A Three Unit Warm and Cold Standby System Model of Discrete Parametricmarkov Chain

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Abstract

The paper deals with cost benefit analysis of a threeidentical unit cold and warm standby system model. Each unit has two modes- normal (N) and total failure (F). The warm standby unit becomes operative instantaneously upon the failure of an operative unit, whereas cold standby unit needs activation to become operative or to be warm standby. A single repairman is always available to repair a failed unit. The activation of a cold standby unit is made by the operator itself. The distributions of time to failure, time to repair and activation time are taken as independent random variables of discrete nature having geometric distributions with different parameters.

Keywords: Regenerative point, reliability, MTSF, availability, geometric distribution, Markov-Chain.

I. Introduction

Two-unit cold standby redundant system models have been analyzed widely in the literature of reliability by various authors including [1,3,5,8]. In these system models the authors have assumed that the standby unit starts operation instantaneously with the help of a switching device when the operative unit fails. In real life, the situations arise in many times when the standby unit does not work instantaneously and take a significant time to be operative and this time may be called activation time of cold standby unit. Gupta et al. [6] analyzed a two-unit standby system model assuming that the cold standby unit goes for activation before starting its operation on line. During the activation time of cold standby unit the system remains down and no output is obtained by the system till the activation is completed and standby unit starts working. To avoid the situation of down time of the system due to activation, the warm standby redundancies have been considered in the literature of reliability as a warm standby unit becomes operative instantaneously without any activation. But the drawback of the warm standby unit is that it can fail during its standby state. So, one is to prefer a warm standby over the cold standby when during the down period of the system due to activation of cold standby there is a great unbearable loss.

Some authors including [4,7,9,10] analyzed the two-unit warm standby redundant system models using different concepts. All the above system models have been analyzed by considering continuous distributions of all the random variables involved.

The purpose of the present paper is to consider both types of standbys warm and cold simultaneously in a single system i.e. a three unit redundant system. As soon as the operating unit fails, the warm standby unit becomes operative instantaneously whereas the cold standby unit needs activation before coming into operation or warm standby unit. The warm standby unit may also fail while it is in standby position whereas the cold standby unit can't fail during its standby state. This system model is based on discrete parametric Markov-Chain. Gupta and Varshney [2] introduced the concept of discrete parametric Markov-Chain in analyzing the system models in the field of reliability modeling. The following economic related measures of system effectiveness are obtained by using regenerative point technique-

i) Transition probabilities and mean sojourn times in various states.
ii) Reliability and mean time to system failure.
iii) Point-wise and steady-state availability of the system during time (0, t-1).
iv) Expected busy period of repairman during time (0, t-1).
v) Net expected profit incurred by the system during a finite and steady-state are obtained.

II. Model Description and Assumptions

1. The system consists of three identical units.Initially one unit is operative and rests two are kept in spare as cold and warm standbys.
2. Each unit has two modes- Normal (N) and Total failure (F).
3. Upon failure of an operating unit, the warm standby unit becomes operative instantaneously whereas the cold standby unit requires activation time before coming into operation/warm standby.
4. A switching device is used to start the activation of cold standby unit and to put a warm standby into operation which is always perfect and instantaneous.
5. A single repairman is always available with the system to repair a failed unit and a repaired unit becomes either operative, warm standby or cold standby as per the situations.
6. The activation action of a cold standby unit is carried out by the operator itself and there is no need for a separate human being at the system for this purpose. After completion of activation unit becomes operative or warm standby as the requirement.
7. The time to failure, time to activation and repair time follow geometric distributions with different parameters.

III. Notations and States of the System

a) Notations :

pq^x : p.m.f. of failure time of a unit; $p+q=1$.

rs^x : p.m.f. of repair time byrepairman of failed unit and $r+s=1$.

cd^x : p.m.f. of time to activate of a cold standby unitrespectively; $c+d=1$.

$q_{ij}(\bullet), Q_{ij}(\bullet)$: p.m.f. and C.d.f. of one step or direct transition time from state S_i to S_j.

p_{ij} : steady state transition probability from state S_i to S_j.

$$p_{ij} = Q_{ij}(\infty)$$

$Z_i(t)$: probability that the system sojourn in state S_i up to epoch (t-1).

ψ_i : mean sojourn time in state S_i.

$*, h$: symbol and dummy variable used in geometric transform e. g.

$$GT\left[q_{ij}(t)\right] = q_{ij}^*(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)$$

b) Symbols for the States of the System:

$N_0 / N_{ws} / N_{cs}$:unit in normal mode and operative/warm standby/cold standby.

N_{ca} : the cold standby unit in normal mode and under activation.

F_r / F_{wr} : unit in total failure (F) mode and under repair/waits for repair.

The transition diagram of the system model is shown in fig. 1.

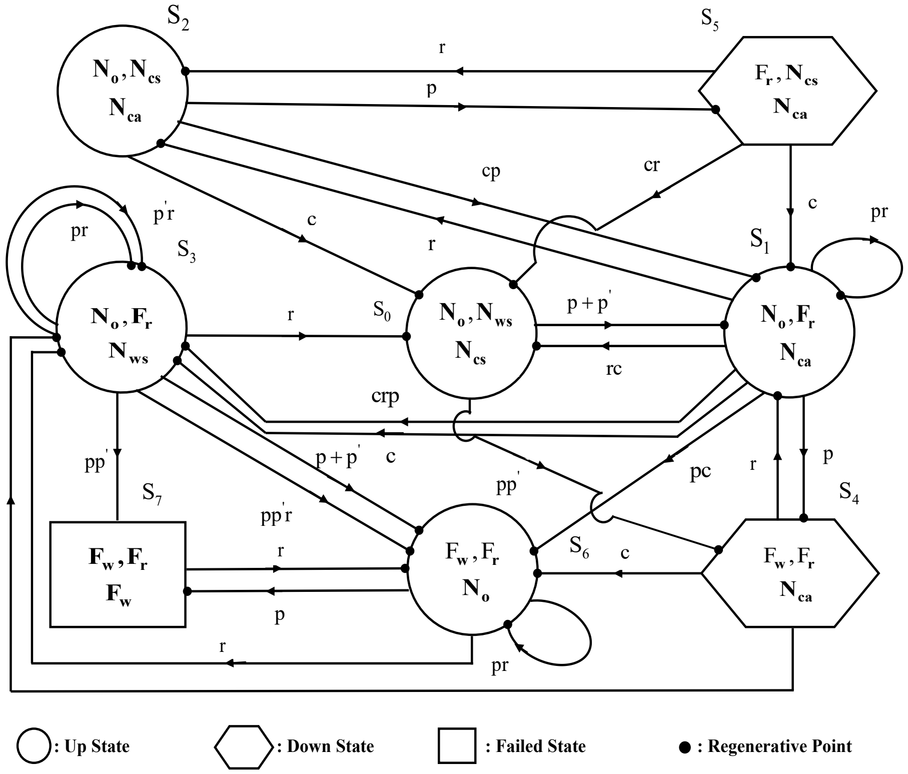

Figure 1

With the help of above symbols the possible states of the system are:

$$S_0 \equiv \begin{pmatrix} N_o, N_{ws} \\ N_{cs} \end{pmatrix}, \quad S_1 \equiv \begin{pmatrix} N_o, F_r \\ N_{ca} \end{pmatrix}, \quad S_2 \equiv \begin{pmatrix} N_o, N_{cs} \\ N_{ca} \end{pmatrix}, \quad S_3 \equiv \begin{pmatrix} N_o, F_r \\ N_{ws} \end{pmatrix}$$

$$S_4 \equiv \begin{pmatrix} F_w, F_r \\ N_{ca} \end{pmatrix}, \quad S_5 \equiv \begin{pmatrix} F_r, N_{cs} \\ N_{ca} \end{pmatrix}, \quad S_6 \equiv \begin{pmatrix} F_w, F_r \\ N_o \end{pmatrix}, \quad S_7 \equiv \begin{pmatrix} F_w, F_r \\ F_w \end{pmatrix}$$

The states S_0, S_1, S_2, S_3, S_6 are up states; S_4, S_5 are down states and S_7 is failed state.

IV. Transition Probabilities

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\left[T_{ij} \le t\right]$$

By using simple probabilistic arguments we have,

$$Q_{01}(t) = \frac{(pq' + p'q)}{1-(qq')}\left[1-(qq')^{(t+1)}\right], \qquad Q_{04}(t) = \frac{pp'}{1-(qq')}\left[1-(qq')^{(t+1)}\right]$$

$$Q_{10}(t) = \frac{rcq}{1-sdq}\left[1-(sdq)^{t+1}\right], \qquad Q_{11}(t) = \frac{prd}{1-qds}\left[1-(qds)^{t+1}\right]$$

$$Q_{12}(t) = \frac{rdq}{1-sdq}\left[1-(sdq)^{t+1}\right] \qquad Q_{13}(t) = \frac{c(rp+qs)}{1-dqs}\left[1-(dqs)^{t+1}\right]$$

$$Q_{14}(t) = \frac{pds}{1-qds}\left[1-(qds)^{t+1}\right], \qquad Q_{16}(t) = \frac{pcs}{1-qds}\left[1-(qds)^{t+1}\right]$$

$$Q_{20}(t) = \frac{cq}{1-qd}\left[1-(qd)^{t+1}\right], \qquad Q_{21}(t) = \frac{cp}{1-dq}\left[1-(dq)^{t+1}\right]$$

$$Q_{25}(t)=\frac{pd}{1-qd}\left[1-(qd)^{t+1}\right], \qquad Q_{30}(t)=\frac{rqq'}{1-sqq'}\left[1-(sqq')^{t+1}\right]$$

$$Q_{33}(t)=\frac{r(pq'+p'q)}{1-sqq'}\left[1-(sqq')^{t+1}\right], \qquad Q_{36}(t)=\frac{s(pq'+p'q)}{1-sqq'}\left[1-(sqq')^{t+1}\right]$$

$$Q_{37}(t)=\frac{spp'}{1-sqq'}\left[1-(sqq')^{t+1}\right], \qquad Q_{41}(t)=\frac{rd}{1-ds}\left[1-(ds)^{t+1}\right]$$

$$Q_{43}(t)=\frac{cr}{1-ds}\left[1-(ds)^{t+1}\right], \qquad Q_{46}(t)=\frac{cs}{1-ds}\left[1-(ds)^{t+1}\right]$$

$$Q_{50}(t)=\frac{cr}{1-ds}\left[1-(ds)^{t+1}\right], \qquad Q_{51}(t)=\frac{cs}{1-ds}\left[1-(ds)^{t+1}\right]$$

$$Q_{52}(t)=\frac{rd}{1-ds}\left[1-(ds)^{t+1}\right], \qquad Q_{63}(t)=\frac{qr}{1-qs}\left[1-(qs)^{t+1}\right]$$

$$Q_{66}(t)=\frac{pr}{1-qs}\left[1-(qs)^{t+1}\right], \qquad Q_{67}(t)=\frac{ps}{1-qs}\left[1-(qs)^{t+1}\right]$$

$$Q_{76}(t)=1-s^{t+1} \tag{1-26}$$

The steady state transition probabilities from state S_i to S_j can be obtained from (1-26) by taking $t\to\infty$, as follows:

$$p_{01}=\frac{(pq'+p'q)}{1-(qq')}, \quad p_{04}=\frac{pp'}{1-(qq')}, \quad p_{10}=\frac{rcq}{1-sdq}, \quad p_{11}=\frac{rdp}{1-sdq}$$

$$p_{12}=\frac{rdq}{1-sdq}, \quad p_{13}=\frac{c(rp+sq)}{1-sdq}, \quad p_{14}=\frac{sdp}{1-sdq}, \quad p_{16}=\frac{scp}{1-sdq}$$

$$p_{20}=\frac{cq}{1-dq}, \quad p_{21}=\frac{cp}{1-dq}, \quad p_{25}=\frac{pd}{1-qd}, \quad p_{30}=\frac{rqq'}{1-sqq'}$$

$$p_{33}=\frac{r(pq'+p'q)}{1-sqq'}, \quad p_{36}=\frac{s(pq'+p'q)}{1-sqq'}, \quad p_{37}=\frac{spp'}{1-sqq'}, \quad p_{41}=\frac{rd}{1-sd}$$

$$p_{43}=\frac{rc}{1-sd}, \quad p_{46}=\frac{sc}{1-sd}, \quad p_{50}=\frac{rc}{1-sd}, \quad p_{51}=\frac{sc}{1-sd}$$

$$p_{52}=\frac{rd}{1-sd}, \quad p_{63}=\frac{rq}{1-sq}, \quad p_{66}=\frac{pr}{1-sq}, \quad p_{67}=\frac{sp}{1-sq}$$

$$p_{76}=1-s^{t+1}$$

We observe that the following relations hold-

$$p_{76}=1, \quad p_{01}+p_{04}=1, \quad p_{10}+p_{11}+p_{12}+p_{13}+p_{14}+p_{16}=1$$

$$p_{20}+p_{21}+p_{25}=1, \quad p_{30}+p_{33}+p_{36}+p_{37}=1, \quad p_{41}+p_{43}+p_{46}=1$$

$$p_{50}+p_{51}+p_{52}=1 \quad p_{63}+p_{66}+p_{67}=1 \tag{26-34}$$

V. Mean SojournTimes

Let T_i be the sojourn time in state S_i (i=0-7) then ψ_i mean sojourn time in state S_i is given by

$$\psi_i=\sum_{t=1}^{\infty}P\left[T\geq t\right]$$

In particular,

$$\psi_0=\frac{qq'}{1-qq'}, \quad \psi_1=\frac{sdq}{1-sdq}, \quad \psi_2=\frac{cq}{1-dq}, \quad \psi_3=\frac{sqq'}{1-sqq'}$$

$$\psi_4 = \frac{ds}{1-ds}, \qquad \psi_5 = \frac{ds}{1-ds}, \qquad \psi_6 = \frac{qs}{1-qs}, \qquad \psi_7 = \frac{s}{r} \qquad (35\text{-}42)$$

VI. Methodology for Developing Equations

In order to obtain various interesting measures of system effectiveness we developed the recurrence relations for reliability, availability and busy period of repairman as follows-

a) Reliability of the system-

Here we define $R_i(t)$ as the probability that the system does not fail up to epochs 0, 1, 2,.., (t-1) when it is initially started from up state S_i. To determine it, we regard the failed states S_7 as absorbing state. Now, the expression for $R_i(t)$; i=0, 1, 2, 3, 4,5,6; we have the following set of convolution equations.

$$R_0(t) = (qq')^t + \sum_{u=0}^{t-1} q_{01}(u) R_1(t-1-u) + \sum_{u=0}^{t-1} q_{04}(u) R_4(t-1-u)$$
$$= Z_0(t) + q_{01}(t-1) © R_1(t-1) + q_{04}(t-1) © R_4(t-1)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{10}(t-1) © R_0(t-1) + q_{11}(t-1) © R_1(t-1) + q_{12}(t-1) © R_2(t-1)$$
$$+ q_{13}(t-1) © R_3(t-1) + q_{14}(t-1) © R_4(t-1) + q_{16}(t-1) © R_6(t-1)$$
$$R_2(t) = Z_2(t) + q_{20}(t-1) © R_0(t-1) + q_{21}(t-1) © R_1(t-1) + q_{25}(t-1) © R_5(t-1)$$
$$R_3(t) = Z_3(t) + q_{30}(t-1) © R_0(t-1) + q_{33}(t-1) © R_3(t-1) + q_{36}(t-1) © R_6(t-1)$$
$$R_4(t) = Z_4(t) + q_{41}(t-1) © R_1(t-1) + q_{43}(t-1) © R_3(t-1) + q_{46}(t-1) © R_6(t-1)$$
$$R_5(t) = Z_5(t) + q_{50}(t-1) © R_0(t-1) + q_{51}(t-1) © R_1(t-1) + q_{52}(t-1) © R_2(t-1)$$
$$R_6(t) = Z_6(t) + q_{63}(t-1) © R_3(t-1) + q_{66}(t-1) © R_6(t-1) \qquad (43\text{-}49)$$

Where,

$$Z_1(t) = (sdq)^t, \qquad Z_2(t) = (qd)^t, \qquad Z_3(t) = (sqq')^t, \qquad Z_4(t) = (sd)^t$$
$$Z_5(t) = (sd)^t, \qquad Z_6(t) = (sq)^t$$

b) Availability of the System-

Let $A_i(t)$ be the probability that the system is up at epoch (t-1), when it initially started from state S_i. Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $A_i(t)$; i=0 to 7.

$$A_0(t) = Z_0(t) + q_{01}(t-1) © A_1(t-1) + q_{04}(t-1) © A_4(t-1)$$
$$A_1(t) = Z_1(t) + q_{10}(t-1) © A_0(t-1) + q_{11}(t-1) © A_1(t-1) + q_{12}(t-1) © A_2(t-1)$$
$$+ q_{13}(t-1) © A_3(t-1) + q_{14}(t-1) © A_4(t-1) + q_{16}(t-1) © A_6(t-1) + q_{17}(t-1) © A_7(t-1)$$
$$A_2(t) = Z_2(t) + q_{20}(t-1) © A_0(t-1) + q_{21}(t-1) © A_1(t-1) + q_{25}(t-1) © A_5(t-1)$$
$$A_3(t) = Z_3(t) + q_{30}(t-1) © A_0(t-1) + q_{33}(t-1) © A_3(t-1) + q_{36}(t-1) © A_6(t-1)$$
$$+ q_{37}(t-1) © A_7(t-1)$$

$$A_4(t) = q_{41}(t-1)©A_1(t-1) + q_{43}(t-1)©A_3(t-1) + q_{46}(t-1)©A_6(t-1)$$
$$A_5(t) = q_{50}(t-1)©A_0(t-1) + q_{51}(t-1)©A_1(t-1) + q_{52}(t-1)©A_2(t-1)$$
$$A_6(t) = Z_6(t) + q_{63}(t-1)©A_3(t-1) + q_{66}(t-1)©A_6(t-1) + q_{67}(t-1)©A_7(t-1)$$
$$A_7(t) = q_{76}(t-1)©A_6(t-1) \quad (50\text{-}57)$$

Where, The values of $Z_i(t)$; i=0 to 3 are same as given in section 6(a). $Z_6(t) = q^t s^t$

c) Busy Period of Repairman

Let $B_i(t)$ be the probability that the repairman is busy in the repairof a failed unit at epoch t-1, when it initially started from state S_i. Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $B_i(t)$; i=0 to 7.

$$B_0(t) = q_{01}(t-1)©B_1(t-1) + q_{04}(t-1)©B_4(t-1)$$
$$B_1(t) = Z_1(t) + q_{10}(t-1)©B_0(t-1) + q_{11}(t-1)©B_1(t-1) + q_{12}(t-1)©B_2(t-1) + q_{13}(t-1)©B_3(t-1)$$
$$+q_{14}(t-1)©B_4(t-1) + q_{16}(t-1)©B_6(t-1) + q_{17}(t-1)©B_7(t-1)$$
$$B_2(t) = q_{20}(t-1)©B_0(t-1) + q_{21}(t-1)©B_1(t-1) + q_{25}(t-1)©B_5(t-1)$$
$$B_3(t) = Z_3(t) + q_{30}(t-1)©B_0(t-1) + q_{33}(t-1)©B_3(t-1) + q_{36}(t-1)©B_6(t-1)$$
$$+q_{37}(t-1)©B_7^I(t-1)$$
$$B_4(t) = Z_4(t) + q_{41}(t-1)©B_1(t-1) + q_{43}(t-1)©B_3(t-1) + q_{46}(t-1)©B_6(t-1)$$
$$B_5(t) = Z_5(t) + q_{50}(t-1)©B_0(t-1) + q_{51}(t-1)©B_1(t-1) + q_{52}(t-1)©B_2(t-1)$$
$$B_6(t) = Z_6(t) + q_{63}(t-1)©B_3(t-1) + q_{66}(t-1)©B_6(t-1) + q_{67}(t-1)©B_7(t-1)$$
$$B_7(t) = Z_7(t) + q_{76}(t-1)©B_6(t-1) \quad (58\text{-}65)$$

Where, $Z_7(t) = s^t$.

VII. Analysis of Reliability and MTSF

Taking geometric transform of (43-46) and simplifying the resulting set of algebraic equations for $R_0^*(h)$ we get

$$R_0^*(h) = \frac{N_1(h)}{D_1(h)} \quad (66)$$

Where,

$$N_1(h) = [(1-hq_{11}^*)(1-h^2q_{25}^*q_{52}^*)(1-hq_{33}^*-h^2q_{36}^*q_{63}^*) - h^2q_{12}^*q_{21}^*(1-hq_{33}^*) - h^3q_{12}^*q_{25}^*q_{51}^*(1-hq_{33}^*-h^2q_{36}^*q_{63}^*)$$
$$+hq_{14}^*(1-hq_{33}^*)(hq_{41}^*+hq_{46}^*) + h^2q_{14}^*q_{36}^*(1-hq_{46}^*) + h^2q_{13}^*q_{36}^* + h^3q_{14}^*q_{25}^*q_{52}^*(h^3q_{36}^*q_{41}^*q_{63}^*$$
$$-hq_{46}^*(1-hq_{33}^*)) - h^3q_{16}^*q_{25}^*q_{52}^*(1-q_{33}^*)]Z_0^* + [hq_{01}^*(1-h^2q_{25}^*q_{52}^*)(1-hq_{33}^*-h^2q_{36}^*q_{63}^*) - hq_{04}^*$$
$$(1-hq_{33}^*)(1-h^2q_{25}^*q_{52}^*)]Z_1^* + [h^2q_{01}^*q_{12}^*(1-hq_{33}^*-h^2q_{36}^*q_{63}^*) - h^2q_{04}^*q_{12}^*\{h^2q_{51}^*q_{63}^*+hq_{51}^*(1-hq_{33}^*)\}]Z_2^*$$
$$+[hq_{01}^*\{hq_{13}^*-hq_{14}^*(hq_{43}^*-h^2q_{46}^*q_{63}^*) - h^3q_{14}^*q_{25}^*q_{52}^*(hq_{43}^*-h^2q_{46}^*q_{63}^*)\} + hq_{16}^*(1-h^2q_{25}^*q_{52}^*)$$
$$-hq_{04}^*(hq_{63}^*(1-hq_{11}^*)(1-h^2q_{25}^*q_{52}^*) - hq_{12}^*(h^2q_{21}^*q_{63}^*+(1-hq_{33}^*) + hq_{41}^*q_{63}^*(hq_{14}^*+h^2q_{25}^*q_{52}^*)]Z_3^*$$
$$+[h^2q_{01}^*q_{14}^*(1-h^2q_{25}^*q_{52}^*)\{h^2q_{36}^*q_{63}^*(1-h^2q_{25}^*q_{52}^*) + (1-q_{33}^*)\} + hq_{14}^*(1-hq_{33}^*)(1-h^2q_{25}^*q_{52}^*)]Z_4^*$$
$$+[h^3q_{01}^*q_{12}^*q_{25}^*(1-hq_{33}^*-h^2q_{36}^*q_{63}^*)]Z_5^* + [h^2q_{01}^*q_{36}^*(1-h^2q_{25}^*q_{52}^*)(hq_{13}^*+h^2q_{14}^*q_{43}^*) - h^2q_{36}^*q_{63}^*]$$
$$-h^3q_{01}^*q_{12}^*q_{25}^*(1-hq_{33}^*)(h^2q_{14}^*q_{46}^*+hq_{16}^*) + hq_{01}^*(1-hq_{33}^*)(hq_{16}^*+h^2q_{14}^*q_{46}^*)$$

$$D_1(h) = [(1-hq_{11}^*)(1-h^2q_{25}^*q_{52}^*)(1-hq_{33}^*-h^2q_{36}^*q_{63}^*) - h^2q_{12}^*q_{21}^*(1-hq_{33}^*) - h^3q_{12}^*q_{25}^*q_{51}^*(1-hq_{33}^*-h^2q_{36}^*q_{63}^*)$$
$$+hq_{14}^*(1-hq_{33}^*)(hq_{41}^*+hq_{46}^*) + h^2q_{14}^*q_{36}^*(1-hq_{46}^*) + h^2q_{13}^*q_{36}^* + h^3q_{14}^*q_{25}^*q_{52}^*(h^3q_{36}^*q_{41}^*q_{63}^*$$

$$-hq_{46}^*(1-hq_{33}^*))-h^3q_{16}^*q_{25}^*q_{52}^*(1-q_{33}^*)]-hq_{10}^*[hq_{01}^*(1-h^2q_{25}^*q_{52}^*)(1-hq_{33}^*-q_{36}^*q_{63}^*)-h^2q_{04}^*$$
$$(1-hq_{33}^*)(1-h^2q_{25}^*q_{52}^*)]-q_{20}^*[h^2q_{01}^*q_{12}^*(1-hq_{33}^*-h^2q_{36}^*q_{63}^*)-h^2q_{04}^*q_{12}^*\{h^2q_{51}^*q_{63}^*+hq_{51}^*(1-hq_{33}^*)\}]$$
$$-q_{30}^*[hq_{01}^*\{hq_{13}^*-hq_{14}^*(hq_{43}^*-h^2q_{46}^*q_{63}^*)-h^3q_{14}^*q_{25}^*q_{52}^*(hq_{43}^*-h^2q_{46}^*q_{63}^*)\}+hq_{16}^*(1-h^2q_{25}^*q_{52}^*)$$
$$-hq_{04}^*(hq_{63}^*(1-hq_{11}^*)(1-h^2q_{25}^*q_{52}^*)-hq_{12}^*(h^2q_{21}^*q_{63}^*+(1-hq_{33}^*)+h^2q_{41}^*q_{63}^*(hq_{14}^*+h^2q_{25}^*q_{52}^*)]$$
$$-q_{50}^*[h^3q_{01}^*q_{12}^*q_{25}^*(1-hq_{33}^*-h^2q_{36}^*q_{63}^*)]$$

Collecting the coefficient of h^t from expression (66), we can get the reliability of the system $R_0(t)$. The MTSF is given by-

$$E(T)=\lim_{h\to 1}\sum_{t=1}^{\infty}h^tR(t)=\frac{N_1(1)}{D_1(1)}-1 \quad (67)$$

$$N_1(1)=[(1-p_{11})(1-p_{25}p_{52})(1-p_{33}-p_{36}p_{63})-p_{12}p_{21}(1-p_{33})-p_{12}p_{25}p_{51}(1-p_{33}-p_{36}p_{63})$$
$$+p_{14}(1-p_{33})(p_{41}+p_{46})+p_{14}p_{36}(1-p_{46})+p_{13}p_{36}+p_{14}p_{25}p_{52}\{p_{36}p_{41}p_{63}$$
$$-p_{46}p_{61}(1-p_{33})\}-p_{16}p_{25}p_{52}(1-p_{33})]\psi_0+[p_{01}(1-p_{25}p_{52})(1-p_{33}-p_{36}p_{63})-p_{04}$$
$$(1-p_{33})(1-p_{25}p_{52})]\psi_1+[p_{01}p_{12}(1-p_{33}-p_{36}p_{63})-p_{04}p_{12}\{p_{51}p_{63}+p_{51}(1-p_{33})\}]\psi_2$$
$$+[p_{01}\{p_{13}-p_{14}(p_{43}-p_{46}p_{63})-p_{14}p_{25}p_{52}(p_{43}-p_{46}p_{63})\}+p_{16}(1-p_{25}p_{52})$$
$$-p_{04}\{p_{63}(1-p_{11})(1-p_{25}p_{52})-p_{12}p_{21}p_{63}+(1-p_{33})+p_{12}p_{41}p_{63}(p_{14}+p_{25}p_{52})\}]\psi_3$$
$$+[p_{01}p_{14}((1-p_{25}p_{52})(p_{36}p_{63}(1-p_{25}p_{52})+(1-p_{33}))+p_{14}(1-p_{33})(1-p_{25}p_{52})]\psi_4$$
$$+[p_{01}p_{12}p_{25}(1-p_{33}-p_{36}p_{63})]\psi_5+[p_{01}p_{36}(1-p_{25}p_{52})(p_{13}+p_{14}p_{43})-p_{36}p_{63}]$$
$$-p_{12}p_{21}-p_{14}p_{41}(1+p_{25}p_{51})\}]\psi_6$$

$$D_1(1)=[(1-p_{11})(1-p_{25}p_{52})(1-p_{33}-p_{36}p_{63})-p_{12}p_{21}(1-p_{33})-p_{12}p_{25}p_{51}(1-p_{33}-p_{36}p_{63})$$
$$+p_{14}(1-p_{33})(p_{41}+p_{46})+p_{14}p_{36}(1-p_{46})+p_{13}p_{36}+p_{14}p_{25}p_{52}\{p_{36}p_{41}p_{63}$$
$$-p_{46}(1-p_{33})\}-p_{16}p_{25}p_{52}(1-p_{33})]-p_{10}[p_{01}(1-p_{25}p_{52})(1-p_{33}-p_{36}p_{63})-p_{04}$$
$$(1-p_{33})(1-p_{25}p_{52})]-p_{20}[p_{01}p_{12}(1-p_{33}-p_{36}p_{63})-p_{04}p_{12}\{p_{51}p_{63}+p_{51}(1-p_{33})\}]$$
$$-p_{30}[p_{01}\{p_{13}-p_{14}(p_{43}-p_{46}p_{63})-p_{14}p_{25}p_{52}(p_{43}-p_{46}p_{63})\}+p_{16}(1-p_{25}p_{52})$$
$$-p_{04}(p_{63}(1-p_{11})(1-p_{25}p_{52})-p_{12}(p_{21}p_{63}+(1-p_{33})+p_{41}p_{63}(p_{14}+p_{25}p_{52})]$$
$$-p_{50}[p_{01}p_{12}p_{25}(1-p_{33}-p_{36}p_{63})]$$

VIII. Availability Analysis

On taking geometric transform of (50-57) and simplifying the resulting equations for we get,

$$A_0^*(h)=\frac{N_2(h)}{D_2(h)} \quad (68)$$

Where,

$$N_2(h)=\begin{vmatrix} Z_0 & -hq_{01}^* & 0 & 0 & -hq_{04}^* & 0 & 0 & 0 \\ Z_1 & 1-hq_{11}^* & -hq_{12}^* & -hq_{13}^* & -hq_{14}^* & 0 & -hq_{16}^* & 0 \\ Z_2 & -hq_{21}^* & 1 & 0 & 0 & -hq_{25}^* & 0 & 0 \\ Z_3 & 0 & 0 & -hq_{33}^* & 0 & 0 & -hq_{36}^* & -hq_{37}^* \\ 0 & -hq_{41}^* & 0 & -hq_{43}^* & 1 & 0 & -hq_{46}^* & 0 \\ 0 & -hq_{51}^* & -hq_{52}^* & 0 & 0 & 1 & 0 & 0 \\ Z_6 & 0 & 0 & -hq_{63}^* & 0 & 0 & 1-hq_{66}^* & -hq_{67}^* \\ 0 & 0 & 0 & 0 & 0 & 0 & -hq_{76}^* & 1 \end{vmatrix}$$

and

$$D_2(h) = \begin{vmatrix} 1 & -hq_{01}^* & 0 & 0 & -hq_{04}^* & 0 & -hq_{06}^* & 0 \\ -hq_{10}^* & 1-hq_{11}^* & -hq_{12}^* & -hq_{13}^* & -hq_{14}^* & 0 & -hq_{16}^* & 0 \\ -hq_{20}^* & -hq_{21}^* & 1 & 0 & 0 & -hq_{25}^* & 0 & 0 \\ -hq_{30}^* & 0 & 0 & 1-hq_{33}^* & 0 & 0 & -hq_{36}^* & -hq_{37}^* \\ 0 & -hq_{41}^* & 0 & -hq_{43}^* & 1 & 0 & -hq_{46}^* & 0 \\ -hq_{50}^* & -hq_{51}^* & -hq_{52}^* & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -hq_{63}^* & 0 & 0 & 1-hq_{66}^* & -hq_{67}^* \\ 0 & 0 & 0 & 0 & 0 & 0 & -hq_{76}^* & 1 \end{vmatrix}$$

The steady state availabilities of the system due to operation of unit -

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{h\to 1}(1-h)\frac{N_2(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$A_0 = -\frac{N_2(1)}{D_2'(1)} \tag{69}$$

Where,

$$N_2(1) = u_0\psi_0 + u_1\psi_1 + u_2\psi_2 + u_3\psi_3 + u_6\psi_6$$

and

$$D_2'(1) = -\left[u_0\psi_0 + u_1(\psi_1 + p_{12}\psi_2 + p_{14}\psi_4) + u_3\psi_3 + u_5\psi_5 + u_6(\psi_6 + p_{76}\psi_7)\right]$$

Where,

$u_i = U_i^*(0)$ and $U_i^*(h)$; i=0, 1,…, 7 are the minors of the elements of first column of $D_2(h)$.

Now the expected uptime of the system due to operative unit upto epoch (t-1) are given by

$$\mu_{up}(t) = \sum_{x=0}^{t-1} A_0(x)$$

So that

$$\mu_{up}^*(h) = \frac{A_0^*(h)}{(1-h)} \tag{70}$$

IX. Busy Period Analysisof Repairman

On taking geometric transforms of (58-65) and simplifying the resulting equations,we get

$$B_0^*(h) = \frac{N_4(h)}{D_2(h)} \tag{71}$$

Where,

$$N_3(h) = h\left[U_1^*\left(Z_1^* + q_{12}^*Z_2^* + q_{14}^*Z_4^*\right) + U_3^*Z_3^* + U_5^*Z_5^* + U_6^*\left(Z_6^* + q_{76}^*Z_7\right)\right]$$

and $D_2(h)$ is same as in availability analysis.

In the long run the respective probabilities that the repairman is busy in the repair of a failed unit are given by-

$$B_0 = \lim_{t\to\infty} B_o(t) = \lim_{h\to 1}(1-h)\frac{N_3(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0 = -\frac{N_3(1)}{D_2'(1)} \tag{72}$$

Where,

$$N_3(1) = \left[u_1\left(\psi_1 + p_{12}\psi_2 + p_{14}\psi_4\right) + u_3\psi_3 + u_5\psi_5 + u_6\left(\psi_6 + p_{76}\psi_7\right)\right]$$

and $D_2'(1)$ is same as in availability analysis.

Now the expected busy period of the repairman in repair of a failed unit up to epoch (t-1) are respectively given by-

$$\mu_b(t) = \sum_{x=0}^{t-1} B_0(x), \tag{73}$$

X. Profit Function Analysis

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier section. Let us consider,

$K_0 =$ revenue per-unit time by the system due to operative unit.

$K_1 =$ cost per-unit time when repairman is busy in repair of failed unit.

Then, the net expected profit incurred up to epoch (t-1) is given by

$$P(t) = K_0\mu_{up}(t) - K_1\mu_b(t) \tag{74}$$

The expected profit per unit time in steady state is given by-

$$P = \lim_{t\to\infty}\frac{P(t)}{t} = \lim_{h\to 1}(1-h)^2 P^*(h)$$

$$= K_0 \lim_{h\to 1}(1-h)^2 \frac{A_0^*(h)}{(1-h)} - K_1 \lim_{h\to 1}(1-h)^2 \frac{B_0^*(h)}{(1-h)}$$

$$= K_0A_0 - K_1B_0 \tag{75}$$

XI. Graphical Representation

The curves for MTSF and profit function have been drown for different values of failure parameters. Fig. 2 depicts the variation in MTSF with respect to failure rate (p) for different values of repair rate (r) of a unit and activation rate (c) when values of other parameters are kept fixed as $p' = 0.99$ and $q' = 0.01$. From the curves we conclude that expected life of the system decrease with increase in p. Further, increases as the values of r and c increases.

Similarly, Fig. 3 reveals the variations in profit (P) with respect to p for varying values of r and c, when other parameters are kept fixed as $p' = 0.99$, $q' = 0.01$, $K_0 = 50$, and $K_1 = 450$. From the figure it is clearly observed from the smooth curves, that the system is profitable if the value of parameter p is smaller than 0.062, 0.102 and 0.112 respectively for $r = 0.1$, 0.15 and 0.2 for fixed value of $c = 0.95$. From dotted curves, we conclude that system is profitable only if value of parameter p is smaller than 0.052, 0.078 and 0.118 respectively for $r = 0.1$, 0.15 and 0.2 for fixed value of $c = 0.94$.

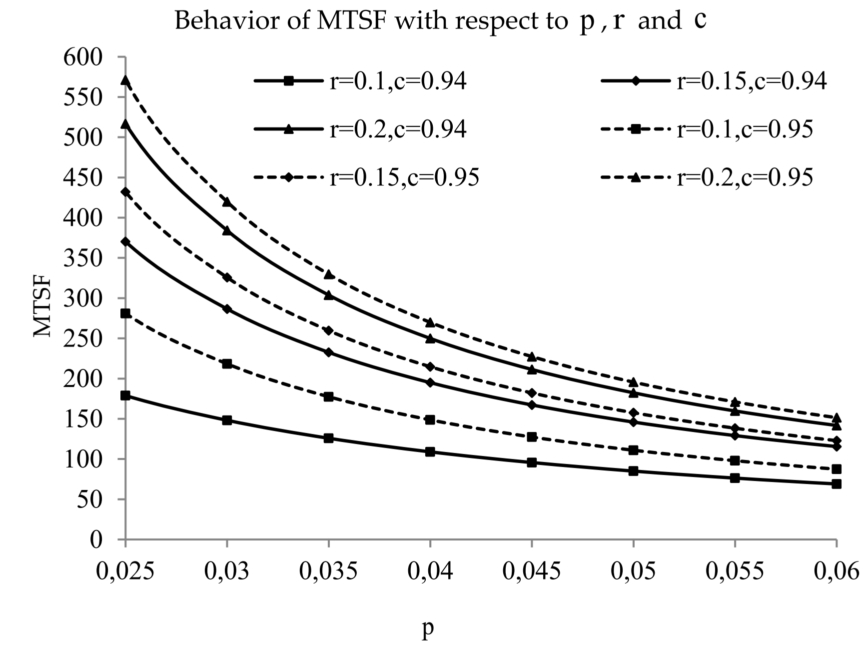

Figure 2

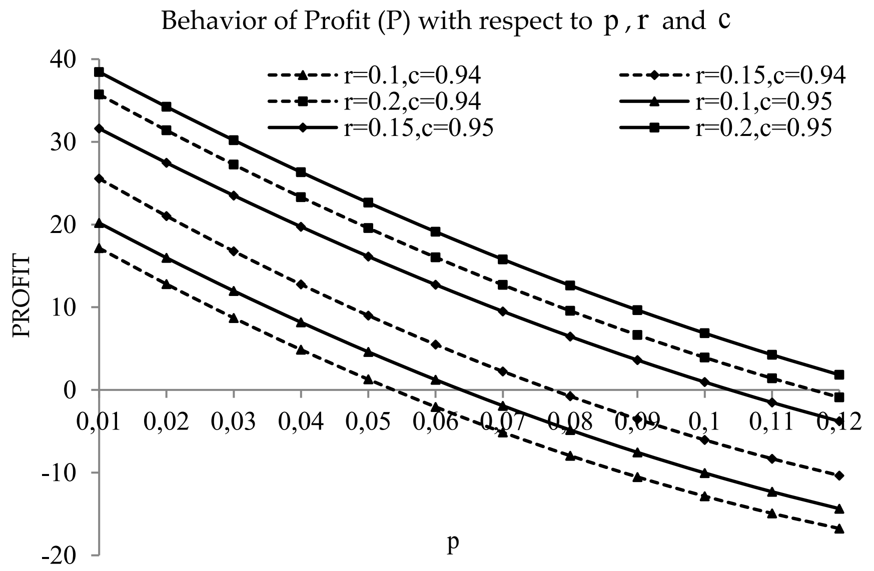

Figure 3

XII. Conclusions

1. It is indicated in fig.2 that we can easily obtain the upper limit of "p" to achieve at least a particular value of MTSF. As an illustration to get at least MTSF 150 unit, the failure rate "p" must be less than 0.025, 0.031 and 0.037 respectively for repair rate r = 0.1, 0.15 and 0.2 when activation rate is kept fixed as c = 0.94. Similarly, when c = 0.95 is kept fixed as "p" must be less than 0.253, 0.034 and 0.040 corresponding to r = 0.1, 0.15 and 0.20.
2. In fig. 3 it is reveled from the dotted curves that the system is profitable only if failure rate "p" is greater than 0.052, 0.078 and 0.118 respectively for r = 0.1, 0.15 and 0.2 for fixed value of c =

0.94. From smooth curves, we conclude that system is profitable only if value of parameter "p" is greater than 0.062, 0.102 and 0.112 respectively for r = 0.1, 0.15 and 0.2 for fixed value of c = 0.95.

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