Sensitivity Analysis of a Complex Engineering System with the Application of Wait in Line Theory

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Abstract

In this bloodthirsty scenario of competition, rapid and cost-effective production is a key obligation for endurance. To attain this objective the thought of production area is fetching people now days. In this sector, non- identical equipment is arranged rationally to execute desired procedure to convert unprocessed materials into the processed material. Sometimes during the manufacturing, the problem of waiting line arises because of some unpredictable reasons, so here this paper reveals the same problem with its effects on system reliability and availability, also system sensitive nature is analyzed with respect to unexpected failures. Supplementary variable technique and copula method is used to solve the system.

Keywords: Markov processes, steady state behavior, supplementary variable technique, reliability, wait in line etc.

1. Introduction

Wait in line theory has an important role in almost all analysis of repair services. Wait in line models are very helpful tool for calculating the performance of various repair systems like automated systems, production systems, computer systems, telecommunication systems, networking systems like computer networking or communication networking, and flexible manufacturing systems. Conventional wait in models forecast the system performance on the basis of the assumption that all service facilities offer failure-free service

With reference to the above facts, here we have discussed the behavior analysis of an engineering system having three-subsystems 1, 2 and 3, connected in series, having waiting line for repair. First subsystem has two units one is key unit and another is active superfluous. Second subsystem has one key Unit and another is cold superfluous while Third subsystem consists of two units' connected in parallel arrangement. The system can completely fail due to failure of any of the subsystems [3]. In the beginning when the system starts working, the key units of subsystem 1 and 2 and both units of the subsystem 3 are fully operational. When the main units of the subsystems 1 and 2 fail, the supporting units are switched on automatically and failed units are sent for repair to repairing section [1, 2]. Here, a realistic situation is taken into the consideration that when main units and supporting units of both subsystems 1 and 2 are failed and they are sent for repair to repairing section, they are in waiting line because repairmen is busy somewhere else. In this case a wait in line is there at repair section. So the main concern of the study is that all the four units are waiting for repair [4]. Transition state diagram is shown by Figures 1. Table 1 shows the state specification of the system.

Assumptions

- In the beginning the system is in good operating state.
- All Subsystems are connected in series.
- System has two states only good and failed not degraded.
- Catastrophic failure is also responsible for system failure in the study also they require constant and exponential repair. So, copula technique is used for finding probability distribution [5].
- Repair facility which follows general time distribution is there for the service of both the subsystems of unit 3 and also failure are exponential in both cases.
- For the subsystems 1 and 2 failure and repairs both are exponential.

Table 1: *State specification of the system*

States	Description	System State
S0	The system is in good working state	G
S1	The system is in working state when key unit is failed.	G
S2	The system is in failed state because of failure of superfluous unit.	F
S3	When all four units are in waiting at repair section, system is in failed state.	F
S4	The system is in working state when superfluous unit of subsystem 1 is failed.	G
S5	The system is in failed state due to the failure of key unit of subsystem 1.	F
S6	The system is in working condition when key unit subsystem 2 is failed.	G
S7	The system is in failed state when superfluous unit of subsystem 2 is failed.	F
S8	The system is in operable condition when key unit of subsystem 3 failed.	G
S9	The system is in failed state from the state S_8 due to failure of superfluous unit of subsystem 3.	FR
S10	The system is in operable condition when superfluous unit of subsystem 3 is failed.	G
S11	The system is in failed state from the state S_{10} due to failure of key unit of subsystem 3.	FR
S12	The system is in failed state from the state S_1 due to failure of subsystem 3.	FR
S13	The system is in failed state from the state S_6 due to failure of subsystem 3.	FR
S14	System is failed state because of catastrophic failure.	FR

G: Good state; F: Failed State; FR= Failed state and under repair.

2. Notations

Pr	Probability
$P_0(t)$	Pr (at time t system is in good state S0)
$P_i(t)$ t},	Pr {the system is in failed state due to the failure of the i[th] subsystem at time where i=2, 5, 7, 14.
λ_i	Failure rates of subsystems, where i=a_1, a_2, b_1, b_2, c_1, c_2, CSF.
ψ	Arrival rate of all four units of subsystems 1 and 2 to the repair section named as a_1, a_2, b_1, b_2.
μ	Repair rate of unit's a_1, a_2, b_1, b_2.
$\phi_i(k)$	General repair rate of i[th] system in the time interval (k, k+Δ), where i= c_1, c_2, (names for

the units of subsystem 3) CSF and k=v, g, r, l.

$P_3(t)$	Pr (at time t there is a queue (a_1, a_2, b_1, b_2) in the maintenance section due to servicing of some other unit and all four machines are waiting for repair.
$P_i(j,k,t)$	Pr (at time t system is in failed state due to the failure of j[th] unit when k[th] unit has been already failed, where i=9, 11. j=g, v. and k=v, g.
K1, K2	Profit cost and service cost per unit time respectively.

Let $u_1 = e^l$ and $u_2 = \phi_{CSF}(l)$ then the expression for joint probability according to Gumbel-Hougaard family of copula is given as $\phi_{CSF}(l) = \exp[l^{\theta} + (\log\phi_{CSF}(l))^{\theta})^{1/\theta}]$

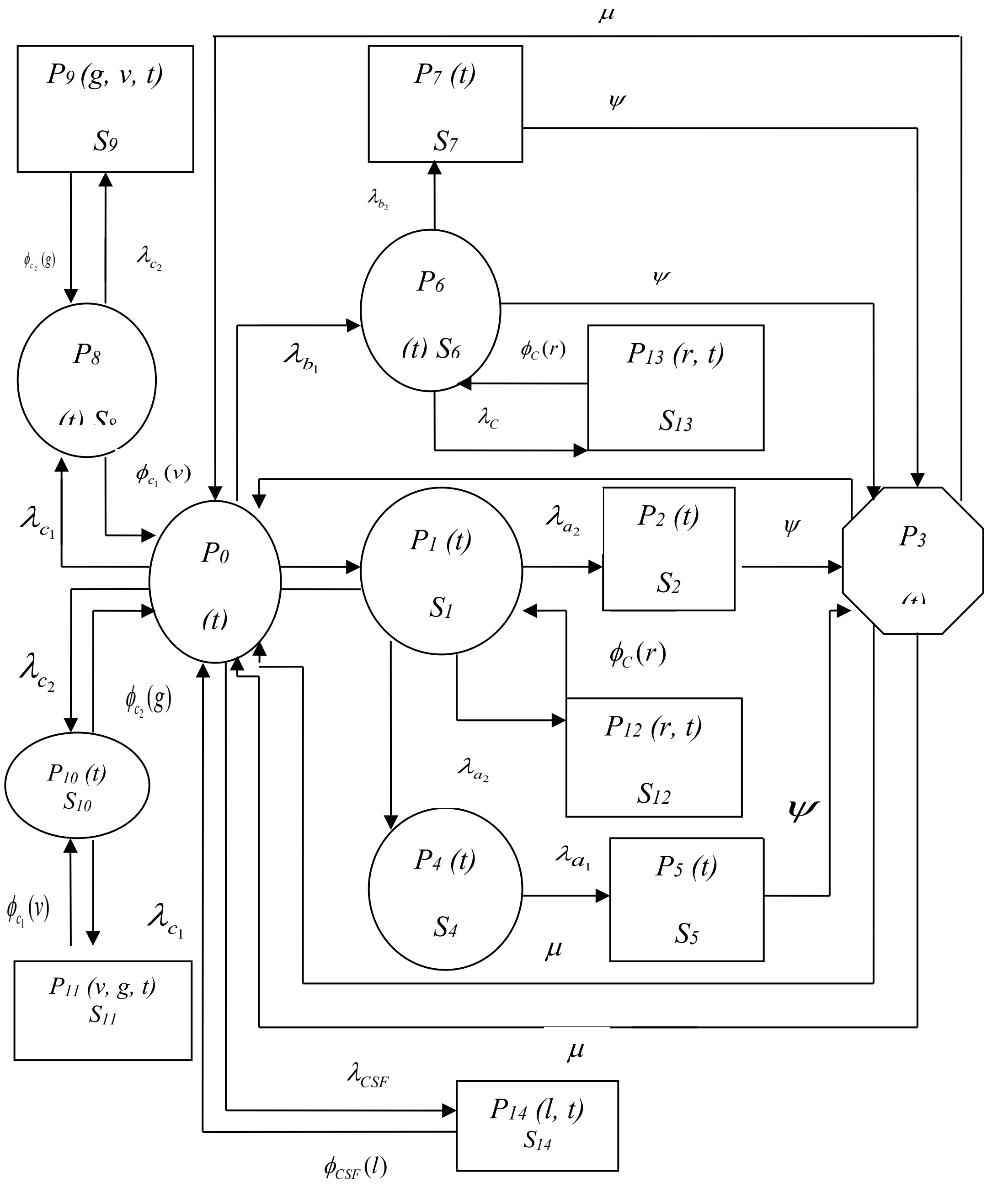


Figure 1: *Transition state diagram*

3. Formulation of the mathematical model

The following differential equations have been obtained by considering limiting procedures and different probability constraints which satisfying the model:

$$\left[\frac{d}{dt}+\lambda_{a_1}+\lambda_{a_2}+\lambda_{b_1}+\lambda_{b_2}+\lambda_{c_1}+\lambda_{c_2}+\lambda_{CSF}\right]P_0(t)=\int_0^\infty \mu\ (i)P_3(t)di+\phi_{c_1}P_8(t)+\phi_{c_2}P_{10}(t)+\int_0^\infty \phi_{CSF}(l)P_{14}(l,t)dl \quad ...(1)$$

$$\left[\frac{\partial}{\partial t}+\lambda_{a_2}+\lambda_c\right]P_1(t)=\lambda_{a_1}P_0(t)+\int_0^\infty \phi_C(r)P_{12}(r,t)dr \quad ...(2)$$

$$\left[\frac{\partial}{\partial t}+\psi\right]P_2(t)=\lambda_{a_2}P_1(t) \quad ...(3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial i}+(\mu+\psi)\right]P_3(t)=\psi[P_2(t)+P_5(t)+P_6(t)+P_7(t)]+\frac{(\psi t)^3e^{-\psi t}}{6} \quad ...(4)$$

$$\left[\frac{\partial}{\partial t}+\lambda_{a_1}\right]P_4(t)=\lambda_{a_2}P_0(t) \quad ...(5)$$

$$\left[\frac{\partial}{\partial t}+\psi\right]P_5(t)=\lambda_{a_1}P_4(t) \quad ...(6)$$

$$\left[\frac{\partial}{\partial t}+\lambda_{b_2}+\psi\right]P_6(t)=\lambda_{b_1}P_0(t)+\int_0^\infty \phi_C(r)P_{13}(r,t)dr \quad ...(7)$$

$$\left[\frac{\partial}{\partial t}+\psi\right]P_7(t)=\lambda_{b_2}P_6(t) \quad ...(8)$$

$$\left[\frac{\partial}{\partial t}+\phi_{c_1}(v)+\lambda_{c_2}\right]P_8(t)=\lambda_{c_1}P_0(t)+\int_0^\infty \phi_{c_2}(g)P_{11}(g,v,t)dg \quad ...(9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial g}+\phi_{c_2}(g)\right]P_9(g,v,t)=0 \quad ...(10)$$

$$\left[\frac{\partial}{\partial t}+\phi_{c_2}(g)+\lambda_{c_1}\right]P_{10}(t)=\lambda_{c_2}P_0(t)+\int_0^\infty \phi_{c_1}(v)P_{11}(v,g,t)dv \quad ...(11)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial v}+\phi_{c_1}(v)\right]P_{11}(v,g,t)=0 \quad ...(12)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\phi_C(r)\right]P_{12}(r,t)=0 \quad ...(13)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\phi_C(r)\right]P_{13}(r,t)=0 \quad ...(14)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial l}+\phi_{CSF}(l)\right]P_{14}(l,t)=0 \quad ...(15)$$

Boundary Conditions:

$$P_3(i=0,t)=\psi[P_2(t)+P_3(t)+P_6(t)+P_7(t)] \quad ...(16)$$

$$P_8(0,t)=\lambda_{c_1}P_0(t) \quad ...(17)$$

$$P_9(0,v,t)=\lambda_{c_2}P_8(t) \quad \ldots (18)$$

$$P_{10}(0,t)=\lambda_{c_2}P_0(t) \quad \ldots (19)$$

$$P_{11}(0,g,t)=\lambda_{c_1}P_{10}(t) \quad \ldots (20)$$

$$P_{12}(0,t)=\lambda_C P_1(t) \quad \ldots (21)$$

$$P_{13}(0,t)=\lambda_C P_6(t) \quad \ldots (22)$$

$$P_{14}(0,t)=\lambda_{CSF}P_0(t) \quad \ldots (23)$$

Initial condition:

$P_0(0)=1$, otherwise zero.

Solving equations (1) through (15) by taking Laplace transform and by using initial and boundary conditions we obtained following probabilities of system is in up) and down states at time *t*,

$$\overline{P}up=\overline{P}_0(s)+\overline{P}_1(S)+\overline{P}_4(s)+\overline{P}_6(s)+\overline{P}_8(s)+\overline{P}_{10}(s)$$

$$=\frac{1}{K(s)}[\,1+\frac{\lambda_{a_1}}{[s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]}+\frac{\lambda_{a_2}}{[s+\lambda_{a_1}]}+\frac{\lambda_{b_1}}{[s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}+$$
$$\frac{\lambda_{c_1}}{[s+\lambda_{c_2}+\phi_{c_1}(v)-\lambda_{c_2}\overline{S}_{\phi_{C_2}}(s)]}+\frac{\lambda_{c_2}}{[s+\lambda_{c_1}+\phi_{c_2}(g)-\lambda_{c_1}\overline{S}_{\phi_{c1}}(s)]}\,] \quad \ldots (24)$$

$$\overline{P}down=\overline{P}2(s)+\overline{P}5(s)+\overline{P}7(s)+\overline{P}9(s)+\overline{P}11(s)+\overline{P}12(s)+\overline{P}13(s)+\overline{P}14(s)$$

$$=\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]}\frac{1}{K(s)}+\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_1}]}\frac{1}{K(s)}+$$
$$\frac{\lambda_{b_1}\lambda_{b_2}}{[s+\psi][s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}\frac{1}{K(s)}+$$
$$\frac{\lambda_{c_1}\lambda_{c_2}D_{\phi_{c2}}(s)}{[s+\lambda_{c_2}+\phi_{c_1}(v)-\lambda_{c_2}\overline{S}_{\phi_{c2}}(s)]}\frac{1}{K(s)}+\frac{\lambda_{c_1}\lambda_{c_2}D_{\phi_{c1}}(s)}{[s+\lambda_{c_1}+\phi_{c_2}(g)-\lambda_{c_1}\overline{S}_{\phi_{c1}}(s)]}\frac{1}{K(s)}+$$
$$\frac{\lambda_C\lambda_{a_1}D_{\phi_c}(s)}{[s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]}\frac{1}{K(s)}+\frac{\lambda_C\lambda_{b_1}D_{\phi_c}(s)}{[s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}\frac{1}{K(s)}+$$
$$\frac{\lambda_{CSF}D_{\phi_{CSF}}(s)}{K(s)} \quad \ldots (25)$$

where,

$$K(s)=s+\lambda_{a_1}+\lambda_{a_2}+\lambda_{b_1}+\lambda_{b_2}+\lambda_{c_1}+\lambda_{c_2}+\lambda_{CSF}-\psi\{[\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]}$$
$$+\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_1}]}+\frac{\lambda_{b_1}}{[s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}+\frac{\lambda_{b_1}\lambda_{b_2}}{[s+\psi][s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}]D_\mu(s)$$
$$+\frac{\psi^3}{(s+\psi)^4}\}-\frac{\lambda_{c_1}\phi_{c_1}(v)}{[s+\lambda_{c_2}+\phi_{c_1}(v)-\lambda_{c_2}\overline{S}_{\phi_{C_2}}(s)]}-\frac{\lambda_{c_2}\phi_{c_2}(g)}{[s+\lambda_{c_1}+\phi_{c_2}(g)-\lambda_{c_1}\overline{S}_{\phi_{c1}}(s)]}$$
$$-\lambda_{CSF}\overline{S}_{\phi_{CSF}}(s)$$

$$M(s)=\psi\{[\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_2}+\lambda_C-\lambda_C\overline{S}_{\phi_C}(s)]}+\frac{\lambda_{a_1}\lambda_{a_2}}{[s+\psi][s+\lambda_{a_1}]}+\frac{\lambda_{b_1}}{[s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}$$

$$+\frac{\lambda_{b_1}\lambda_{b_2}}{[s+\psi][s+\lambda_{b_2}+\psi-\lambda_C\overline{S}_{\phi_C}(s)]}]D_\mu(s)++\frac{\psi^3}{(s+\psi)^4}\}$$

… (26)

$$D_\mu(s)=\frac{1-\overline{S}_\mu(s)}{s+\psi} \qquad \text{… (27)}$$

$$\phi_{CSF}(l)=\exp[l^\theta+(\log\phi_{CSF}(l))^\theta)^{1/\theta}] \qquad \text{… (28)}$$

Also,

$$\overline{P}_{up}(s)+\overline{P}_{down}(s)=\frac{1}{s}$$

… (29)

Steady state behavior of the system By Abel's lemma we have,

$$\lim_{s\to 0}\{s\overline{F}(s)\}=\lim_{t\to\infty}F(t)$$

In equations (24) and (25) we get,

$$\overline{P}_{up}(s)=\frac{1}{K(0)}[1+\frac{\lambda_{a_1}}{\psi\lambda_{a_2}}+\frac{\lambda_{a_2}}{\lambda_{a_1}}+\frac{\lambda_{b_1}}{\lambda_{b_2}+\psi-\lambda_c}+\frac{\lambda_{c_1}}{\phi_{c_1}(v)}+\frac{\lambda_{c_2}}{\phi_{c_2}(g)}] \qquad \text{… (30)}$$

$$\overline{P}_{down}=\frac{1}{K(0)}[\frac{\lambda_{a_1}}{\psi}+M(0)+\frac{\lambda_{a_2}}{\psi}+\frac{\lambda_{b_1}\lambda_{b_2}}{\psi(\lambda_{b_2}+\psi-\lambda_C)}+\frac{\lambda_{c_1}\lambda_{c_2}M_{\phi_{c2}}}{\phi_{c_1}(v)}+\frac{\lambda_{c_1}\lambda_{c_2}M_{\phi_{c1}}}{\phi_{c_2}(g)}+$$

$$\frac{\lambda_c\lambda_{a_1}M_{\phi_C}}{\lambda_{a_2}}+\frac{\lambda_C\lambda_{b_1}M_{\phi_C}}{\lambda_{b_2}+\psi-\lambda_C}+\lambda_{CSF}M_{\phi_{CSF}}] \qquad \text{… (31)}$$

where,

$$M(0)=\lim_{s\to 0}M(s)$$

… (32)

$$M_{\phi_i}=\lim_{s\to 0}\frac{1-\overline{S}_{\phi_i}(s)}{s}$$

… (33)

$$\mathrm{S}_{\phi_i}(s)=\frac{\phi_1}{\mathrm{s}+\phi_1} \qquad \text{… (34)}$$

Sensitivity analysis:

Here we have done sensitivity analysis of the system for catastrophic failure rate and key unit of subsystem 1.

S = -t*exp (-(Lcsf + La1 + La2 + Lb1 + Lb2 + Lc1 + Lc2)*t)

5. Results and Discussion

Here reliability, availability and sensitivity analysis with respect to catastrophic failure and key unit of subsystem 1 is done for the considered system by employing Supplementary variables technique and Copula methodology. The Figure 2 shows the movement of reliability of the system against time for fixed values of failure and repair rates. From the graph we conclude that the reliability of the system reduces hastily with passage of time because of waiting line for repair in the repair section.

Figure 3 talks about the availability of the system which says that the availability reduces approximately in a constant manner as time increases.

The sensitivities of the system reliability R (t) with respect to system parameters like catastrophic failure and key unit of subsystem 1 are shown in figures-4 and 5. It can easily be observed that there is very negligible impact of both the parameters on system sensitivity.

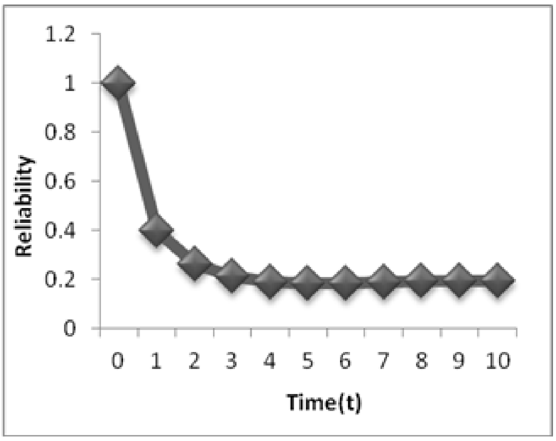


Figure 2: *Reliability against time*

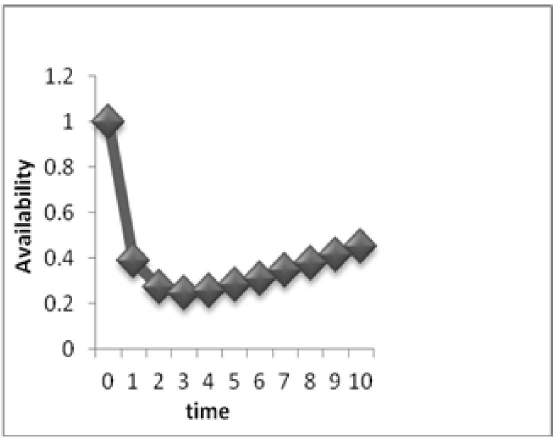
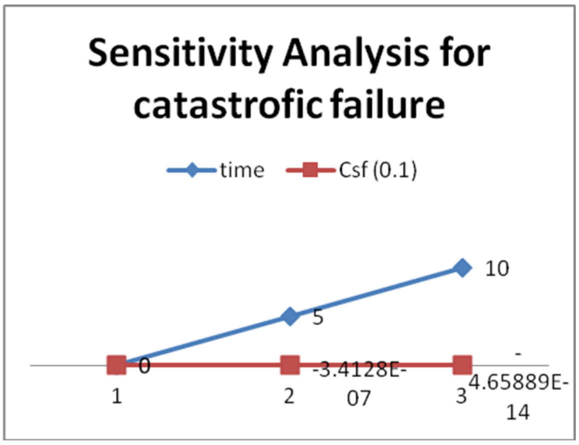


Figure 3: *Availability against time*



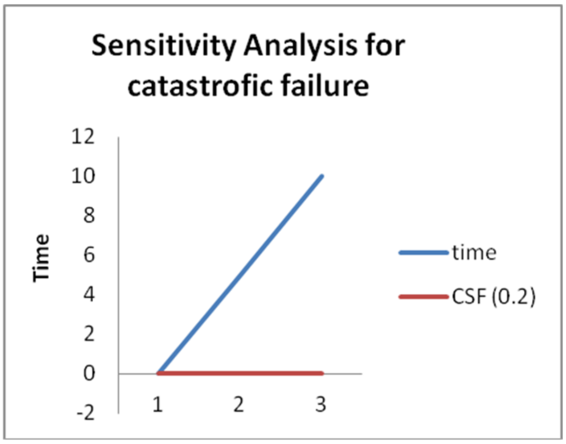


Figure 4: *Sensitivity Analysis for catastrophic failure*

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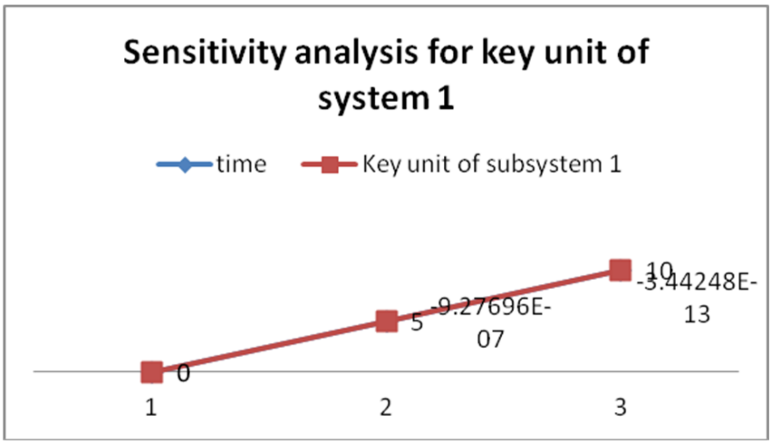

Figure 5: *Sensitivity Analysis for key unit of system1*

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