Sensitivity Analysis of Three Different Series – Parallel Dynamo Configurations

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Abstract

This paper deals with the sensitivity analysis of three configurations arranged in series-parallel. Configuration I consist of six units in which four are on operation while two are on standby. Configuration II consist of seven units with three of the units are on standby while the remaining four are on operation. Configuration III comprises of two subsystems C and D with three unit in each subsystem with a unit on standby. Units in each configuration provide 25MW. Both the failure and repair time are assumed exponentially distributed. System of first order linear differential difference equations is obtained using the transition diagram. Explicit expressions of the system availability, Mean Time To Failure (MTTF), busy period due to partial failure, busy period due to complete failure and profit were derived. Furthermore results of sensitivity of the system availability, MTTF and profit were determined. The obtained results were analyzed and compared, configuration I was found to be the optimal configuration.

Keywords: Sensitivity, Reliability, Dynamo, Availability, Configurations, Series - Parallel.

I. Introduction

Every manufacturer expects the performance of his/her engines with full efficiency within the designed limits. However, in real life users have the tendency to operate the system beyond even their control limits and such situations are termed as abnormal condition. In the system design, redundancy is found in almost all types of systems that plays an important role for improvement of reliability and availability of the system. Sometimes, it is difficult to keep a high cost identical unit in standby situation; therefore, a duplicate unit may be kept as spare for use in emergency and to provide services to the customers for a considerable period. Each unit can perform same kind of functions, but their degree of reliability and desirability may differ from unit to unit, Kumar *et al.* (2020). High system reliability and availability of electrical system plays a vital role towards industrial growth as the profit is directly dependent on production volume which depends upon system performance. Due to their prevalence in domestic, manufacturing, and industrial systems, many researchers have studied reliability and availability problem of different electrical systems.

A great number of models have been introduced to describe the behaviour and performance of electrical system that is subject to failure. For this reason, many researchers have studied reliability problem of different electrical systems.Redundancy technique is widely used to improve system reliability. However, in the real world situation, many systems are load-sharing, such as electric generators sharing an electrical load in a power plant, cables in a suspension bridge, and valves or pumps in a hydraulic system, Chunbo *et al.* (2015). To cite few, Chauhan and Malik (2017) focused on the evaluation of reliability and MTSF of a parallel system with Weibull failure laws. Abdul Kareem and Singh (2019) worked on cost assessment of complex repairable system consisting two subsystems in series configuration using Gumbel Hougaard family copula. Rajesh *et al.* (2018) have studied the reliability and availability for a three unit gas turbine power generating system with seasonal effect and FCFS repair pattern. Dalip *et al.* (2014) have also studied reliability and economic analysis of a power generating system comprising one gas and one gas steam turbine with random inspection. However, situations may be there, where the two units may be dissimilar but the nature of the work done by them is the same. Such a situation was discussed by Singh and Taneja (2013) and (2014) for a gas turbine power plant. However, they did not consider the parameter 'Temperature' which also affects the working/function and efficiency of a gas turbine system. One such situation was discussed by Rajesh *et al.* (2018) where effects of temperature on production of a system comprising one gas turbine and one steam turbine have been taken into account. Such a system necessarily goes to down mode on failure of gas turbine irrespective of operability of steam turbine, as steam turbine cannot work without working of gas turbine. However, this problem can be overcome to some extent if number of gas turbine is increased, i.e., redundancy is introduced._Yusuf (2016) presented an article on reliability evaluation of parallel system with two types of preventive maintenance. Ram M. and Kumar (2015) discussed on performability/performance analysis of a system under 1-out-of -2: G scheme with perfect reworking, Wang *et al.* (2003) have studied cost benefit analysis of series systems with warm standby components, Tseng *et al.* (2013) studied comparative analysis of three systems with imperfect coverage and standby switching failures and Wang and Chin (2006) also discussed on cost benefit analysis of series systems with cold standby components and a repairable service station. In their research paper Wang and Chin (2006) considered three configurations as follows:

The first configuration is a serial system of one primary 30 MW component with one cold standby 30MW component. The second configuration is a serial system of two primary 15MW components and one cold standby 15MW component. The last configuration is a serial system of three primary 10MW components with two interchangeable cold standby 10MW components. Each standby unit can replace either one of the failed components and the total of 30MW is expected in all the three configurations. Lastly, Wang and Kuo (2018) have studied cost benefit analysis of three systems with imperfect coverage and standby switching failures. In the paper, data center require a 30MW power electricity, and they assumed that the electricity generation capacity of generators is available in units of 30MW, 15MW, and 10MW. To provide reliable and stable power supply, there are standby generators, and all the active and standby generators are continuously monitored by a fault detecting device to identify if they fail. They also assumed that standby generators are allowed to fail while inactive before they are put into full operation. Goyal *et al*. (2017) published a research work on Sensitivity analysis of a three unit series system under k-out of-n redundancy. Considering reliability, as one of the performance measure, the authors have designed a complex system which consists of three subsystems, namely, A, B and C in series configuration. The subsystem A consists of n numbers of units which are arranged in parallel configuration, subsystem B consists of two sub-subsystems X and Y align parallel to one another, where X is a type of 1-out-of-n. Failure and repair rates are assumed to follow the general distribution.

In this research work, some relevant literature related to reliability analysis and performance evaluation of dynamo system configurations were reviewed which mostly focused on the cost benefit analysis of the system. Relevant literature that has to do with system modeling and how the model would be applied to solved practical system and improved efficiency as studied by many scholars were reviewed. This research paper further enhanced the work of the previous researchers. 100MW was considered as the total output and the three configurations have uniform of 25MW in all the units of the configurations. Furthermore, some practical applications are also addressed.

II.Notation, Assumption and Description of Three Configurations

Notations

λ : $Failure\ rate$
μ : $Repair\ rate$
$Av_{i;}$ i = 1,2,3 Availability of system
$MTTF_{i;}$ i = 1,2,3 Mean time to failure
Q(t) = Probability row vector

Assumptions

1. Systems have redundant standby units
2. Repair is immediate
3. Switching from standby level to operation stage is perfect

Description of the three configurations

Configuration I consists of six units each of the unit has 25MW arranged in series-parallel. Out of the six units four are on operational stage while two are on standby. The failure of A_1 or A_6 causes the complete failure of the system. Configuration II has seven sub-components/ units with 25MW each arranged in series–parallel, three of the units are on standby while the remaining four are on operation stages, the failure of the system is said to have occur if B_2 and A_1 or A_2 fails. Configuration III comprises of two subsystems C and D with three units in each subsystem and out three units there is one standby with 25MW in each unit. Out of the six units in total four are on operation while two are on standby. The system will collapse if C_1 and C_2 or C_5 and C_6 fail. The parameter λ represents the failure rate in all the three configurations. Whenever active unit fails, it will immediately be replaced by a standby and the failed unit is taken for repair which is represented by μ.

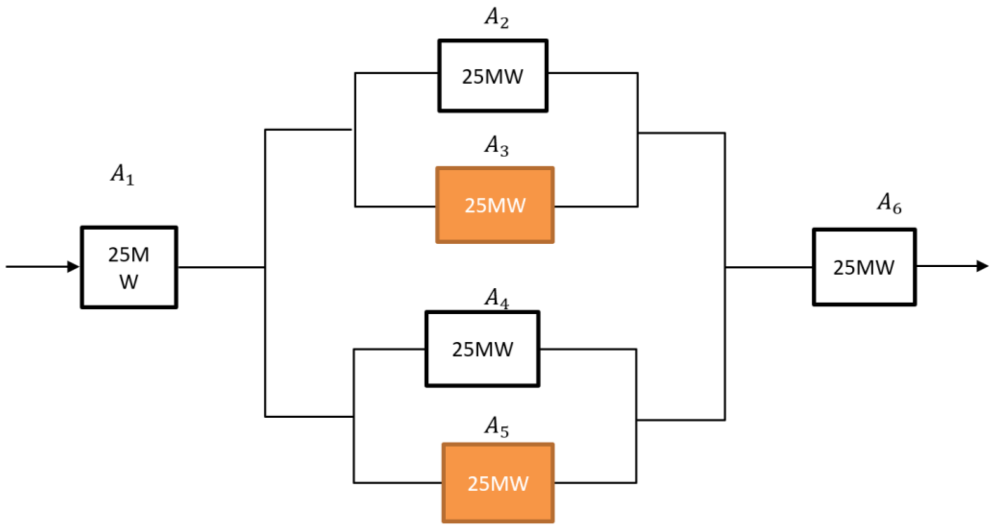

Figure 1: Block diagram of Configuration I

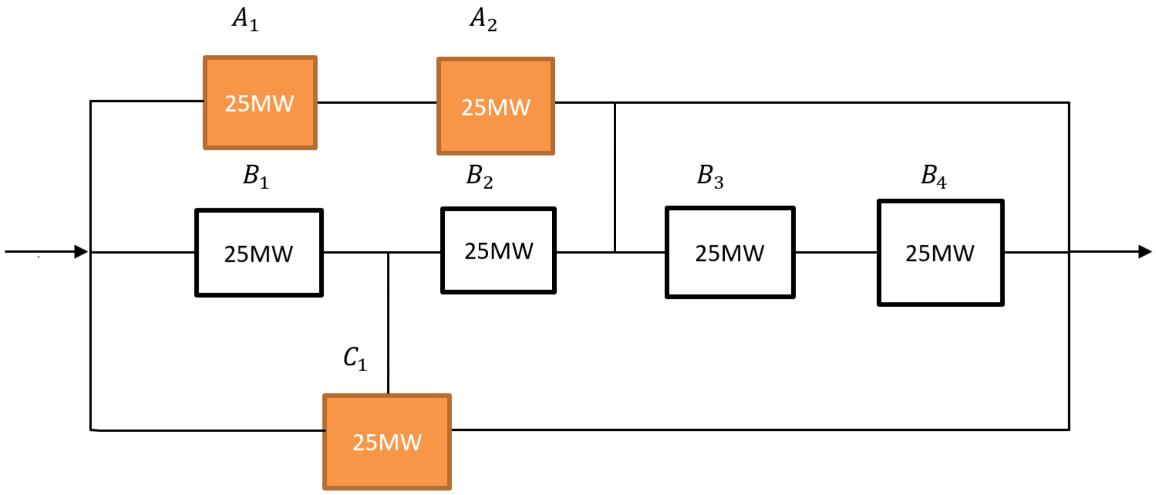

Figure 2: Block diagram of Configuration II

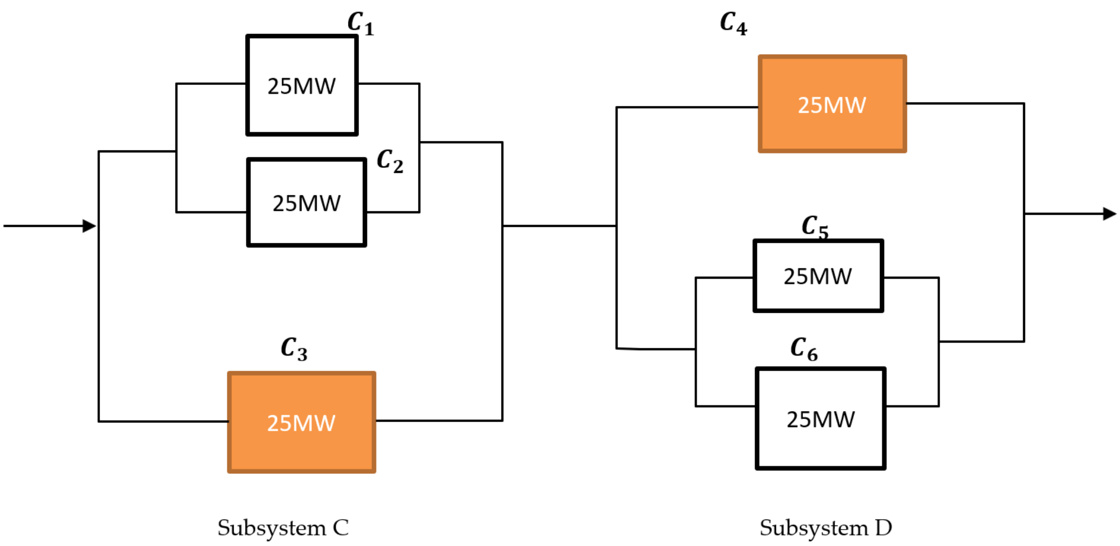

Figure 3: Block diagram of Configuration III

III. Models Formulation

Availability and Meantime to Failure of Configuration I

According to Wang *et al.* (2006), let $Q\ (t)$ be the probability that at time t there are n components working in the system. Then the initial conditions for this problem are stated as follows:

$$Q(0) = [Q_0(0), Q_1(0), Q_2(0), \dots, Q_{16}(0)] = [1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$$

The following differential equations are obtained:

$$\begin{aligned}
Q_0^I(t) &= -4\lambda Q_0(t) + \mu Q_1(t) + \mu Q_2(t) + \mu Q_7(t) + \mu Q_8(t)\\
Q_1^I(t) &= -(4\lambda+\mu)Q_1(t) + \lambda Q_0(t) + \mu Q_3(t) + \mu Q_4(t) + \mu Q_9(t) + \mu Q_{10}(t)\\
Q_2^I(t) &= -(4\lambda+\mu)Q_2(t) + \lambda Q_0(t) + \mu Q_5(t) + \mu Q_6(t) + \mu Q_{11}(t) + \mu Q_{12}(t)\\
Q_3^I(t) &= -(\lambda+\mu)Q_3(t) + \lambda Q_1(t) + \mu Q_{13}(t)\\
Q_4^I(t) &= -(\lambda+\mu)Q_4(t) + \lambda Q_1(t) + \mu Q_{14}(t)\\
Q_5^I(t) &= -(\lambda+\mu)Q_5(t) + \lambda Q_2(t) + \mu Q_{15}(t)\\
Q_6^I(t) &= -(\lambda+\mu)Q_6(t) + \lambda Q_2(t) + \mu Q_{16}(t)\\
Q_7^I(t) &= -\mu Q_7(t) + \lambda Q_0(t)\\
Q_8^I(t) &= -\mu Q_8(t) + \lambda Q_0(t)\\
Q_9^I(t) &= -\mu Q_9(t) + \lambda Q_1(t)\\
Q_{10}^I(t) &= -\mu Q_{10}(t) + \lambda Q_1(t)\\
Q_{11}^I(t) &= -\mu Q_{11}(t) + \lambda Q_2(t)\\
Q_{12}^I(t) &= -\mu Q_{12}(t) + \lambda Q_2(t)\\
Q_{13}^I(t) &= -\mu Q_{13}(t) + \lambda Q_3(t)\\
Q_{14}^I(t) &= -\mu Q_{14}(t) + \lambda Q_4(t)\\
Q_{15}^I(t) &= -\mu Q_{15}(t) + \lambda Q_5(t)\\
Q_{16}^I(t) &= -\mu Q_{16}(t) + \lambda Q_6(t)
\end{aligned} \tag{1}$$

The differential equations in (1) above can be written in the matrix form as

$$Q^I(t) = T_1 Q(t) \tag{2}$$

where

$$\mathrm{T}_1 = \begin{bmatrix}
-4\lambda & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\lambda & -y_1 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0\\
\lambda & 0 & -y_1 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0\\
0 & \lambda & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 & 0 & 0\\
0 & \lambda & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 & 0\\
0 & 0 & \lambda & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0\\
0 & 0 & \lambda & 0 & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu\\
\lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0\\
0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0\\
0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu
\end{bmatrix}$$

$y_1 = (4\lambda + \mu)$ and $y_2 = (\lambda + \mu)$

Equation (2) above can be written in the matrix form as:

$$\begin{pmatrix} Q_0^I(t) \\ Q_1^I(t) \\ Q_2^I(t) \\ Q_3^I(t) \\ Q_4^I(t) \\ Q_5^I(t) \\ Q_6^I(t) \\ Q_7^I(t) \\ Q_8^I(t) \\ Q_9^I(t) \\ Q_{10}^I(t) \\ Q_{11}^I(t) \\ Q_{12}^I(t) \\ Q_{13}^I(t) \\ Q_{14}^I(t) \\ Q_{15}^I(t) \\ Q_{16}^I(t) \end{pmatrix} = \begin{pmatrix} -4\lambda & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & -y_1 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & -y_1 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{pmatrix}$$

In the steady state all the derivative equal to zero, thus from equation (2) above, $T_1Q(\infty) = 0$ is obtained.

$$\begin{pmatrix} -4\lambda & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & -y_1 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & -y_1 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu \end{pmatrix} \begin{pmatrix} Q_0(\infty) \\ Q_1(\infty) \\ Q_2(\infty) \\ Q_3(\infty) \\ Q_4(\infty) \\ Q_5(\infty) \\ Q_6(\infty) \\ Q_7(\infty) \\ Q_8(\infty) \\ Q_9(\infty) \\ Q_{10}(\infty) \\ Q_{11}(\infty) \\ Q_{12}(\infty) \\ Q_{13}(\infty) \\ Q_{14}(\infty) \\ Q_{15}(\infty) \\ Q_{16}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (3)$$

Using the normalizing condition

$$\sum_{i=0}^{16} Q_i(\infty) = 1 \quad (4)$$

Following Wang et al (2006) Equation (4) is substituted in the last row of (3) to obtain

$$\begin{pmatrix} -4\lambda & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & -y_1 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & -y_1 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & -y_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\mu & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} Q_0(\infty) \\ Q_1(\infty) \\ Q_2(\infty) \\ Q_3(\infty) \\ Q_4(\infty) \\ Q_5(\infty) \\ Q_6(\infty) \\ Q_7(\infty) \\ Q_8(\infty) \\ Q_9(\infty) \\ Q_{10}(\infty) \\ Q_{11}(\infty) \\ Q_{12}(\infty) \\ Q_{13}(\infty) \\ Q_{14}(\infty) \\ Q_{15}(\infty) \\ Q_{16}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad (5)$$

System of linear differential equations given in equation (5) above was solved using MATLAB package to obtain the explicit solution of $Q_0(\infty)$, $Q_1(\infty)$,$Q_2(\infty)$,…, $Q_{16}(\infty)$

$$AV_1(\infty) = Q_0(\infty) + Q_1(\infty) + Q_2(\infty) + Q_3(\infty) + Q_4(\infty) + Q_5(\infty) + Q_6(\infty) = \frac{\mu^3 + 2\lambda\mu^2 + 4\lambda^2\mu}{4\lambda^3 + 8\lambda^2 + 4\lambda\mu^2 + \mu^3}$$

Now to evaluate the MTTF1, the rows and column of the absorbing (failure) state were deleted and the new matrix M1 was transposed as given in the equation (6) below, Wang *et al.* (2006):

$$E[T_{Q(0) \to Q(absorbing)}] = Q(0)\,(-M_1^{-1})[1,1,1,1,1,1,1]^T \quad (6)$$

Where,

$$M_1 = \begin{pmatrix} -4\lambda & \lambda & \lambda & 0 & 0 & 0 & 0 \\ \mu & -y_1 & 0 & \lambda & \lambda & 0 & 0 \\ \mu & 0 & -y_1 & 0 & 0 & \lambda & \lambda \\ 0 & \mu & 0 & -y_2 & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 & -y_2 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 & -y_2 & 0 \\ 0 & 0 & \mu & 0 & 0 & 0 & -y_2 \end{pmatrix}.$$

From equation (6) we have:

$$E[T_{Q(0) \to Q(absorbing)}] = MTTF_1 = \frac{(\lambda + \mu) + 2\lambda}{8\lambda^2 + 5\lambda\mu + \mu^2} + \frac{4\lambda^2 + 3\lambda\mu + \mu^2}{2\lambda(8\lambda^2 + 5\lambda\mu + \mu^2)}$$

Availability and Meantime to Failure of Configuration II

To further investigate the availability of configuration II, $Q_i(t), i = 0,1,2,3,\dots,10$ were defined to be the probabilities that the system at time t ≥ 0 is in state S_i. Let Q(t) be the probability row vector at time t ≥ 0. The initial condition for this problem is

$Q(0) = [Q_0(0), Q_1(0), Q_2(0), \dots, Q_{10}(0)] = [1,0,0,0,0,0,0,0,0,0,0]$.

Then the following differential equations are obtained:

$$\frac{dQ_0}{dt}(t) = -8\lambda Q_0(t) + \mu Q_1(t) + \mu Q_2(t)$$
$$\frac{dQ_1}{dt}(t) = -(8\lambda + \mu)Q_1(t) + 4\lambda Q_0(t) + \mu Q_3(t) + \mu Q_4(t)$$
$$\frac{dQ_2}{dt}(t) = -(8\lambda + \mu)Q_2(t) + 4\lambda Q_0(t) + \mu Q_5(t) + \mu Q_6(t)$$
$$\frac{dQ_3}{dt}(t) = -(8\lambda + \mu)Q_3(t) + 4\lambda Q_1(t) + \mu Q_7(t) + \mu Q_8(t)$$
$$\frac{dQ_4}{dt}(t) = -(8\lambda + \mu)Q_4(t) + 4\lambda Q_1(t) + \mu Q_9(t) + \mu Q_{10}(t)$$

$$\frac{dQ_5}{dt}(t) = -\mu Q_5(t) + 4\lambda Q_2(t)$$
$$\frac{dQ_6}{dt}(t) = -\mu Q_6(t) + 4\lambda P_2(t)$$
$$\frac{dQ_7}{dt}(t) = -\mu Q_7(t) + 4\lambda Q_3(t)$$
$$\frac{dQ_8}{dt}(t) = -\mu Q_8(t) + 4\lambda Q_3(t)$$
$$\frac{dQ_9}{dt}(t) = -\mu Q_9(t) + 4\lambda Q_4(t)$$
$$\frac{dQ_{10}}{dt}(t) = -\mu Q_{10}(t) + 4\lambda Q_4(t) \quad (7)$$

With initial conditions $Q(0) = [Q_0(0), Q_1(0), Q_2(0), \ldots, Q_{10}(0)] = [1,0,0,0,0,0,0,0,0,0,0]$. Equation (7) could be written in the form of matrix as given in equation (8) below:

$$Q^I(t) = T_2 Q(t) \quad (8)$$

$$\begin{pmatrix} \frac{dQ_0}{dt}(t) \\ \frac{dQ_1}{dt}(t) \\ \frac{dQ_2}{dt}(t) \\ \frac{dQ_3}{dt}(t) \\ \frac{dQ_4}{dt}(t) \\ \frac{dQ_5}{dt}(t) \\ \frac{dQ_6}{dt}(t) \\ \frac{dQ_7}{dt}(t) \\ \frac{dQ_8}{dt}(t) \\ \frac{dQ_9}{dt}(t) \\ \frac{dQ_{10}}{dt}(t) \end{pmatrix} = \begin{pmatrix} -8\lambda & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4\lambda & -y_3 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 4\lambda & 0 & -y_3 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 \\ 0 & 4\lambda & 0 & -y_3 & 0 & 0 & 0 & \mu & \mu & 0 & 0 \\ 0 & 4\lambda & 0 & 0 & -y_3 & 0 & 0 & 0 & 0 & \mu & \mu \\ 0 & 0 & 4\lambda & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4\lambda & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & 0 & 0 & -\mu \end{pmatrix} \begin{pmatrix} Q_0(t) \\ Q_1(t) \\ Q_2(t) \\ Q_3(t) \\ Q_4(t) \\ Q_5(t) \\ Q_6(t) \\ Q_7(t) \\ Q_8(t) \\ Q_9(t) \\ Q_{10}(t) \end{pmatrix}$$

Where, $y_3 = (8\lambda + \mu)$

To calculate the state probabilities, all derivatives of state are equal to zero. This will enable us to compute steady state availability by equating the left hand side of equation (8) to zero. Now we have

$$T_2 Q(\infty) = 0 \quad (9)$$

Thus, equation (9) above could be written in matrix form as:

$$\begin{pmatrix} -8\lambda & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4\lambda & -y_3 & 0 & \mu & \mu & 0 & 0 & 0 & 0 & 0 & 0 \\ 4\lambda & 0 & -y_3 & 0 & 0 & \mu & \mu & 0 & 0 & 0 & 0 \\ 0 & 4\lambda & 0 & -y_3 & 0 & 0 & 0 & \mu & \mu & 0 & 0 \\ 0 & 4\lambda & 0 & 0 & -y_3 & 0 & 0 & 0 & 0 & \mu & \mu \\ 0 & 0 & 4\lambda & 0 & 0 & -\mu & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4\lambda & 0 & 0 & 0 & -\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & 0 & 0 & -\mu \end{pmatrix} \begin{pmatrix} Q_0(t) \\ Q_1(t) \\ Q_2(t) \\ Q_3(t) \\ Q_4(t) \\ Q_5(t) \\ Q_6(t) \\ Q_7(t) \\ Q_8(t) \\ Q_9(t) \\ Q_{10}(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the normalizing condition below, it follows that

$$\sum_{i=0}^{10} Q_i(\infty) = 1 \tag{10}$$

Solving equation (10) above to obtain the explicit solution of $Q_0(\infty)$, $Q_1(\infty)$, $Q_2(\infty)$, …, $Q_{10}(\infty)$, the explicit equation for steady state availability is therefore obtained as follows:

$$AV_2(\infty) = Q_0(\infty) + Q_1(\infty) + Q_2(\infty) + Q_3(\infty) + Q_4(\infty) = \frac{\mu^3 + 8\lambda\mu^2 + 32\lambda^2\mu}{256\lambda^3 + 64\lambda^2 + 8\lambda\mu^2 + \mu^3} \tag{11}$$

Now to evaluate the MTTF for configuration II, following Wang and Kuo (2000) and Wang *et al.* (2006), the MTTF of the system could be obtained by deleting the rows and column of the absorbing (failure) state and transposing the new matrix M_2. The expected time to reach an absorbing state is given in equation (12) below:

$$E[T_{Q(0)\rightarrow Q(absorbing)}] = Q(0)\,(-M_2^{-1})[1,1,1,1,1]^T \tag{12}$$

Where

$$M_2 = \begin{pmatrix} -8\lambda & 4\lambda & 4\lambda & 0 & 0 \\ \mu & -y_3 & 0 & 4\lambda & 4\lambda \\ \mu & 0 & -y_3 & 0 & 0 \\ 0 & \mu & 0 & -y_3 & 0 \\ 0 & \mu & 0 & 0 & -y_3 \end{pmatrix}$$

Now for the second system, the explicit expression/equation of $MTTF_2$ is given by equation (13) below:

$$E[T_{Q(0)\rightarrow Q(absorbing)}] = MTTF_2 = \frac{8\lambda + \mu}{128\lambda^2 + 16\lambda\mu + \mu^2} + \frac{128\lambda^2 + 24\lambda\mu + 2\mu^2}{8(128\lambda^3 + 16\lambda^2\mu + \lambda\mu^2)} + \frac{512\lambda^3 + 128\lambda^2\mu + 16\lambda\mu^2 + \mu^3}{32\lambda(128\lambda^3 + 16\lambda^2\mu + \lambda\mu^2)} \tag{13}$$

Availability and Meantime to Failure of Configuration III

For the analysis of availability case of configuration III, $Q_i(t), i = 0,1,2,3,\ldots,7$ are defined to be the probability that the system at time t ≥ 0 is in the state S_i. Let Q(t) also be the probability row vector at time t ≥ 0. The initial condition for this problem is:

$Q(0) = [Q_0(0), Q_1(0), Q_2(0), \ldots, Q_7(0)] = [1,0,0,0,0,0,0,0]$.

$$\frac{dQ_0}{dt}(t) = -8\lambda Q_0(t) + \mu Q_1(t) + \mu Q_2(t)$$

$$\frac{dQ_1}{dt}(t) = -(8\lambda + \mu)Q_1(t) + 4\lambda Q_0(t) + \mu Q_3(t) + \mu Q_4(t)$$

$$\frac{dQ_2}{dt}(t) = -(8\lambda + \mu)Q_2(t) + 4\lambda Q_0(t) + \mu Q_3(t) + \mu Q_5(t)$$

$$\frac{dQ_3}{dt}(t) = -(8\lambda + 2\mu)Q_3(t) + 4\lambda Q_1(t) + 4\lambda Q_2(t) + \mu Q_6(t) + \mu Q_7(t)$$

$$\frac{dQ_4}{dt}(t) = -\mu Q_4(t) + 4\lambda Q_1(t)$$

$\frac{dQ_5}{dt}(t) = -\mu Q_5(t) + 4\lambda Q_2(t)$
$\frac{dQ_6}{dt}(t) = -\mu Q_6(t) + 4\lambda Q_3(t)$
$\frac{dQ_7}{dt}(t) = -\mu Q_7(t) + 4\lambda Q_3(t)$ (14)

Equation (14) is rewritten in the matrix form as presented in equation (15) below:

$$Q^I(t) = T_3 Q(t) \quad (15)$$

Where,

$$\mathrm{T}_3 = \begin{bmatrix} -8\lambda & \mu & \mu & 0 & 0 & 0 & 0 & 0 \\ 4\lambda & -y_3 & 0 & \mu & \mu & 0 & 0 & 0 \\ 4\lambda & 0 & -y_3 & \mu & 0 & \mu & 0 & 0 \\ 0 & 4\lambda & 4\lambda & -y_4 & 0 & 0 & \mu & \mu \\ 0 & 4\lambda & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 4\lambda & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 4\lambda & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & -\mu \end{bmatrix}$$

Where $y_4 = (8\lambda + 2\mu)$

Initial conditions are considered as given in the following equation: $Q(0) = [Q_0(0), Q_1(0), Q_2(0), \dots, Q_7(0)] = [1,0,0,0,0,0,0,0]$.

To obtain the steady state probabilities, right hand side of (15) is equated to zero such that

$$T_3 Q(\infty) = 0 \quad (16)$$

Thus, (16) can be written in matrix form as follows:

$$\begin{pmatrix} -8\lambda & \mu & \mu & 0 & 0 & 0 & 0 & 0 \\ 4\lambda & -y_3 & 0 & \mu & \mu & 0 & 0 & 0 \\ 4\lambda & 0 & -y_3 & \mu & 0 & \mu & 0 & 0 \\ 0 & 4\lambda & 4\lambda & -y_4 & 0 & 0 & \mu & \mu \\ 0 & 4\lambda & 0 & 0 & -\mu & 0 & 0 & 0 \\ 0 & 0 & 4\lambda & 0 & 0 & -\mu & 0 & 0 \\ 0 & 0 & 0 & 4\lambda & 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & 4\lambda & 0 & 0 & 0 & -\mu \end{pmatrix} \begin{pmatrix} Q_0(\infty) \\ Q_1(\infty) \\ Q_2(\infty) \\ Q_3(\infty) \\ Q_4(\infty) \\ Q_5(\infty) \\ Q_6(\infty) \\ Q_7(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Solving (16) using normalizing condition

$$\sum_{i=0}^{7} Q_i(\infty) = 1 \quad (17)$$

$Q_0(\infty)$, $Q_1(\infty)$, $Q_2(\infty)$, $Q_3(\infty)$, $Q_4(\infty)$, $Q_5(\infty)$, $Q_6(\infty)$, $Q_7(\infty)$ are obtained. Therefore, the explicit expression/equation of $AV_3(\infty)$ is given by

$$AV_3(\infty) = Q_0(\infty) + Q_1(\infty) + Q_2(\infty) + Q_3(\infty) = \frac{\mu^3 + 8\lambda\mu^2 + 16\lambda^2\mu}{128\lambda^3 + 48\lambda^2 + 8\lambda\mu^2 + \mu^3} \quad (18)$$

To compute MTTF for configuration III, follow similar argument used in configurations I and II. The rows and column of the absorbing states of the matrix T_3 are therefore deleted and take the transpose to obtain a new matrix M_3

$$E[T_{Q(0)\rightarrow Q(absorbing)}] = Q(0)\,(-M_3^{-1})[1,1,1,1]^T \quad (19)$$

Where, $M_3 = \begin{pmatrix} -8\lambda & 4\lambda & 4\lambda & 0 \\ \mu & -y_3 & 0 & 4\lambda \\ \mu & 0 & -y_3 & 4\lambda \\ 0 & \mu & \mu & -y_4 \end{pmatrix}$

The explicit expression/equation of MTTF3 is therefore obtained as follows:

$$E[T_{Q(0)\rightarrow Q(absorbing)}] = MTTF_3 = \frac{1}{2(8\lambda+\mu)} + \frac{4\lambda+\mu}{4(8\lambda^2+ +\lambda\mu)} + \frac{32\lambda^2 + 8\lambda\mu + \mu^2}{32\lambda(8\lambda^2 + \lambda\,\mu)} \quad (20)$$

IV. Discussion

Sensitivity Analysis of Three Configurations

In this section, numerical comparisons for the result of sensitivity analysis for all the developed models were presented. Computer software, MATLAB is used to compute the three configurations in terms of their sensitivity analysis. From the results of system one it has been observed that configuration I is far better than all the remaining configurations as we can observed in table 1 through 2 below. It can be see that availability of configuration I is compared with that of configuration II and configuration III in terms of failure rate λ and repair rate μ. Furthermore, virtually all the configurations were compared with configuration I in terms of their **MTTF** with effect of failure rate λ and repair rate μ that is table 5 to table 6 below. It is also observed that configuration I retain its optimality.

Similarly, configuration I was compared with all the remaining configurations and turn to be the best in terms of Profit and table 3 to table 4 below clearly justify that the configuration I is the optimal. However, in the sensitivity results obtained from table 1 through table 6 with the help of Bar chat (i.e. Figure 1 – 18) below, availability versus failure and repair rate, Profit versus failure and repair rates and **MTTF** versus failure and repair, it can be justified that configuration I was the best because as one can observed from all tables below. Despite the fact that failure increases in table 1 for instance configuration I has the maximum availability and similarly with repair it shows more increasing trends which is far better than the remaining configurations and this bridge the practical gap that remains untouched.

Table 1: Variation of Availability with respect to μ for the three Configurations for different values of λ

μ	λ = 0.1			λ = 0.5			λ = 0.9		
	Configuration			Configuration			Configuration		
	I	II	III	I	II	III	I	II	III
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1111	0.4316	0.1383	0.1507	0.1668	0.0278	0.0285	0.1042	0.0154	0.0157
0.2222	0.5664	0.2710	0.3017	0.2681	0.0555	0.0581	0.1804	0.0309	0.0317
0.3333	0.6473	0.3907	0.4331	0.3377	0.0833	0.0886	0.2389	0.0463	0.0481
0.4444	0.7030	0.4935	0.5398	0.3899	0.1109	0.1196	0.2856	0.0617	0.0649
0.5556	0.7439	0.5788	0.6239	0.4316	0.1383	0.1507	0.3240	0.0771	0.0818
0.6667	0.7750	0.6483	0.6897	0.4663	0.1656	0.1818	0.3566	0.0925	0.0989
0.7778	0.7995	0.7044	0.7411	0.4961	0.1925	0.2126	0.3847	0.1078	0.1161
0.8889	0.8193	0.7497	0.7817	0.5222	0.2191	0.2430	0.4094	0.1231	0.1334
1.0000	0.8356	0.7864	0.8140	0.5455	0.2453	0.2727	0.4316	0.1383	0.1507

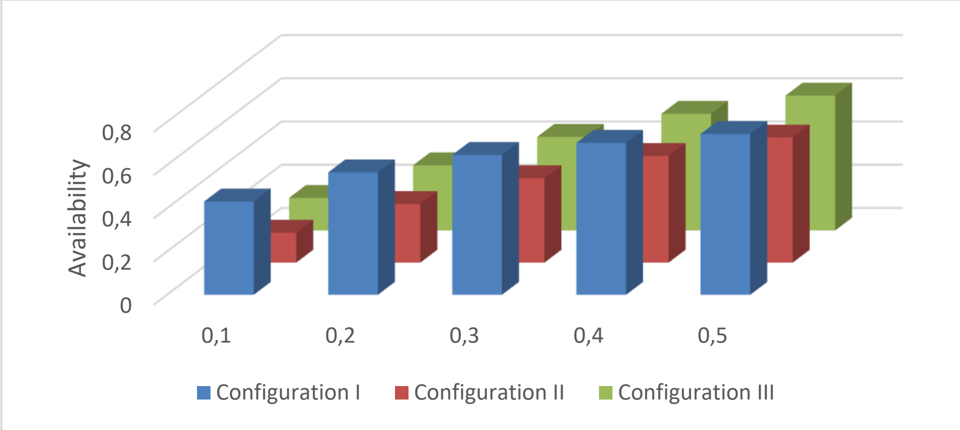

Figure 1: Availability against μ for λ= 0.1

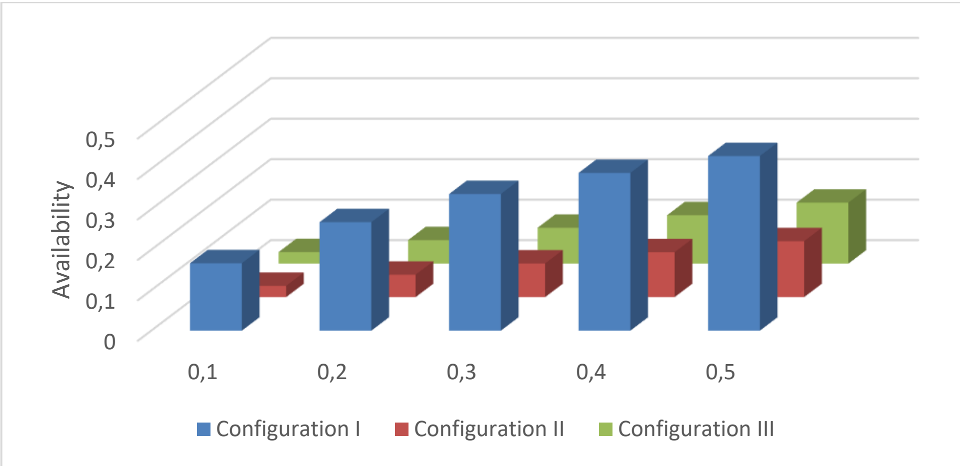

Figure 2: Availability against μ for λ = 0.5

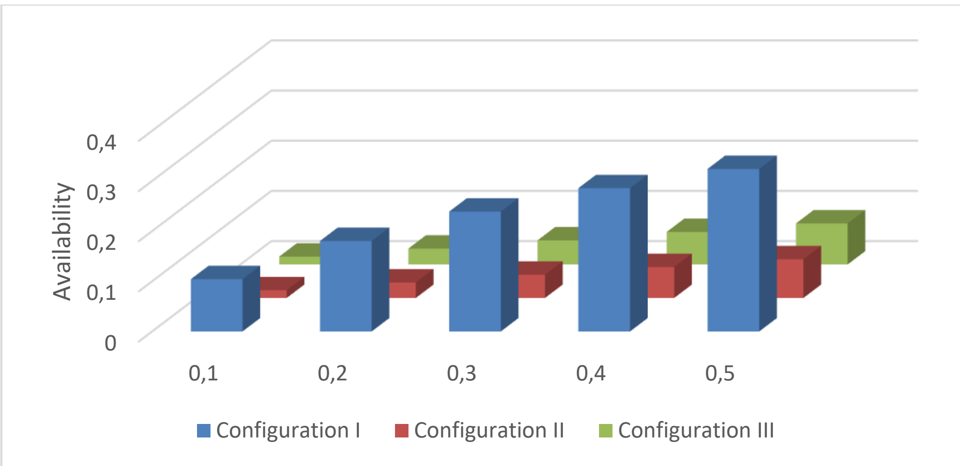

Figure 3: Availability against μ for λ = 0.9

Table 2: Variation of Availability with respect to λ for the three Configurations for different values of μ

λ	μ = 0.3			μ = 0.6			μ = 0.9		
	Configuration			Configuration			Configuration III		
	I	II	III	I	II	III	I	II	III
0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.1111	0.6053	0.3244	0.3611	0.7388	0.5679	0.6133	0.8057	0.7186	0.7539
0.2222	0.4686	0.1676	0.1841	0.6053	0.3244	0.3611	0.6853	0.4591	0.5047
0.3333	0.3922	0.1122	0.1211	0.5246	0.2217	0.2460	0.6053	0.3244	0.3611
0.4444	0.3399	0.0843	0.0898	0.4686	0.1676	0.1841	0.5479	0.2482	0.2760
0.5556	0.3008	0.0675	0.0712	0.4262	0.1345	0.1464	0.5040	0.2003	0.2215
0.6667	0.2701	0.0562	0.0589	0.3922	0.1122	0.1211	0.4686	0.1676	0.1841
0.7778	0.2452	0.0482	0.0502	0.3639	0.0963	0.1032	0.4392	0.1440	0.1572
0.8889	0.2247	0.0422	0.0437	0.3399	0.0843	0.0898	0.4141	0.1262	0.1369
1.0000	0.2073	0.0375	0.0388	0.3191	0.0749	0.0794	0.3922	0.1122	0.1211

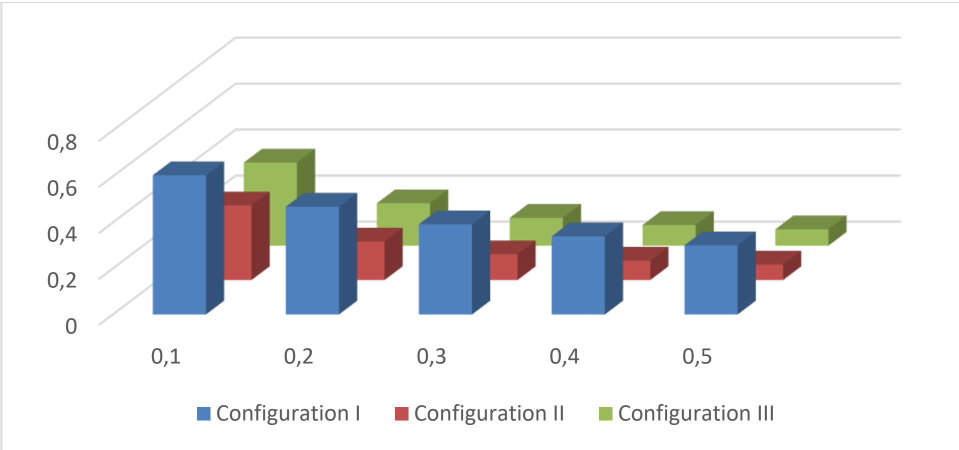

Figure 4: Availability against λ for μ = 0.3

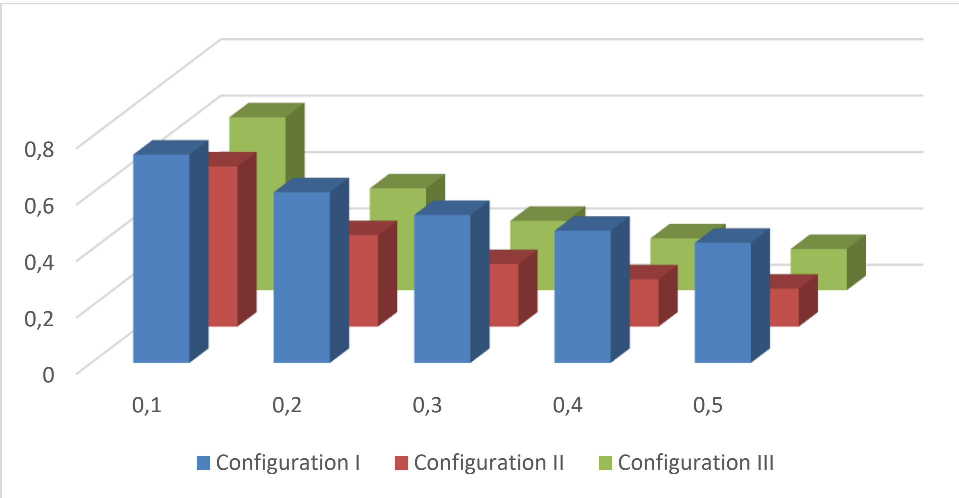

Figure 5: Availability against λ for μ = 0.6

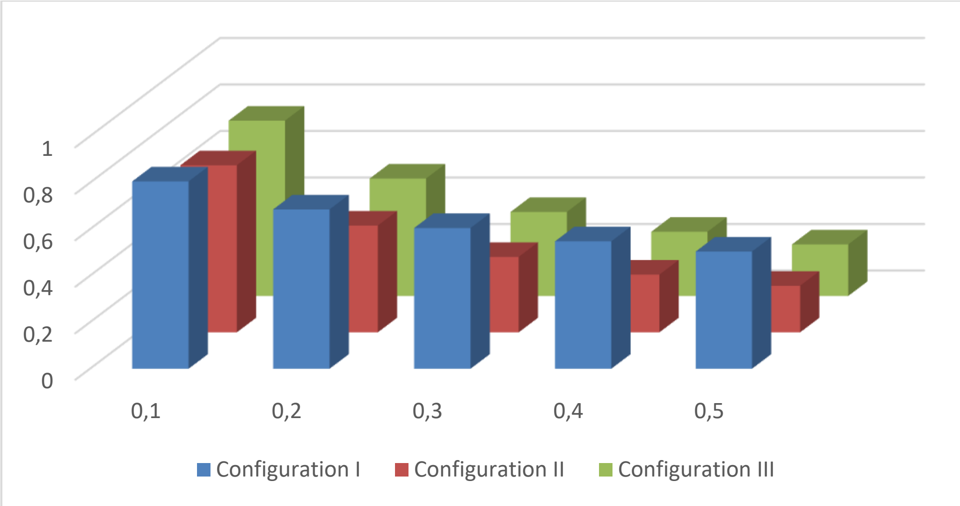

Figure 6: Availability against λ for $\mu = 0.9$

Table 3: Variation of Profit*10^6 with respect to μ for the three Configurations for different values of λ

μ	$\lambda = 0.1$			$\lambda = 0.5$			$\lambda = 0.9$		
	Configuration			Configuration			Configuration		
	I	II	III	I	II	III	I	II	III
0.1111	2.1574	0.6912	0.7532	0.8337	0.1384	0.1420	0.7785	0.7671	0.5205
0.2222	2.8318	1.3545	1.5082	1.3399	0.2773	0.2903	1.582	1.5387	0.9016
0.3333	3.2362	1.9534	2.1650	1.6879	0.4158	0.4426	2.403	2.3100	1.1941
0.4444	3.5149	2.4673	2.6986	1.9491	0.5539	0.5974	3.2382	3.0808	1.4275
0.5556	3.7191	2.8939	3.1193	2.1574	0.6912	0.7532	4.0849	3.8506	1.6198
0.6667	3.8749	3.2413	3.4480	2.3310	0.8274	0.9087	4.9401	4.6192	1.7825
0.7778	3.9976	3.5220	3.7053	2.4799	0.9621	1.0628	5.8013	5.3860	1.9231
0.8889	4.0964	3.7484	3.9082	2.6106	1.0951	1.2146	6.666	6.1505	2.0469
1.0000	4.1778	3.9316	4.0696	2.7270	1.2260	1.3632	7.5319	6.9121	2.1574

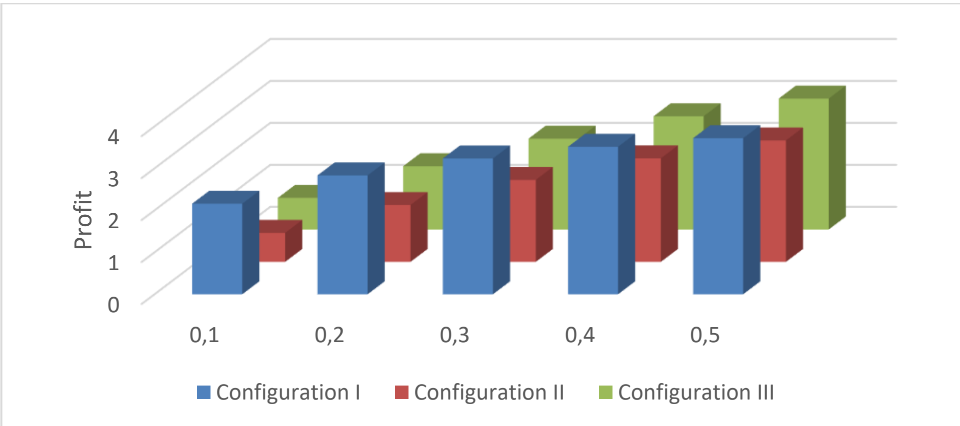

Figure 7: Profit against μ for $\lambda = 0.1$

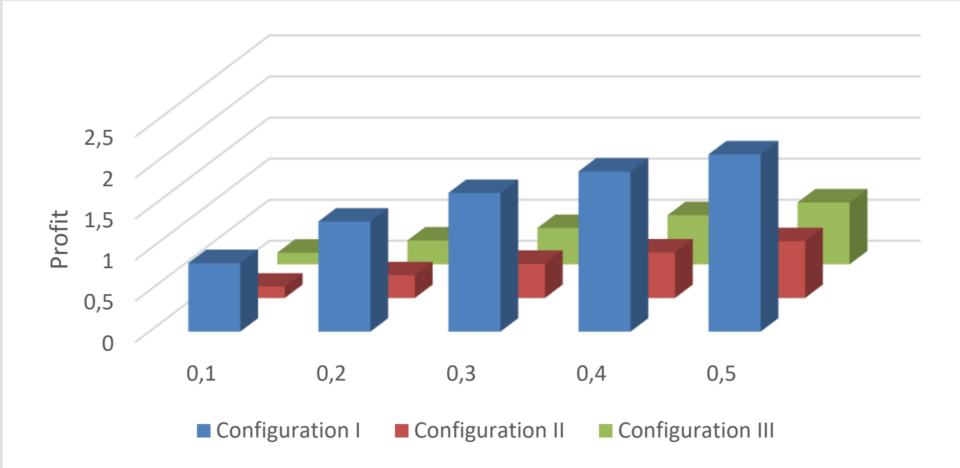

Figure 8: Profit against μ for $\lambda = 0.5$

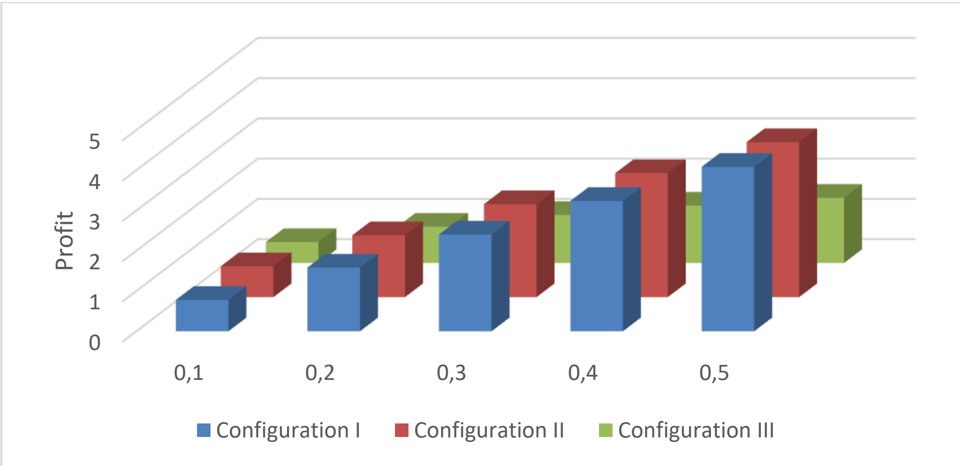

Figure 9: Profit against μ for $\lambda = 0.9$

Table 4: Variation of Profit*10^6 with respect to λ for the three Configurations for different values of μ

λ	μ = 0.3			μ = 0.6			μ = 0.9		
	Configuration			Configuration			Configuration		
	I	II	III	I	II	III	I	II	III
0.1111	3.0264	1.6216	1.8054	3.6939	2.8392	3.0664	4.0284	3.5927	3.7691
0.2222	2.3429	0.8376	0.9203	3.0264	1.6216	1.8054	3.4264	2.2950	2.5230
0.3333	1.9605	0.5608	0.6052	2.6228	1.1083	1.2296	3.0264	1.6216	1.8054
0.4444	1.6990	0.4210	0.4484	2.3429	0.8376	0.9203	2.7393	1.2406	1.3798
0.5556	1.5035	0.3369	0.3554	2.1306	0.6720	0.7314	2.5195	1.0009	1.1071
0.6667	1.3501	0.2807	0.2940	1.9605	0.5608	0.6052	2.3429	0.8376	0.9203
0.7778	1.2259	0.2406	0.2506	1.8193	0.4810	0.5154	2.1958	0.7195	0.7855
0.8889	1.1229	0.2105	0.2183	1.6990	0.4210	0.4484	2.0701	0.6304	0.6840
1.0000	1.0361	0.1870	0.1933	1.5949	0.3743	0.3966	1.9605	0.5608	0.6052

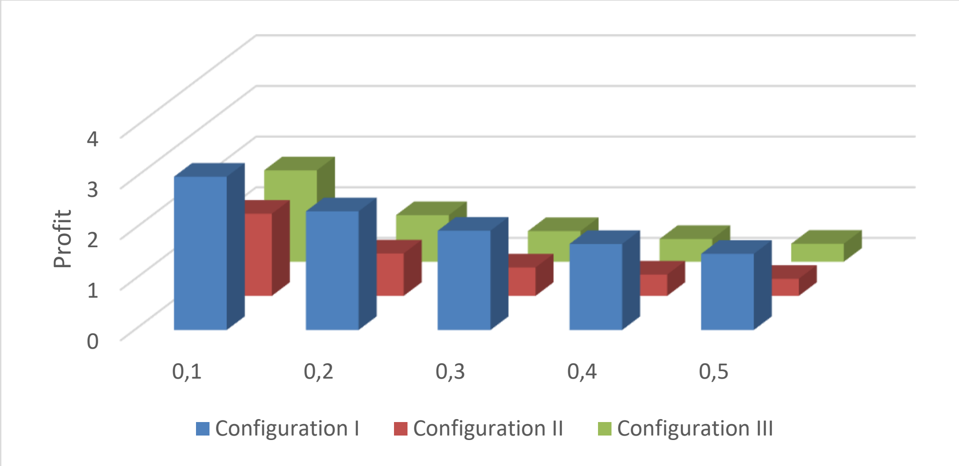

Figure 10: Profit against λ for μ = 0.3

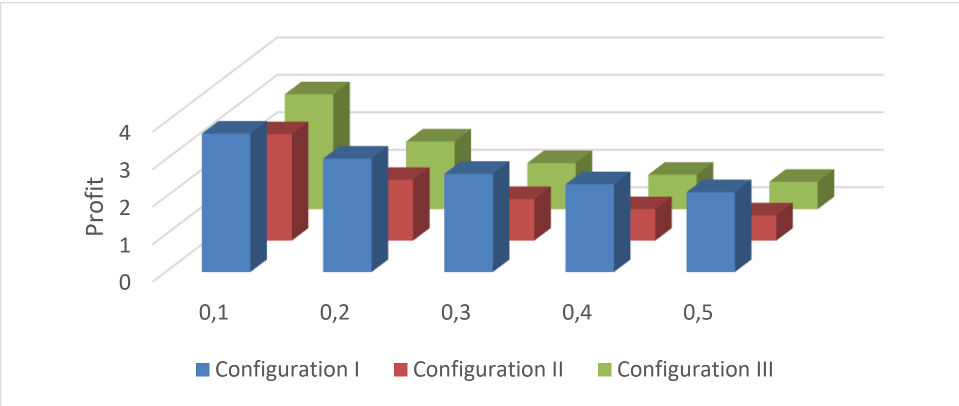

Figure 11: Profit against λ for μ = 0.6

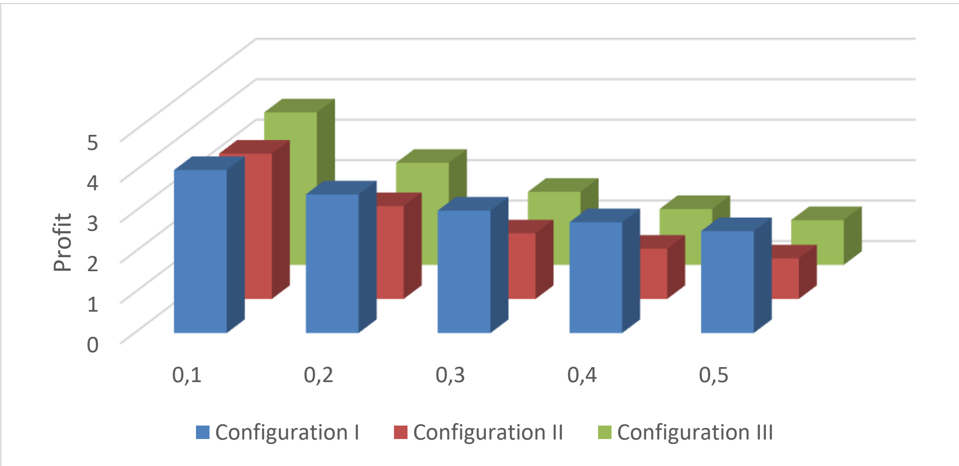

Figure 12: Profit against λ for μ = 0.9

Table 5: Variation of MTTF with respect to μ for the three Configurations for different values of λ

μ	λ = 0.1			λ = 0.5			λ = 0.9		
	Configuration			Configuration			Configuration		
	I	II	III	I	II	III	I	II	III
0.1111	5.6761	3.3914	3.3960	1.2183	0.6355	0.6355	0.6843	0.3504	0.3505
0.2222	5.4158	3.6692	3.6836	1.1919	0.6460	0.6462	0.6751	0.3537	0.3537
0.3333	5.2795	3.9574	3.9828	1.1698	0.6567	0.6571	0.6668	0.3569	0.3570
0.4444	5.2001	4.2544	4.2907	1.1511	0.6674	0.6681	0.6593	0.3602	0.3603
0.5556	5.1501	4.5590	4.6050	1.1352	0.6783	0.6792	0.6524	0.3635	0.3637
0.6667	5.1166	4.8700	4.9242	1.1216	0.6892	0.6905	0.6462	0.3668	0.3671
0.7778	5.0931	5.1862	5.2475	1.1099	0.7002	0.7019	0.6406	0.3701	0.3705
0.8889	5.0761	5.5069	5.5738	1.0998	0.7114	0.7134	0.6354	0.3735	0.3739
1.0000	5.0633	5.8312	5.9028	1.0909	0.7226	0.7250	0.6307	0.3768	0.3773

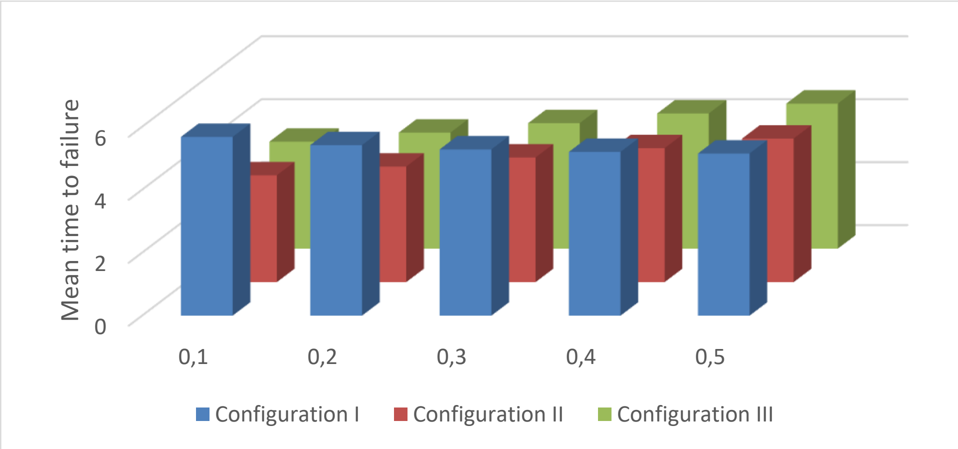

Figure 13: Profit against μ for λ = 0.1

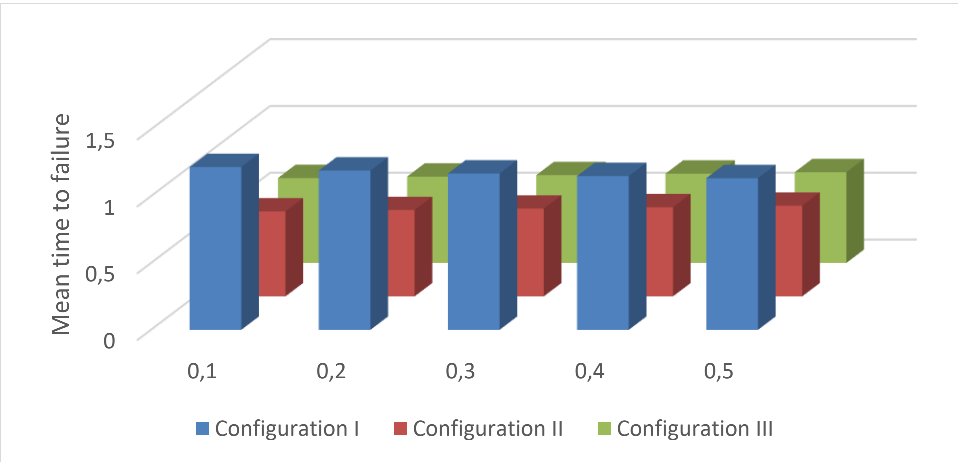

Figure 14: Profit against μ for λ = 0.5

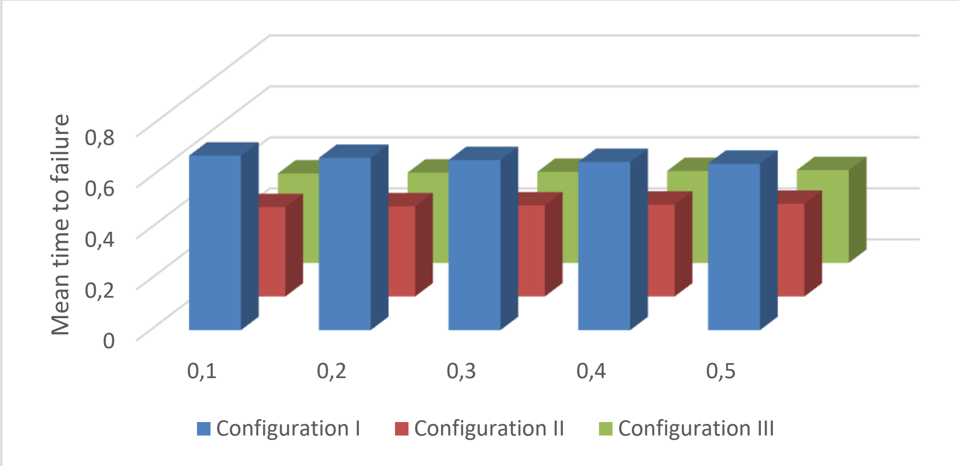

Figure 15: Profit against μ for λ = 0.9

Table 6: Variation of MTTF with respect to λ for the three Configurations for different values of μ

λ	μ= 0.3			μ= 0.6			μ= 0.9		
	Configuration			Configuration			Configuration		
	I	II	III	I	II	III	I	II	III
0.1111	4.8126	3.4128	3.4299	4.6403	4.0644	4.1046	4.5789	4.7509	4.8076
0.2222	2.5215	1.5526	1.5555	2.4063	1.7064	1.7150	2.3508	1.8666	1.8813
0.3333	1.7254	1.0020	1.0029	1.6482	1.0687	1.0718	1.6042	1.1376	1.1433
0.4444	1.3152	0.7392	0.7396	1.2608	0.7763	0.7777	1.2262	0.8143	0.8171
0.5556	1.0638	0.5855	0.5858	1.0236	0.6091	0.6099	0.9961	0.6331	0.6347
0.6667	0.8935	0.4847	0.4849	0.8627	0.5010	0.5015	0.8405	0.5175	0.5185
0.7778	0.7704	0.4135	0.4136	0.7461	0.4254	0.4257	0.7279	0.4375	0.4381
0.8889	0.6773	0.3605	0.3606	0.6576	0.3696	0.3698	0.6424	0.3788	0.3793
1.0000	0.6043	0.3196	0.3196	0.5880	0.3267	0.3269	0.5751	0.3340	0.3343

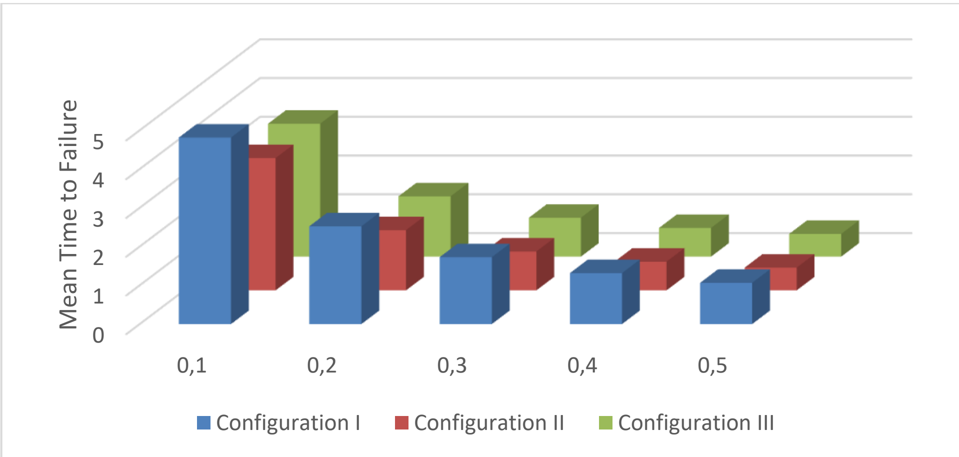

Figure 16: Profit against λ for μ= 0.3

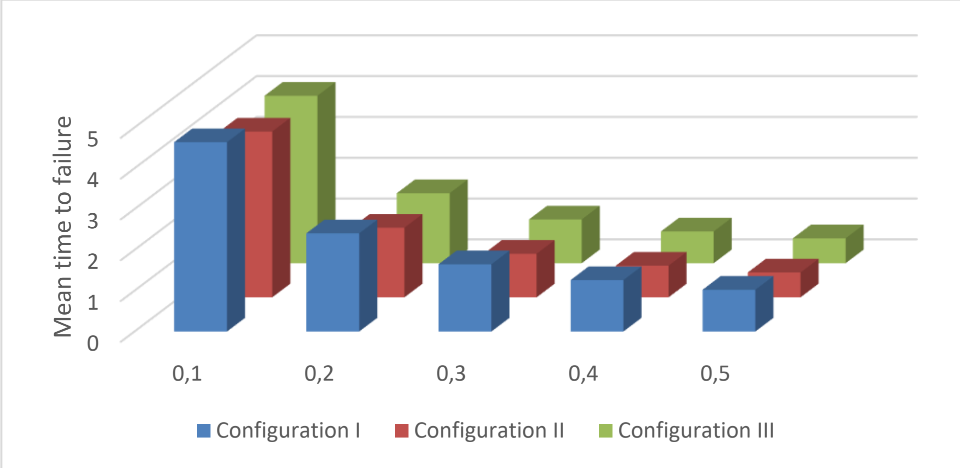

Figure 17: Profit against λ for μ= 0.6

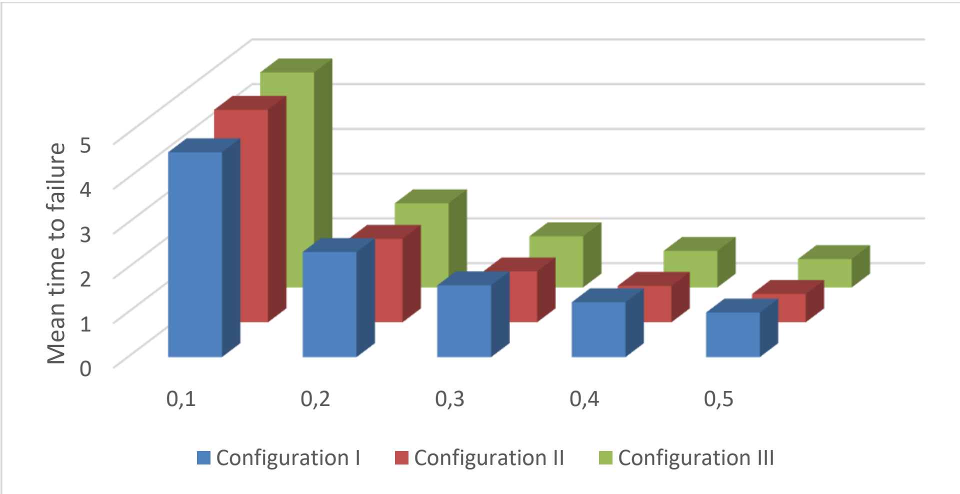

Figure 18: Profit against λ for μ= 0.

Conclusion

In this paper, three different series - parallel dynamo configurations were constructed with standby in each of the configuration and a repairable service station to study the sensitivity analysis of the three configurations under probability. The explicit expressions/equations for **MTTF** and Availability for the three configurations were developed and performed a sensitivity analysis base on the numerical values fixed. It was found out that the optimal configuration using the sensitivity analysis by fixing both λ and μ is configuration I.

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