2	∂(MTTF)/ β_3	∂(MTTF)/ β_4
0.01	-65.5677	-32.4004	-68.4440	-78.8498
0.02	-57.2902	-33.9081	-53.4467	-66.2544
0.03	-49.6020	-32.4031	-43.1042	-56.9357
0.04	-42.9726	-29.8976	-35.6883	-49.6544
0.05	-37.3926	-27.1880	-30.1944	-43.8144
0.06	-32.7291	-24.5889	-26.0096	-39.0221
0.07	-28.8269	-22.2133	-22.7442	-35.0194
0.08	-25.5471	-20.0894	-20.7423	-31.6289
0.09	-22.7743	-18.2092	-28.7236	-28.7236

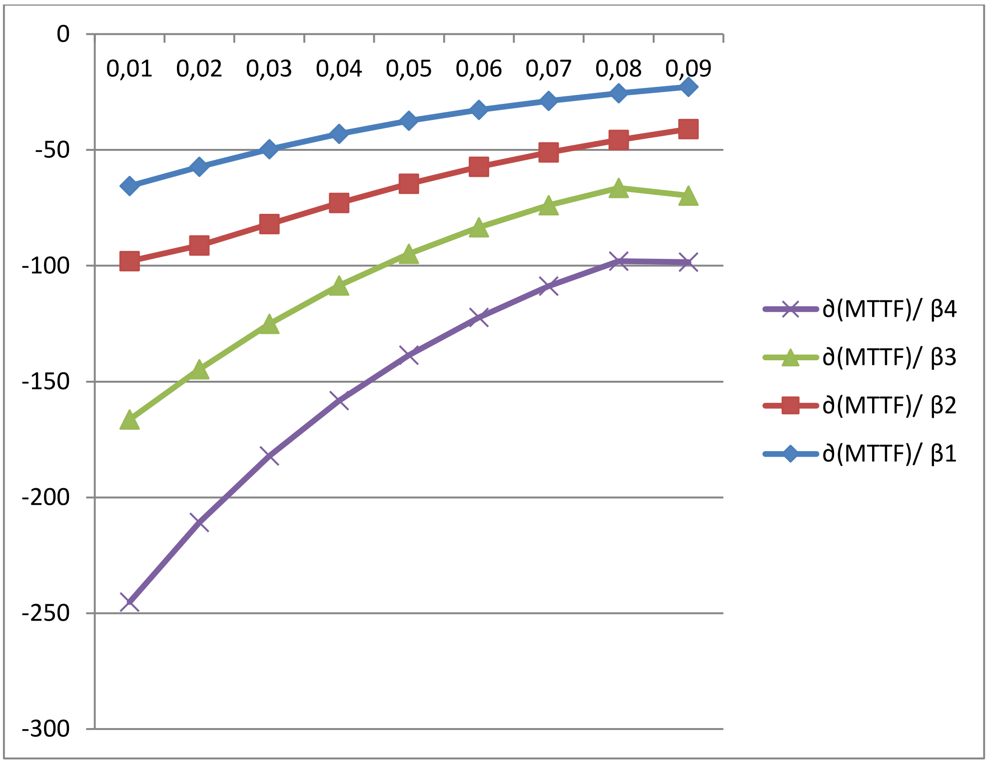

Figure 5: MTTF sensitivity as function of time

Cost Analysis

$$E_P(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{65}$$

Equation (65) can be used to attain expected profit of the system in the interval [0,t), if service facilities is always available.

Using (61) and (65), we obtain:

$$E_P(t) = K_1(-0.01371345485e^{-2.838301077t} + 0.04313147859e^{-1.489776405t} + 0.001878413822e^{-1.2833327761t} - 0.0001907069004e^{-1.229731692t} - 60.37376699e^{-0.017163064600t} + 0.001768793892e^{-1.150000000t} + 0.005995767776e^{-1.110000000t} + 60.335) - K_2 t \tag{66}$$

Table 5 and Figure 6 express the expected profit by setting $K_1 = 1$and $K_2 =$ 0.6, 0.5, 0.4, 0.3, 0.2 and 0.1 respectively and varying time t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 in (66).

Table 5: Expected profit as function of time

Time(t)	Ep(t);K2=0.6	Ep(t);K2=0.5	Ep(t);K2=0.4	Ep(t);K2=0.3	Ep(t);K2=0.2	Ep(t);K2=0.1
0	0	0	0	0	0	0
2	0.8016	0.0016	1.2016	1.4016	1.6016	1.8016
4	1.5672	1.9672	2.3672	2.7672	3.1672	3.5672
6	2.2690	2.8690	3.4690	4.0690	4.6690	5.2690
8	2.9069	3.7069	4.5069	5.3069	6.1069	6.9069
10	3.4828	4.4828	5.4828	6.4828	7.4828	8.4828
12	3.9987	5.1987	6.3987	7.5987	8.7987	9.9987
14	4.4567	5.8567	7.2567	8.6567	10.0567	11.4567
16	4.8588	6.4588	8.0588	9.6588	11.2588	12.8588
18	5.2069	7.0069	8.8069	10.6069	12.4068	14.2069

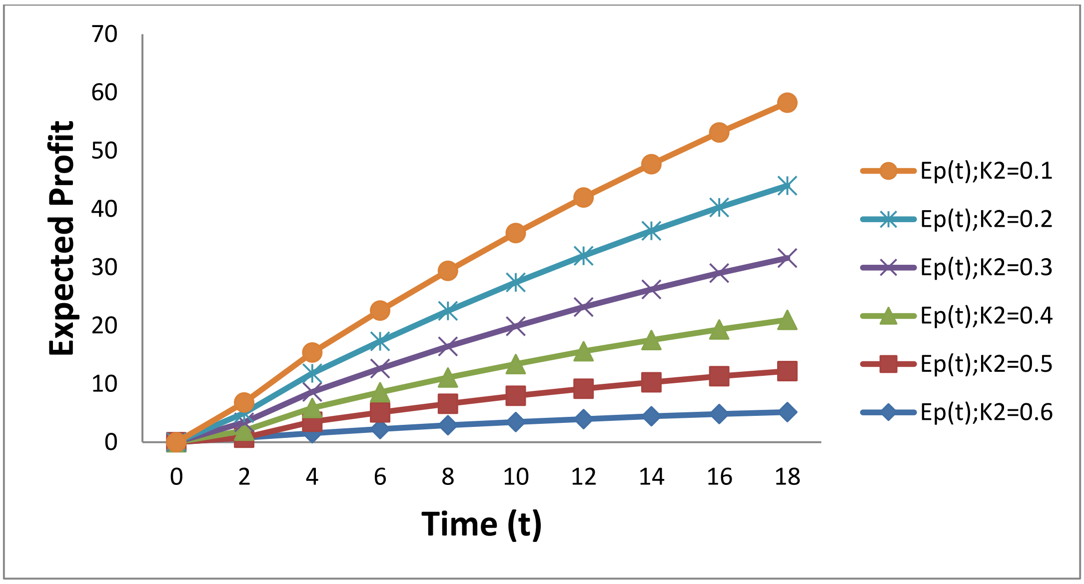

Figure 6: Expected profit as function of time

5. Conclusions

To make a conclusion on the performance of the system in this study, a study of the reliability measures on repair and failure rates with different values has been made. A provision of information on the availability of the system changing with time at fixed failure rates with different values has been made in table 1 and figure 2. It has been evidently found that gently and slowly the availability of system decreases and the probability of the failure increases at fixed failure rates $\beta_1 = 0.03, \beta_2 = 0.02, \beta_3 = 0.05, \beta_4 = 0.06$, and at a long run it will become steady to zero value. Therefore, at any chosen time and set of parametric values, one can easily say the future behavior of the system.

Setting repairs to zero, an analysis of the system reliability has been provided in table 2 and figure 3. By studying the availability and reliability of the system, one can conclude that providing repair to the system is better than replacement.

With other parameters considered as constant, mean-time-to-failure (MTTF) in variation of $\beta_1, \beta_2, \beta_3$ *and* β_4 respectively has been provided in figure 4. It also signifies that the variation of $\beta_1, \beta_2, \beta_3$ *and* β_4 are responsible for the better performance of the system.

Figure 5 shows the sensitivity analysis of system.

Figure 6 and table 5 shows the calculations of the profit at fixed revenue cost $K_1 = 1$ per unit time and service cost$K_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1. The result shows that when the service cost $K2$ fixed at minimum value 0.1, the expected profit increases with respect to the time. In the end, one can observe that profit decrease whenever service cost increase.

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