Fatigue of Unidirectional Fiber Composite and Static Strength of its Components. Simplified Daniels-Epsilon Sequence

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Abstract

A simple method is proposed for obtaining such a description of the average fatigue curve (and residual static strength) of a unidirectional fiber composite (UFC), which is directly related to the parameters of the static strength distribution of its components (SSDC), which are the longitudinal items bearing the main longitudinal load. This description makes it possible to predict changes in the fatigue life of the UFC when the SSDC changes. The method is based on a Daniels_epsilon_sequence (DeS), which is a modification of Daniels_sequence (DS) which takes into account the short-term damaging effects of one separate cycle of fatigue loading. Here we use a specific version of it in which the number of components of a critical link of UFC is equal to infinity. We call this version as simplified DeS. The concept of a DeS_fatigue equivalent distribution (DeS_FED) of local static strength of LI is introduced. The DS the calculations of fatigue life using the DeS_FED coinside with the test data. The simplified DeS model studied in this article should be used for preliminary analysis of the mean SN curve. For the more detailed analysis should be used the models considered by author some earlier which include the use of the theory of Markov processes and the Monte Carlo method, which allows modeling and statistical aspects of the problems under consideration, but require much more time-consuming calculations. At the end of the paper a numerical example of processing the fatigue test data and prediction a new fatigue life at some SSDC changes are given.

Keywords: Composite, Daniels'_epsilon_sequence, fatigue life, residual strength.

1. Introduction

In our previous publications [1, 2] it was shown that the use of the Daniels' sequence and Daniels'_epsilon_sequence (DeS) allows

1) to describe the process of step-by-step growth of local stresses in a weak link of theUFC. This process is similar to the well-known S-shaped change in some physical parameters during the fatigue tests (Fig. 1);

2) to relate directly the number of the DeS items (the calculated local stresses) at which the value DeS item tend to infinity with the composite DS-fatigue life (DS_FLf) which is a function of the parameters of the SSDC; this functon can be used for the regression analysys of the fatigue test data and the prediction of the composite fatigue life changes at some its component static strength changes; it is the main specific feature and advantage of the DeS models;

3) to explain the specific features of the residual strength: a long period of very gradual degradation of strength is suddenly replaced by a sharp drop to zero;

4) to explain the existence of an infinite calculated fatigue life and fatigue limit.

The main drawback of the considered previously DS model is this: if the cycle loads only slightly exceeds the fatigue limit, the predicted DS_FL is very small. In the work [1], this drawback is eliminated by the use the theory of Markov processes. In the work [2] the use of a Daniels'_epsilon_sequence (DeS) was proposed. Some additional parameter which is denoted by the symbol ε, was introduced in order makes it easy to control the value of the calculated DeS_FLf. This parameter takes into account that only a part of the components whose strength is less than the applied load is destroyed during one fatigue loading cycle. The value ε, apparently, depends on the maximum load in the current cycle, on the SSDC, on the frequency of loading, on the structure of the UFC and on the other circumstances.

In the paper [2], the initial number of components n in the considered critical link of UFC is assumed to be equal to some finite number. Assumption that the strength of each of these components is a random variable allows to model the statistical characteristics of the fatigue life of the UFC. But in this article, we make an additional simplification. We assume that this number is equal to infinity. It is clear that this assumption is acceptable only if we are interested in analyzing the average values of the fatigue life, but just this case is considered in this article. This assumption greatly simplifies calculations and is often appropriate in the applications. As it was told already, a detailed analysis involving the theory of Markov processes and Monte-Carlo method is described in our previous work [1].

Short history of the investigation of the fatigue of composite is considered in large number of the publications. Here we mention only most significant papers. The first scientific publication devoted to this problem appears to be the Peirce's work [3]. Peirce gives an approximate formula for the average strength of a bundle of LIs (fibers, bundles, strands) forming the UFC. The normal approximation of the strength distribution law of a LI parallel system was shown by Daniels [4, 5]. His result was refined by Smith [6] already with a reference to the series-parallel system (SPS), definition of which was earlier proposed in [7]. A detail review of the residual strength a given in [8, 9].

The term "Daniels Sequence" first appeared in work [10], but the first calculation of such a sequence took place much earlier [11, 12]. Later, some specific versions of DS, the Daniels_epsilon_Sequences (DeS) appears and was discussed in [1, 2]. But the models studiet in these papers take into account too many factors and required preliminary estimations of too large number of parameters. The main purpose of this paper is to offer the model which is much easier to use for the construction and the description of the SN fatigue curve and residual strength.

In section 2 of the paper the general mathematical definition of the DeS, in sections 3 and 4 its application to the the processing of the fatigue test data is studied. In the following section the prediction of the fatigue life after some changes of the statical strength of the components of the UFC and the following conclusions are discussed.

2. The Daniel's epsilon sequence

2.1. The Daniel's sequence

The UFC can be considered as the series-parallel system (SPS) with series of n_L links and n_C parallel items in every link. But in this section, we consider only one link of SPS that has n items. The strength of such link was studied by Daniels [4,5]. I t is called now the "classical model of bundle of n parallel fibers stretched between two clamps". In general case, strands or some set of strands can be considered instead of fibers. Here for all structural items of these types we'll use more general terms : „longitudional item" (LI) or just „component". The connection of the fatigue characteristic of one link and the SPS as a whole is described in [11,12]. We assume that the total load on the link in question is uniformly distributed among all functional LIs both at the beginning of the test and after the destruction of some of them. The state of the UFC is defined by the number of undisturbed workable LIs.

First, we will recall the general definition of DS with some corrections. The DS as some random process is defined by two components: a vector $X_{L,1:n}=(X_{L,1},X_{L,2},...,X_{L,n})$, whose components are mutually independent random variables with the same cumulative distribution function (cdf), and an infinite sequence $S^+_{0:\infty}=\{S^+_0,S^+_1,S^+_2,...\}$, whose components are random variables with a known law of joint probability distribution. In the description of the fatigue test $X_{L,1},X_{L,2},...,X_{L,n}$ are the strengths of the components of UFC, $S^+_0,S^+_1,S^+_2,...$ are the values of the maximum stress in the sequence of fatigue loading cycles. Specific realization of the *the Daniels sequence (DS)* for specific loading sequence, $s^+_{0:\infty}=\{s^+_0,s^+_1,s^+_2,...\}$, $s^+_0\le s^+_1\le s^+_2,...$ is defined by equation

$$s_0=s^+_0\ ,\quad s_{i+1}=s^+_{i+1}/(1-\nu(s_i)/n_c)\quad i=0,1,2,... \tag{1}$$

where $s^+_{0:\infty}=\{s^+_0,s^+_1,s^+_2,...\}$ is realization of the $S^+_{0:\infty}=\{S^+_0,S^+_1,S^+_2,...\}$; $\nu(s_i)$ is the number of of elements of vector $x_{L,1:n}=(x_{L,1},x_{L,2},...,x_{L,n})$, realization of $X_{L,1:n}=(X_{L,1},X_{L,2},...,X_{L,n})$, which are lower than s^+_i. The transition from s_i to s_{i+1} is what we call *the step of DS*.

We call the *Daniels_epsilon_Sequence (DeS)* the introdused in [2] modification of DS, which is defined by the equation

$$s_0=s^+_0\ ,\quad s_{i+1}=s^0_{i+1}\varepsilon+s_i(1-\varepsilon),\ \ i=0,1,2,... \tag{2}$$

where $s^0_{i+1}=s^+_{i+1}/(1-\nu(s_i)/n)\quad i=0,1,2,...$; $0<\varepsilon\le 1$,

As it was told already, the parameter ε defines the rate of accumulation of the fatigue damages into one cycle and takes into account the fact that the destruction of all $\nu(s_i)$ LIs requires both time and sufficient energy supply. The part of them the fracture of which takes place in one cycle depends on the frequency of loading and the other factors.

The ratio $\nu(s_i)/n_c$ is the empirical estimate, $\hat{F}(x)$, of the cdf, $F(x)$. For considered here simplified version of the DeS we assume that the dimension of the vector $X_{i:n}$ (number of LIs in the considered link) is so much that instead of $\hat{F}(x)$ in (1) can be used $F(x)$.

In the fatigue test to study the SN curve we are interested only in the case when $s^+_1=s^+_2=...=s^+$ where s^+ is some specific level of load. Then the equation (2) can be rewriten in this way

$$s_0=s^+_0\ ,\quad s_{i+1}=s^0_{i+1}\varepsilon+s_i(1-\varepsilon),\ \ i=0,1,2,... \tag{3}$$

where $s^0_{i+1}=s^+/(1-F(s_i))\quad i=0,1,2,...$; $0<\varepsilon\le 1$,

There are two types of the DeS (see Fig.1). The **first type** of it takes place if the parameter s^+ is large enough and items of DS has tendency to grow up to infinity.

Let for the fixed s^+ and some ε the integer function $n_{DeS}(s;s^+,\varepsilon),\ n=1,2,...$, $s=s_1,s_2,...$ determines the number of steps at which the sequence DeS reaches the value s. The event when s_i reaches some large enough critical value s_C we consider as fatigue failure and define the *DeS Fatigue Life (DeS_FLf)* to be equal to

$$n_{DeS.s_C}=n_{DeS}(s_C;s^+,\varepsilon),\ \ n=1,2,... \tag{4}$$

Then for $s=s_C$ and $s=\infty$ the DeS_FLfs, the corresponding n_{DeS,s_C} and $n_{DeS,\infty}$, are equal to the values of this function.

We also introduce the concept of *the Daniels residual function* as a function that determines the residual strength after the DeS reaches the value s. We define it by the equation

$$r_{DeS}(s)=\max_{x>s} x(1-F(x)) \ . \tag{5}$$

Now it can be introduced another definition of fatigue life, $n_{DeS,R}$, which is determined by the moment when the residual strength of the tested specimens decreases below a certain critical level r_C, $r_C \geq 0$. We define it by equation

$$n_{DeS,R}=1+\max(i: r_{DeS}(s_i)>r_C,\ i=0,1,2,... \tag{6}$$

The **second type** of DeS takes place if the stress s^+ is small enough. Then after some number of steps the increasing of the value of DeS items almost ceases in spite of increasing the number of steps up to infinity. The maximal value of cycle parameter s^+ for which this fenomenon takes place we considered as *the Daniels fatigue strength (DFSt)* or *the Daniels fatigue limit (DFLt),* s_D. The fatigue limit is determined by the maximum load value s^+ at which equality $s_{i^*+1}=s_{i^*}$ can occur for some i^*.

In [2] it is shown that regardless of the value of ε for load level, s^+, more than $\max x(1-F(x))$ the event $s_{i^*+1}=s_{i^*}$ can not take place but at every $i=1,2,...$, the inequality $s_{i+1}>s_i$ takes place; the items of the DeS grow up to infinity, all LIs will be destroyed, the value of the DeS_FLf is final. So the value

$$s_D=\max x(1-F(x)) \ , \tag{7}$$

can be used as the definition of *the Daniels fatigue limit* **(***DFLt***).**

It is necessary to note the obvious. The values of s_D in (7) and $r_{DeS}(0)$ in (5) coincide with the value of the static strength of the "classical model of bundle of n parallel fibers stretched between two clamps" predicted by Daniels [4,5]. The difference between these values takes place only if there is a difference in the distribution functions of the strength of LIs, $F(x)$.

The mathematics of the above analysis is valid for any parallel system for which equations (1) and (2) are valid. The connection of this mathematics with the fatigue phenomenon of an UFC occurs if instead of the distribution of static strength, $F(x)$, we use some other cdf $F_L(x)$ of the local strength of the element working as part a the" weak segment " of the UFC. The length of the LI in the "weak segment" (the link in the considered here model) does not match the length of LI for laboratory static strength tests, the structure of the "weak segment" has a special support conditions, … So, in the following we will usually use the $F_L(x)$ instead of $F(x)$.

Later on it will be shown that processing the test data we can find the function $\varepsilon=\varepsilon(s^+)$ in such a way that the the DeS_FLfs, n_{DeS,s_C} or $n_{DeS,\infty}$, wil be equal to the corresponding test fatigue lifes. This function together with the equations (3) and (7) gives the description of fatigue curve which is directly related to the parameters of the static strength distribution of its components (SSDC).

Next, we will consider two examples of using the considered mathematical apparatus (as we will say, using the DeS approach) to process the results of fatigue tests in order to establish a connection between the description of the fatigue curve of a composite and the static strength of its components. In the second example, the analysis of residual strength will also be considered.

3. Application to the fatigue curve analysis. Numerical example 1

In this part of the article, we will review the analysis of data on fatigue tests of composite samples and on the static strength of its components presented in (13).

3.1. The cdf of the local tensile strength

Two simple assumptions help to explain (and to model) why during fatigue tests the composite collapses at the load significantly lower than its static strength:
A) The local strength in the considered specificweak link k_L time lower than in other links, $k_L \geq 1$;
B) in the considered specific link the local stress k_C time greater than in other links , $k_C \geq 1$.

Let's consider the difference between the influence of coefficients k_C and k_L on the results of analysis of fatigue test data. For simplicity we assume that ε is equal to 1. And instead of the symbol DeS, we'll use the DS symbol. Here we study the case of the lognormal distribution with the cdf $F(x) = \Phi\left((\log(x) - \theta_0)/\theta_1\right)$, where $\Phi(.)$ is cdf of the standard normal distribution, but all the following will be true in more general case (for example, for Weibull distributions with cdf $F(x) = 1 - \exp\left(-\exp\left((\log(x) - \theta_0)/\theta_1\right)\right)$). Note that if the explanation B of the local stress concentration is accepted, then instead of equation (3), we should use its corrected version - equation (3 b)

$$s_0 = s_0^+, \quad s_{i+1} = s_{i+1}^0\varepsilon + s_i(1-\varepsilon), \quad i = 0,1,2,\ldots, \text{ where } s_{i+1}^0 = k_C s^+ / (1 - F_L(s_i)) \quad i = 0,1,2,\ldots; \; 0 < \varepsilon \leq 1, \tag{3b}$$

It is clear that the use of $k_C s^+$ instead of s^+ increases the values of DeS curve by k_C times and reduces DeS_FLs, $n_{DeS;sC}$. But it can be shown that fatigue limit s_D remains unchanged (see [1]). Now, if we use simultaneously k_C and k_L then $F_L(k_C x) = P(X / k_L < k_C x) = \Phi\left((\log(k_C k_L x) - \theta_0)/\theta_1\right)$ and in equation (7) we use $F_L(x) = F(k_C x)$ instead of $F(x)$ then the fatigue limit is determined by the product of $k_L k_C$. And if $k_L = 1$ then instead of equation (7), we should use its corrected version - equation (7a)

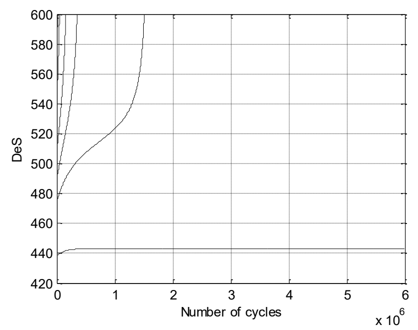
$$s_D = \max x(1 - F_L(x)) / k_C \,, \tag{7a}$$

It is also clear that the values of the residual strength determined by the equation (5) do not depend on the k_C, but depend on the k_L and determine the **local residual strength** in the critical link under consideration. And equation (5) defines the residual static strength of the whole composite only in the case when $k_L = 1$. For this reason, some later in section 4 we use the explanation B for processing the data of residual strength.

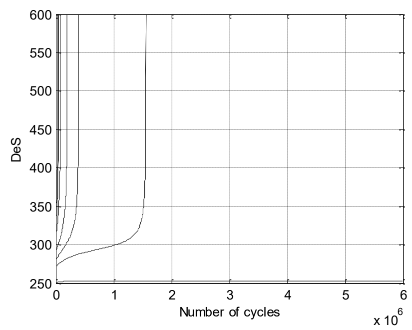
In order to see the difference between the influence of k_C and k_L under different values ε the calculation of DeS curves and the corresponding DeS_FLfs for two pairs: ($k_C = 1.75$, $k_L = 1.0$ **)** and **(**

$k_C = 1.0$, $k_L = 1.75$ **)** with two epsilon values: $\varepsilon = \mathbf{0}.0000\mathbf{1}$ and $\varepsilon = \mathbf{0}.00000\mathbf{1}$ were made. In the work [13] in the Table 2.1 the results of the carbon fiber strand specimen tensile test, in the table 2.11 of the results of the fatigue test at an approximately pulsed ($s_{\min} / s_{\max} = 0.1$) load on CFRP specimens are presented. The results of processing the data from static strength tests of carbon fiber strand specimens, including testing hypotheses about the type of distribution law using OSPPT and ρ criteria [11,14,15] show that the hypothesis about the lognormal distribution ("normal" on a logarithmic scale) was more plausible than the Weibull distribution. For the cdf $F(x) = \Phi((\log(x) - \theta_0)/\theta_1)$ the following parameter estimates are received $\theta_0 = 6.48$, θ_1=0.168 (where $\Phi(.)$ is the cdf of the standard normal distribution).

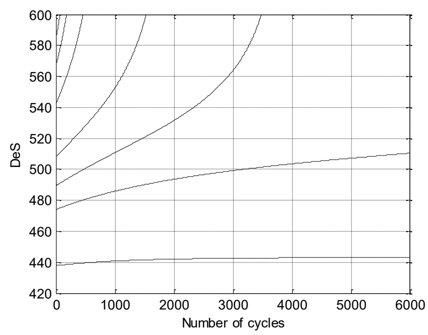
The results of calculation DeS curves and $n_{DeS}(s_C; s^+, \varepsilon)$ for $s_C = 600$ are shown on Fig. 1. It turned out that at high stress loads there is some small difference in the values of DeSFLf for two pairs: ($k_C = 1.75$, $k_L = 1.0$ **)** and **(** $k_C = 1.0$, $k_L = 1.75$ **)**. But this difference disappears and for $\varepsilon = 0.00001$ and for $\varepsilon = 0.000001$ if the load values are small**.** So if we need to know only the fatigue curve, there is no big difference : to use the k_C or k_L parameter, but if the residual strength data is also analyzed at the same time, it is more convenient to use parameter k_C assuming parameter k_L to be 1 because , as it was told already the equation (5) defines local residual stress corresponding to cdf $F_L(.)$.



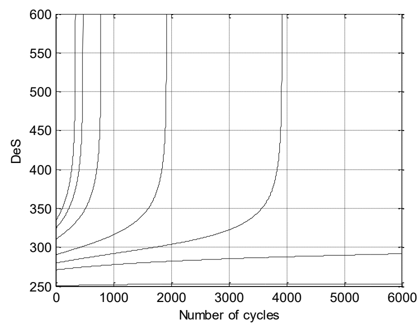
$\varepsilon = 0.00001$**;** $k_C = 1.75$, $k_L = 1.0$ DeS_DFLs : 8000 19000 47000 154000 349000 1507000 >6000000



$\varepsilon = 0.00001$**;** $k_C = 1.0$, $k_L = 1.75$ DeS_DFLs : 37000 50000 82000 195000 396000 1558000 >6000000



$\varepsilon = 0.000001$, $k_C = 1.75$, $k_L = 1.0$ DeS_DFLs : 71000 182000 461000 1531000 3483000 >6000000 >6000000



$\varepsilon = 0.000001$, $k_C = 1.0$, $k_L = 1.75$ DeS_DFLs : 343000 473000 785000 1919000 3920000 >6000000 >6000000

Figure 1: DeS and $n_{DeS}(s_C; s^+, \varepsilon)$ for load levels: 333.5 323.7 309.7 290.1 279.6 270.8 250.2 MPa

There is a need to investigate the influence of the parameter ε more closely. In first and second lines of Table 1 the results of the fatigue test [13] are shown. The results of calculating the corresponding $n_{DeS}(600; s^+, \varepsilon)$ for $k_C = 1.75$ and different ε are presented.

Table 1: Comparison of test data and calculations of DeS_FLfs

S	333.5	323.7	309.7	290.1
Test (cycles)	4928	115733	373199	1004800
ε	DeS_FLf			
0.1	1	3	5	16
0.01	8	19	47	154
0.00001	7038	18129	46031	153012

The analysis of the Table shows that for relatively small value of ε, $\varepsilon < 0.01$, DeS_FLf is approximately proportional to the $1/\varepsilon$ and to some specific value $N_{DeS}(s^+, \varepsilon_{DeS})$ of the DeS_FLf which is defined by some function $N_{DeS}(s^+, \varepsilon)$ for some specific value of $\varepsilon = \varepsilon_{DeS}$ .The values of DeS_FLf for some other ε, $\varepsilon < 0.01$, is defined by equation

$$N_{DeS}(s^+, \varepsilon) = (\varepsilon_{DeS} / \varepsilon) N_{DeS}(s^+, \varepsilon_{DeS}) . \tag{8}$$

Now we see that in order to get reasinoble fitting of the test SN curve by the use of The equation (8) we should fine the function $\varepsilon(s)$ corresponding to equation

$$N_{DeS}(s^+, \varepsilon(s^+)) = N_T(s^+) , \tag{9}$$

where $N_T(s^+)$ mean test fatigue life as function of s^+ which define SN curve.

By the way, let us note, the tedious calculations with a very small ε value in some cases can be replaced by faster calculations with a larger ε value. However, this rule does not take place for $\varepsilon > 0.01$.

3.2. The DeS fatigue equivalent distribution of the local strength

Using the corresponding value of the parameter ε we can obtain arbitrarily large calculated fatigue life . But the natural question appears: is there such a cdf, we denote it by $F_{DeS}(.)$, that the calculations using the equation (3) and a pair (ε , $F_L(.)$) give the same results as when using the same equation but pair (1 , $F_{DeS}(.)$) (using value $\varepsilon = 1$). The function $F_{DeS}(.)$ should correspond to the equation

$$(1-\varepsilon)s + \varepsilon s^+ / (1 - F_L(s)) = s^+ / (1 - F_{DeS}(s)) , \qquad s^+ \le s < \infty .$$

It is easy to get the following solution for $F_{DeS}(.)$

$$F_{DeS}(s)) = 1 + s^+(1 - F_L(s)) / ((1 - F_L(s))(1-\varepsilon)s + \varepsilon s^+) , \qquad s^+ \le s < \infty . \tag{10}$$

The function $F_{DeS}(.)$ we will call *the DeS fatigue equivalent distribution (DeS_FD) of local strength*. The fatigue life calculated using this function and DS approach (instead of DeS approach) would coincide with the data of fatigue tests.

3.3 Approximation

The way to calculate the DeS_FLf is the use of requrent formule (3) and its modifications. It is easy if the DeS_FLf is not too large. In other case it is very exhausting work. The following approximation can be used.

Let us consider i as continuous variable. If i is very large then the difference $(s_{i+1}-s_i)$ is very small and in subsequent calculations it can be used as a derivative $ds/di=\varepsilon(-s+k_C s^+/(1-F_L(s_i))$. Then the value of step corresponding to increasing s_i from s_0 to s

$$i(s_0,s,\varepsilon)=(1/\varepsilon)K(s_o,s), \tag{11}$$

where

$$K(s_0,s)=\int_{s_0}^{s}(1/(-x+k_C s^+/(1-F_L(x)))dx\,. \tag{12}$$

If $s_0=k_C s^+$, $s=s_C$ then we can get the approximate estimate of $n_{DeS}(s_C;s^+,\varepsilon)$.

For example, in Table 1 we see the values $n_{DeS}(600;s^+,0.00001)$: 7038; 18129; 46031; 153012 for s^+ =333.5, 323.7, 309.7, 290.1. Corresponding calculations using equation (9) give the very similar results : 7108 ; 18312 ; 46503 ; 154498 .

Let us note that the integral $K(s_0,s)$ can be used for approximate calculation of the $n_{DeS}(s_C;s^+,\varepsilon)$ and then for calculation of the approximate function $\varepsilon(s)$ which is necessary for fitting real test fatigue lives.

4. Numerical example 2. Residual strength

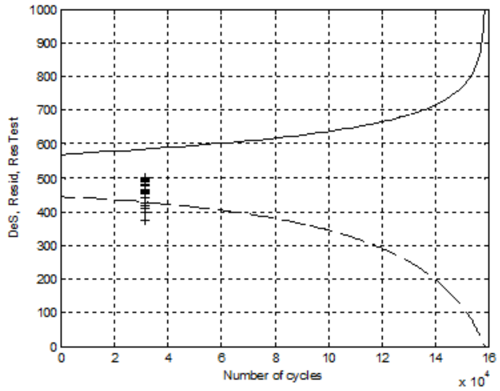
Now we consider the processing of the result of the fatigue test in which the data not only about the fatigue life but about the residual strength was obtained. The test data was taken from Tables 1-3 in Ref. [16] concerned T300/934 graphite/epoxy laminates with $[0/45/90-45_2/90/45/0]_2$ lay-up. In Table 1 of this paper the static strength of 25 specimens, in Table 2 numers of cycles to failure at three different stress levels (namely: for $\sigma_{\max}$ = 400, 380 and 290 MPa, $R=\sigma_{\min}/\sigma_{\max}=0$) and in Table 3 two sets of residual strength data are reported for 15 and 18 specimens subjected to cyclic loading up to 3,640,000 and 31,400 cycles at a maximum stress, $\sigma_{\max}$ = 290 and 345 MPa respectively.

The tested specimens are not the UFC. But we suppose that the failure of this composite takes the place after the failure of some weak microvolume (WMV) which is the bundle of the n parallel LIs. We make (enough rough) assumption that this WMV is a *UFC - equivalent* which has the same distribution of fatigue strength. In work [16], there is no information about the static strength of the composite components. We will use the data which we have used already in the previous example in which carbon fiber longitudinal elements were also used for the test specimens. This, of course, means that the following should be considered as an example of the application of the technique in question, and not as a study of a specific experiment. We accept also the lognormal distribution $F(x)=\Phi((\log(x)-\theta_0)/\theta_1)$ of the static strength of LIs with the same values of parameters $\theta_0=6.475$, $\theta_1=0.168$ as in the section 3.1.

After transformation of equation (11) we can calculate the value of ε

$$\varepsilon=K(s_o,s^+)/N_T(s^+), \tag{13}$$

which should ensure the equality of the calculated DeS_FL and the corresponding test value for specific load level. For two load levels, 345 and 290 MPa, we have got the corresponding values ε : 0.00000242 and 0.00000464. Then using these values further in equations (3 b), (5) and (8) , we get the results shown in Fig. 2. In the left part of this figure, for the load level of 345 MPa, the following is shown : a1) DeS curves (-), calculated (--) and test residual strengths (+); a2) cdf $F_L(.)$ and cdf $F_{DeS}(.)$; a3) "the derivatives" of these functions (df=$(F_L(s_{i+1})-F_L(s_i))/(s_{i+1}-s_i)$; def= $(F_{DeS}(s_{i+1})-F_{DeS}(s_i))/(s_{i+1}-s_i)$, $i=1,2,...$) .



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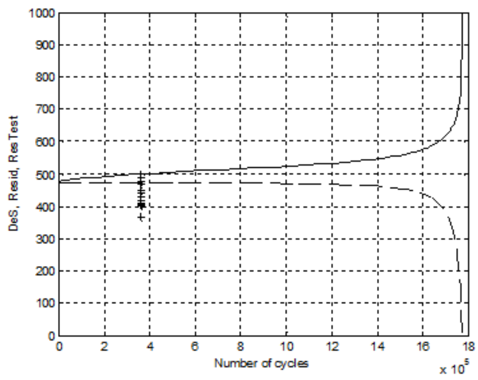

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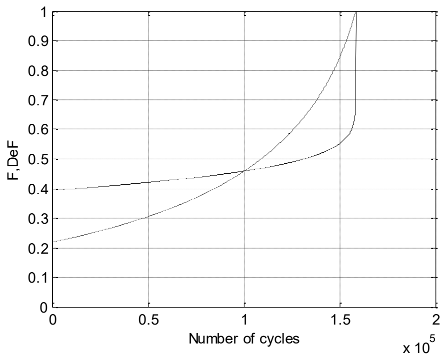

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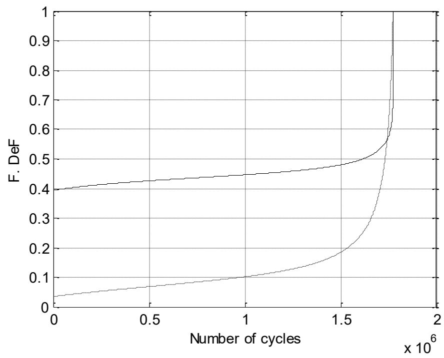

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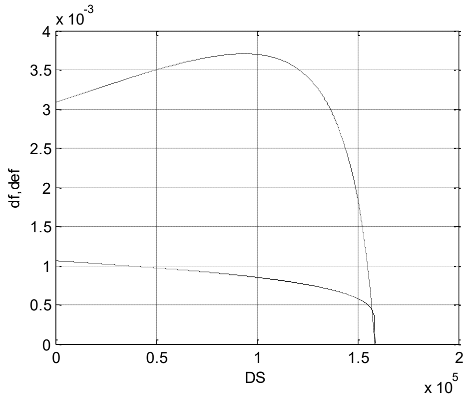
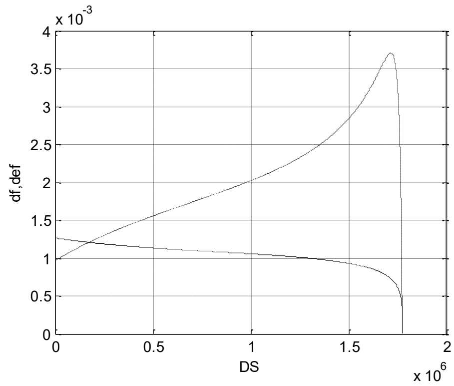


a 3



a 4

Figure.2. Daniels' sequences (DeS) , calculated (--) and test residual strength (+) (a1, b1); cdf $F_L(.)$ and Daniels'- equivalent cdf $F_{DeS}(.)$ (a2, b2) and the "derivatives" of these functions (a3, b3) for two load levels 345(a) and 290(b) MPa.

The right part of the figure shows similar results for the load level of 290 MPa. Let us note that the Daniels fatigue equivalent distribution, $F_{DeS}(.)$, provides another measurement of differences in the static strength distributions of separate longitudinal components, $F(.)$, and inside the structure of the composite.

On the sub-plots a1 and b1, we see that equation (5) for the assumed distribution function $F_L(.)$ gives a plausible description of the residual strength. It is useful to note: the DeS curve rushes to infinity and the calculated residual strength rushes to zero at the same number of cycles. Since equation (10) is suitable only for relatively low load levels, for levels 400 and 380 MPa, the values of ε were selected from the equation of the coincidence of the calculated and tested values of durability: 0.0000181 and 0.00000166. The final calculated fatigue curve is shown in Fig. 3.

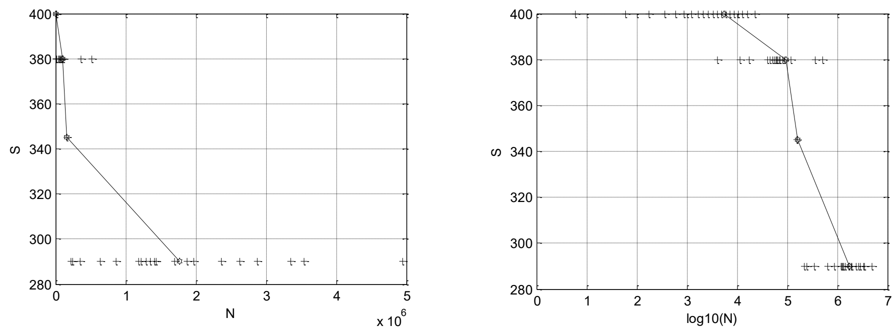


Figure 3. DeS_SN, log10DeS_SN (-) and tast data

Let us clarify that the description of the fatigue curve is obtained by selecting the coefficient $k_C = 1.6$, using equation (3b) and preliminary calculation values of ε: 0.00000242 and 0.00000464 using equation (10) for loading levels 345 and 290 MPa and by direct selection 0.0000181 and 0.00000166 for levels 400 and 380 MPa.

5. Prediction

The study of the possibility and accuracy of predicting changes in the fatigue life of a composite with changes in the static strength of its components using the DeS approach requires a volumetric experiment. And this is the task of subsequent research. Here we will limit ourselves to analyzing the effect of reducing the spread of static strength, more precisely, the parameter θ_1 on the calculated average fatigue life at the level of loads considered in the last example. A comparison of the results of the the calculations DeS_FLf for the two different values θ_1 but the same all the other parameters is shown in Table 2.

Table 2. ***Comparison of the calculations of DeS_FLfs for two values*** θ_1

s^+	480	380	345	290
Test (cycles)	5313	95760	158489	1772812
θ_1	DeS_FLf			
0.168	5500	95900	158600	1772800
0.015	4800	88100	164300	>4000000

We see a significant increase in DeS_FLf when the load is close to the fatigue limit. But with a relatively high load, the decrease led to a slight decrease in DeS_FLf. Recall that the same effect applies to calculating the average static strength. For the studied here lognormal distribution the average static strength is equal to $\exp(\theta_0 + (\theta_1)^2/2)$. These conclusions should be taken into account in the desine of a new composite which is similar to the studied here.

Conclusions

It is shown that a simplified version of DeS can be used for such description of the average fatigue curve (and residual static strength) of an UFC, which is directly related to the parameters of the static strength distribution of its components (SSDC). This description is very desirable because it makes it possible to predict changes in the fatigue characteristics of the UFC when the static strength characteristics of its components change. A numerical example of processing the test data on fatigue life and residual strength of a carbon-fiber reinforced composite confirms the reasonable coincidence of the calculation result and test data. The offered type of description of fatigue curve

The concept of a DeS fatigue equivalent distribution (DeS_FED) of local static strength of LI is introduced. The fatigue life calculated using this distribution and the basic Daniels sequence (DS) would coincide with the data of fatigue tests. Comparison of the DeS_FED with the real SSDC shows the specific behavior of the UFC components under short-term loading during a single fatigue loading cycle, as opposed to loading during static tests.

The application of the proposed method for processing fatigue tests data of composite material samples that differ in structure from the UFC will allow us to judge the influence of its features on the fatigue curve.

Randomization of the considered version of the DeS, using the Monte Carlo method allows to analyze the scatter of the fatigue life. But this time the search of the parameters of the corresponding nonlinear regression is a difficult task an example of this analysis is given in [2]. Against all the odds, we think that, in due course, the structure of the models suggested will be of the interest not only for the graduation theses of the students but also for the engineering applications, in particular, for the prediction of the variations fatigue life of the UFC after the changes in the parameters of their components.

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