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Statistical Analysis and Optimum Step Stress Accelerated Life Test Design for Nadarajah- Haghighi Distribution

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Abstract

In this study we have considered step stress accelerated life testing plan for complete data. The lifetimes of the failure items are assumed to follow Nadarajah-Haghighi distribution which is an extension of exponential distribution and has all the properties like Weibull, gamma exponentiated exponential distribution. The maximum likelihood estimates of the parameters and accelerated factors have been estimated and confidence intervals of these parameters are also obtained. Newton-Raphson iterative procedure is used to solve the non-linear equations which are not in closed form. Later, a simulation study has been performed to check the performance of the parameters and hence the theory of the paper.

Keywords: Nadarajah-Haghighi distribution, step-stress accelerated life testing plan, maximum likelihood estimation, simulation, R.

I. Introduction

In modern advanced technologies era, there is a very high competition among companies to maintain the value and honor in the market for their products. Every manufacturer and producer are trying their best to produce an item of high reliability that could stay longer and perform better which makes the lifetime of the products very high. Therefore, it is not only a very tedious but also a very time consuming hence the costly job for the researcher to predict the exact lifetime of the items in terms of hour, days, months or years. The analysis of the life and quality of the product must be done before the launch; therefore, they do not have sufficient time to obtain the failure lifetime of the selected specimens and analyze on the basis of them. So, to obtain the lifetime in quick span of time, the experimenter accelerates the process and obtains the failure time.

Step stress accelerated life test is one of the very important methods to accelerate the process to obtain failure times quickly. In this test, first we put the testing units at some stress (higher than use or normal stress). At a specific time point we observe the failed units and increase the stress level to higher than the previous one, and again we count the failed units and so on.Studies in [7] and [5] suggested that the cumulative effect of the applied stresses should be reflected by the life-stress model when dealing with data from accelerated tests with time-varying stresses. Based on this idea,[7, 8] proposed a cumulative damage (exposure) model which had gained acceptance in the reliability engineering field. Later [1] extended the results of [5] to the case where a prescribed censoring time is involved. Since then many researchers such as [9, 2, 3, 4, 10] studied SSALT with different censoring schemes and distributions.

In this paper we have considered that the lifetimes of the items follow Nadarjah-Haghighi (NH) lifetime distribution. In second section model and testing methods have been discussed. Maximum likelihood estimation (MLE) technique is used to estimate the parameters and acceleration factor and discussed in section 3. In section 4, approximate confidence intervals for the parameters are obtained. Section 5 talks about optimality criterion for the stress change time or the optimum time at what the stress have been changed or increased. Simulation study has been performed to validate the assumptions made in this study and is in section 6.

II. Model and Methods

Nadarajah and Haghighi (2011) proposed that a random variable X is said to follow the NH distribution with the probability density function (PDF) is given by

$$f(x;\alpha,\beta) = \alpha\beta(1+\beta x)^{\alpha-1} exp[1-(1+\beta x)^{\alpha}] \tag{1}$$

Where β is scale parameter and α is the shape parameter. The corresponding, cumulative distribution function (CDF), survival function (SF) and hazard rate function (HRF) are given by

$$F(x;\alpha,\beta) = 1 - exp[1-(1+\beta x)^{\alpha}] \tag{2}$$

$$S(x) = exp[1-(1+\beta x)^{\alpha}]$$

$$h(x) = \alpha\beta(1+\beta x)^{\alpha-1}$$

For $\alpha = 1$, NH distribution is reduced to the exponential distribution. This distribution is an alternative to the Weibull, gamma and exponentiated exponential distributions with an attractive feature of always having the zero mode. NH distribution has closed form of survival and hazard rate functions like Weibull distribution, so it is a good choice for the lifetime data analyst.

Basic assumptions

1. In this test only two stress levels S_1 and $S_2 (S_1 < S_2)$ are used.
2. A random sample of n identical products is placed on the test initially under at stress level S_1 and run until time τ, then the stress is changed to S_2 and the test is continued until all products fail.
3. The lifetimes of the products are i.i.d. according to NH distribution at each level of stress.
4. The scale parameter β is a log-linear function of stress given by $log(\beta_i) = a + bS_i$, $i = 1,2$. where $a$and $b$are unknown parameters depending on the nature of the product and the test method.
5. The cumulative exposure model given by equation (3) holds to reflect the effect of the applied stresses. For more detail reader may refer to Nelson (1990).

$$F(x) = \begin{cases} F_1(x), & 0 < x < \tau \\ F_2\left(\frac{\beta_2}{\beta_1}\tau + x - \tau\right), & \tau \le x < \infty \end{cases} \tag{3}$$

The PDF of (3) is obtained as

$$f(x) = \begin{cases} f_1(x), & 0 < x < \tau \\ f_2\left(\frac{\beta_2}{\beta_1}\tau + x - \tau\right), & \tau \le x < \infty \end{cases} \tag{4}$$

Now using equations (1), (2), (3) and (4) he CDF and PDF for the test are given by

$$F(x) = \begin{cases} 1 - exp[1 - (1 + \beta_1 x)^{\alpha}], & 0 < x < \tau \\ 1 - exp\left[1 - \left\{1 + \beta_2\left(x - \tau\langle 1 - \frac{\beta_2}{\beta_1}\rangle\right)\right\}^{\alpha}\right], & \tau \le x < \infty \end{cases} \tag{5}$$

$$f(x) = \begin{cases} \alpha\beta_1(1 + \beta_1 x)^{\alpha-1} exp[1 - (1 + \beta_1 x)^{\alpha}], \quad 0 < x < \tau \\ \alpha\beta_2\left\{1 + \beta_2\left(x - \tau\langle 1 - \frac{\beta_2}{\beta_1}\rangle\right)\right\}^{\alpha-1} exp\left[1 - \left\{1 + \beta_2\left(x - \tau\langle 1 - \frac{\beta_2}{\beta_1}\rangle\right)\right\}^{\alpha}\right], \quad \tau \le x < \infty \end{cases} \tag{6}$$

III. Point Estimates of the Parameters using Maximum Likelihood Method

The ML method is used to determine the parameters that maximize the probability of the sample data. This method is considered to be more robust (with some exceptions) and yields estimates with good statistical properties. Also, it is an efficient method for quantifying uncertainty through confidence bounds. The MLE methods are versatile and are applicable to most of the models and to different types of data. However, the methodology for maximum likelihood estimation is simple, the implementation is mathematically intense. Since these estimators do not exist in closed form, numerical techniques are used to compute them.

For obtaining the MLE of the model parameters, let $x_{ij}, j = 1, 2, 3, \dots n_i, i = 1,2$ be the observed failure times of a test unit $j$under stress level i, where n_1 denotes the number of units failed at the low stress S_1 and $n_2$denotes the number of units failed at higher stress S_2.

In this paper, the lifetime of the test item is assumed to follow the NH distribution with scale parameter β and shape parameter α. Therefore, the likelihood function can be written in the form

$$L(\beta_1, \beta_2, \alpha) = \prod_{j=1}^{n_1} \alpha\beta_1\left(1 + \beta_1 x_{1j}\right)^{\alpha-1} exp\left[1 - \left(1 + \beta_1 x_{1j}\right)^{\alpha}\right] \prod_{j=1}^{n_2} \alpha\beta_2\left\{1 + \beta_2\left(x_{2j} - \tau\langle 1 - \frac{\beta_2}{\beta_1}\rangle\right)\right\}^{\alpha-1} exp\left[1 - \left\{1 + \beta_2\left(x_{2j} - \tau\langle 1 - \frac{\beta_2}{\beta_1}\rangle\right)\right\}^{\alpha}\right] \tag{7}$$

The log-likelihood function corresponding to the above equation can be rewritten as

$$log\, L = nlog\alpha + n_1 log\beta_1 + n_2 log\beta_2 + (\alpha - 1)\sum_{j=1}^{n_1} log\left(1 + \beta_1 x_{1j}\right) + \sum_{j=1}^{n_1}\left[1 - \left(1 + \beta_1 x_{1j}\right)^{\alpha}\right] + (\alpha - 1)\sum_{j=1}^{n_2} log\left\{1 + \beta_2\left(x_{2j} - \tau\langle 1 - \frac{\beta_2}{\beta_1}\rangle\right)\right\} + \sum_{j=1}^{n_2}\left[1 - \left\{1 + \beta_2\left(x_{2j} - \tau\langle 1 - \frac{\beta_2}{\beta_1}\rangle\right)\right\}^{\alpha}\right] \tag{8}$$

Where, $n_1 + n_2 = n$

Now by using the life stress relationship $log(\beta_i) = a + bS_i$, $i = 1, 2$ in equation (8), the log-likelihood function ir deduced to the following equation:

$$\log L = l = n \log\alpha + na + (n_1 S_1 + n_2 S_2)b + (\alpha - 1)\sum_{j=1}^{n_1} \log\left[1 + x_{1j}e^{a+bS_1}\right] + \sum_{j=1}^{n_1}\left[1 - \left(1 + x_{1j}e^{a+bS_1}\right)^{\alpha}\right] + (\alpha - 1)\sum_{j=1}^{n_2} \log\left[1 + \left\{x_{2j} - \tau\left(1 - e^{b(S_2 - S_1)}\right)\right\}e^{a+bS_2}\right] + \sum_{j=1}^{n_2}\left[1 - \left\{1 + \left(x_{2j} - \tau\langle 1 - e^{b(S_2 - S_1)}\rangle\right)e^{a+bS_2}\right\}^{\alpha}\right] \tag{9}$$

Differentiating (9) partially w.r.t.a, b and α, we get

$$\frac{\partial l}{\partial a} = n + (\alpha-1)\sum_{j=1}^{n_1}\frac{x_{1j}e^{a+bS_1}}{[1+x_{1j}e^{a+bS_1}]} + \alpha\sum_{j=1}^{n_1} x_{1j}e^{a+bS_1}\left(1+x_{1j}e^{a+bS_1}\right)^{\alpha-1} + (\alpha-1)\sum_{j=1}^{n_2}\frac{\{x_{2j}-\tau(1-e^{b(S_2-S_1)})\}e^{a+bS_2}}{[1+\{x_{2j}-\tau(1-e^{b(S_2-S_1)})\}e^{a+bS_2}]} + \alpha\sum_{j=1}^{n_2}\left(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\right)e^{a+bS_2}\left\{1+\left(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\right)e^{a+bS_2}\right\}^{\alpha-1} \quad (10)$$

$$\frac{\partial l}{\partial b} = n_1S_1 + n_2S_2 + (\alpha-1)\sum_{j=1}^{n_1}\frac{x_{1j}S_1e^{a+bS_1}}{[1+x_{1j}e^{a+bS_1}]} + \alpha\sum_{j=1}^{n_1} x_{1j}S_1\, e^{a+bS_1}\left(1+x_{1j}e^{a+bS_1}\right)^{\alpha-1} + (\alpha-1)\sum_{j=1}^{n_2}\frac{S_2(x_{2j}-\tau)e^{a+bS_2}-\tau\,(2S_2-S_1)e^{[a-b(2S_2-S_1)]}}{[1+\{x_{2j}-\tau(1-e^{b(S_2-S_1)})\}e^{a+bS_2}]} + \alpha\sum_{j=1}^{n_2} S_2\left(x_{2j}-\tau\right)e^{a+bS_2}-\tau\, e^{[a+b(2S_2-S_1)]}\left\{1+\left(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\right)e^{a+bS_2}\right\}^{\alpha-1} \quad (11)$$

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + \sum_{j=1}^{n_1}\log\left[1+x_{1j}e^{a+bS_1}\right] - \sum_{j=1}^{n_1}\left(1+x_{1j}e^{a+bS_1}\right)^{\alpha}\log\left(1+x_{1j}e^{a+bS_1}\right) + \sum_{j=1}^{n_2}\log\left[1+\left\{x_{2j}-\tau\left(1-e^{b(S_2-S_1)}\right)\right\}e^{a+bS_2}\right] + \sum_{j=1}^{n_2}\left\{1+\left(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\right)e^{a+bS_2}\right\}^{\alpha}\log\left\{1+\left(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\right)e^{a+bS_2}\right\} \quad (12)$$

From (12) the MLE of α is given by the following equation:

$$\frac{n}{\alpha} + n_1[\log(\psi_1) - \psi_1^{\alpha}\log(\psi_1)] + n_2[\log(\psi_2) + \psi_2^{\alpha}\log(\psi_2)] = 0$$

$$\hat{\alpha} = \frac{n}{-n_1[\log(\psi_1) - \psi_1^{\alpha}\log(\psi_1)] - n_2[\log(\psi_2) + \psi_2^{\alpha}\log(\psi_2)]}$$

where,

$$\psi_1 = \left[1 + x_{1j}e^{a+bS_1}\right]$$

and $\psi_2 = \left[1 + \left\{x_{2j} - \tau\left(1 - e^{b(S_2-S_1)}\right)\right\}e^{a+bS_2}\right]$

IV. The approximate confidence intervals for the parameters

The observed Fisher-information matrix can be written as follows:

$$F = -\begin{bmatrix} \frac{\partial^2 l}{\partial a^2} & \frac{\partial^2 l}{\partial a \partial b} & \frac{\partial^2 l}{\partial a \partial \alpha} \\ \frac{\partial^2 l}{\partial b \partial a} & \frac{\partial^2 l}{\partial b^2} & \frac{\partial^2 l}{\partial b \partial \alpha} \\ \frac{\partial^2 l}{\partial \alpha \partial a} & \frac{\partial^2 l}{\partial \alpha \partial b} & \frac{\partial^2 l}{\partial \alpha^2} \end{bmatrix}$$

for large samples, the point estimates of the parameters obtained by maximum likelihood method follow approximately normal distribution with mean (a, b, α) and varianceF^{-1}, therefore, $(\hat{a}, \hat{b}, \hat{\alpha}) \sim N(a, b, \alpha), F^{-1})$. Then the two sided $100(1-\gamma)\%$ approximate confidence interval for the parameter of (a, b, α) can be written as

$\hat{a} \pm Z_{\gamma/2}\sqrt{var(\hat{a})}\ ;\ \hat{b} \pm Z_{\gamma/2}\sqrt{var(\hat{b})}\ ;\ \hat{\alpha} \pm Z_{\gamma/2}\sqrt{var(\hat{\alpha})}$

Where $Z_{\gamma/2}$ is the $(1-\gamma/2)th$ quantile of a standard normal distribution and $\sqrt{\mathrm{var}(\hat{\alpha})}$, $\sqrt{var(\hat{a})}$ and $\sqrt{var(\hat{b})}$ is obtained by taking the square root of the diagonal elements of F^{-1}.

The elements of the information matrix Fcan be expressed by the following equations:

$$\frac{\partial^2 l}{\partial a^2} = (\alpha-1)\sum_{j=1}^{n_1}\frac{x_{1j}e^{a+bS_1}}{[1+x_{1j}e^{a+bS_1}]^2} + \alpha\sum_{j=1}^{n_1}\Big[x_{1j}e^{a+bS_1}\big(1+x_{1j}e^{a+bS_1}\big)^{\alpha-1} + (\alpha-1)\big(x_{1j}e^{a+bS_1}\big)^2\big(1+x_{1j}e^{a+bS_1}\big)^{\alpha-2}\Big] + (\alpha-1)\sum_{j=1}^{n_2}\frac{\{x_{2j}-\tau(1-e^{b(S_2-S_1)})\}e^{a+bS_2}}{[1+\{x_{2j}-\tau(1-e^{b(S_2-S_1)})\}e^{a+bS_2}]^2} + \alpha\sum_{j=1}^{n_2}\Big[\{\big(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\big)e^{a+bS_2}\}\{1+\big(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\big)e^{a+bS_2}\}^{\alpha-1} + (\alpha-1)\{\big(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\big)e^{a+bS_2}\}^2\{1+\big(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\big)e^{a+bS_2}\}^{\alpha-2}\Big] \quad (13)$$

$$\frac{\partial^2 l}{\partial b^2} = (\alpha-1)\sum_{j=1}^{n_1}\frac{S_1^2x_{1j}e^{a+bS_1}}{[1+x_{1j}e^{a+bS_1}]^2} + \alpha\sum_{j=1}^{n_1}\begin{bmatrix}\big(S_1^2x_{1j}e^{a+bS_1}\big)\big(1+x_{1j}e^{a+bS_1}\big)^{\alpha-1}\\ +(\alpha-1)S_1^2\big(x_{1j}e^{a+bS_1}\big)^2\big(1+x_{1j}e^{a+bS_1}\big)^{\alpha-2}\end{bmatrix} + (\alpha-1)\sum_{j=1}^{n_2}\frac{S_2^2(x_{2j}-\tau)e^{a+bS_2}+\tau(2S_2-S_1)^2e^{a+b(2S_2-S_1)}+\tau(x_{2j}-\tau)(9S_2^2+S_1^2-6S_1S_2)e^{a+b(3S_2-S_1)}}{[1+\{x_{2j}-\tau(1-e^{b(S_2-S_1)})\}e^{a+bS_2}]^2} + \alpha\sum_{j=1}^{n_2}\Big\{\Big(\big[S_2^2\big(x_{2j}-\tau\big)e^{a+bS_2}+\tau(2S_2-S_1)^2e^{a+b(2S_2-S_1)}\big]\big[1+\{x_{2j}-\tau\big(1-e^{b(S_2-S_1)}\big)\}e^{a+bS_2}\big]^{\alpha-1}\Big) + (\alpha-1)\big[\{S_2\big(x_{2j}-\tau\big)e^{a+bS_2}\}+\tau(2S_2-S_1)e^{a+b(2S_2-S_1)}\big]^2\big[1+\big(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\big)e^{a+bS_2}\big]^{\alpha-2}\Big\} \quad (14)$$

$$\frac{\partial l}{\partial \alpha^2} = -\frac{n}{\alpha^2} - \sum_{j=1}^{n_1}\Big(\big[1+x_{1j}\mathrm{e}^{a+bS_1}\big]\big[\log\big(1+x_{1j}\mathrm{e}^{a+bS_1}\big)\big]^2\Big) - \sum_{j=1}^{n_2}\Big(\big[1+\{x_{2j}-\tau\big(1-\mathrm{e}^{b(S_2-S_1)}\big)\}\mathrm{e}^{a+bS_2}\big]^{\alpha}\big[\log\{1+\big(x_{2j}-\tau\langle 1-\mathrm{e}^{b(S_2-S_1)}\rangle\big)\mathrm{e}^{a+bS_2}\}\big]^2\Big) \quad (15)$$

$$\frac{\partial^2 l}{\partial a\partial b} = \frac{\partial^2 l}{\partial b\partial a} = (\alpha-1)\sum_{j=1}^{n_1}\frac{S_1x_{1j}e^{a+bS_1}}{[1+x_{1j}e^{a+bS_1}]^2} + \alpha\sum_{j=1}^{n_1}\begin{bmatrix}\big(S_1x_{1j}e^{a+bS_1}\big)\big(1+x_{1j}e^{a+bS_1}\big)^{\alpha-1}\\ +(\alpha-1)S_1\big(x_{1j}e^{a+bS_1}\big)^2\big(1+x_{1j}e^{a+bS_1}\big)^{\alpha-2}\end{bmatrix} + (\alpha-1)\sum_{j=1}^{n_2}\frac{[S_2(x_{2j}-\tau)e^{a+bS_2}+\tau(2S_2-S_1)e^{a+b(2S_2-S_1)}]}{[1+\{x_{2j}-\tau(1-e^{b(S_2-S_1)})\}e^{a+bS_2}]^2} + \alpha\sum_{j=1}^{n_2}\Big\{\Big(\big[S_2\big(x_{2j}-\tau\big)e^{a+bS_2}+\tau(2S_2-S_1)e^{a+b(2S_2-S_1)}\big]\big[1+\{x_{2j}-\tau\big(1-e^{b(S_2-S_1)}\big)\}e^{a+bS_2}\big]^{\alpha-1}\Big) + (\alpha-1)\big[\{x_{2j}-\tau\big(1-e^{b(S_2-S_1)}\big)\}e^{a+bS_2}\big]\big[S_2\big(x_{2j}-\tau\big)e^{a+bS_2}+\tau\,(2S_2-S_1)e^{a+b(2S_2-S_1)}\big]\big[1+\big(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\big)e^{a+bS_2}\big]^{\alpha-2}\Big\} \quad (16)$$

$$\frac{\partial^2 l}{\partial a\partial \alpha} = \frac{\partial^2 l}{\partial \alpha\partial a} = \sum_{j=1}^{n_1}\frac{x_{1j}e^{a+bS_1}}{[1+x_{1j}e^{a+bS_1}]^2} + \sum_{j=1}^{n_1}\begin{matrix}\big(x_{1j}e^{a+bS_1}\big)\big(1+x_{1j}e^{a+bS_1}\big)^{\alpha-1}\\ +\big[1+\alpha\,log\big(1+x_{1j}e^{a+bS_1}\big)\big]\end{matrix} + \sum_{j=1}^{n_2}\frac{[(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle)e^{a+bS_2}]}{[1+\{x_{2j}-\tau(1-e^{b(S_2-S_1)})\}e^{a+bS_2}]^2} + \sum_{j=1}^{n_2}\Big\{\Big(\big[\big(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\big)e^{a+bS_2}\big]\big[1+\{x_{2j}-\tau\big(1-e^{b(S_2-S_1)}\big)\}e^{a+bS_2}\big]\Big) + \Big[1+\alpha\,log\Big(\big(x_{2j}-\tau\langle 1-e^{b(S_2-S_1)}\rangle\big)e^{a+bS_2}\Big)\Big]\Big\} \quad (17)$$

$$\frac{\partial^2 l}{\partial b \partial \alpha} = \frac{\partial^2 l}{\partial \alpha \partial b} = \sum_{j=1}^{n_1} \frac{S_1 x_{1j} e^{a+bS_1}}{[1+x_{1j}e^{a+bS_1}]} + \sum_{j=1}^{n_1} \left[\left(S_1 x_{1j} e^{a+bS_1}\right)\left(1+x_{1j}e^{a+bS_1}\right)^{\alpha-1}\left\{1+\alpha\, log\left(1+ x_{1j}e^{a+bS_1}\right)\right\}\right] + \sum_{j=1}^{n_2} \frac{[S_2(x_{2j}-\tau)e^{a+bS_2}+\tau(2S_2-S_1)e^{a+b(2S_2-S_1)}]}{[1+\{x_{2j}-\tau(1-e^{b(S_2-S_1)})\}e^{a+bS_2}]} + \sum_{j=1}^{n_2} \left\{\left[S_2(x_{2j}-\tau)e^{a+bS_2}+\tau(2S_2-S_1)e^{a+b(2S_2-S_1)}\right]\left[1+\left\{x_{2j}-\tau\left(1-e^{b(S_2-S_1)}\right)\right\}e^{a+bS_2}\right]^{\alpha-1} + \left[1+\alpha\, log\left(1+\left(x_{2j}-\tau\langle 1- e^{b(S_2-S_1)}\rangle\right)e^{a+bS_2}\right)\right]\right\} \quad (18)$$

V. Estimation of Optimal Stress Change Time

I. Asymptotic variance of MLEs of the model parameters

The asymptotic variance of $\hat{a}, \hat{b}$ and $\hat{\alpha}$ is given by the diagonal elements of the inverse of Fisher information matrix.

II. Generalized asymptotic variance of MLEs of the model parameters

The generalized asymptotic variance of$\hat{a}, \hat{b}$ and $\hat{\alpha}$ is obtained by the reciprocal of the determinant of Fisher information matrix.

i.e. $G_e A_s Var(\hat{a}, \hat{b}, \hat{\alpha}) = \frac{1}{det(F)} or \frac{1}{|F|}$

First, we obtain the optimum value of the stress change time τ either by minimizing the asymptotic or the generalized asymptotic variance. After that we would estimate the values of a, b and α by using the optimum value of τ and by maximizing the log likelihood function of the distribution. We obtain the optimum value of τ using the *optim()* function in R software. This function has several methods to minimise and gives the global minima of the objective function. The available methods in *optim()* are Nelder-Mead, BFGS, L-BFGS-B, CG, SANN and Brent.

VI. Simulation Study

Simulation study has been used to examine and validate the assumptions made in the study. The study has been performed using R-software/language. Here, in this study, point and confidence interval have been estimated along with their root mean square(s) and mean absolute error(s). Monte-Carlo simulation technique is used to perform simulation study as per the detailed steps presented below:

1. The random samples of sizes 30, 50, 75, 100, 125, 150 and 200 from are generated from NH distribution. To generate the random number from NH distribution, CDF inverse transformation method is used.
2. Two stress levels are fixed, $S_1$and S_2, and their respective values are 2 and 3.
3. First put all the testing units to stress $S_1$and run until the optimum stress change time τ=1.2 is attained. Then changed the level of stress to next level that is S_2at prefixed stress change time τ=1.2 and run the experiment.
4. For each sample, the acceleration factor and the parameters of the model are estimated in SSALT.
5. The above procedure from step 1-4 is repeated 10,000 times to avoid the randomness.
6. The Newton–Raphson method was used for solving the nonlinear equations given in ....

7. The RMSEs and MAEs of the estimators for acceleration factor and other parameters for all sample sizes are tabulated.
8. The confidence limit with confidence level γ=0.95 and γ =0.99 of the acceleration factor and other two parameters were constructed.
9. The results are summarized in Tables 1, 2 and 3. Table 1 presents the Estimates, RMSEs and MAEs of the estimators. The approximated confidence limits at 95% and 99% for the parameters and acceleration factor are presented in Table 2. Optimum value of stress change time is tabulated in Table 3.

Table 1: ***The maximum likelihood estimates of parameters and their RMSEs and MAEs***

N	Parameters	Estimate	RMSE	MAE
30	$\hat{\alpha}$	2.5869	0.4424	0.0347
	$\hat{a}$	2.2382	0.6177	0.1391
	$\hat{b}$	-1.1074	0.1915	0.0189
50	$\hat{\alpha}$	2.5064	0.3259	0.0359
	$\hat{a}$	2.1863	0.2361	0.1254
	$\hat{b}$	-1.1509	0.1266	0.0463
75	$\hat{\alpha}$	2.5730	0.3974	0.0292
	$\hat{a}$	2.2180	0.4185	0.1469
	$\hat{b}$	-1.1209	0.1260	0.0190
100	$\hat{\alpha}$	2.5124	0.5276	0.0049
	$\hat{a}$	2.2944	0.3612	0.1175
	$\hat{b}$	-1.1360	0.1033	0.0327
125	$\hat{\alpha}$	2.5038	0.3201	0.0246
	$\hat{a}$	2.3044	0.3101	0.1057
	$\hat{b}$	-1.1459	0.1087	0.0373
150	$\hat{\alpha}$	2.5417	0.2343	0.0167
	$\hat{a}$	2.2318	0.2816	0.1415
	$\hat{b}$	-1.1218	0.0907	0.0198
200	$\hat{\alpha}$	2.5473	0.2141	0.0147
	$\hat{a}$	2.3031	0.1052	0.1879
	$\hat{b}$	-1.1686	0.0572	0.0624

Table 2: *Confidence interval of the estimators*

N	Confidence level	$\hat{\alpha}$		$\hat{a}$		$\hat{b}$	
		LCL	UCL	LCL	UCL	LCL	UCL
30	95%	1.8592	3.3145	1.2221	3.2542	-1.4223	-0.7924
	99%	1.5577	3.6160	0.8012	3.6751	-1.5528	-0.6619
50	95%	1.9703	3.0424	1.7979	2.5746	-1.3591	-0.9426
	99%	1.7482	3.2645	1.6370	2.7355	-1.4454	-0.8563
75	95%	1.9193	3.2266	1.5296	2.9063	-1.3281	-0.9136
	99%	1.6485	3.4974	1.2444	3.1915	-1.4140	-0.8277
100	95%	1.9735	3.0512	1.7002	2.8885	-1.3059	-0.9660
	99%	1.7502	3.2745	1.4541	3.1346	-1.3763	-0.8956
125	95%	1.9772	3.0303	1.7943	2.8144	-1.3246	-0.9671
	99%	1.7591	3.2484	1.5830	3.0258	-1.3987	-0.8930
150	95%	2.1563	2.9270	1.7686	2.6949	-1.2709	-0.9726
	99%	1.9966	3.0867	1.5767	2.8869	-1.3328	-0.9108
200	95%	2.1951	2.8994	2.1300	2.4761	-1.2626	-1.0745
	99%	2.0492	3.0453	2.0583	2.5478	-1.3016	-1.0355

Table 3: *Result of optimal design of step-stress ALT for different sample sizes*

n	nG.A.V.	$\hat{\tau}$	$\hat{\tau}'$
30	0.006035	1.2	1.25
50	0.000644	1.2	1.23
75	0.000401	1.2	1.24
100	0.000146	1.2	1.21
125	0.000128	1.2	1.20
150	0.000036	1.2	1.19
200	0.000089	1.2	1.21

VII. Conclusion

In this paper we have studied NH distribution under step stress model with complete data. First the testing units have been placed on test to obtain the failure times of these items and then using these data we have analysed the lifetimes of the items on normal stress condition or general use conditions. We have calculated MLEs of parameters, their respective RMSEs and MAEs and then approximate confidence intervals of these parameters were also derived using the MLEs of these parameters.

The simulation study shows that all our assumptions are true. We see that as the sample sizes increases the RMSE and MAE are getting smaller and confidence intervals are also getting narrower. Here optimality criteria for changing the stress time are also checked and at that time the estimation technique has been used to obtain the numerical value of the parameters.

Bayesian aspects of this study may be considered as future work or one also may use different censoring schemes with classical or Bayesian approach.

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