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RAM Analysis of E_r/M/1/N Phase-Type Queueing System with Working and Working-Breakdown States

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Abstract

In this paper, the Reliability, Availability and Maintainability (RAM) analysis for the finite capacity Erlangian Phase-type Queueing model is studied with regard to failure and recovery rates. The arrival process of the machines to repair is assumed to follow Erlang distribution and the service process is exponentially distributed in FCFS discipline. Apart from the multi-phases in the queueing system two different environmental states such as the working and working-breakdown states were also taken into consideration. The transient state differential-difference equations for the general case and for the special case of N=5 is obtained. The results are presented numerically and graphically along with some special metrics such as MTBF and MTTR. The sensitivity analysis is also performed to find changes in different parametric values for the model.

Keywords: Availability, Erlang distribution, Multi-phase queueing system, Maintainability, Reliability, Sensitivity Analysis.

I. Introduction

Queueing theory was developed in order to provide models to predict the behaviour of systems that aims to provide service for randomly arising demands. Any system in which arrivals place demands upon a finite capacity resource may be determined as queueing system. In order to describe the queueing of systems more effectively it is necessary to understand Erlang theory. The main assumption of the Erlang in queueing model is that the calls arrive as a Poisson process and when there is more than one inter-related Poisson process occurring in phases it is considered to be Phase-Type distribution with continuous variable. Thus, in Phase-Type theory Erlang distribution is considered to be a special case. The phase-type distribution and phase-type renewal processes were introduced by Neuts [7], who formed the substrata for the definition of the N-process and the Markov-modulated Poisson process (MMPP). Binkowski and Carragher [3] employed an Er/Ek/1/N queuing system to model the operation of a stockyard mining. Baba [2] studied GI/M/1 queue with working vacations by using the matrix analytic method and subsequently, for the queue with working vacation and vacation interruption. Plumchitchom and Thomopoulos [8] made a study on a single-server queuing system with Erlang distributed inter-arrival and service times, Li et al. [5] studied the GI/M/1 queue such that the vacation time follows an exponential distribution.

Along with Queueing of the systems, it is also important to analyze the performance of the industrial systems by using the most important metric such as the Reliability. In order to receive effective results in the Industrial systems it is proved to analyze the Availability and Maintainability of the machines along with Reliability. Performance modelling and availability analysis are applied by many researchers on different industrial systems such as the paper plant, paint, and thermal power plant Industry etc., Singh, and Goyal, [11] developed a methodology to study the transient behaviour of repairable mechanical biscuit shaping system on a biscuit manufacturing plant for determining the availability of the system based on Markov modelling. Lin, et al. [6] made a study on reliability using both classical and Bayesian semi-parametric frame-works, they illustrated modelled a wheel- set's degradation data and analyzed to ease the calculation of system reliability during applying preventive maintenance. The differential equations have been solved using Laplace Transforms. These Laplace Transform are commonly used in the transient state to obtain the state probabilities. Aggarwal, et. al. [1] presented a model using Markov birth-death process with the concept of fuzzy reliability and availability assuming that the failure and repair rates of each subsystem as exponential distribution.

In this paper RAM analysis of the Er/M/1 finite space queueing model for different environmental states such as the Working state and the Working-Breakdown state is studied. The differential-difference equations for the model are formed and a special case of N=5 is considered. The transient equations are solved using Fourth-Order Runge Kutta numerical method. The results are shown numerically and graphically for reliability, availability and maintainability analysis for the queueing system. The Sensitivity Analysis is also carried out for the changes in different parametric values involved in the model.

II. Assumptions and Notations

The following are the assumptions that are used in this model:

- The arrival of machines for repair to the queueing system is independent according to the Erlang process with a constant parameter λ
- The service process is exponentially distributed with First Come First Service (FCFS) queue discipline
- When the system is in the working state (i.e., there should be at least one machine) failure occurs at the interarrival phase which is also exponentially distributed and once the failure occurs in the system the process is moved to the working-breakdown state where the it is performed at a low rate
- Whenever working-breakdown occurs in the system, it is immediately recovered in the recovery state which is also exponentially distributed. Once the system recovers it performs its activity at a normal arrival rate
- All inter-arrival times and the service times are independent of each other

The following are the notations that are used in this paper:

N(t) : Total no of machines in the system at any time t

Er : Erlang distribution with r identical phases

S(t) : The environmental state at any instant of time t which is given by

$$S(t) = \begin{cases} 0, \text{ if the server is in the working environment state for Phase 1 \& 2} \\ 1, \text{ if the server is in the working breakdown state for Phase 1 \& 2} \end{cases}$$

λ : Arrival rate

$\mu 1$: Service rate for working state

$\mu 2$: Service rate for working-breakdown state ($\mu 1 > \mu 2$).

α : Failure rate of the queueing system

β : Recovery rate of the queueing system

The transient state-probabilities that are used in this model:

P0,0,0(t): Probability of arrival of a machine (i.e., at least one machine) in the system

Pn, i, j(t): Probability that there are (n-1) machines in the system with i (i=0, 1, …, r) phases and j (j=1,2) states

III. Description of the model

The RAM analysis of an Erlang phase type arrival and single server queue with finite capacity queueing system is considered. The arrival of machines to repair follows Erlang distribution with the parameter $r\lambda$ is used for this model. Two different service mechanisms are exponentially distributed with parameters $\mu 1$ and $\mu 2$ are considered for this model based on the environmental states namely, working and working break-down states respectively. When the system is not empty (i.e., at least one machine in the system) failure occurs in the arrival process of the system with the failure rate α. Therefore, whenever failure occurs it is immediately recovered in the recovery state with rate β. The failure and recovery rate are assumed to be exponentially distributed. The state-transition diagram for the RAM analysis of the phase type Erlang queuing model is presented in Figure 1:

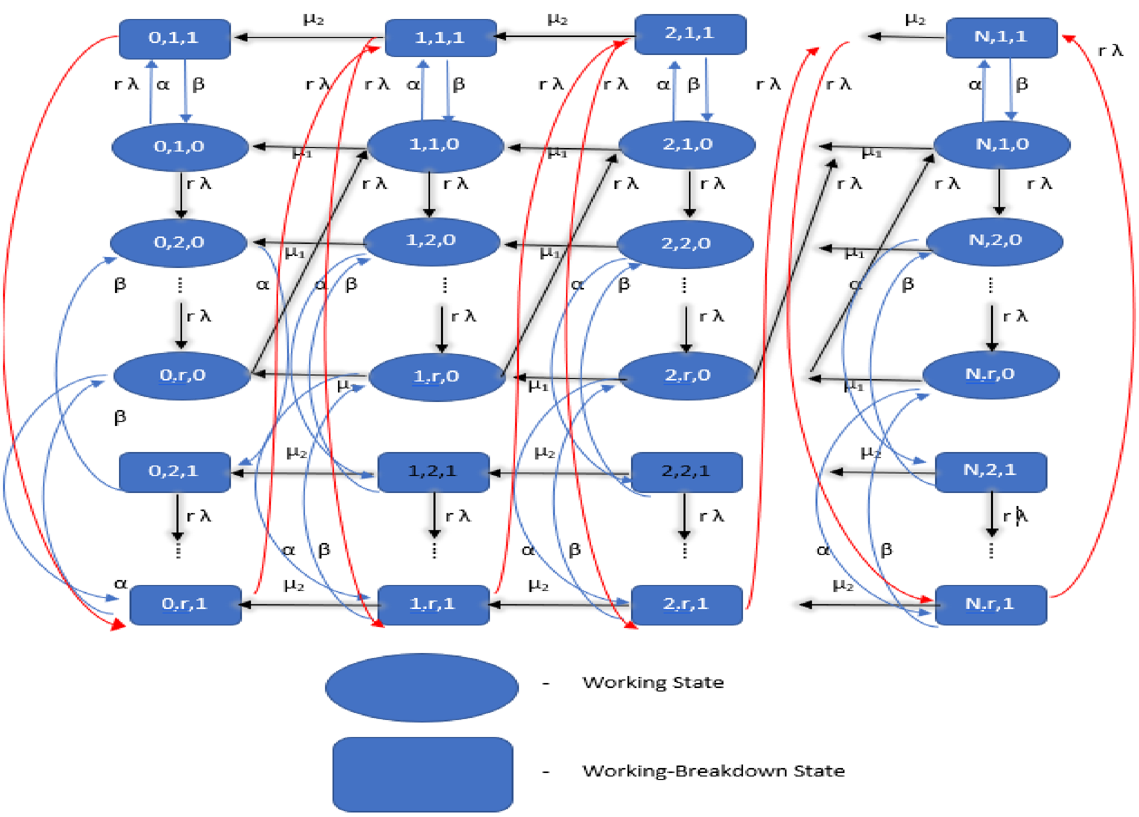

Figure 1: State-Transition Diagram of E_r/M/1

By using the state-transition diagram, the transient state differential-difference equations are formed for the Erlangian Phase-type queueing model with working and working-breakdown states.

WORKING STATE

$$\frac{dP_{0,1,0}(t)}{dt} = \mu_1 P_{n+1,1,0}(t) + \beta P_{0,1,0}(t) - (r\lambda + \alpha)P_{0,1,0}(t), \quad n=0 \quad (3.1)$$

$$\frac{dP_{n,1,0}(t)}{dt} = r\lambda P_{n-1,r,0}(t) + \mu_1 P_{n+1,1,0}(t) + \beta P_{n,1,1}(t) - (r\lambda + \mu_1 + \alpha)P_{n,1,0}(t), \quad n \geq 1 \quad (3.2)$$

$$\frac{dP_{N,1,0}(t)}{dt} = r\lambda P_{N-1,1,0}(t) + \beta P_{N,1,1}(t) - (\mu_1 + r\lambda + \alpha_1)P_{N,1,0}(t), \quad n=N \quad (3.3)$$

$$\frac{dP_{0,r,0}(t)}{dt} = \mu_1 P_{n+1,r,0}(t) + \beta P_{0,r,0}(t) + r\lambda P_{0,r-1,0}(t) - (r\lambda + \alpha)P_{0,1,0}(t), \quad n=0 \quad (3.4)$$

$$\frac{dP_{n,r,0}(t)}{dt} = r\lambda P_{n-1,r-1,0}(t) + \mu_1 P_{n+1,r,0}(t) + \beta P_{n,r,1}(t) - (r\lambda + \mu_1 + \alpha)P_{n,r,0}(t), \quad n \geq 1 \quad (3.5)$$

$$\frac{dP_{N,r,0}(t)}{dt} = r\lambda P_{N-1,r,0}(t) + \beta P_{N,r,1}(t) - (\mu_1 + \alpha)P_{N,r,0}(t), \quad n=N \quad (3.6)$$

WORKING - BREAKDOWN STATE

$$\frac{dP_{0,1,1}(t)}{dt} = \mu_2 P_{1,1,1}(t) + \alpha P_{0,1,0}(t) - (r\lambda + \beta)P_{0,1,1}(t), \quad n=0 \quad (3.7)$$

$$\frac{dP_{n,1,1}(t)}{dt} = r\lambda P_{n-1,r,1}(t) + \mu_2 P_{n+1,r-1,1}(t) + \alpha P_{n,1,0}(t) - (r\lambda + \mu_2 + \beta)P_{n,1,1}(t), \quad n \geq 1 \quad (3.8)$$

$$\frac{dP_{N,1,1}(t)}{dt} = r\lambda P_{N-1,r,1}(t) + \alpha P_{N,1,1}(t) - (\mu_2 + r\lambda + \beta)P_{N,1,1}(t), \quad n=N \quad (3.9)$$

$$\frac{dP_{0,r,1}(t)}{dt} = \mu_2 P_{n+1,r,1}(t) + \alpha P_{0,r,0}(t) + r\lambda P_{0,r-1,1}(t) - (r\lambda + \beta)P_{0,1,1}(t), \quad n=0 \quad (3.10)$$

$$\frac{dP_{n,r,1}(t)}{dt} = r\lambda P_{n-1,r,1}(t) + \mu_2 P_{n+1,r,1}(t) + \alpha P_{n,r,0}(t) - (r\lambda + \mu_2 + \beta)P_{n,r,1}(t), \quad n \geq 1 \quad (3.11)$$

$$\frac{dP_{N,r,1}(t)}{dt} = r\lambda P_{N-1,r,1}(t) + \alpha P_{N,r,0}(t) - (\mu_2 + \beta)P_{N,r,1}(t), \quad n=N \quad (3.12)$$

without loss of generality the initial state *conditions* are given by
$P_{0,0,0}(0) = 0, P_{n,i,j}(0) = 0, \forall n = 1,2,...,N;$ i=1,2; j=0,1

The system reliability at time t is calculated as follows:

$$R(t) = \sum_{n=0}^{N}\sum_{i=1}^{2}\sum_{j=0}^{1} P_{n,i,j} \quad (3.13)$$

The system Availability at time t is calculated by considering all the working states is as follows:

$$A(t) = \sum_{n=0}^{N}\sum_{i=1}^{2}\sum_{j=0} P_{n,i,j} \quad (3.14)$$

The system Maintainability at time t is calculated by considering working-breakdown state which is calculated as follows:

$$M(t) = \sum_{n=0}^{N}\sum_{i=1}^{2}\sum_{j=1} P_{n,i,j} \quad (3.15)$$

Apart from the RAM, the special metrics such as MTBF (Mean time between failures) and MTTR (Mean Time till Recovery) are also calculated as follows:

$$MTBF = \frac{1}{\alpha}$$

$$MTTR = \frac{1}{\beta}$$

IV. Special case

The differential-difference equations for N=5 is formed for the transient state of the Reliability model for the Erlangian Phase-type queueing system. The equations for the working and working-breakdown states are given below:

WORKING STATE

$$\frac{dP_{0,1,0}(t)}{dt} = \mu_1 P_{1,1,0}(t) + \beta P_{0,1,1}(t) - (2\lambda + \alpha) P_{0,1,0}(t) \quad (3.1.1)$$

$$\frac{dP_{1,1,0}(t)}{dt} = 2\lambda P_{0,2,0}(t) + \mu_1 P_{2,1,0}(t) + \beta P_{1,1,1}(t) - (2\lambda + \mu_1 + \alpha) P_{1,1,0}(t), \quad (3.1.2)$$

$$\frac{dP_{2,1,0}(t)}{dt} = 2\lambda P_{1,2,0}(t) + \mu_1 P_{3,1,0}(t) + \beta P_{2,1,1}(t) - (2\lambda + \mu_1 + \alpha) P_{2,1,0}(t), \quad (3.1.3)$$

$$\frac{dP_{3,1,0}(t)}{dt} = 2\lambda P_{2,2,0}(t) + \mu_1 P_{4,1,0}(t) + \beta P_{3,1,1}(t) - (2\lambda + \mu_1 + \alpha) P_{3,1,0}(t), \quad (3.1.4)$$

$$\frac{dP_{4,1,0}(t)}{dt} = 2\lambda P_{3,2,0}(t) + \beta P_{4,1,1}(t) - (2\lambda + \mu_1 + \alpha) P_{4,1,0}(t), \quad (3.1.5)$$

$$\frac{dP_{0,2,0}(t)}{dt} = \mu_1 P_{1,2,0}(t) + \beta P_{0,2,1}(t) + 2\lambda P_{0,1,0}(t) - (2\lambda + \alpha) P_{0,2,0}(t) \quad (3.1.6)$$

$$\frac{dP_{1,2,0}(t)}{dt} = \mu_1 P_{2,2,0}(t) + \beta P_{1,2,1}(t) + 2\lambda P_{1,1,0}(t) - (2\lambda + \mu_1 + \alpha) P_{1,2,0}(t), \quad (3.1.7)$$

$$\frac{dP_{2,2,0}(t)}{dt} = 2\lambda P_{2,1,0} + \mu_1 P_{3,2,0}(t) + \beta P_{2,2,1}(t) - (2\lambda + \mu_1 + \alpha) P_{2,2,0}(t), \quad (3.1.8)$$

$$\frac{dP_{3,2,0}(t)}{dt} = 2\lambda P_{3,1,0} + \mu_1 P_{4,2,0}(t) + \beta P_{3,2,1}(t) - (2\lambda + \mu_1 + \alpha) P_{3,2,0}(t), \quad (3.1.9)$$

$$\frac{dP_{4,2,0}(t)}{dt} = 2\lambda P_{4,1,0}(t) + \beta P_{4,2,1}(t) - (\mu_1 + \alpha) P_{4,2,0}(t), \quad (3.1.10)$$

WORKING - BREAKDOWN STATE

$$\frac{dP_{0,1,1}(t)}{dt} = \mu_2 P_{1,1,1}(t) + \alpha P_{0,1,0}(t) - (2\lambda + \beta) P_{0,1,1}(t), \quad (3.1.11)$$

$$\frac{dP_{1,1,1}(t)}{dt} = 2\lambda P_{0,2,1}(t) + \mu_2 P_{2,1,1}(t) + \alpha P_{1,1,0}(t) - (2\lambda + \mu_2 + \beta) P_{1,1,1}(t), \quad (3.1.12)$$

$$\frac{dP_{2,1,1}(t)}{dt} = 2\lambda P_{1,2,1}(t) + \mu_2 P_{3,1,1}(t) + \alpha P_{2,1,0}(t) - (2\lambda + \mu_2 + \beta) P_{2,1,1}(t), \quad (3.1.13)$$

$$\frac{dP_{3,1,1}(t)}{dt} = 2\lambda P_{2,2,1}(t) + \mu_2 P_{4,1,1}(t) + \alpha P_{3,1,0}(t) - (2\lambda + \mu_2 + \beta) P_{3,1,1}(t), \quad (3.1.14)$$

$$\frac{dP_{4,1,1}(t)}{dt} = 2\lambda P_{3,2,1}(t) + \alpha P_{4,1,0}(t) - (\mu_2 + 2\lambda + \beta) P_{4,1,1}(t), \quad (3.1.15)$$

$$\frac{dP_{0,2,1}(t)}{dt} = \mu_2 P_{1,2,1}(t) + \alpha P_{0,2,0}(t) + 2\lambda P_{0,1,1}(t) - (2\lambda + \beta) P_{0,2,1}(t), \quad (3.1.16)$$

$$\frac{dP_{1,2,1}(t)}{dt} = \mu_2 P_{2,2,1}(t) + \alpha P_{1,2,0}(t) + 2\lambda P_{0,1,1}(t) - (2\lambda + \mu_2 + \beta) P_{1,2,1}(t), \quad (3.1.17)$$

$$\frac{dP_{2,2,1}(t)}{dt} = 2\lambda P_{1,1,1}(t) + \mu_2 P_{3,2,1}(t) + \alpha P_{2,2,0}(t) - (2\lambda + \mu_2 + \beta) P_{2,2,1}(t), \quad (3.1.18)$$

$$\frac{dP_{3,2,1}(t)}{dt} = 2\lambda P_{2,1,1}(t) + \mu_2 P_{4,2,1}(t) + \alpha P_{3,2,0}(t) - (2\lambda + \mu_2 + \beta) P_{3,2,1}(t), \quad (3.1.19)$$

$$\frac{dP_{4,2,1}}{dt} = 2\lambda P_{3,1,1}(t) + \alpha P_{4,2,0}(t) - (\mu_2 + \beta) P_{4,2,1}(t), \quad (3.1.20)$$

V. Numerical illustration

The transient behaviour of the Reliability, Availability and Maintainability for the Erlangian Phase-Type queueing model of N=5, has been analyzed and are solved by using Fourth-Order Runge-Kutta numerical method. Assuming the time range from t=0 to t=200 (in hours) and the parametric values as λ=0.6, μ1=1.0, μ2=0.7, α=0.05, β=0.03, the values of Pn(t), the transient probabilities are obtained by solving the system of equations 3.1.1- 3.1.20.

Figure 2 shows the probability distribution, Pn(t), time-dependent total system size for the queueing system. The probability curves are displayed to understand the distribution trend of the system probabilities over the specified time interval.

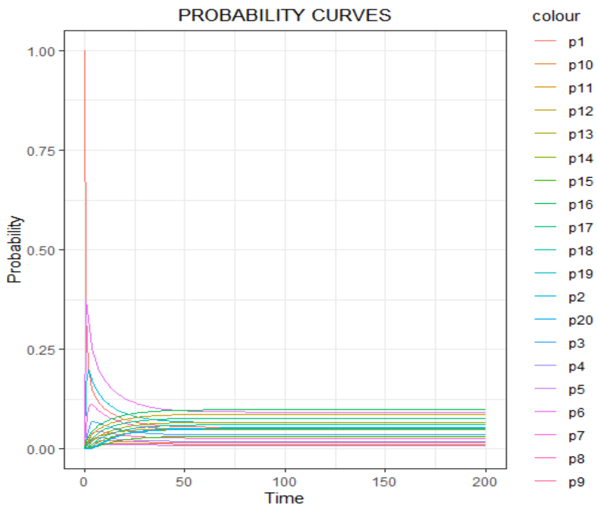

Figure 2

Figure 3, represents the Reliability of the system of the Erlangian Phase-Type queueing model. It is found out that as time increases the reliability of the system decreases. The reliability of the system is found out to be 38% after 200 hours. Figure 4, shows the Availability of the system and it is found out that as time increases the availability of the system decreases. Figure 5, depicts the maintainability of the system of the Erlangian Phase-Type queueing model. It is seen that as time increases the maintainability of the system increases. It is found out that the Maintainability of the system is 62% after 200 hours. The values of MTBF (Mean Time Between Failures) and MTTR (Mean Time till Repair) for the Erlangian Phase-Type Queueing system are found to be 20 hours/failure and 33 hours/recovery.

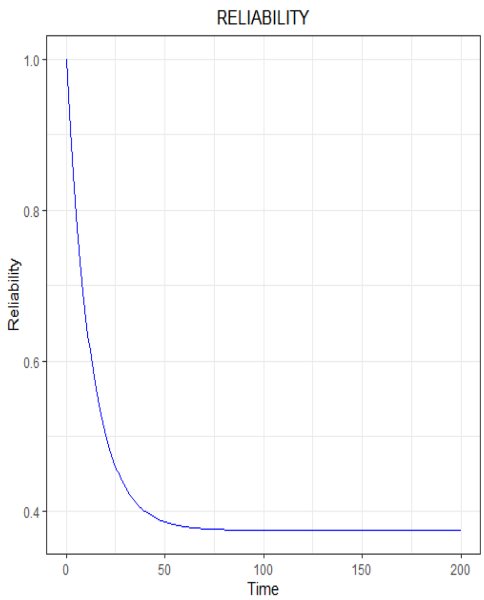

Figure 3

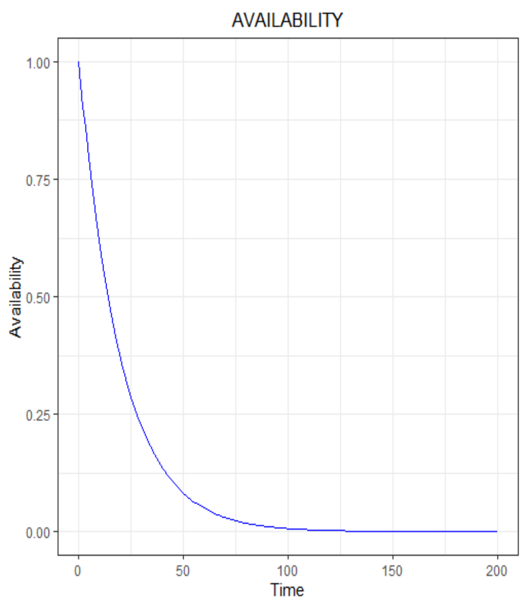

Figure 4

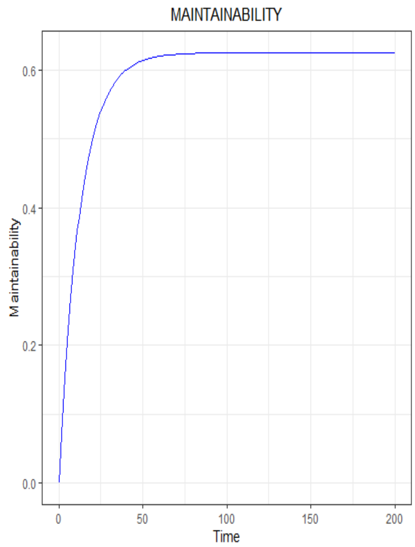

Figure 5

VI. Sensitivity analysis

For different parametric values sensitivity analysis has been carried out for RAM model for the Erlangian Phase-Type queueing model. Figures 6,7 and 8 shows the Reliability, Availability and Maintainability for different sets of Failure rates (0.05,0.06,0.07). By keeping other parameters constant, it is observed that as the failure rate value increases Reliability and Availability of the system decreases, whereas Maintainability of the system increases.

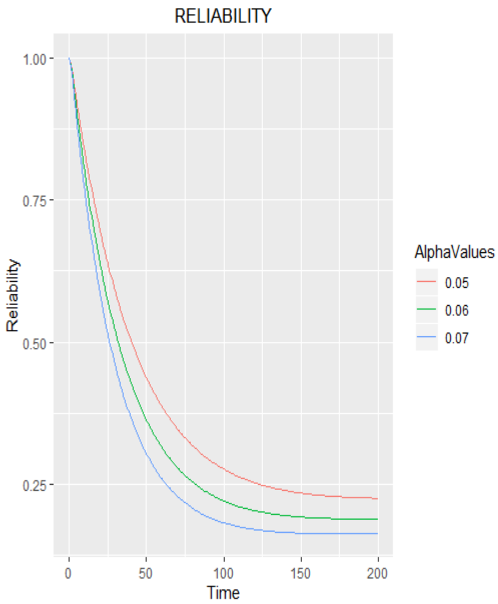

Figure 6

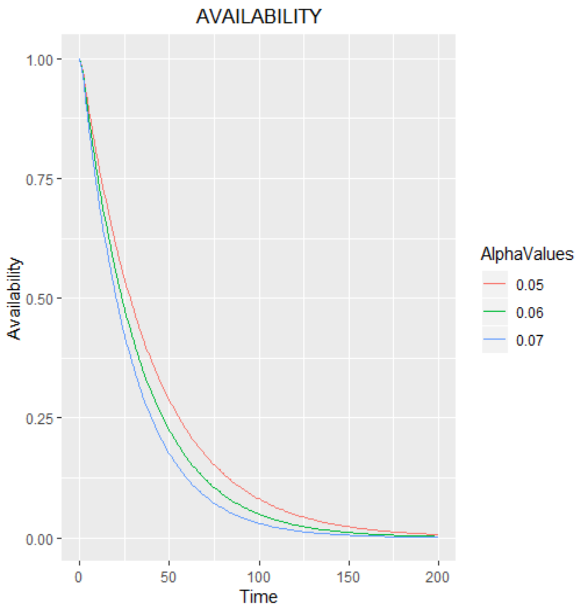

Figure 7

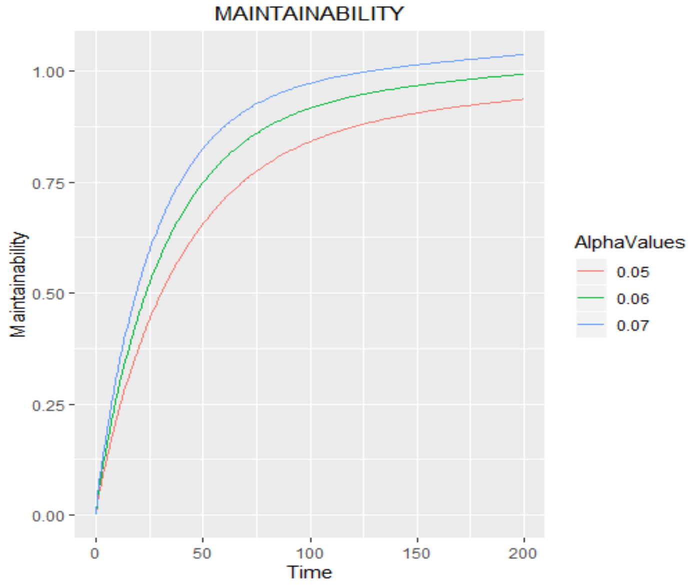

Figure 8

Figures 9 and 10 illustrates Reliability and Maintainability of the system for different Recovery rates (0.03,0.04,0.05) by keeping the other parameters constant. It can be seen from the graph that as the recovery rate value increases the Reliability of the system increases whereas Maintainability decreases.

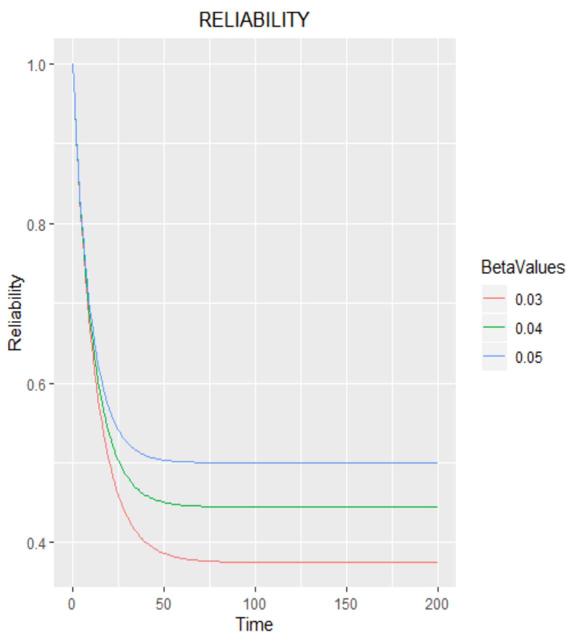

Figure 9

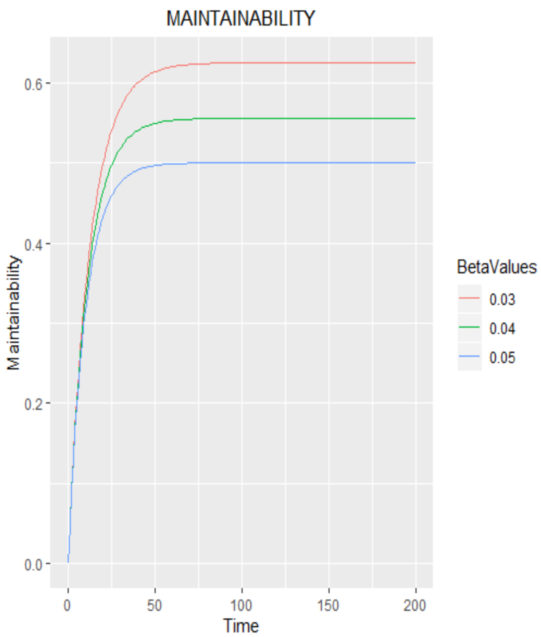

Figure 10

Table 1, represents the changes in the Reliability, Availability and Maintainability of the system for different values of arrival rates and failure rates by keeping the other parameters constant. It is found out that as the failure rate value increases keeping the arrival rate constant Reliability and Availability of the system decreases whereas the Maintainability of the system increases. It is also found that after 100 hours the Reliability, Availability and Maintainability of the system becomes constant.

Table 1: Sensitivity Analysis for the change of Arrival (0.1,0.2,0.3) and Failure (0.03,0.04,0.05) rate values

ARRIVAL RATE Vs FAILURE RATE										
TIME		**λ=0.1**			**λ=0.2**			**λ=0.3**		
		R(t)	**M(t)**	**A(t)**	**R(t)**	**M(t)**	**A(t)**	**R(t)**	**M(t)**	**A(t)**
	α1=0.03	0.4014	0.5986	0.3012	0.4116	0.6004	0.3112	0.4218	0.6115	0.3213
40	α1=0.04	0.3083	0.6917	0.2019	0.3185	0.7019	0.2120	0.3287	0.7121	0.2221
	α1=0.05	0.2423	0.7577	0.1353	0.2525	0.7679	0.1455	0.2627	0.7781	0.1556
60	α1=0.03	0.3180	0.6820	0.1653	0.3283	0.6922	0.1755	0.3386	0.7024	0.1857
	α1=0.04	0.2398	0.7602	0.0907	0.2400	0.7704	0.1009	0.2502	0.7806	0.1911
	α1=0.05	0.1894	0.8106	0.0498	0.1996	0.9007	0.0599	0.2008	0.9109	0.06
80	α1=0.03	0.2806	0.7194	0.0907	0.2907	0.7295	0.0948	0.3008	0.7310	0.1909
	α1=0.04	0.2147	0.7853	0.0408	0.2248	0.7954	0.0509	0.2349	0.8057	0.061
	α1=0.05	0.1735	0.8265	0.0183	0.1836	0.8367	0.0285	0.1937	0.8469	0.0387
100	α1=0.03	0.2637	0.7363	0.0498	0.2738	0.7466	0.0599	0.2839	0.7578	0.0611
	α1=0.04	0.2054	0.7946	0.0183	0.3055	0.8047	0.0285	0.4056	0.8149	0.0387
	α1=0.05	0.1687	0.8313	0.0067	0.1788	0.8415	0.0168	0.1889	0.8517	0.0269
120	α1=0.03	0.2562	0.7438	0.0273	0.2562	0.7438	0.0273	0.2562	0.7438	0.0273
	α1=0.04	0.2020	0.7980	0.0082	0.2020	0.7980	0.0082	0.2220	0.7980	0.0082
	α1=0.05	0.1673	0.8327	0.0025	0.1673	0.8327	0.0025	0.1673	0.8327	0.0025

Table 2, shows the changes in the Reliability, Availability and Maintainability of the system for different values of failure rates and service rates for the Working State by keeping the other parameters constant. As the Failure rates increases keeping the service rate constant, Reliability and Availability of the system decreases but the Maintainability of the system increases.

Table 2: Sensitivity Analysis for the change of Working State in Service rate (0.7,0.8,0.9) and Failure (0.03,0.04,0.05) rate values

FAILURE RATE Vs SERVICE RATE										
TIME		μ1=0.7			μ1=0.8			μ1=0.9		
		R(t)	**M(t)**	**A(t)**	**R(t)**	**M(t)**	**A(t)**	**R(t)**	**M(t)**	**A(t)**
40	α1=0.04	0.3083	0.6917	0.2019	0.3185	0.6815	0.2007	0.3287	0.6713	0.1995
	α1=0.05	0.2423	0.7577	0.1353	0.2525	0.7475	0.1251	0.2627	0.7373	0.1149
	α1=0.06	0.1950	0.8050	0.0907	0.2042	0.8028	0.0890	0.2144	0.7996	0.0800
60	α1=0.04	0.2398	0.7602	0.0607	0.2400	0.7580	0.0595	0.2502	0.7499	0.05
	α1=0.05	0.1894	0.8106	0.0498	0.1996	0.8004	0.0396	0.2098	0.7992	0.0294
	α1=0.06	0.1557	0.8443	0.0273	0.1659	0.8341	0.0171	0.1761	0.8239	0.0069
80	α1=0.04	0.2147	0.7853	0.0408	0.2248	0.7752	0.0307	0.2349	0.7651	0.0206
	α1=0.05	0.1735	0.8265	0.0183	0.1836	0.8164	0.0172	0.1937	0.8063	0.0163
	α1=0.06	0.1460	0.8540	0.0082	0.1561	0.8439	0.0069	0.1662	0.8338	0.0040
100	α1=0.04	0.2054	0.7946	0.0183	0.2155	0.7845	0.0082	0.2256	0.7744	0.0020
	α1=0.05	0.1687	0.8313	0.0067	0.1788	0.8212	0.0056	0.1889	0.8111	0.0035
	α1=0.06	0.1436	0.8564	0.0025	0.1537	0.8463	0.0008	0.1638	0.8362	0.0002
120	α1=0.04	0.2020	0.7980	0.0082	0.2020	0.7980	0.0082	0.2020	0.7980	0.0082
	α1=0.05	0.1673	0.8327	0.0025	0.1673	0.8327	0.0025	0.1673	0.8327	0.0025
	α1=0.06	0.1430	0.8570	0.0007	0.1430	0.8570	0.0007	0.1430	0.8570	0.0007

Table 3, depicts the changes in the Reliability and Maintainability of the system for different set of values of Arrival rate and Recovery rate by keeping the other parameters constant. As the recovery rate value increases by keeping the Arrival rate constant it is found that the Reliability of the system increases but the Maintainability of the system decreases.

Table 3: Sensitivity Analysis for the change of Arrival (0.1,0.2,0.3) and Recovery (0.01,0.02,0.03) rate values

ARRIVAL RATE Vs RECOVERY RATE							
TIME		λ=0.1		λ=0.2		λ=0.3	
		R(t)	**M(t)**	**R(t)**	**M(t)**	**R(t)**	**M(t)**
	β1=0.01	0.4014	0.5986	0.4117	0.5883	0.4222	0.5779
40	β1=0.02	0.4812	0.5188	0.4917	0.5084	0.5021	0.4890
	β1=0.03	0.5454	0.4546	0.5554	0.4443	0.5655	0.4340
60	β1=0.01	0.3180	0.6820	0.3288	0.6719	0.3395	0.6616
	β1=0.02	0.4299	0.5701	0.4301	0.5698	0.4411	0.5597
	β1=0.03	0.5137	0.4863	0.5239	0.4762	0.5341	0.4661
80	β1=0.01	0.2806	0.7194	0.2909	0.7092	0.3013	0.7009
	β1=0.02	0.4110	0.5890	0.4211	0.5787	0.4314	0.5685
	β1=0.03	0.5041	0.4959	0.5144	0.4858	0.5249	0.4757
100	β1=0.01	0.2637	0.7363	0.2741	0.7262	0.2845	0.7161
	β1=0.02	0.4040	0.5960	0.4142	0.5859	0.4244	0.5757
	β1=0.03	0.5012	0.4988	0.5113	0.4886	0.5214	0.4784
120	β1=0.01	0.2562	0.7438	0.2562	0.7438	0.2562	0.7438
	β1=0.02	0.4015	0.5985	0.4015	0.5985	0.4015	0.5985
	β1=0.03	0.5004	0.4996	0.5004	0.4996	0.5004	0.4996

Table 4, illustrates the changes in the Reliability and Maintainability of the system for different set of values of service rates and recovery rates keeping the other parameters constant. The table shows that as the Recovery rate values increases by keeping Service rate constant it is found that Reliability of the system increases whereas the Maintainability of the system decreases.

Table 4: Sensitivity Analysis for the change of Recovery rate (0.7,0.8,0.9) and Service rate (0.01,0.02,0.03) values

RECOVERY RATE Vs SERVICE RATE							
TIME		μ1=0.7		μ1=0.8		μ1=0.9	
		R(t)	**M(t)**	**R(t)**	**M(t)**	**R(t)**	**M(t)**
	β1=0.01	0.4014	0.5986	0.4115	0.5884	0.4216	0.5782
40	β1=0.02	0.4812	0.5188	0.4913	0.5086	0.5014	0.5004
	β1=0.03	0.5454	0.4546	0.5557	0.4444	0.5659	0.4342
60	β1=0.01	0.3180	0.6820	0.3283	0.6718	0.3285	0.6616
	β1=0.02	0.4299	0.5701	0.4300	0.5699	0.4311	0.5597
	β1=0.03	0.5137	0.4863	0.5239	0.4761	0.5341	0.4659
80	β1=0.01	0.2806	0.7194	0.2709	0.7092	0.2612	0.7009
	β1=0.02	0.4110	0.5890	0.4213	0.5789	0.4315	0.5688
	β1=0.03	0.5041	0.4959	0.5144	0.4858	0.5247	0.4757
100	β1=0.01	0.2637	0.7363	0.2739	0.7262	0.2841	0.7161
	β1=0.02	0.4040	0.5960	0.4142	0.5859	0.4244	0.5758
	β1=0.03	0.5012	0.4988	0.5113	0.4887	0.5214	0.4786
120	β1=0.01	0.2562	0.7438	0.2562	0.7438	0.2562	0.7438
	β1=0.02	0.4015	0.5985	0.4015	0.5985	0.4015	0.5985
	β1=0.03	0.5004	0.4996	0.5004	0.4996	0.5004	0.4996

Conclusion

RAM analysis of Er/M/1/N Queueing model with two different environmental states are studied in this paper. The state-transition diagram for the transient state of the r phase Erlangian queueing model is formed from which the differential-difference equations are obtained. A special case of N=5 is solved using Fourth-Order Runge-Kutta numerical method. It is observed that as time increases Reliability and Availability decreases, whereas Maintainability increases. In order to find the failure rate and the recovery rate, MTBF and MTTR of the Erlangian Queueing model was calculated. Sensitivity values becomes constant after 100 hours for different parametric values.

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