# Modified Group Lottery Scheduling Algorithm for Ready Queue Mean Time Estimation in Multiprocessor Environment

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**Abstract**

*The problem of ready queue mean time estimation in the multiprocessor environment was discussed by Shukla et. al. [5] and several others. In recent years, most of the existing and relating contributions assume that all processes in the ready queue might have been completed before a particular instant of time occur like a sudden failure or interrupt. Due to this, data of time consumed by processes remain available. The idea of improvement in this paper is to assume that at the instant of occurrence of breakdown, some processes are partially completed and remaining is completely processed. Under this situation, the time computation and allocation strategies need to be re-designed. Therefore this has been taken into account in this paper with a proposal of a modified scheme. It contains arbitrary, Type-A, and Type- B allocations of sample units to the processors. Confidence intervals for the sample mean values are calculated and simulated over many samples using cumulative probabilities. It was found that Type-A allocation has the lowest variance.*

**Keywords:** CPU, Scheduling, Lottery Scheduling, Estimation, Sampling, Probability, Allocation, Simulation.

## I. Introduction

The challenging task of an operating system is CPU scheduling algorithms where various non-probabilities based traditional schemes are operational. These can simply be handled easily by processors while probabilistic scheduling schemes have to face the difficulty of resource management, system performance, and low system overhead. Lottery scheduling is one such probability-based scheme first introduced by Carl A. Waldspurger [12]. Shukla, Jain, and Choudhary [4] have initiated the problem of estimation of ready queue processing time by suggesting SL scheduling algorithm in a multiprocessor environment. The contribution contains a sample-based estimation of ready queue mean time which likely to be spent while completes exhaust of ready queue occurs. It reveals the approach of systematic sampling which has some limitations in terms of efficiency of the predicted value. Shukla et. al. [6] extended similar problem under the approach of lottery scheduling. Content of contribution stands for randomly selected processes from the ready queue for forecasting the sample-based mean time. The limitation of lottery scheduling appears due to the reason that processes happen to be of any size may appear in any order before multiprocessors. Shukla and Jain [7] extended the ready queue processing time estimation approach to the care of probability proportional to size-dependent lottery scheduling which provides better prediction than earlier. Following the similar approach, Shukla and Jain [8] used factor type estimation method for estimating mean ready queue processing time in setup of lottery scheduling under a multiprocessor environment. Shukla and Jain [9] extended approach using ratio type estimation method and advocated for better efficiency under constraints. A similar approach adopted in Jain and Shukla [10] and Shukla and Jain [11] with additive features. An exhaustive review of the problem of ready queue mean time estimation is due to Shukla and More [1] and some suggestive contributions are due to Shukla and More [2] [3]. Sampling technique concepts and applications are in Cochran [13].

Shukla D., Jain, and Choudhary [5] discussed GL scheduling which assumes the processes present in all processors in the time session (0-T) have been completely processed at instant T and their compound predictive estimate of average processing time could be obtained. Such an estimate is useful for forecasting the expected time required to vacate the entire ready queue. This helps in backup management while sudden failure (or disaster) occurs. But it doesn't cover the case when a sudden failure occurs during the processing of these jobs (processes). How estimation will be in a situation when the last process is partially processed and kept on hold. This paper takes into account this problem and provides a solution

## II. GL Scheduling Scheme (due to Shukla, Jain, and Choudhary [5]):

**Step 1**: Assume multiple processors $Q_1$, $Q_2$, $Q_3$……$Q_r$, each draws random samples of jobs from corresponding ready queues. Processes in the $i^{th}$ ready queue are homogeneous concerning certain characteristics whereas in the usual waiting queue they are present in any order of size measure.

**Step 2**: The CPU restricts a session of time duration T. All N ready queue processes are divided into r groups each of size containing $N_i$ processes ( $\sum N_i = N$). This division is based on size measure.

**Step 3**: All N processes are allotted token of numbers and each processor draws a random number. If the random number of $i^{th}$ processor matches the allotted random number to the $j^{th}$ process of the $i^{th}$ group then it is selected for processing (i=1, 2, 3….r, j=1, 2, 3…..$N_i$).

**Step 4**: Let $k_1$ processes received from the first group, $k_2$ processes from the second group, and so on, the $k_r^{th}$ received processes from $r^{th}$ group in a random manner using lottery procedure [ $\sum k_i = k$] in a session of fixed time T where k is the total sample size.

**Step 5**: At the end of a session, the CPU provides processed time data for $k_1$, $k_2$, $k_3$….$k_r$ jobs as ($t_{11}$, $t_{12}$, $t_{13}$…... $t_{21}$, $t_{22}$, $t_{23}$…., .$t_{i1}$, $t_{i2}$, $t_{i3}$…) where $t_{ij}$ are the time consumed by $j^{th}$ job.

## III. Modified Group Lottery Scheduling (MGLS) Scheme

The proposed contribution is an extension of the previous algorithm suggested by Shukla et. al. [5], with the idea of improvement to include the processing time of those processes that remained partially processed due to sudden system breakdown or occurrence of an interrupt. Following are steps of the proposed scheme:

**Step 1**: Assume r processors $Q_1$, $Q_2$, $Q_3$, $Q_4$……….$Q_r$, in a system each, receives random samples from corresponding linked ready queues. Processes in corresponding ready queues are of homogeneous concerning a specific characteristic. If any event wait appears, that process moves to a waiting/blocked/suspended queue.

**Step 2**: Total N processes assumed present in the system are divided into r groups of ready queues with the assumption that $i^{th}$ group (or ready queue) has $N_i$ processes ($\sum N_i = N$).

**Step 3**: All N processes in the system are assigned token of numbers. Processors generate random numbers whose matching occurs with token assigned to processes. If $i^{th}$ processor random number matches to the token number of $j^{th}$ process then $j^{th}$ assigns to $i^{th}$ processor.

**Step 4:** Using (3), suppose total $k_r$ processes selected from $r^{th}$ group of the ready queue in a random manner and assigned to $Q_r$th processor. The total sample size is $k = \sum k_i$ where $i = 1,2,3,\ldots\ldots.r$, $j = 1,2,3,\ldots\ldots\ldots N_i$

**Step 5**: Let $t_{ij}$ denote time consumed by the jth process assigned to ith processor.

**Step 6**: At instant time T, out of total $k_i$ processes present in ith processor, assume $k_{i-1}$ have completely processed but the last one is partially processed with time $t_i^*$ in all $Q_1$, $Q_2$, $Q_3$ …$Q_r$. The set of time ($t_1^*$, $t_2^*$, $t_3^*$.......$t_r^*$) is the time consumed by partially processed jobs.

**Step 7:** Processes within the processor are divided into two parts. The Part A being sub-group of completely processed and part B for unprocessed ($t_i^*$)

**Step 8**: Overall mean time, $\overline{mt} = \frac{1}{N}\sum\sum t_{ij}$, $\overline{mt}_i = \frac{1}{N_i}\sum_{j=1}^{N_i}(t_{ij})$ (for ith ready queue), $S_i^2 = \frac{1}{N_{i-1}}\sum_{j=1}^{N_i}(t_{ij} - \overline{mt}_i)^2$ (for ith ready queue) and $S^2 = \frac{1}{N-1}\sum_{i=1}^{r}\sum_{j=1}^{N_i}(t_{ij} - \overline{mt})^2$ under assumption while all N completely processed before occurring T but under step (6) it does not happen.

**Note: The steps 5, 6, and 7 are the idea of improvement in this paper over the Shukla et. al. [5].**

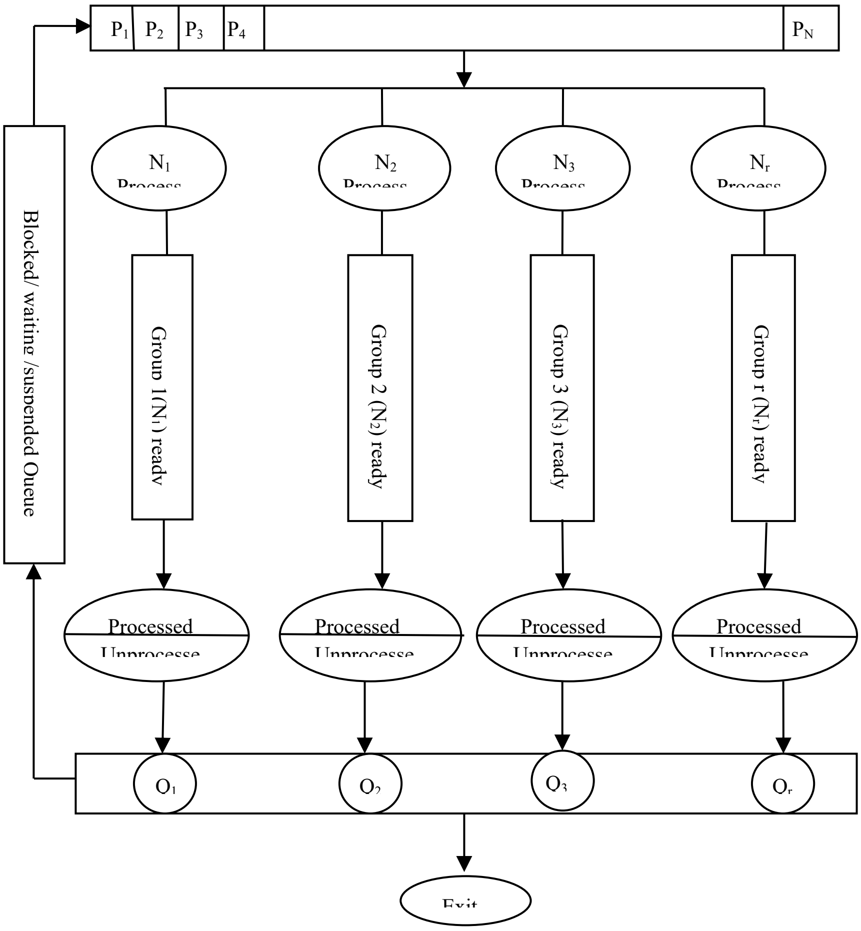



**Figure 1:** *Setup of ready queue and multiprocessor environment*

## IV. Estimation Procedure under Arbitrary Allocation

The Modified Group Lottery Scheduling algorithm (MGLS) provides the estimation of mean time likely to consume by the N processes in the ready queue while occurrences of time T. For ith ready queue (group), the mean time is spited into:

(a) $\bar{t}_i' = \left(\frac{1}{(k_i-1)}\right)\sum_{j=1}^{k_i-1}(t_{ij})$ (for processed part A of sample not including unprocessed)

(b) $\bar{t}^* = \frac{1}{r}\sum_{j=1}^{r}(t_i^*)$ (for unprocessed part B jobs in all r samples)

(c) The mean time estimator is $\overline{u} = [\sum_{i=1}^{r} w_i \overline{t}_i{}' + \overline{t}^*] / 2$ where $w_i = \frac{N_i}{N}$

(d) The mean square of time $\bar{t}_i$ for $i^{th}$ group is $S_i^2 = \frac{1}{(N_i-1)} \sum_{j=1}^{N_i}(t_{ij} - \bar{t}_i)^2 = \left(\frac{1}{(N_i-1)}\right) \sum_{j=1}^{N_i}(t_{ij} - \overline{mt}_i)^2$ Where $\bar{t}_i = \frac{1}{N_i} \sum_{j=1}^{N_i} t_{ij}$

(e) $S^2 = \frac{1}{(N-1)} \sum_{i=1}^{r} \sum_{j=1}^{N_i}(t_{ij} - \bar{t})$ where $\bar{t} = \frac{1}{N} \sum_{i=1}^{r} \sum_{j=1}^{N_i}(t_{ij}) = \overline{mt}$

(f) Variance of estimator $\overline{u}$ is $V(\bar{u})_{arbit} = V[\sum_{i=1}^{r} w_i \bar{t}_i' + \bar{t}^*] = \sum_{i=1}^{r} w_i^2 V(\bar{t}_i') + V(\bar{t}^*)$

$$= \sum_{i=1}^{r}\left(\frac{1}{(k_i-1)} - \frac{1}{N_i}\right) w_i^2 S_i^2 + \left[\left(\frac{1}{r} - \frac{1}{N}\right) S^2\right] \tag{4.1}$$

This estimator $\overline{u}$ and variance $V(\bar{u})_{arbit}$ is based on arbitrary allocation of processes to the processors.

# V. Types of Allocations:

## Type-A Allocation: Based on prior information of processor speed

The choice of $k_i$ depends on the speed of processors. A fast processor can randomly pick a larger number of jobs from the group of ready queue samples. Let priority known processor speed are $S_1^*$, $S_2^*$, $S_3^*$.......$S_r^*$ for $Q_1, Q_2, Q_3$ ….$Q_r$ respectively, and $\sum_{i=1}^{r} S_i^* = S^*$ holds.

Let $k_i \ \alpha \ S_i^*$, $k_i = MS_i^*$, $\sum k_i = \sum MS_i^*$, $k = M S^*$, $M = (k/s^*)$, $k_i = (\frac{k}{S^*})S_i^*$ (M is any constant) (5.1) Substituting (5.1) in (4.1) one can get

$$V(\bar{u})_I = \sum_{i=1}^{r}\left[\left(\frac{1}{\left(\left(\frac{k}{S^*}\right)S_i^* - 1\right)} - \frac{1}{N_i}\right) w_i^2 S_i^2\right] + \left[\left(\frac{1}{r} - \frac{1}{N}\right) S^2\right] = \sum_{i=1}^{r}\left[\left(\frac{S^*}{(k S_i^* - S^*)} - \frac{1}{N_i}\right) w_i^2 S_i^2\right] + \left[\left(\frac{1}{r_*} - \frac{1}{N}\right) S^2\right]$$

$$V(\bar{u})_I = \sum_{i=1}^{r}\left[\left(\frac{S^*(w_i^2 S_i^2)}{(k S_i^* - S^*)}\right)\right] - \frac{1}{N} \sum_{i=1}^{r} w_i S_i^2 + \left[\left(\frac{1}{r} - \frac{1}{N}\right) S^2\right] \tag{5.2}$$

## Type-B Allocation: Based on prior information of variation ($S_i^2$) in ready queue:

The $S_i^2$ for $i^{th}$ group is defined in section 4.0 as under

$$S_i^2 = \sum_{j=1}^{N_i} \frac{1}{(N_i-1)} (t_{ij} - \bar{t}_i) = \left(\frac{1}{(N_i-1)}\right) \sum_{j=1}^{N_i}(t_{ij} - \overline{mt}_i)^2$$

Consider $k_i \ \alpha \ S_i^*$ and $k_i \ \alpha \ S_i$ together where $S_i$ refers to variability among processes in $i^{th}$ queue related to a characteristic (e.g. expected time of process) and assumed known.

Then, $k_i \ \alpha \ S_i^* S_i$, $k_i = M^* S_i^* S_i$ where M is constant $\sum k_i = M^* \sum S_i^* S_i$,

$M^* = \frac{k}{\sum S_i^* S_i}$ and $k_i = \left[\frac{k}{\sum S_i^* S_i}\right] S_i^* S_i$ (5.3)

The variance under Type-B allocation could be obtained by substituting (5.3) in expression (4.1)

$$V(\bar{u})_{II} = \sum_{i=1}^{r}\left[\left(\frac{k S_i^* S_i - \sum S_i^* S_i}{\sum S_i^* S_i}\right) w_i^2 S_i^2\right] - \left[\frac{1}{N} \sum_{i=1}^{r} w_i S_i^2\right] + \left[\left(\frac{1}{r} - \frac{1}{N}\right) S^2\right] \tag{5.4}$$

# VI. Numerical Illustration:

Consider a small data setup with 30 processes in the ready queue whose expected processing time ($t_{ij}$) are given in table 1. This numerical table 1 is to justify the computations, expressions, results.

**Table 1:** *Total Processes Data*

| *Total Processes Data* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Process | CPU Time | Process | CPU Time | Process | CPU Time | Process | CPU Time | Process | CPU Time | Process | CPU Time |
| $Proc_1$ | 30 | $Proc_6$ | 60 | $Proc_{11}$ | 138 | $Proc_{16}$ | 89 | $Proc_{21}$ | 143 | $Proc_{26}$ | 79 |
| $Proc_2$ | 20 | $Proc_7$ | 33 | $Proc_{12}$ | 43 | $Proc_{17}$ | 123 | $Proc_{22}$ | 29 | $Proc_{27}$ | 46 |
| $Proc_3$ | 142 | $Proc_8$ | 43 | $Proc_{13}$ | 109 | $Proc_{18}$ | 67 | $Proc_{23}$ | 147 | $Proc_{28}$ | 59 |
| $Proc_4$ | 40 | $Proc_9$ | 101 | $Proc_{14}$ | 26 | $Proc_{19}$ | 58 | $Proc_{24}$ | 94 | $Proc_{29}$ | 72 |
| $Proc_5$ | 59 | $Proc_{10}$ | 69 | $Proc_{11}$ | 138 | $Proc_{16}$ | 89 | $Proc_{21}$ | 143 | $Proc_{26}$ | 79 |

Assume there are three processors $Q_1$, $Q_2$, $Q_3$ (r=3) having known processing speed $S_1^*$, $S_2^*$, $S_3^*$ respectively. Ready queues are divided into three groups as under as in Table 2, Table 3 and 4.

Table 2: First Group Data (below 50 CPU time)

| Ready Queue Group 1 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Process | Proc1 | Proc2 | Proc4 | Proc7 | Proc8 | Proc12 | Proc14 | Proc22 | Proc27 | Proc30 |
| CPUTime | 30 | 20 | 40 | 33 | 43 | 43 | 26 | 29 | 46 | 22 |

Table 3: Second Group Data (above 50 but below 100 CPU time)

| Ready Queue Group 2 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Process | Proc5 | Proc6 | Proc10 | Proc15 | Proc16 | Proc18 | Proc19 | Proc20 | Proc24 | Proc26 | Proc28 | Proc29 |
| CPUTime | 59 | 60 | 69 | 74 | 89 | 67 | 58 | 84 | 94 | 79 | 59 | 72 |

Table 4: Third Group Data (above 100 CPU time)

| Ready Queue Group 3 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Process | Proc3 | Proc9 | Proc11 | Proc13 | Proc17 | Proc21 | Proc23 | Proc25 |
| CPUTime | 112 | 101 | 138 | 109 | 123 | 143 | 147 | 131 |

Table 5: Available Speed of the Processor

| Processor's Speeds | | | | |
|---|---|---|---|---|
| Processors | $Q_1$ | $Q_2$ | $Q_3$ | Total available speed |
| Speed | $S_1^*=2.5$ | $S_2^*=3.0$ | $S_3^*=5.5$ | 11.0 |

Table 6: Parameters of all N Processes in System

| Parameters of all N Processes in System | | | |
|---|---|---|---|
| **Complete N** | **Group 1 (Table 6.2)** | **Group 2 (Table 6.3)** | **Group 3 (Table 6.4)** |
| Mean time $\bar{t} = \frac{1}{k_i}\sum_{i=1} t_{ij}$ = 73.33 | $w_1 = \frac{N_1}{N} = 0.33$ | $w_2 = \frac{N_2}{N} = 0.4$ | $w_3 = \frac{N_3}{N} = 0.26$ |
| Mean square $S^2 = 1461.8484$ | Mean time $(\overline{mt_1}) = \bar{t}_1 = 33.20$ | Mean time $(\overline{mt_2}) = \bar{t}_2 = 72.0$ | Mean time $(\overline{mt_3}) = \bar{t}_3 = 125.50$ |
| | Square of mean time $(\overline{mt_1})^2 = 1102.24$ | Square of mean time $(\overline{mt_2})^2 = 5184$ | Square of mean time $(\overline{mt_3})^2 = 15750.25$ |
| | Total sum of square $\sum_{j=1}^{N_i} t_{1j}^2 = 11804$ | Total sum of square $\sum_{j=1}^{N_i} t_{2j}^2 = 63890$ | The total sum of square $\sum_{j=1}^{N_i} t_{3j}^2 = 128018$ |
| | Mean square $S_1^2 = 86.8444$ and $S_1 = 9.32$ | Mean square $S_2^2 = 152.9090$ and $S_2 = 12.37$ | Mean square $S_3^2 = 288$ and $S_3 = 16.97$ |

## VII. Calculation for Arbitrary Allocation

Table 6 reveals parametric values of all three queues assuming if all N have been processed before occurrences of instant breakdown T. Parameters $S_i^2$, $S^2$, $\overline{t_1}$, $\overline{t_2}$, $\overline{t_3}$, and $\bar{t}$ have been calculated at the entire level. Moving on at the sample level, the arbitrary allocation $k_1$, $k_2$, $k_3$ is adopted for sample size $k = \sum k_i = 12$. In table 7, sample values $k_1 = 4$, $k_2 = 4$, $k_3 = 4$ considered for total random sample size k=12 drawn from N=30.

Variance of estimator $\bar{u}$ is $V(\bar{u})_{arbit} = V\left[\sum_{i=1}^{r} w_i \bar{t}_i' + \bar{t}^*\right] = \sum_{i=1}^{r} w_i^2 V(\bar{t}_i') + V(\bar{t}^*)$

$$= \sum_{i=1}^{r}\left(\frac{1}{(k_i-1)} - \frac{1}{N_i}\right) w_i^2 S_i^2 + \left[\left(\frac{1}{r} - \frac{1}{N}\right) S^2\right]$$

Table 7: Variances Calculation under Arbitrary Allocations ($S_i^2$ and $S^2$ known)

| Variance under Arbitrary Allocation |
|---|
| $k_1 = 4$, $k_2 = 4$, $k_3 = 4$ |
| $V(\bar{u})_{arbit} = 446.442$ |

**Calculation for Type-A and Type-B allocations:**

Consider following available data for variability and processor speed, both are assumed priory known. Table 8 has similar content relating to $S_i^*$

**Table 8: Prior knowledge of Speed and Variability**

| Prior knowledge of Speed and Variability | | | |
|---|---|---|---|
| **Processors** | **Speed ($S_i^*$)** | **Variability ($S_i$)** | **$S_i^* S_i$** |
| Processor 1 | $S_1^* = 2.5$ | $S_1 = 9.3$ | 23.25 |
| Processor 2 | $S_2^* = 3.0$ | $S_2 = 12.3$ | 36.9 |
| Processor 3 | $S_3^* = 5.5$ | $S_3 = 16.9$ | 92.95 |
| Total | $(S^*) = 11.0$ | | $\sum S_i^* S_i = 153.1$ |

**Case 1:** For Type-A allocation using (5.1), $k_i = (k/S^*) S_i^*$, $S^* = \sum S_i^*$, $k = \sum k_i$, For pre-fixed k = 12, its division in three parts is in table 9 .

**Table 9: Allocation under Type -A**

| Allocation under Type -A | | | |
|---|---|---|---|
| $k_1$ | $= (k/S^*) S_1^*$ | = 2.72 | = 3 (from first ready queue) |
| $k_2$ | $= (k/S^*) S_2^*$ | = 3.27 | = 3 (from second ready queue) |
| $k_3$ | $= (k/S^*) S_1^3$ | = 6.0 | = 6 (from third ready queue) |
| Total $k = (k_1+k_2+k_3)$ | | | k = 12 |

**Case 2:** For Type-B allocation using (5.3), $k_i = \left[\frac{k}{\sum S_i^* S_i}\right](S_i^* S_i)$, and k = 12 is divided in three parts as shown in table 10.

**Table 10: Allocation under Type- B**

| Allocation under Type-B | | | |
|---|---|---|---|
| $k_1$ | = | $[k/(\sum S_i^* S_i)]$= 2.20 | = 2 (from first ready queue) |
| $k_2$ | = | $[k/(\sum S_i^* S_i)]$= 1.98 | = 2 (from second ready queue) |
| $k_3$ | = | $[k/(\sum S_i^* S_i)]$= 7.87 | = 8 (from third ready queue) |
| Total k = $(k_1+k_2+k_3)$ | | | k = 12 |

**Calculation of Variance under Type-A allocation:**

$$V(\bar{u})_I = \sum_{i=1}^{r} [S^* (w_i^2 S_i^2) / (kS_i^* - S^*)] - \frac{1}{N}\sum w_i S_i^2 + \left(\frac{1}{r} - \frac{1}{N}\right) S^2$$
$$= S^*\{[w_1^2 S_1^2/ (kS_1^* - S^*)] + [w_2^2 S_2^2/ (kS_2^* - S^*)] + [w_3^2 S_3^2/ (kS_3^* - S^*)]\} - \frac{1}{N}[w_1 S_1^2 + w_2 S_2^2 + w_3 S_3^2]$$
$$+ \left(\frac{1}{r} - \frac{1}{N}\right) \frac{1}{N-1} [\sum_{i=1}^{r}\sum_{j=1}^{N_i}(t_{ij} - \bar{t})] \text{ when } r = 3 \quad (7.1)$$

**Calculation of Variance under Type-B allocation:**

$$V(\bar{u})_{II} = \sum_{i=1}^{r}[(kS_i^* S_i - \sum S_i^* S_i)/\sum S_i^* S_i] w_i^2 S_i^2 - \frac{1}{N}\sum_{i=1}^{r} w_i S_i^2 + \left[\left(\frac{1}{r} - \frac{1}{N}\right) S^2\right]$$
$$= [(kS_1^* S_1 - \sum S_1^* S_1)/ \sum S_1^* S_1] w_1^2 S_1^2 + [(kS_2^* S_2 - \sum S_2^* S_2)/ \sum S_2^* S_2] w_2^2 S_2^2 + [(kS_3^* S_3 - \sum S_3^* S_3)/ \sum S_3^* S_3]$$
$$w_3^2 S_3^2 - \frac{1}{N}[w_1 S_1^2 + w_2 S_2^2 + w_3 S_3^2] + \left(\frac{1}{r} - \frac{1}{N}\right)\frac{1}{N-1} [\sum_{i=1}^{r}\sum_{j=1}^{N_i}(t_{ij} - \bar{t})] \text{ when } r = 3 \quad (7.2)$$

**Table 11: Comparison of Variances under different Allocations**

| Comparison of Variances under different Allocations | | |
|---|---|---|
| **Variance under Type-A Allocation** | **Variance under Type-B Allocation** | **Variance under Arbitrary Allocation** |
| $k_1$ =3, $k_2$= 3, $k_3$= 6 | $k_1$ =2, $k_2$= 2, $k_3$= 8 | $k_1$ =4, $k_2$= 4, $k_3$= 4 |
| $V(\bar{u})_I$ = 442.08 | $V(\bar{u})_{II}$ = 611.452 | $V(\bar{u})_{arbit}$= 446.442 |

Table 8 contains the assumption that three $S_i^2$ (i = 1, 2, 3) are priory known (or guessed) and so the variance $V(\bar{u})_I$ is lowest under the type-A allocation (while $S_i^2$ and $S^2$ known) in comparison to Type-B and Arbitrary allocation.

**Estimate of Variance :**

The value $S_i^2 = \left(\frac{1}{(N_i-1)}\right) \sum_{j=1}^{N_i}(t_{ij} - \bar{t}_i)^2$ suppose not known then they are to be replaced by sample value estimates. The sample based estimate of $S^2$ and $S_i^2$ are defined like $(es)^2$ and $(es_i)^2$ with expressions are as under:

$$(es_i)^2 = \left(\frac{1}{(k_i-1)}\right) \sum_{j=1}^{k_i-1}(t_{ij} - \bar{t}_i) \quad \text{and } (es)^2 = \left(\frac{1}{[(k-r)-1]}\right) \sum_{i=1}^{r}\sum_{j=1}^{[k-r-1]}(t_{ij} - \bar{t}_i)^2 \quad (7.3.1)$$

$$Est[V(\bar{u})_{arbit}] = \sum_{i=1}^{r}\left(\frac{1}{(k_i-1)} - \frac{1}{N_i}\right) w_i^2 (es_i)^2 + [(\frac{1}{r} - \frac{1}{N}) (es)^2] \quad (7.3.2)$$

$$Est[V(\bar{u})_I] = \sum_{i=1}^{r}[S^* (w_i^2 (es)^2) /( kS_i^* - S^*)] - \frac{1}{N}\sum w_i (es_i)^2 + \left(\frac{1}{r} - \frac{1}{N}\right) (es)^2 \quad (7.3.3)$$

$$Est[V(\bar{u})_{II}] = [( \sum_{i=1}^{r}[k S_i^* (es_i) - \sum S_i^* (es_i))/ \sum S_i^* (es_i) ]w_i^2 (es_i)^2 - \frac{1}{N}\sum_{i=1}^{r} w_i(es_i)^2 + \left[\left(\frac{1}{r} - \frac{1}{N}\right) (es)^2\right] \quad (7.3.4)$$

Calculations of estimated values are in table 7.6 and 7.7 on the 10 samples.

**Table 12: Calculations of Sample Mean and Estimate of Variance under Arbitrary Allocation (Section 4.0) in 10 samples (when $S_i^2$ and $S^2$ unknown)**

(*Partially processed job containing a part of the processing time and unprocessed due time)

| Calculations of Sample Mean and Estimate of Variance under Arbitrary Allocation | | | | | | | |
|---|---|---|---|---|---|---|---|
| Random Sample No. | Sampled Selected with Processing Time (k=9) | | | Processed $\sum w_i \bar{t}_i$ | Unprocessed $(t_1^*+t_2^*+t_3^*)/3$ $es^2=\frac{1}{(r-1)}\sum_{i=1}^{r}(t_i^*-\bar{t}^*)^2$ | Sample Mean $(\bar{u})$ | $V(\bar{u})_{arbit}$ |
| | Group1 $K_1$=4 | Group2 $K_2$=4 | Group3 $K_3$=4 | | | | |
| **1.** | 30,43,33,30* Mean=35.33 $t_1$*=25 $(es_1)^2$=46.33 | 60,84,67,59* Mean=70.33 $t_2$*=39 $(es_2)^2$=152.33 | 138,112,109,101* Mean=119.6 $t_3$*=61 $(es_3)^2$=254.33 | 70.88 | 41.6 $(es)^2$=37.66 | 56.24 | 112.478 |
| 2. | 33,46,40,20* Mean=39.6 $t_1$*=15 $(es_1)^2$=50.26 | 69,58,59,60* Mean=62 $t_2$*=35 $(es_2)^2$= 37 | 109,101,112,143* Mean=107.33 $t_3$*=88S $(es_3)^2$= 32.33 | 65.77 | 46 $(es)^2$ =1423 | 55.88 | 430.07 |
| 3. | 20,46,30,40* Mean=32 $t_1$*=25 $(es_1)^2$=172 | 59,72,79,69* Mean=70 $t_2$*=39 $(es_2)^2$=103 | 147,138,101,123* Mean=128.6 $t_3$*=56 $(es_3)^2$=594.33 | 71.99 | 40 $(es)^2$ =241 | 55.99 | 86.66 |
| 4. | 40,22,26,33* Mean=29.33 $t_1$* =23 $(es_1)^2$=89.33 | 74,84,60,58* Mean=72.66 $t_2$*=29 $(es_2)^2$=146.79 | 131,109,123,112* Mean=121 $t_3$*=67 $(es_3)^2$=124 | 70.20 | 39.77 $(es)^2$ =557 | 54.98 | 176.44 |
| 5. | 43,29,30,20* Mean=34 $t_1$*= 15 $(es_1)^2$ = 61 | 79,67,58,60* Mean=68 $t_2$*=35 $(es_2)^2$= 111 | 123,143,112,101* Mean= 126 $t_3$*=65 $(es_3)^2$=247 | 71.18 | 38.33 $(es)^2$ =634 | 54.75 | 198.63 |
| 6. | 20,22,29,43* Mean=23.66 $t_1$*=28 $(es_1)^2$=22.80 | 59,72,84,67* Mean=71.66 $t_2$*=47 $(es_2)^2$=156.33 | 101,109,123,131* Mean= 111 $t_3$*=81 $(es_3)^2$=124 | 65.33 | 52 $(es)^2$ =721 | 58.66 | 224.36 |
| 7. | 30,29,20,26* Mean=26.33 $t_1$*=19 $(es_1)^2$=30.33 | 59,69,72,58* Mean=66.66 $t_2$*=38 $(es_2)^2$=46.33 | 101,147,109,112* Mean=119 $t_3$*=66 $(es_3)^2$=604 | 66.29 | 41 $(es)^2$=559 | 53.64 | 176.34 |
| 8. | 30,26,33,29* Mean=29.66 $t_1$* =24 $(es_1)^2$= 12.33 | 72,58,74,60* Mean=68 $t_2$*=44 $(es_2)^2$=76 | 112,131,101,123* Mean=114.66 $t_3$*=68 $(es_3)^2$=230.33 | 66.79 | 45.33 $(es)^2$ =486 | 56.06 | 151.44 |
| 9. | 40,29,30,46* Mean=33 $t_1$*=26 $(es_1)^2$=37 | 60,58,67,79* Mean=61.66 $t_2$*= 49 $(es_2)^2$=23.57 | 109,112,131,101* Mean= 117.33 $t_3$*=79 $(es_3)^2$= 142.33 | 66.05 | 51.33 $(es)^2$ =707 | 58.69 | 215.38 |
| 10. | 20,43,40,22* Mean=34.33 $t_1$*=16 $(es_1)^2$=156.5 | 79,58,60,59* Mean=65.66 $t_2$*=34 $(es_2)^2$=134.33 | 123,101,112,143* Mean= 112 $t_3$*=73 $(es_3)^2$ =121 | 66.71 | 41 $(es)^2$ =849 | 53.85 | 265.19 |

**Table 13: Estimated values of Variances over 10 samples as per table 6.7 (when $S_i^2$ and $S^2$ are unknown)**

| Sample Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Sample Mean ($\bar{u}$) | 56.24 | 55.88 | 55.99 | 54.98 | 54.75 | 58.66 | 53.64 | 56.06 | 58.69 | 53.85 |
| Est[$V(\bar{u})_{arbit}$] | 112.478 | 430.07 | 86.66 | 176.44 | 198.63 | 224.36 | 176.34 | 151.44 | 215.38 | 265.19 |
| Est[$V(\bar{u})_I$] | 113.65 | 431.86 | 90.26 | 180.95 | 201.02 | 227.11 | 175.22 | 151.93 | 216.11 | 271.09 |
| Est[$V(\bar{u})_{II}$] | 242.29 | 453.07 | 333.11 | 261.55 | 317.58 | 308.78 | 405.65 | 253.46 | 273.94 | 349.22 |

## Calculation of Confidence Interval (CI):

**A.** The 95% Confidence Interval of the sample mean $\bar{\mathbf{u}}$ is defined as:
Probability [$(\bar{\mathbf{u}}) \pm \mathbf{1.96}\sqrt{\mathbf{v}(\bar{\mathbf{u}})}$ ] = 0.95. The interpretation of C.I. is that it is an interval where the chance of laying the unknown true value of mean time is 95%.

**B.** In another way, the 95% chance is that unknown mean processing time of all N processes will lie in the confidence interval.

**C.** Table 8, 9, and 10 present the computation of confidence intervals for different types of allocations. When $S_i^2$, $S^2$ treated unknown.

**Table 14: Confidence Interval Calculation under Arbitrary Allocation [using Table 6 and 7]**

| Sample Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Sample Mean ($\bar{u}$)** | 56.24 | 55.88 | 55.99 | 54.98 | 54.75 | 58.66 | 53.64 | 56.06 | 58.69 | 53.85 |
| **Est.[ $V(\bar{u})_{arbit}$]** | 112.478 | 430.07 | 86.66 | 176.44 | 198.63 | 224.36 | 176.34 | 151.44 | 215.38 | 265.19 |
| **Estimate of Confidence Interval for Est[ $V(\bar{u})_{arbit}$ ]** | (35.45, 77.02) | (15.23, 81.28) | (37.74, 74.23) | (28.94, 81.01) | (27.12, 82.37) | (29.30, 88.01) | (27.61, 79.66) | (31.94, 80.17) | (29.92, 87.45) | (21.93, 85.76) |

**Table 15: Confidence Interval Calculation for Type-A Allocation [using Table 9 and 10]**

| Sample Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Sample Mean ($\bar{u}$)** | 56.24 | 55.88 | 55.99 | 54.98 | 54.75 | 58.66 | 53.64 | 56.06 | 58.69 | 53.85 |
| **Est.$V(\bar{u})_I$** | 113.65 | 431.86 | 90.26 | 180.95 | 201.02 | 227.11 | 175.22 | 151.93 | 216.11 | 271.09 |
| **Estimate of Confidence Interval for Est[ $V(\bar{u})_I$ ]** | (35.34, 77.13) | (15.14, 96.61) | (37.36, 74.61) | (28.61, 81.34) | (26.96, 82.53) | (29.12, 88.19) | (27.69, 79.58) | (31.90, 80.21) | (29.87, 87.5) | (21.57, 86.12) |

**Table 16: Confidence Interval Calculation for Type-B Allocation [using Table 11 and 12]**

| Sample Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Sample Mean ($\bar{u}$)** | 56.24 | 55.88 | 55.99 | 54.98 | 54.75 | 58.66 | 53.64 | 56.06 | 58.69 | 53.85 |
| **Est.[$V(\bar{u})_{II}$]** | 242.29 | 453.07 | 333.11 | 261.55 | 317.58 | 308.78 | 405.65 | 253.46 | 273.94 | 349.22 |
| **Estimate of Confidence Interval for Est[ $V(\bar{u})_{II}$]** | (25.73, 86.74) | (14.16, 97.59) | (20.21, 91.76) | (23.28, 86.67) | (19.82, 89.67) | (24.21, 93.1) | (14.16, 93.11) | (24.85, 87.26) | (26.24, 91.13) | (17.22, 90.47) |

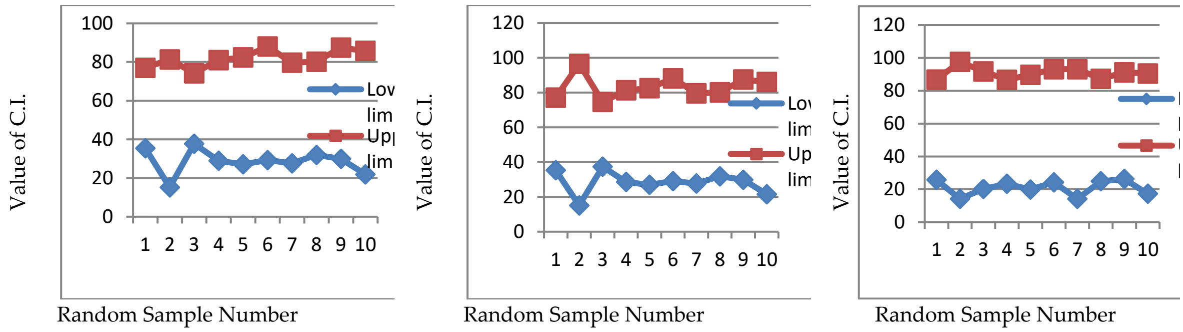



**Fig. 2: Fig. 3: & Fig 4: Graphical Representation of Estimated CI under Arbitrary, Type-A and Type-B Allocation over 10 samples**

The graphical representation in Fig. 2, 3, 4 shows wide gap between the upper and lower limit. The Fig 2 shows the smallest length interval.

### 8.1 Simulation of Confidence Interval under Arbitrary Allocation:

#### 8.1.1 Simulation Algorithm:

**Step I:** Draw a random sample of size k.

**Step II**: Compute the lower limit and upper limit of confidence interval (CI) under three allocations.

**Step III:** Repeat step I and II for d times (here d =200 considered)

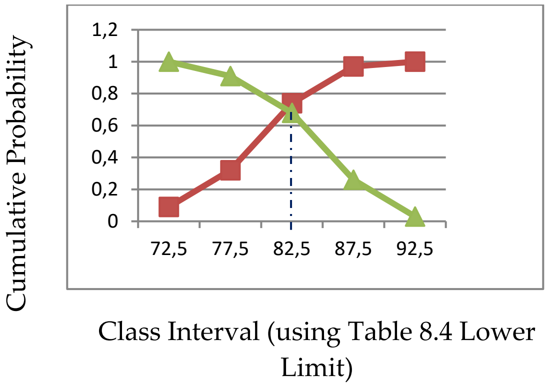
**Step IV:** Let $f_i$ be the frequency of $i^{th}$ class interval for lower limit (LL) of CI over d=200 samples. Calculate probabilities pi = ($f_i$/d) = (frequency of class interval /Total frequency d). Similar is for upper limit (UL) CI.

**Step V:** Compute the Less than Type (LTT) and more than Type (MTT) cumulative probabilities overall d samples for lower limit (LL) and upper limit (UL) of confidence intervals.

**Step VI:** Plot data of step IV on the graph. The perpendicular from point of intersection on the x-axis is the simulated value of lower limit and upper limit of a confidence interval for unknown parameters required to be estimated.

**Table 17: Cumulative Probability-based Simulation for Arbitrary Allocation (over d=200)**

| **The lower limit of Confidence Interval** | | | | | **The upper limit of Confidence Interval** | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **Class Interval (LL)** | **Mid-value of class interval** | **Probability $P_i$** | **Cumulative probabilities** | | **Class Interval (UL)** | **Mid-value of class interval** | **Probability $P_i$** | **Cumulative probabilities** | |
| | | | **LTT** | **MTT** | | | | **LTT** | **MTT** |
| 10-15 | 12.5 | 0.01 | 0.01 | 1 | 70-75 | 72.5 | 0.09 | 0.09 | 1 |
| 15-20 | 17.5 | 0.12 | 0.13 | 0.99 | 75-80 | 77.5 | 0.23 | 0.32 | 0.91 |
| 20-25 | 22.5 | 0.15 | 0.28 | 0.87 | 80-85 | 82.5 | 0.42 | 0.74 | 0.68 |
| 25-30 | 27.5 | 0.43 | 0.71 | 0.72 | 85-90 | 87.5 | 0.23 | 0.97 | 0.26 |
| 30-35 | 32.5 | 0.18 | 0.89 | 0.29 | 90-95 | 92.5 | 0.03 | 1.00 | 0.03 |
| 35-40 | 37.5 | 0.10 | 0.99 | 0.01 | Total | | 1.00 | | |
| 40-45 | 42.5 | 0.01 | 1.00 | 0 | | | | | |
| Total | | 1.00 | | | | | | | |



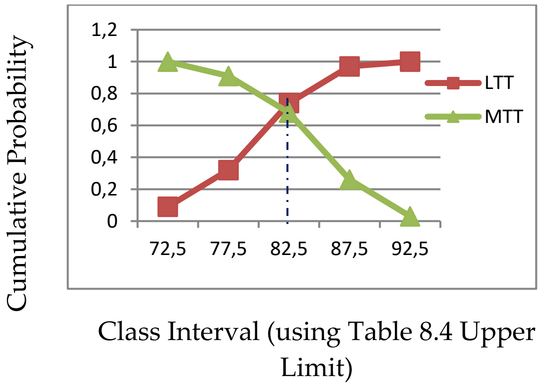




**Fig 5: & Fig 6: Graphical representation for LTT & MTT for Arbitrary Allocation**

**Table 18: Simulated values of C I under Arbitrary Allocation (using Table 12, Fig 5 & Fig. 6)**

| Simulated values of Lower Limit of C I | Simulated values of Upper Limit of C I |
|---|---|
| 24.5 | 79.5 |

Fig. 5.and Fig. 6 is revealing point of intersection of two curves. The final value is determined by perpendicular drawn on the X-axis. The table 18 contains the estimated value, based on perpendicular, which is (24.5, 79.5).

**Simulation of Confidence Interval under Type-A Allocation:**

**Table 19 Sample mean and variance calculation for Type-A allocation (over 10 samples)**

| Sample Number | Sampled Selected with Processing Time (k=9)<br>Group1 $K_1$=(3) | <br>Group2 $K_2$=(3) | <br>Group3 $K_3$=(6) | Processed $\sum w_i \bar{t}_{i'}$ | Unprocessed $(t_1^*+t_2^*+t_3^*)/3$<br>$es^2=\frac{1}{(r-1)}\sum_{i=1}^{r}(t_i^* - \bar{t}^*)^2$ | Sample Mean $(\bar{u})$ | $V(\bar{u})_I$ |
|---|---|---|---|---|---|---|---|
| 1. | 30,43,33*<br>Mean=36.5<br>$t_1$*=25<br>$(es_1)^2$= 42.25 | 60,84,67*<br>Mean=72<br>$t_2$*=37<br>$(es_2)^2$= 144 | 138,112,109, 101,143,123*<br>Mean=120.6<br>$t_3$*=83<br>$(es_3)^2$= 279.44 | 72.19 | 48.33<br>$(es)^2$=937.8 | 60.26 | 293.31 |
| 2. | 33,46,40*<br>Mean=39.5<br>$t_1$*=20<br>$(es_1)^2$= 42.25 | 69,58,59*<br>Mean=63.5<br>$t_2$*=34<br>$(es_2)^2$= 30.25 | 109,101,112, 143,147,131*<br>Mean=122.4<br>$t_3$*=81<br>$(es_3)^2$= 355.04 | 70.25 | 45<br>$(es)^2$= 1021 | 57.62 | 312.19 |
| 3. | 20,46,30*<br>Mean=33<br>$t_1$*=20<br>$(es_1)^2$= 169 | 59,72,79*<br>Mean=65.5<br>$t_2$*=49<br>$(es_2)^2$= 42.25 | 147,138,101, 123,112,109*<br>Mean=124.2<br>$t_3$*=59<br>$(es_3)^2$=279.76 | 68.91 | 42.66<br>$(es)^2$= 274.12 | 55.78 | 94.93 |
| 4. | 40,22,26*<br>Mean=31<br>$t_1$* =20<br>$(es_1)^2$=81 | 74,84,60*<br>Mean=79<br>$t_2$*=31<br>$(es_2)^2$= 25 | 131,109,123, 112,101,143*<br>Mean=115.2<br>$t_3$*=100<br>$(es_3)^2$=112.19 | 71.78 | 50.33<br>$(es)^2$=1880.83 | 61.05 | 570.43 |
| 5. | 43,29,30*<br>Mean=39<br>$t_1$*= 15<br>$(es_1)^2$ = 176 | 79,67,58*<br>Mean=73<br>$t_2$*=35<br>$(es_2)^2$= 36 | 123,143,112, 101,109,147*<br>Mean=117.6<br>$t_3$*=75<br>$(es_3)^2$=211.04 | 72.64 | 41.66<br>$(es)^2$=934.16 | 57.15 | 292.54 |
| 6. | 20,22,29*<br>Mean=21 | 59,72,84*<br>Mean=65.5 | 101,109,123, 131,143,112* | 64.69 | 52<br>$(es)^2$=964 | 58.34 | 356.33 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | $t_1$*=20<br>$(es_1)^2$= 1 | $t_2$*=54<br>$(es_2)^2$= 42.25 | Mean121.4<br>$t_3$*=82<br>$(es_3)^2$=226.24 | | | | |
| 7. | 30,29,20*<br>Mean=29.5<br>$t_1$*=25<br>$(es_1)^2$= 0.25 | 59,69,72*<br>Mean=64<br>$t_2$*=42<br>$(es_2)^2$= 25 | 101,147,109,<br>112,138,123*<br>Mean=121.4<br>$t_3$*=73<br>$(es_3)^2$=317.84 | 66.89 | 46.66<br>$(es)^2$=593.26 | 56.77 | 192.63 |
| 8. | 30,26,33*<br>Mean=28<br>$t_1$* =22<br>$(es_1)^2$= 4 | 72,58,74*<br>Mean=65<br>$t_2$*=50<br>$(es_2)^2$= 49 | 112,131,101,<br>123,109,131*<br>Mean=115.2<br>$t_3$*=90<br>$(es_3)^2$=112.16 | 65.19 | 54<br>$(es)^2$=1168 | 59.59 | 353.95 |
| 9. | 40,29,30*<br>Mean=34.5<br>$t_1$*=21<br>$(es_1)^2$=<br>30.25 | 60,58,67*<br>Mean=59<br>$t_2$*= 47<br>$(es_2)^2$= 1 | 109,112,131,<br>123,143,101*<br>Mean=123.6<br>$t_3$*=79<br>$(es_3)^2$=155.84 | 67.11 | 49<br>$(es)^2$= 844 | 58.05 | 255.55 |
| 10. | 20,43,40*<br>Mean=31.5<br>$t_1$*=30<br>$(es_1)^2$=<br>132.25 | 79,58,60*<br>Mean=68.5<br>$t_2$*=35<br>$(es_2)^2$ =<br>110.25 | 123,101,112,<br>143,147,138*<br>Mean=125.2<br>$t_3$*=78<br>$(es_3)^2$ = 311.36 | 66.12 | 47.66<br>$(es)^2$=697.28 | 56.89 | 223.97 |

**Table 20: Confidence Interval for Type-A Allocation (using Table 19)**

| Confidence Interval for Type-A Allocation | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Sample Number** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Sample Mean ($\bar{u}$ )** | 60.26 | 57.62 | 55.78 | 61.05 | 57.15 | 58.34 | 56.77 | 59.59 | 58.05 | 56.89 |
| **Est.[V($\bar{u}$)$_I$ ]** | 293.31 | 312.19 | 94.93 | 570.43 | 292.54 | 356.33 | 192.63 | 353.95 | 255.55 | 223.97 |
| **Estimate of confidence interval for Est[ V($\bar{u}$)$_I$ ]** | (26.69, 93.82) | (22.98, 92.25) | (36.68, 74.87) | (14.23, 106.61) | (23.62, 90.67) | (21.34, 95.33) | (29.56, 83.97) | (22.71, 96.46) | (26.71, 89.38) | (27.55, 86.22) |

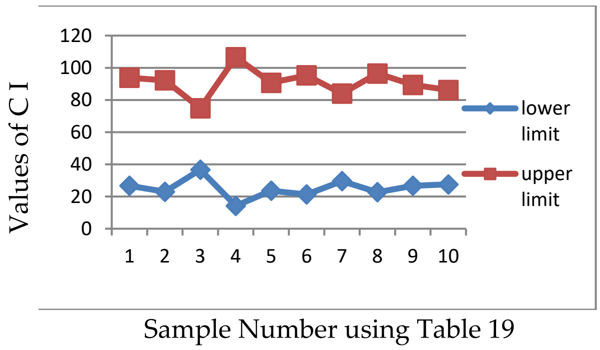
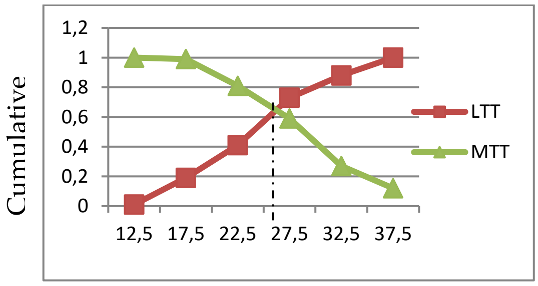


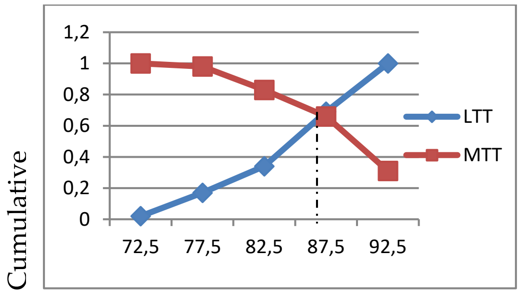

**Fig 7: Graphical Representation of Confidence Interval for Type-A Allocation**

**Table 21: Cumulative Probabilities Simulation for Type-A Allocation (over d=200)**

| The lower limit of Confidence Interval | | | | | The upper limit of Confidence Interval | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Class Interval (LL) | Mid-value of class interval | Probability $P_i$ | Cumulative probabilities LTT | MTT | Class Interval (UL) | Mid-value of class interval | Probability $P_i$ | Cumulative Probabilities LTT | MTT |
| 10-15 | 12.5 | 0.01 | 0.01 | 1 | 70-75 | 72.5 | 0.02 | 0.02 | 1 |
| 15-20 | 17.5 | 0.18 | 0.19 | 0.99 | 75-80 | 77.5 | 0.15 | 0.17 | 0.98 |
| 20-25 | 22.5 | 0.22 | 0.41 | 0.81 | 80-85 | 82.5 | 0.17 | 0.34 | 0.83 |
| 25-30 | 27.5 | 0.32 | 0.73 | 0.59 | 85-90 | 87.5 | 0.35 | 0.69 | 0.66 |
| 30-35 | 32.5 | 0.15 | 0.88 | 0.27 | 90-95 | 92.5 | 0.31 | 1.00 | 0.31 |
| 35-40 | 37.5 | 0.12 | 1.00 | 0.12 | Total | | 1.00 | | |
| Total | | 1.0 | | | | | | | |




Class Interval (using Table 21 Lower Limit)




Class Interval (using Table 21 Upper Limit)

**Fig 8: & Fig 9: Graphical representation for Lower limit & Upper limit for Type-A allocation**

**Table 22: Simulated values of CI under Type-A Allocation (using Table 9, Fig 8 & Fig. 9)**

| Simulated values of Lower Limit of C I | Simulated values of Upper Limit of C I |
|---|---|
| 23.5 | 83.5 |

Fig. 8 and Fig. 9 are revealing point of intersection of two curves. The final value is determined by perpendicular drawn on the X-axis. Table 22 contains the estimated value, based on the perpendicular, which is (23.5, 83.5).

**Simulation of Confidence Interval for Type-B Allocation:**

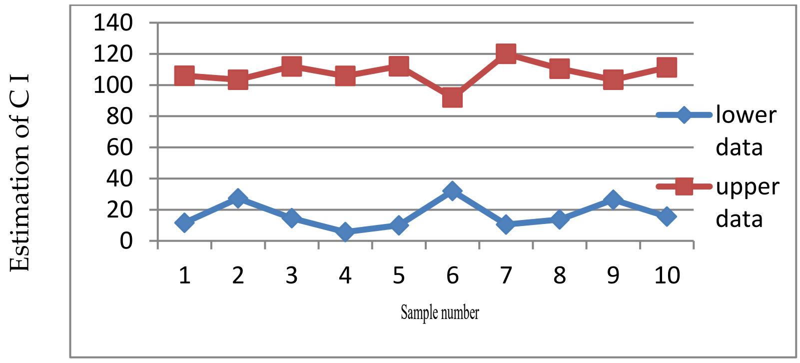
**Table 23: Sample Mean and Variance Calculation for Type-B Allocation (over 10 samples)**

| Random sample | Sampled Selected with Processing Time (k=9) | | | Processed $\sum w_i \bar{t}_{i'}$ | Unprocessed $(t_1^*+t_2^*+t_3^*)/3$ $es^2=\frac{1}{(r-1)}\sum_{i=1}^{r}(t_i^{\ *}-\bar{t}^{\ *})^2$ | Sample Mean $(\bar{u})$ | $V(\bar{u})_{II}$ |
|---|---|---|---|---|---|---|---|
| | Group1 $K_1$=(2) | Group2 $K_2$=(2) | Group3 $K_3$=(8) | | | | |
| 1. | 30,20* <br> Mean=30 <br> $t_1^*$=20 <br> $(es_1)^2$=30 | 59,60* <br> Mean=59 <br> $t_2^*$=60 <br> $(es_2)^2$=59 | 123,101,112,143, 147,138,109,131* <br> Mean=124.71 <br> $t_3^*$=131 <br> $(es_3)^2$=331.48 | 65.92 | 51.66 <br> $(es)^2$=1909.36 | 58.79 | 579.42 |
| 2. | 40,33* <br> Mean=40 <br> $t_1^*$=33 <br> $(es_1)^2$=40 | 69,74* <br> Mean=69 <br> $t_2^*$=74 <br> $(es_2)^2$=69 | 123,101,112,143, 147,138,131,109* <br> Mean=127.85 <br> $t_3^*$=109 <br> $(es_3)^2$= 286.27 | 74.04 | 56.66 <br> $(es)^2$=1234.46 | 65.35 | 377.46 |
| 3. | 43,20* <br> Mean=43 <br> $t_1^*$=20 <br> $(es_1)^2$=43 | 67,58* <br> Mean=67 <br> $t_2^*$=58 <br> $(es_2)^2$=67 | 123,101,112,143, 147,109,131,138* <br> Mean=123.71 <br> $t_3^*$=138 <br> $(es_3)^2$= 306.23 | 73.15 | 53.33 <br> $(es)^2$=2033.86 | 63.24 | 617.62 |
| 4. | 40,29* <br> Mean=40 <br> $t_1^*$=29 <br> $(es_1)^2$=40 | 33,58* <br> Mean=33 <br> $t_2^*$=58 <br> $(es_2)^2$=33 | 123,101,112,143, 138,109,131,147* <br> Mean=122.42 <br> $t_3^*$=147 <br> $(es_3)^2$= 247.95 | 58.22 | 53.33 <br> $(es)^2$=2158.86 | 55.77 | 652.91 |
| 5. | 46,22* <br> Mean=46 <br> $t_1^*$=22 <br> $(es_1)^2$=46 | 58,59* <br> Mean=58 <br> $t_2^*$=59 <br> $(es_2)^2$=58 | 123,101,112,147, 138,109,131,143* <br> Mean= 123 <br> $t_3^*$=143 <br> $(es_3)^2$= 277.66 | 70.36 | 51.66 <br> $(es)^2$=2234.36 | 61.01 | 677.44 |
| 6. | 30,40* <br> Mean=30 <br> $t_1^*$=40 <br> $(es_1)^2$=30 | 59,72* <br> Mean=59 <br> $t_2^*$=72 <br> $(es_2)^2$=59 | 101,143,147,138, 109,131,143,112* <br> Mean=130.28 <br> $t_3^*$=112 <br> $(es_3)^2$= 328.90 | 67.37 | 56.66 <br> $(es)^2$=759.46 | 62.01 | 234.36 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7. | 43,26* Mean=43 $t_1^*=26$ $(es_1)^2=43$ | 59,69* Mean=59 $t_2^*=69$ $(es_2)^2=59$ | 112,143,147,138, 109,131,101,123* Mean= 125.85 $t_3^*=123$ $(es_3)^2=336.90$ | 70.51 | 60 $(es)^2=2575$ | 65.25 | 779.92 |
| 8. | 26,30* Mean=26 $t_1^*=30$ $(es_1)^2=26$ | 69,58* Mean=69 $t_2^*=58$ $(es_2)^2=69$ | 123,101,112,143, 147,138,109,131* Mean=124.71 $t_3^*=131$ $(es_3)^2=331.48$ | 68.60 | 55 $(es)^2=1975$ | 61.8 | 601.96 |
| 9. | 22,29* Mean=22 $t_1^*=29$ $(es_1)^2=22$ | 94,59* Mean=94 $t_2^*=59$ $(es_2)^2=94$ | 123,101,112,143, 147,138, 131,109* Mean= 127.85 $t_3^*=109$ $(es_3)^2=286.27$ | 78.10 | 51.66 $(es)^2=1259.36$ | 64.88 | 385.75 |
| 10. | 20,33* Mean=20 $t_1^*=33$ $(es_1)^2=20$ | 59,79* Mean=59 $t_2^*=79$ $(es_2)^2=59$ | 123,101,112,143, 147,109,131,138* Mean=123.71 $t_3^*=138$ $(es_3)^2=307.47$ | 62.36 | 64 $(es)^2=1948$ | 63.18 | 590.45 |

**Table 24: Confidence Interval for Type-B Allocation (using Table 10.1)**

| **Confidence Interval for Type-B Allocation** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Random sample** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| **Sample Mean ($\bar{u}$)** | 58.79 | 65.35 | 63.24 | 55.77 | 61.01 | 62.01 | 65.25 | 61.8 | 64.88 | 63.18 |
| **Est.[V($\bar{u}$)$_{II}$]** | 579.42 | 377.46 | 617.62 | 652.91 | 677.44 | 234.36 | 779.92 | 601.96 | 385.75 | 590.45 |
| **Estimate of confidence interval for Est.[V($\bar{u}$)$_{II}$]** | (11.61, 105.96) | (27.27, 103.42) | (14.53, 111.94) | (5.68,105.85 ) | (9.99, 112.02) | (32.00, 92.01) | (10.51, 119.98) | (13.71, 110.5) | (26.38, 103.37) | (15.55, 111.26) |

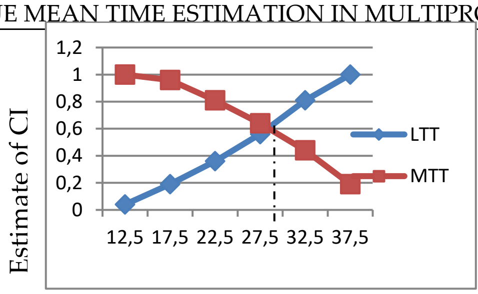



Sample Number using Table 10.1

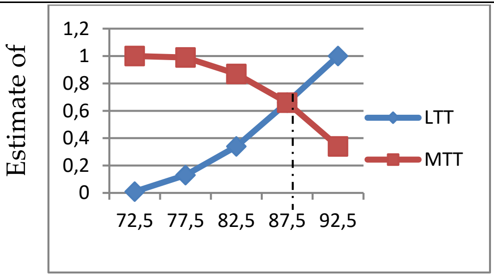
**Fig 10: Graphical Representation for Type-B Allocation**

**Table 25: Cumulative Probabilities Simulation for Type-B Allocation (over d=200)**

| The lower limit of the confidence interval | | | | | The upper limit of the confidence interval | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Class Interval (LL) | Mid-value of class interval | Probability $P_i$ | Cumulative probabilities LTT | MTT | Class Interval (UL) | Mid-value of class interval | Probability $P_i$ | Cumulative probabilities LTT | MTT |
| 10-15 | 12.5 | 0.04 | 0.04 | 1 | 70-75 | 72.5 | 0.01 | 0.01 | 1 |
| 10-15 | 17.5 | 0.15 | 0.19 | 0.96 | 75-80 | 77.5 | 0.12 | 0.13 | 0.99 |
| 15-20 | 22.5 | 0.17 | 0.36 | 0.81 | 80-85 | 82.5 | 0.21 | 0.34 | 0.87 |
| 20-25 | 27.5 | 0.20 | 0.56 | 0.64 | 85-90 | 87.5 | 0.32 | 0.66 | 0.66 |
| 25-30 | 32.5 | 0.25 | 0.81 | 0.44 | 90-95 | 92.5 | 0.34 | 1.00 | 0.34 |
| 30-35 | 37.5 | 0.19 | 1.00 | 0.19 | Total | | 1.00 | | |
| Total | | 1 | | | | | | | |




Class Interval (using Table 10.3 Lower Limit)




Class Interval (using Table 10.3 Upper Limit)

**Fig 11: & Fig 12: Graphical representation for Lower limit & Upper limit Type-B allocation Confidence Interval**

**Table 26: Simulated values of CI under Type-B Allocation**

| Simulated values of Lower Limit of C I | Simulated values of Upper Limit of C I |
|---|---|
| 25.5 | 84.5 |

Fig. 11 and Fig. 12 are revealing point of intersection of two curves. Final value is determined by perpendicular drawn on the X-axis. Table 26 contains the estimated value, based on the perpendicular, which is (25.5, 84.5).

## 11. Results, Discussion and Conclusion:

The comparative analysis is stated in table 27

**Table 27: Comparative Analysis of Variance and Confidence Interval Range**

| Strategy | True Value of Mean | Variance of Mean | 95% Confidence Interval CI |
|---|---|---|---|
| **Arbitrary allocation** | 73.33 | 450.92 | [24.5, 79.5] |
| **Type-A allocation** | 73.33 | 442.08 | [23.5, 83.5] |
| **Type-B allocation** | 73.33 | 611.452 | [25.5, 84.5] |

Algorithm MGLS considers a possibility that some processes remain unprocessed while time instant T occurs which was not considered in GL scheduling [5]. As a consequence, the processes in a sample drawn are divided into two parts A and B. The part A incorporates those who processed and part B has partially processed at the breakdown instant T.

Specific assumption herein is that the last process remains unfinished while T appears in every processor. Estimation procedure proposed herein is such as from whole population of jobs in system, some processes are randomly selected and using the sample estimates mean time and variance of the mean time of processed jobs, as well as the variance of partially processed jobs. The estimation procedure is categorized for arbitrary allocation of sample units to processors.

Further, content has two special cases Type-A allocation and Type-B allocation. The Type-A allocation is based on available prior information of processor speed and Type-B allocation is based on available prior information of variability along with processor speed. In all types of allocations, attempt has been made to find out which allocation will provide the lowest variance (efficient).

For the sake of convenience and simplicity, 30 processes present in system have been considered where groups of ready queues are formed. In particular, three groups Group 1, Group 2, and

Group 3 are formed having some processes according to pre-determined CPU time. Table 5 shows the pre-defined speed of processors. For the arbitrary allocation of sampled processes, the sample mean and variance are calculated with the setup shown in table 12 and subsequently in table 19 and table 23. For the special cases, the processor speed and variability of processors is considered. The variance of the Type-A and Type-B allocation is calculated and compared. This can be seen in Table 4. Table 5 which reveal the comparison between them relating to variance of allocations.

The simulation procedure is proposed and the confidence intervals Prob.[$(\bar{u}) \pm 1.96\sqrt{V}(\bar{u})$] are calculated and represented in graphical form. Over a large number of samples, the confidence interval of Type-A and Type-B allocation are calculated and displayed in graphical representation. For obtaining a single-valued result, it has been introduced the calculation of cumulative probabilities and the LTT and MTT probabilities of lower and upper limits of the confidence interval are measured. Observing all the calculated data and the final table, one can conclude that the Type-A allocation is an efficient scheme to find out the predictive estimate and it is the best one among all who tested.

It was found that estimation of mean times lies within the length of the confidence interval. The improvement suggests over [5] is fruitful and provides better results. The sample-based procedure of estimation of the mean time is more efficient under the Type-A allocation scheme. Such estimates are useful when the system fails suddenly and the system manager needs time estimation for processing the remaining jobs in the queue. This approach helps in the immediate arrangement of resources while disaster management required.

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