On Age Replacement Policy of a System Involving Minimal Repair

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Abstract

This paper made a survey on age replacement model involving minimal repair, and this was done by considering a parallel-series system with two subsystems, which are subsystems A and B, and each of the system is formed by three parallel units, therefore, the whole systems consist of six units. We constructed age replacement model involving minimal repair that will determine the optimal replacement time of the parallel-series system based on two different policies (Policy 1 and Policy 2). A numerical example was given to illustrate the characteristics of the age replacement models involving minimal repair constructed. From the results obtained, it was observed that policy 2 extends the optimal replacement time of a multi-component system, when compared to Policy 1.

Keywords: Optimal, Repair, Replacement, Rate, System, Time.

I. Introduction

The activities of maintaining military equipment, transportation, and civil structures requires high costs, for these reasons, this leaded the development of various maintenance policies that seek the optimal decision models for reducing the risk of a catastrophic breakdown of systems. Thus, maintenance has effect on system reliability, because it prolonged the life span of the systems. For most industrial equipment, maintenance policies are provided to reduce the incidence of system frowning to failure.

There is an extensive literature on the age replacement policy, for example, see Barlow and Proschan (1965), Elsayed (1996), Nakagawa (2005) and Pham (2003). Sandev and Aven (1999) studied the optimal replacement problem of a monotone system comprising n components, where the components are ‘’minimally’’ repaired at failures. Jain *et al.* (2002) evaluated the expressions for expected cost for a system with replacement and minimal, and furthermore discussed the maintenance costs of various policies. Ouali and Yacout (2003) developed an optional replacement policy for the maintenance of two non-identical components connected in series configuration, where by each component is replaced correctively whenever it fails and preventively only if its age reaches or exceeds a preventive replacement age T when the other component fails. Chien and Sheu (2006) proposed age replacement policy for an operating system which is subjected to shocks that arrive according to a non-homogeneous Poisson process, and as shocks occur the system has two types of failure: type I failure (minor) or type II failure (catastrophic). Chen (2007) constructed a cache document replacement policy which content can be tailored to the specific requirements of a caching system. Wang *et al.* (2008) presented a condition-based order-replacement policy for a single-unit system, aiming to optimize the condition-based maintenance and the spare order management jointly. Aven and Castro (2008) presented a minimal repair replacement model of a one unit system subjected to two types of failures. Yaun and Xu (2011) studies a cold standby repairable system with two different components and one repairman who can take multiple vacations. Yusuf and Ali (2012) considered two parallel units in which both units operate simultaneously, and the system is subjected to two types of failures. Type I failure is minor and occur with the failure of a single component and is checked by minimal repairs, while type II failure is catastrophic in which both components failed and the system is replaced. Xu *et al.* (2012) investigated on replacement scheduling for non-repairable safety-related systems (SRS) with multiple components and states, and their aim is to determine the cost-minimizing time for replacing SRS while meeting the required safety. Wang *et al.* (2014) introduced a two-level inspection policy model for a single component plant system based on a three-stage failure process, such that the failure process divide the system's life into three stages: good, minor defective and severe defective stages. Zhao *et al.* (2014) answered the problem which replacement is better for continuous and discrete scheduled times. Chang (2014) considered a system which suffers one of two types of failure based on a specific random mechanism: type-I (repairable) failure is rectified by a minimal repair, and type-II (non-repairable) failure is removed by a corrective replacement. Firstly, he considered a modified random and age replacement policy in which the system is replaced at a planned time T, at a random working time, or at the first type-II failure, whichever occurs first. He further considered a system which work continuously for N jobs with random working times. Malki *et al.* (2015) investigated on age replacement policies for two-component parallel system with stochastic dependence. The stochastic dependence considered, is model by a one-sided domino effect. Coria *et al.* (2015) proposed an analytical optimization method for preventive maintenance (PM) policy with minimal repair at failure, periodic maintenance, and replacement for systems with historical failure time data influenced by a current PM policy. Yusuf *et al.* (2015) modified the work of Aven and Castro (2008) by introducing random working time Y. They constructed a modified random and age replacement model, for which the system is replaced at a planned time T, at a random working time Y, or at the first non-repairable type 2 failure whichever occurs first. Where they assumed that, if there is a component which fails and the repairman is on vacation, the failed component will wait for repair until the repairman is available.

The main contributions of this study are to develop age replacement models involving minimal repair for parallel-series system, which is subjected to two types of failures, so as to addressed (1) the problem of sudden failure of a multi-component system (2) avoid rising maintenance cost of a multi-component system, and (3) to provide some characteristics of the age replacement model involving minimal repair. The remainder of this paper is organized as follows: Section 2 discussed the methodology of the study. Section 3 discussed the proposed models. Section 4 presents the numerical results. Finally, section 5 discussed the conclusion and recommendations.

II. Methods

Reliability measures namely reliability function and failure rates are used to obtain the expressions of age replacement models based on some model assumptions. A numerical example was given for the purpose of investigating the characteristics of the models constructed.

Notations used

- $r_{ia}(t)$: Type I failure rate of unit A_i of subsystem A, for $i = 1, 2, 3$.
- $r_{ib}(t)$: Type II failure rate of unit B_i of subsystem B, for $i = 1, 2, 3$.
- $R_{ia}(t)$: reliability function of unit A_i of subsystem A, for $i = 1, 2, 3$.
- C_{ib} : cost of minimal repair of unit B_i of subsystem B due to Type II failure, for $i = 1, 2, 3$.
- C_p : cost of planned replacement of the system at time T.
- C_r : cost of un-planned replacement of the system due to Type I failure.
- T^*: Optimal replacement time of the system based on Policy 1.
- (T^*, τ^*) : Optimal pair replacement time of the system based on Policy 2.

III. Description of the system

A system comprising of two subsystems A and B in series is considered. Subsystem A consist of three active parallel units, which are A_1, A_2 and A_3. While, subsystem B consist of three active parallel units, which are B_1, B_2 and B_3. See figure 1 below. The three units A_1, A_2 and A_3 are subjected to Type I failure. While the three units B_1, B_2 and B_3 are subjected to Type II failure. The system will stop working completely, if it least one of the two subsystems (A and B) failed.

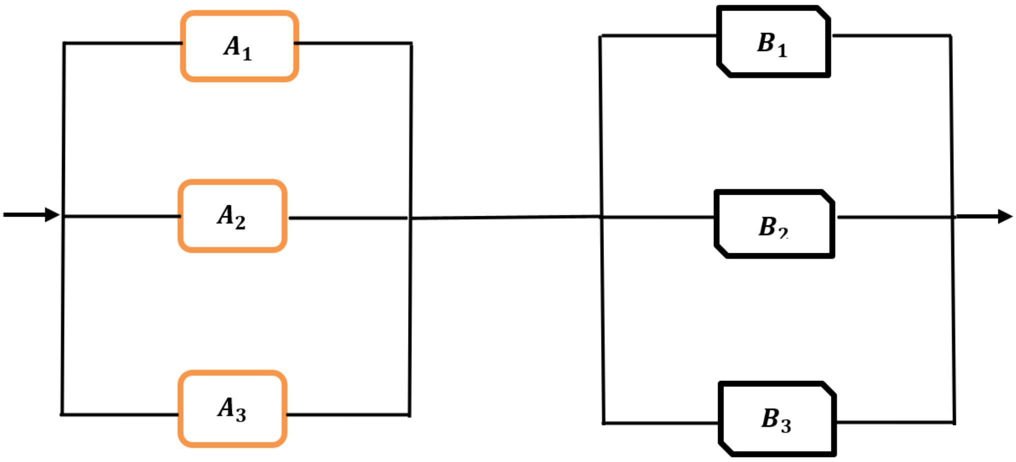

Figure 1. *Reliability block diagram of the system*

IV. Age Replacement Models

This section considers some of the fundamental replacement policies involving minimal repair.

Policy 1

Assumptions for this Policy 1:

1. Type I failure is un-repairable, while Type II failure is repairable.
2. Both the two failures are detected instantaneously.
3. All required resources are available when needed, which means that replacement/minima repair.
4. The system fails due to Type I failure, if all the three units of subsystem A fails due to Type I failure.
5. The system fails due to Type II failure, if all the three units of subsystem B fails due to Type II failure.
6. If the system failed due to Type I failure, the whole system will be replaced completely with new one.

7. If the system failed due to Type II failure, then the system is minimally repair, and allow the system to continue operating from where it stopped.
8. The system is replaced at a planned replacement time $T(T > 0)$ after its installation or Type I failure of the system, whichever occurs first.

Based on the assumptions of Policy 1, we have the probability that the system will be replaced at planned time T before Type I failure occurs, as

$$R(T) = 1 - \big(1 - R_1(T)\big)\big(1 - R_2(T)\big)\big(1 - R_1(T)\big), \quad (1)$$

where

$$R_i(T) = e^{-\int_0^T r_{ia}(t)dt}, \ for \ i = 1, \ 2, \ 3. \quad (2)$$

The cost of unplanned replacement of the system in one replacement cycle is

$$C_r\big(1 - R(T)\big). \quad (3)$$

The cost of planned replacement of the system in one replacement cycle is

$$C_p R(T). \quad (4)$$

The cost of minimal repair of unit B_1 of subsystem B in one replacement cycle is

$$\int_0^T C_{1b} r_{1b}(t)dt. \quad (5)$$

The cost of minimal repair of unit B_2 of subsystem B in one replacement cycle is

$$\int_0^T C_{2b} r_{2b}(t)dt. \quad (6)$$

The cost of minimal repair of unit B_3 of subsystem B in one replacement cycle is

$$\int_0^T C_{3b} r_{3b}(t)dt. \quad (7)$$

Based on this policy 1, we have the total replacement cost rate of the system in one replacement cycle as

$$C(T) = \frac{C_r(1-R(T)) + C_p R(T) + \int_0^T C_{1b} r_{1b}(t)dt + \int_0^T C_{2b} r_{2b}(t)dt + \int_0^T C_{3b} r_{3b}(t)dt}{\int_0^T R(t)dt}. \quad (8)$$

Noting that, $C(T)$ is adopted as the objective function of an optimization problem, and the aim is to determine an optimal replacement time T^* that minimizes$C(T)$.

Policy 2

Assumptions for this Policy 2:

1. Both Type I failure and Type II failure are repairable, where the failure each of the six units is rectify by minimal repair.
2. Both the two failures are detected instantaneously.
3. All required resources are available when needed, which means that there is no waiting time.
4. The system fails due to Type I failure, if all the three units of subsystem A fails due to Type I failure.
5. The system fails due to Type II failure, if all the three units of subsystem B fails due to Type II failure.
6. If the system fails due to Type II failure, we minimally repair the system, and allow the system to continue operating from where it stopped.
7. On the first Type I failure after a given system age τ, an un-planned replacement of the system is carried out. However, if, for given T, such that, $\tau < T$, there is no replacement in $[\tau, T]$, then at time T, a planned replacement of the system is carried out.
8. If the system fails due to Type I failure before a given time τ, we minimally repair the system, and allow the system to continue operating from where it stopped.

Based on the assumptions of Policy 2, we have the probability that the system will be replaced at planned time T before the first Type I failure of the system after a given time τ occurs, as

$$R(T - \tau) = 1 - \big(1 - R_1(T - \tau)\big)\big(1 - R_2(T - \tau)\big)\big(1 - R_1(T - \tau)\big), \quad (9)$$

$$R_i(T - \tau) = e^{-\int_0^{T-\tau} r_{ia}(t)dt}, \ for \ i = 1, \ 2, \ 3. \quad (10)$$

The cost of unplanned replacement of the system in one replacement cycle is

$$C_r\big(1 - R(T-\tau)\big). \tag{11}$$

The cost of planned replacement of the system in one replacement cycle is

$$C_p R(T-\tau). \tag{12}$$

The cost of minimal repair of unit A_1 of subsystem A before given time τ in one replacement cycle is

$$\int_0^{\tau} C_{1a} r_{1a}(t)dt. \tag{13}$$

The cost of minimal repair of unit A_2 of subsystem A before given time τ in one replacement cycle is

$$\int_0^{\tau} C_{2a} r_{2a}(t)dt. \tag{14}$$

The cost of minimal repair of unit A_3 of subsystem A before given time τ in one replacement cycle is

$$\int_0^{\tau} C_{3a} r_{3a}(t)dt. \tag{15}$$

The cost of minimal repair of unit B_1 of subsystem B before planned time T in one replacement cycle is

$$\int_0^{T} C_{1b} r_{1b}(t)dt. \tag{16}$$

The cost of minimal repair of unit B_2 of subsystem B before planned time T in one replacement cycle is

$$\int_0^{\tau} C_{2b} r_{2b}(t)dt. \tag{17}$$

The cost of minimal repair of unit B_3 of subsystem B before planned time T in one replacement cycle is

$$\int_0^{\tau} C_{3b} r_{3b}(t)dt. \tag{18}$$

Based on this policy 2, we have the total replacement cost rate of the system in one replacement cycle as

$$C(T,\tau) = \frac{C_r\big(1 - R(T-\tau)\big) + C_p R(T-\tau) + \int_0^{\tau} C_{1a} r_{1a}(t)dt + \int_0^{\tau} C_{2a} r_{2a}(t)dt + \int_0^{\tau} C_{3a} r_{3a}(t)dt}{\tau + \int_0^{T-\tau} R(t)dt}$$

$$\frac{+\int_0^{T} C_{1b} r_{3b}(t)dt + \int_0^{T} C_{2b} r_{2b}(t)dt + \int_0^{T} C_{3b} r_{3b}(t)dt}{\tau + \int_0^{T-\tau} R(t)dt}. \tag{19}$$

Noting that, $C(T,\tau)$ is adopted as the objective function of an optimization problem, and the aim is to determine the optimal pair replacement time (T^*, τ^*) that minimizes $C(T,\tau)$.

V. Numerical example

Let the rate of Type I failure of the three units of subsystem A follows Weibull distribution:

$$r_{ia}(t) = \lambda_{ia} \propto_{ia} t^{\propto_{ia}-1}, \; t \geq 0, \; i = 1, 2, 3. \tag{20}$$

where $\propto_{ia} > 1$.

Again, let the rate of Type II failure of the three units of subsystem B follows Weibull distribution:

$$r_{ib}(t) = \lambda_{ib} \propto_{ib} t^{\propto_{ib}-1}, \; t \geq 0, \; i = 1, 2, 3. \tag{21}$$

where $\propto_{ib} > 1$.

Let the set of parameters and cost of repair/replacement be used throughout this particular example:

1. $\propto_{ia} = 3$, for i = 1, 2, 3.
2. $\lambda_{ia} = 0.008$, for i = 1, 2, 3.
3. $\propto_{ib} = 3.5$, for i = 1, 2, 3.

4. $\lambda_{ib} = 0.00025$, for i = 1, 2, 3.
5. $C_{ia} = 7$, for i = 1, 2, 3.
6. $C_{ib} = 5$, for i = 1, 2, 3.
7. $C_r = 75$ and $C_p = 50$.

By substituting the parameters in equations 20 and 21, we obtained the failure rates as follows

$$r_{ia}(t) = 0.024t^2, \text{ for } i = 1, 2, 3, \tag{22}$$

and

$$r_{ib}(t) = 0.000875t^{2.5}, \text{ for } i = 1, 2, 3. \tag{23}$$

The tables and the graphs below, are the results obtained by substituting the cost of repair/replacement and equations (22) to (23) in the cost rates C(T) and C(T, τ).

Table 1: *Values of $C(T)$ and C(T, τ) versus planned replacement T*

T	1	2	3	4	5	6	7	8	9	10
C(T)	250.01	125.07	84.05	66.86	64.77	74.12	85.53	89.01	90.39	95.90

Table 2 : *Optimal replacement time of the system from C(T) as C_p decreases.*

C_p	50	40	30	20	10
T^*	5	4	4	4	3

Table 3 : *Optimal replacement time of the system from C(T) as C_r increases.*

C_r	75	85	95	105	115
T^*	5	5	4	4	4

Table 4: *The values of C(T, τ) versus planned replacement T.*

T	T	C(T, τ)
1	0.5	83.36
2	1	41.79
3	1.5	28.08
4	2	21.45
5	2.5	17.85
6	3	15.94
7	3.5	15.00
8	4	14.56
9	4.5	14.40
10	5	14.44
11	5.5	14.64
12	6	14.93
13	6.5	15.30
14	7	15.73
15	7.5	16.22

Table 5 : *Optimal replacement time of the system from C(T, τ) as C_p decreases.*

C_p	50	40	30	20	10
(T^*, τ^*)	(10, 5)	(7, 3.5)	(6, 3)	(5, 2.5)	(4, 2)

Table 6 : *Optimal replacement time of the system from C(T, τ) as C_r increases.*

C_r	75	85	95	105	115
(T^*, τ^*)	(10, 5)	(9, 4.5)	(7, 3.5)	(7, 3.5)	(7, 3.5)

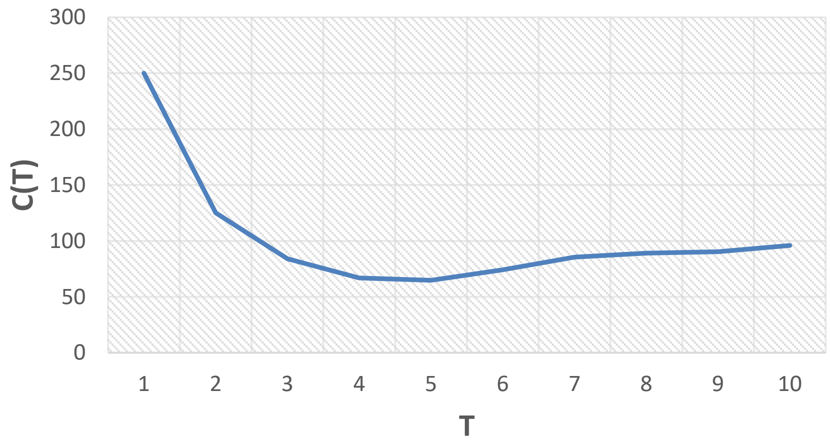

Figure 2. *Plot of C(T) against T*.

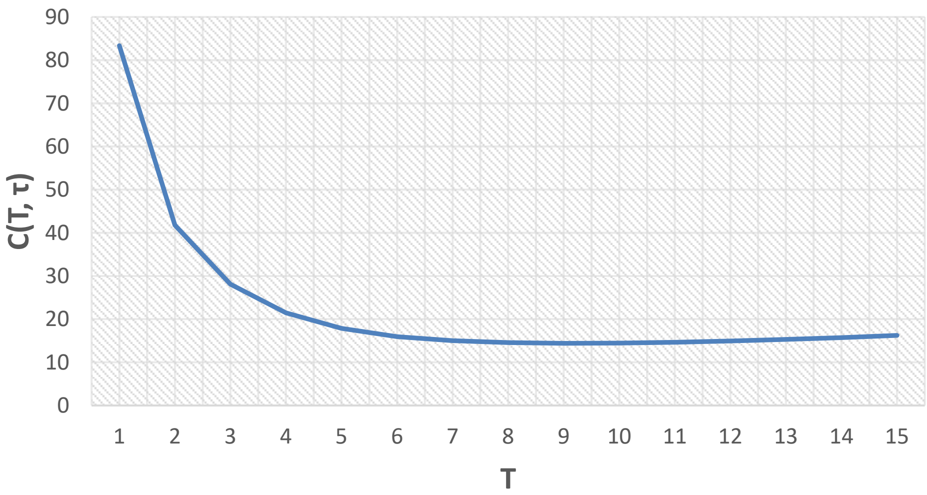

Figure 3. Plot of $C(T,\tau)$ against T.

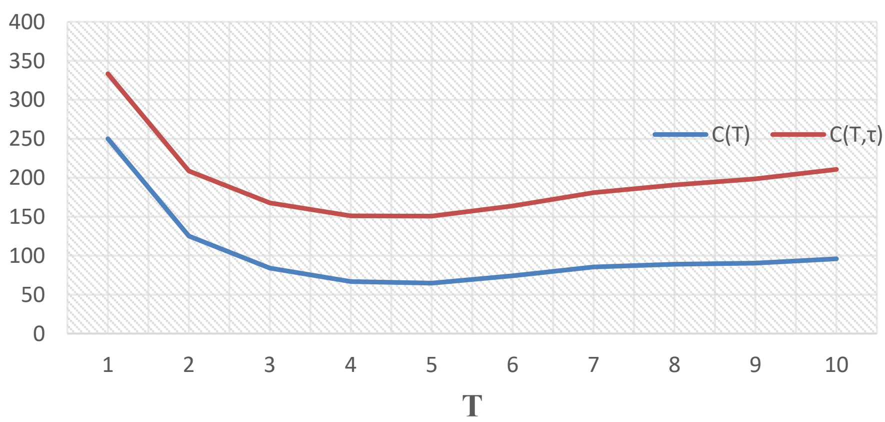

Figure 4: Plot of C(T) and $C(T,\tau)$ agianst T.

Some observations from the results obtained are as follows

1. Observe from table 1, we have the optimal replacement time for the system based on Policy 1 as 5, that is, $T^* = 5$, with minimal cost rate $C(T^* = 5) = 64.77$. See figure 2 below for the plot of $C(T)$ versus T.
2. Observe from table 2, that the optimal replacement time of the system based on Policy 1, sometimes decreases slightly as the cost of planned replacement (C_p) decreases.
3. Observe from table 3, that the optimal replacement time of the system based on Policy 1, sometimes decreases slightly as the cost of un-planned replacement (C_r) increases.
4. Observe from table 4, we have the optimal replacement time for the system based on Policy 2 as (9, 4.5), with minimal cost rate $C(T^* = 9, \tau^* = 4.5) = 14.40$. See figure 3 below for the plot of $C(T, \tau)$ versus T.
5. Observe from table 5, that the optimal replacement time of the system based on Policy 2, sometimes decreases slightly as the cost of planned replacement (C_p) decreases.
6. Observe from table 6, that the optimal replacement time of the system based on Policy 2, sometimes decreases slightly as the cost of un-planned replacement (C_r) increases.
7. Observe from figure 4, we have, $C(T) < C(T, \tau)$.
8. Observe from tables 2, 3, 5 and 6, that the optimal replacement time obtained from Policy 2 is higher than that of Policy 1.

VI. Conclusion and recommendations

This paper gives a survey on some important maintenance policies involving minimal repairs and replacements of multi-component systems. In this paper, we constructed age replacement models with minimal repair for a parallel-series system based on two different policies (Policy 1 and Policy 2), such that the system contained two subsystems, which are subsystem A and subsystem B. We assumed that two subsystems are formed by three units. It was also assumed that subsystem A is subjected to Type I failure and subsystem B is subjected to Type II failure. Finally, a numerical example was given, to investigate the characteristics of the age replacement models with minimal repair constructed for a multi-component system, where from the results, it was also observed that, the optimal replacement time obtained from Policy 2 is higher than the optimal replacement time obtained from Policy 1. The results obtained would be useful for the practical maintenance of multi-component systems.

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