Introducing Probabilistic Models for Cost Analysis of Sachet Water Plant

Nafisatu Muhammad Usman

•

Department of Arts and Humanity, School of General Studies,
Kano State Polytechnic, Kano, Nigeria
mamankhairat2015@gmail.com

Ibrahim Yusuf

•

Department of Mathematical Sciences, Bayero University, Kano, Nigeria
iyusuf.mth@buk.edu.ng

Abstract

The paper deals with modeling and performance assessment of a series-parallel with independent failures using the Markov Birth-Death method and the probabilistic approach. The system consists of five subsystems arranged in series and parallel configurations with three possible states of operation, reduced capacity and failure. First-order systems of ordinary differential equations are developed and recursively resolved using a probabilistic approach via the transition diagram. The state probabilities for the proposed scheme are derived. Using state probabilities, system availability expressions, busy repairman probabilities due to minor and major failures as well as benefit feature are calculated. Profit and availability matrices for each subsystem have been computed to provide various output values for different combinations of parameters. The finding of this paper will boost the efficiency of the system and will be useful for timely maintenance progress, decision-making, preparation and optimization.

Keywords: *Availability, modelling, probability, sachet water plant*

I. Introduction

Reliability, availability and profit are some of the most important factors in any successful sachet water system. Like other systems, sachet water systems are exposed to different types of failures such as common cause, partial, human and complete failure. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. Availability and profit of a sachet water system may be enhanced through adequate maintenance planning, regular inspection, fault tolerant units or subsystems, reliable structural design of the system or subsystem of higher reliability.

Systems are typically analyzed with a view to determining their reliability metrics. High productivity and full income from process plants are important for their survival. In order to do this, the efficiency and reliability of the equipment in the process must be ensured in the highest order. In order to increase the efficiency and reliability of the related development curriculum, more emphasis needs to be put on operational management. The most common weakness of our technical capabilities has been our inability to pay adequate attention to process technology. In the manufacturing phase, inputs are: raw materials, electricity, machinery, information and technology, labor, etc. In order to achieve quality and quantity, efficient plant management is necessary to monitor the conversion process and the variables affecting output. The development of a mathematical model is one of the ways of plant management. The modeling method is commonly used in the technology world. This method is used in the oil and milling industries, etc.

A large volume of literature exists on the issue of predicting performance evaluation of various manufacturing and industrial systems configured as series-parallel system. For instance; Kadiyan et al. (2012) analyzed the reliability and availability of uncaser system of brewery plant. Khanduja et al. (2012) discussed the maintenance planning of bleaching system of paper plant. Gupta and Tewari (2011) focuses on simulation of availability in thermal plant. Garg et al. (2010b) analyzed the availability of crankcase manufacturing of two-wheeler automobile industry. Garg et al. (2010a) analyzed the availability of a cattle feed plant using matrix method. Arvind et al. (2013) dealt with behavioral study of piston manufacturing plant through stochastic models. Aggarwal et al. (2014) presented Markov analysis of urea synthesis system of a fertilizer plant. Aggarwal et al. (2017) focuses on fuzzy availability analysis for serial processes in the crystallization system of a sugar plant. Kumar and Lata (2012) discussed the evaluation of reliability of condensate system using fuzzy Markov Model. Kumar et. al. (2011) dealt with performance modelling of furnace draft air cycle in a thermal plant. Kumar and Tewari (2011) presented modelling and performance optimization of CO2 cooling system of a fertilizer plant. Kumar and Mudgil (2014) presented an optimization of availability of ice cream making unit of milk plant using genetic algorithm.

Mathematical modelling of industrial and manufacturing systems may prove beneficial by analyzing the performance of the system/ subsystems through reliability, availability as well as generated profit and by identifying the combination of the problem that may result in increasing the risk of a complete breakdown which may lead to high corrective maintenance cost, low reliability, availability and profit. Through this mathematical model, the optimal profit level in which the profit is maximum can be identified and the corresponding subsystem that enable the maximum profit in order to lay emphasis on its preventive maintenance as well as the most critical subsystem leading to drop in profit.

One of the key sources of drinking water for low and medium class is sachet water. Knowing that water is an essential resource for the continued life of all living things, including man, sufficient supply of fresh and safe drinking water in abundance is an absolute necessity for all human beings. As such, the implementation of the modeling method in the water sector would play a vital role in ensuring a sufficient supply of fresh and safe water in society. As a result, individuals who can afford water are now sinking holes and selling it, some of them suffering from less efficient machinery to an ever-growing population. In some less developed countries, water is manufactured in a variety of products, such as bottled water, sachet wine, etc. Sachet water is commercially processed water, developed, packaged and distributed for sale in sealed polythene containers for human consumption. The development of sachet water began in the late 1990s, and today the progress of scientific technology has made the development of sachet water one of the fastest growing industries in the less developed countries. Many individuals and corporate bodies are now engaged in packaging water in polythene bags of about 50-60cl, which they sell to the public. Drinking water is therefore commercially available in a bag that is so easy to open.

The marketing and consumption of sachet water has increased enormously. The majority of producers are less concerned about increasing the availability, profit and reliability of their machinery. The continuous increase in the population and the indiscriminate consumption of sachet water demand an increase in production, as it is difficult for the most underprivileged citizens to obtain. Sachet water is seen to be a good addition to other types of packaged water and

can be purchased at a cheaper price. It is a source of drinking water for low and middle class.
The needs of this research are motivated by the increasing pressures on the demand of sachet water industries and their reliability modelling in order to meet the challenges of meeting water demand of the populace.

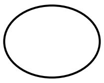
II. Notations and Description of the System

The System consists of five dissimilar subsystems which are:

1. Subsystem A (storage tank): Single units in series whose failure cause complete failure of the entire system.
2. Subsystem B (filter): Consists of two cold standby units. Failure of one unit, the system will work in full capacity. Complete failure occurs when both units failed.
3. Subsystem C (tank): consisting of single unit whose failure cause complete failure of the entire system.
4. Subsystem D (booster):): Consists of two cold standby units. Failure of one unit, the system will work in full capacity. Complete failure occurs when both units failed.
5. Subsystem E: A single unit in series whose failure cause complete failure of the entire system.

Notations

 Indicate the system is in failed state

 Indicate the system is in full working state

A, B, C,D,E: represent full working state of subsystem

B2 denote that the subsystem B is working in reduced capacity

C1 denote subsystem is working on standby unit

a, b, c,d,e represent failed state of subsystem

$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5$ represent failure rates of subsystems A, B,C,D and E

$\mu_1, \mu_2, \mu_3, \mu_4, \mu_5$: represent repair rates of subsystems A,B,C,D and E

$h_m(t), m = 0,1,2,3$: Probability of the system working with full capacity at time t

$h_m(t),\ m = 4,5,6,...,19$: Probability of the system in failed state

$A_V(\infty)$: Steady state availability of the system

$B_{S1}(\infty)$: **Busy period probability of repairman due to type I failure**

$B_{S2}(\infty)$: **Busy period probability of repairman due to type II failure**

$P_F(\infty)$: **Profit function**

k_0 : Total revenue generated

k_1 : Cost due to partial failure

k_2 : Cost due to complete failure

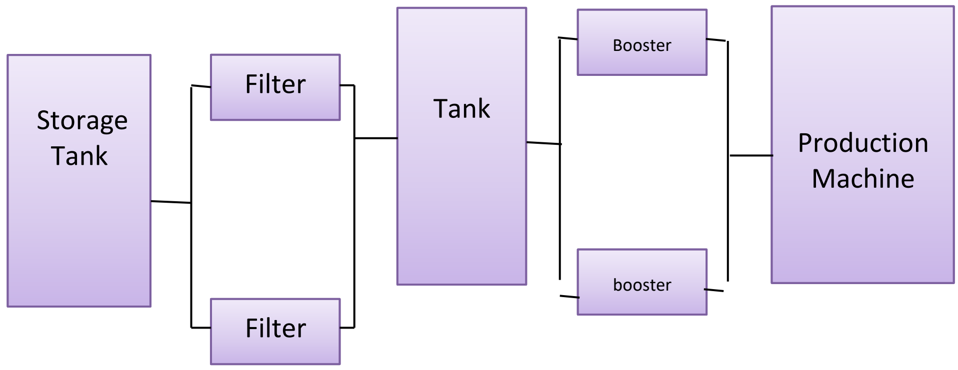


Figure 1: *reliability block diagram of the system*

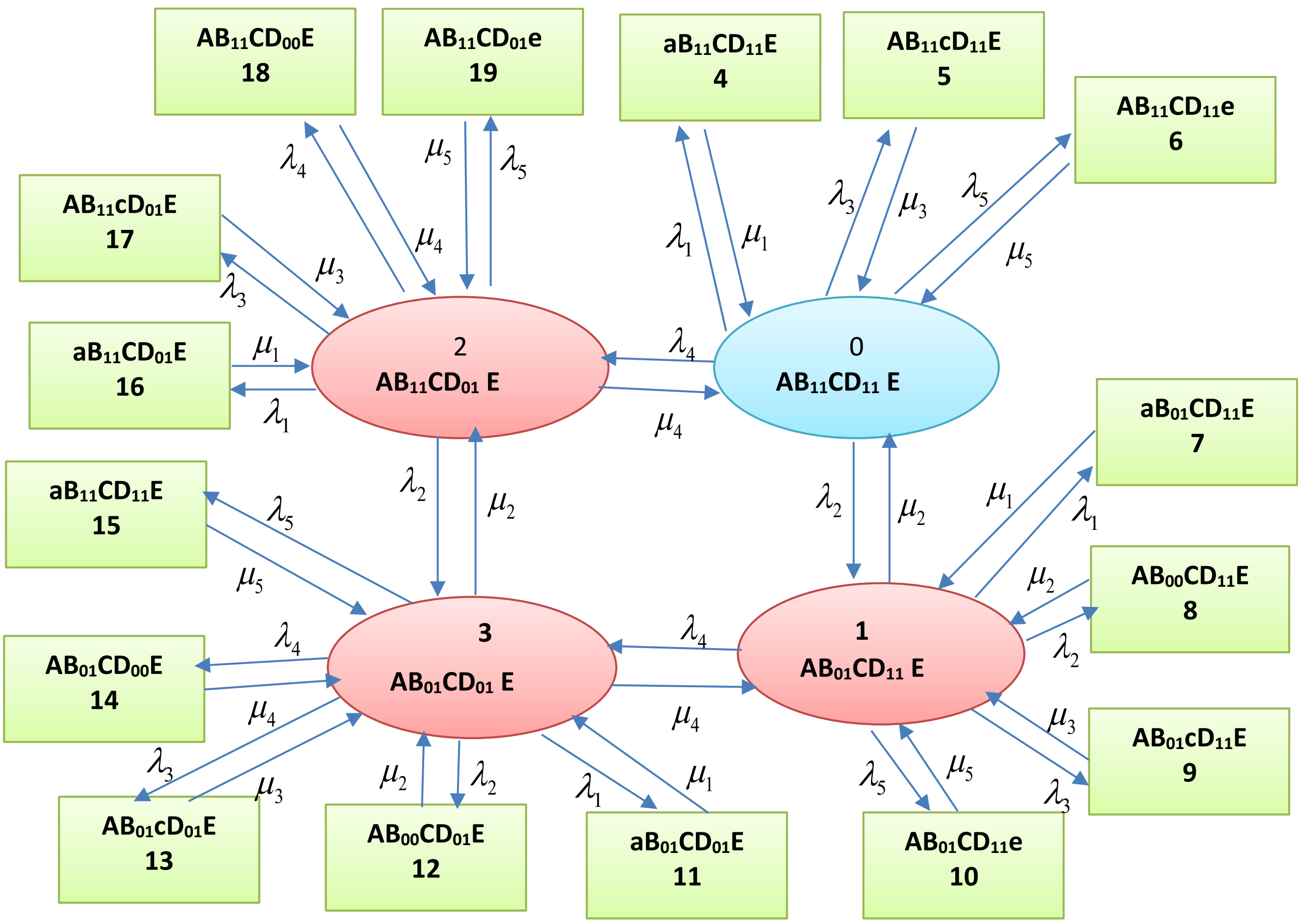


Figure 2: *Transition diagram of the system*

III. Mathematical Model Formulation

System of first order ordinary differential difference equations are derived using Markov birth-death process from Figure 2 above:

$$\left(\frac{d}{dt}+\sum_{k=1}^{5}\lambda_k\right)h_0(t)=\mu_2h_1(t)+\mu_4h_2(t)+\mu_1h_4(t)+\mu_3h_5(t)+\mu_5h_6(t) \quad (1)$$

$$\left(\frac{d}{dt}+\mu_2+\sum_{k=1}^{5}\lambda_k\right)h_1(t)=\lambda_2h_0(t)+\mu_4h_3(t)+\mu_1h_7(t)+\mu_2h_8(t)+\mu_3h_9(t)+\mu_5h_{10}(t) \quad (2)$$

$$\left(\frac{d}{dt}+\mu_4+\sum_{k=1}^{5}\lambda_k\right)h_2(t)=\lambda_4h_0+\mu_2h_3(t)+\mu_1h_{16}(t)+\mu_3h_{17}(t)+\mu_4h_{18}(t)+\mu_5h_{19}(t) \quad (3)$$

$$\left(\frac{d}{dt}+\mu_2+\mu_4+\sum_{k=1}^{5}\lambda_k\right)h_3(t)=\lambda_4h_1(t)+\lambda_2h_2(t)+\mu_1h_{11}(t)+\mu_2h_{12}(t)+\mu_3h_{13}(t)+\mu_4h_{14}(t)+\mu_5h_{15}(t) \quad (4)$$

$$\left(\frac{d}{dt}+\mu_1\right)h_4(t)=\lambda_1h_0(t) \quad (5)$$

$$\left(\frac{d}{dt}+\mu_3\right)h_5(t)=\lambda_3h_0(t) \quad (6)$$

$$\left(\frac{d}{dt}+\mu_5\right)h_6(t)=\lambda_5h_0(t) \quad (7)$$

$$\left(\frac{d}{dt}+\mu_1\right)h_7(t)=\lambda_1h_1(t) \quad (8)$$

$$\left(\frac{d}{dt}+\mu_2\right)h_8(t)=\lambda_2h_1(t) \quad (9)$$

$$\left(\frac{d}{dt}+\mu_3\right)h_9(t)=\lambda_3h_1(t) \quad (10)$$

$$\left(\frac{d}{dt}+\mu_5\right)h_{10}(t)=\lambda_5h_1(t) \quad (11)$$

$$\left(\frac{d}{dt}+\mu_1\right)h_{11}(t)=\lambda_1h_3(t) \quad (12)$$

$$\left(\frac{d}{dt}+\mu_2\right)h_{12}(t)=\lambda_2h_3(t) \quad (13)$$

$$\left(\frac{d}{dt}+\mu_3\right)h_{13}(t)=\lambda_3h_3(t) \quad (14)$$

$$\left(\frac{d}{dt}+\mu_4\right)h_{14}(t)=\lambda_4h_3(t) \quad (15)$$

$$\left(\frac{d}{dt}+\mu_5\right)h_{15}(t)=\lambda_5h_3(t) \quad (16)$$

$$\left(\frac{d}{dt}+\mu_1\right)h_{16}(t)=\lambda_1 h_2(t) \quad (17)$$

$$\left(\frac{d}{dt}+\mu_3\right)h_{17}(t)=\lambda_3 h_2(t) \quad (18)$$

$$\left(\frac{d}{dt}+\mu_4\right)h_{18}(t)=\lambda_4 h_3(t) \quad (19)$$

$$\left(\frac{d}{dt}+\mu_5\right)h_{19}(t)=\lambda_5 h_2(t) \quad (20)$$

With initial condition $h_i(t)=\begin{cases}1, & i=0\\ 0, & i=1,2,3,\ldots,19\end{cases}$ (21)

The steady state availability, busy period due to partial failure and complete failure are respectively given by:

$$A_V(\infty)=h_0(\infty)+h_1(\infty)+h_2(\infty)+h_3(\infty) \quad (22)$$

$$B_{p1}(\infty)=h_1(\infty)+h_2(\infty)+h_3(\infty) \quad (23)$$

$$B_{p2}(\infty)=h_4(\infty)+h_5(\infty)+h_6(\infty)+\ldots+h_{19}(\infty) \quad (24)$$

To compute the states probabilities $h_k(t)$ $k=0,1,2,\ldots,19$, the derivatives of states probabilities are set equal to 0 in (1) to (20) and solving them recursively using (21), the steady state probabilities given Table 1 below:

Table 1: States Probabilities

$h_0(\infty)=\frac{1}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_1(\infty)=\frac{y_2}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_2(\infty)=\frac{y_4}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$
$h_3(\infty)=\frac{y_2 y_4}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_4(\infty)=\frac{y_1}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_5(\infty)=\frac{y_3}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$
$h_6(\infty)=\frac{y_5}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_7(\infty)=\frac{y_1 y_2}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_8(\infty)=\frac{y_2^2}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$
$h_9(\infty)=\frac{y_2 y_3}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_{10}(\infty)=\frac{y_2 y_5}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_{11}(\infty)=\frac{y_1 y_2 y_4}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$
$h_{12}(\infty)=\frac{y_2^2 y_4}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_{13}(\infty)=\frac{y_2 y_3 y_4}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_{14}(\infty)=\frac{y_2 y_2^4}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$
$h_{15}(\infty)=\frac{y_2 y_4 y_5}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_{16}(\infty)=\frac{y_1 y_4}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_{17}(\infty)=\frac{y_3 y_4}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$
$h_{18}(\infty)=\frac{y_4^2}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	$h_{19}(\infty)=\frac{y_4 y_5}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4}$	

Where $m_k=\frac{\lambda_k}{\mu_k}$, $k=1,2,3,4,5$

$\delta_0=y_2^2+y_4^2$, $\delta_1=\left(y_1+y_2+y_3+y_4+y_5\right)$, $\delta_2=\left(1+y_2 y_4\right)$, $\delta_3=\left(y_1+y_3+y_5\right)$ and $\delta_4=\left(y_2+y_4\right)$

Equations (22) to (24) are now:

$$A_V(\infty)=\frac{(1+y_2+y_4+y_2y_4)}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4} \tag{25}$$

$$B_{S1}(\infty)=\frac{y_2+y_4+y_2y_4}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4} \tag{26}$$

$$B_{S2}(\infty)=\frac{(y_1+y_3+y_4)(y_2+y_4+y_2y_4)+\delta_0+y_2y_4(y_2+y_4)}{1+\delta_0+\delta_1\delta_2+\delta_3\delta_4} \tag{27}$$

The units/subsystems are exposed to corrective maintenance due to partial and complete failure, while the repairman is busy performing maintenance action to the failed units. Let C_0, C_1 and C_2 be the revenue generated when the system is in working state and no income when in failed state, cost of each repair due to partial and complete failure respectively. The expected total profit of system per unit time incurred to the system in the steady-state is given by:

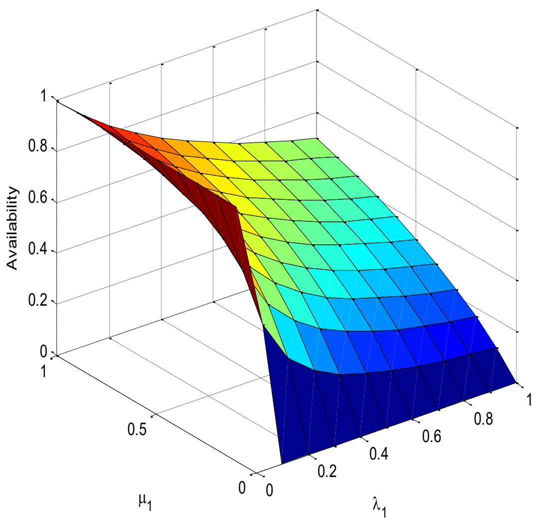
Profit =total revenue generated – cost incurred by the repair man due to partial failure – cost incurred due to complete failure.

$$P_F(\infty)=k_0A_V(\infty)-k_1B_{S1}(\infty)-k_2B_{S2}(\infty) \tag{28}$$

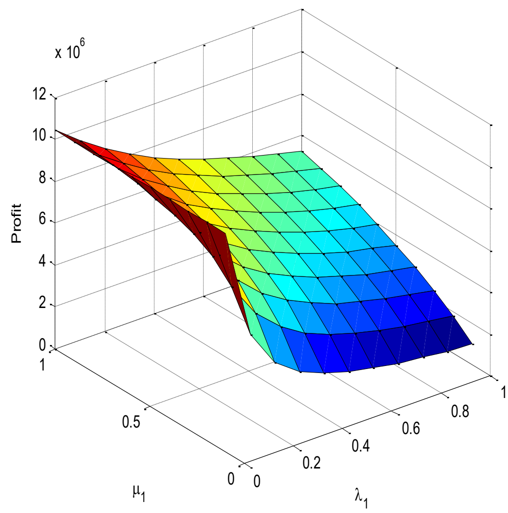
IV. Results and Discussion

In this section, numerical examples are presented using MATLAB package. The following cases are used in the simulations.

The following parameter values are used in this case: $\lambda_1=0.003$; $\lambda_2=0.003$; $\lambda_3=0.001$; $\lambda_4=0.002$; $\lambda_5=0.002$; $\mu_1=0.8$; $\mu_2=0.7$; $\mu_3=0.6$; $\mu_4=0.6$; $\mu_5=0.9$; $k_0=10{,}500{,}000$; $k_1=550$; $k_2=1250$;

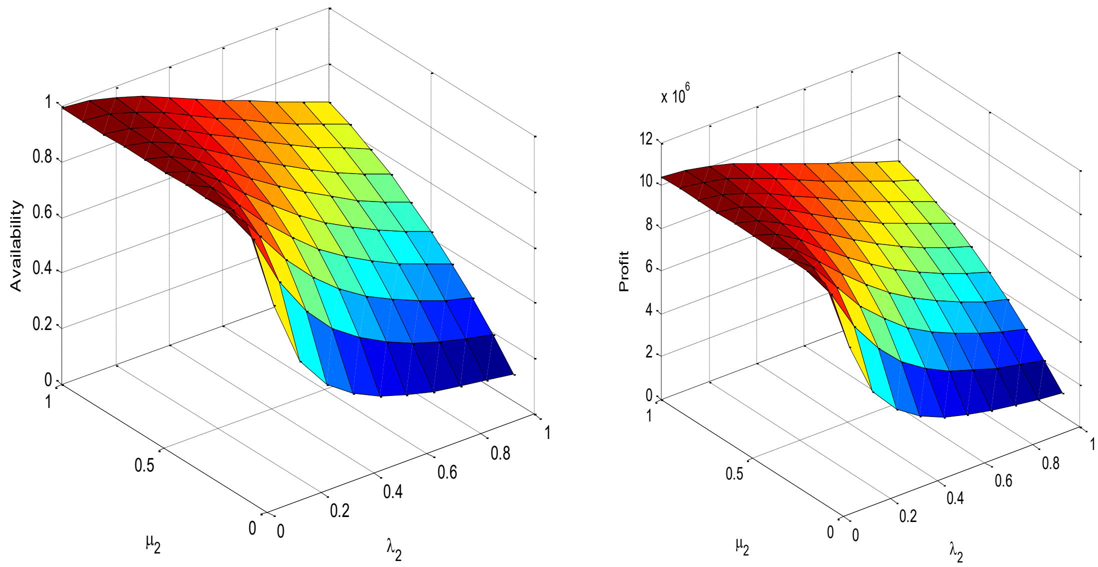


(a)



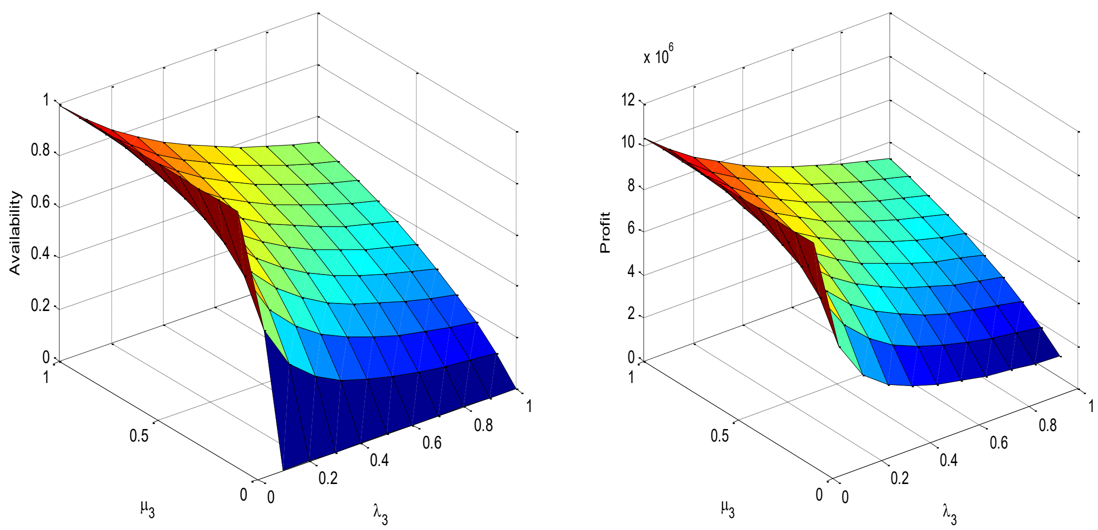
(b)

Figure 3: Availability and Profit with respect to failure and rates of subsystem A



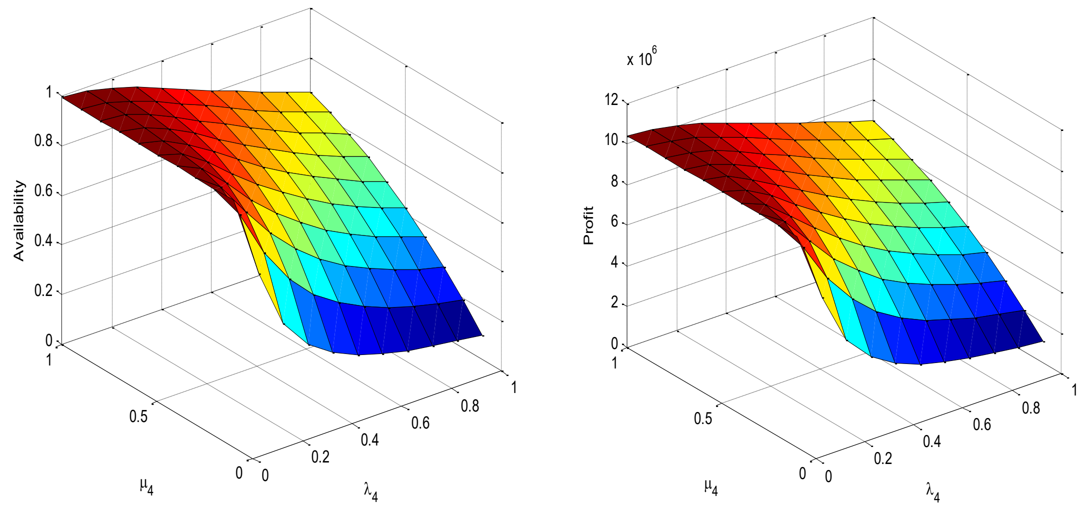
(a) (b)

Figure 4: Availability and Profit with respect to failure and rates of subsystem B



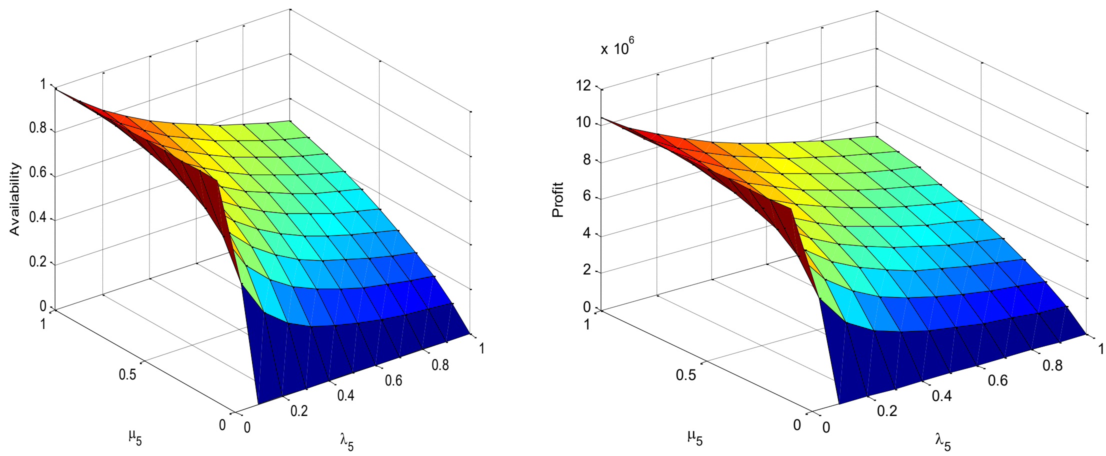
(a) (b)

Figure 5: Availability and Profit with respect to failure and rates of subsystem C



(a) (b)

Figure 6: Availability and Profit with respect to failure and rates of subsystem D



(a) (b)

Figure 7: Availability and Profit with respect to failure and rates of subsystem E

From the surface plots in Figures 3 to 7 availability and profit decreases as the failure rate increases λ_m and increases as repair rate μ_m increases for $m = 1, 2, 3, 4, 5$. These plots indicate that, availability and profit of the system are higher for higher values of μ_m and lower values of λ_m. From these plots it can be observe that reducing the occurrence of failure earlier or practicing perfect repair will lead to higher values of availability and profit as well as the expected life time of the system. This indicate that adequate maintenance action such as inspection, perfect repair or replacement should be practice to enhance the availability and profit.

Table 2: Variation of Availability and Profit with respect to failure and rates of subsystem A

λ_1	Availability						Profit *10^7					
	$\mu_1 \in [0.45:0.05:0.7]$						$\mu_1 \in [0.45:0.05:0.7]$					
0.0005	0.9950	0.9951	0.9952	0.9953	0.9953	0.9954	1.0448	1.0449	1.0450	1.0450	1.0451	1.0452
0.001	0.9939	0.9941	0.9943	0.9945	0.9946	0.9947	1.0436	1.0438	1.0440	1.0442	1.0443	1.0444
0.0015	0.9928	0.9931	0.9934	0.9936	0.9938	0.9940	1.0425	1.0428	1.0431	1.0433	1.0435	1.0437
0.002	0.9917	0.9922	0.9925	0.9928	0.9931	0.9933	1.0413	1.0418	1.0421	1.0425	1.0427	1.0429
0.0025	0.9906	0.9912	0.9916	0.9920	0.9923	0.9926	1.0402	1.0407	1.0412	1.0416	1.0419	1.0422
0.003	0.9895	0.9902	0.9907	0.9912	0.9916	0.9919	1.0390	1.0397	1.0403	1.0407	1.0411	1.0415

Table 3: Variation of Availability and Profit with respect to failure and rates of subsystem B

λ_2	Availability						Profit *10^7					
	$\mu_2 \in [0.5:0.05:0.8]$						$\mu_2 \in [0.5:0.05:0.8]$					
0.03	0.9898	0.9902	0.9906	0.9908	0.9910	0.9912	1.0393	1.0397	1.0401	1.0403	1.0406	1.0408
0.04	0.9878	0.9885	0.9891	0.9896	0.9899	0.9902	1.0372	1.0379	1.0385	1.0390	1.0394	1.0397
0.05	0.9853	0.9864	0.9873	0.9880	0.9885	0.9890	1.0345	1.0357	1.0366	1.0373	1.0379	1.0384
0.06	0.9823	0.9838	0.9851	0.9860	0.9868	0.9875	1.0314	1.0330	1.0343	1.0353	1.0362	1.0369
0.07	0.9788	0.9809	0.9825	0.9838	0.9849	0.9858	1.0277	1.0299	1.0316	1.0330	1.0341	1.0351
0.08	0.9749	0.9776	0.9796	0.9813	0.9827	0.9838	1.0237	1.0264	1.0286	1.0304	1.0318	1.0330

Table 4: Variation of Availability and Profit with respect to failure and rates of subsystem C

λ_3	Availability						Profit *10^7					
	$\mu_3 \in [0.4:0.05:0.65]$						$\mu_3 \in [0.4:0.05:0.65]$					
0.005	0.9818	0.9832	0.9843	0.9851	0.9859	0.9865	1.0309	1.0323	1.0335	1.0344	1.0352	1.0358
0.01	0.9699	0.9726	0.9747	0.9764	0.9778	0.9791	1.0184	1.0212	1.0234	1.0252	1.0267	1.0280
0.015	0.9583	0.9622	0.9653	0.9678	0.9699	0.9718	1.0062	1.0103	1.0135	1.0162	1.0184	1.0203
0.02	0.9470	0.9520	0.9560	0.9594	0.9622	0.9645	0.9943	0.9996	1.0038	1.0073	1.0103	1.0128
0.025	0.9359	0.9420	0.9470	0.9511	0.9545	0.9574	0.9827	0.9891	0.9943	0.9986	1.0022	1.0053
0.03	0.9251	0.9323	0.9381	0.9429	0.9470	0.9504	0.9713	0.9789	0.9850	0.9901	0.9943	0.9980

Table 5: Variation of Availability and Profit with respect to failure and rates of subsystem D

λ_4	Availability						Profit *10^7					
	$\mu_4 \in [0.5:0.05:0.75]$						$\mu_4 \in [0.5:0.05:0.75]$					
0.02	0.9910	0.9913	0.9915	0.9916	0.9917	0.9918	1.0406	1.0409	1.0411	1.0412	1.0413	1.0414
0.045	0.9855	0.9867	0.9876	0.9883	0.9888	0.9893	1.0347	1.0360	1.0369	1.0377	1.0383	1.0388
0.07	0.9763	0.9789	0.9810	0.9826	0.9839	0.9850	1.0251	1.0278	1.0300	1.0317	1.0331	1.0342
0.095	0.9640	0.9686	0.9721	0.9749	0.9772	0.9790	1.0122	1.0170	1.0207	1.0236	1.0260	1.0280
0.12	0.9494	0.9561	0.9613	0.9655	0.9689	0.9717	0.9969	1.0038	1.0093	1.0137	1.0173	1.0203
0.145	0.9329	0.9418	0.9489	0.9546	0.9593	0.9631	0.9795	0.9889	0.9963	1.0023	1.0072	1.0113

Table 6: Variation of Availability and Profit with respect to failure and rates of subsystem E

λ_5	Availability						Profit *10^7					
	$\mu_4 \in [0.5:0.05:0.75]$						$\mu_4 \in [0.5:0.05:0.75]$					
0.0145	0.9600	0.9667	0.9713	0.9745	0.9770	0.9789	1.0080	1.0150	1.0198	1.0232	1.0258	1.0279
0.0170	0.9543	0.9621	0.9673	0.9711	0.9740	0.9763	1.0020	1.0102	1.0157	1.0197	1.0227	1.0251
0.0195	0.9486	0.9575	0.9635	0.9678	0.9711	0.9736	0.9960	1.0053	1.0116	1.0162	1.0196	1.0223
0.022	0.9430	0.9529	0.9596	0.9644	0.9681	0.9710	0.9902	1.0005	1.0076	1.0127	1.0165	1.0195
0.0245	0.9375	0.9484	0.9558	0.9611	0.9652	0.9684	0.9844	0.9958	1.0036	1.0092	1.0135	1.0168
0.027	0.9320	0.9439	0.9520	0.9579	0.9623	0.9658	0.9786	0.9911	0.9996	1.0057	1.0104	1.0141

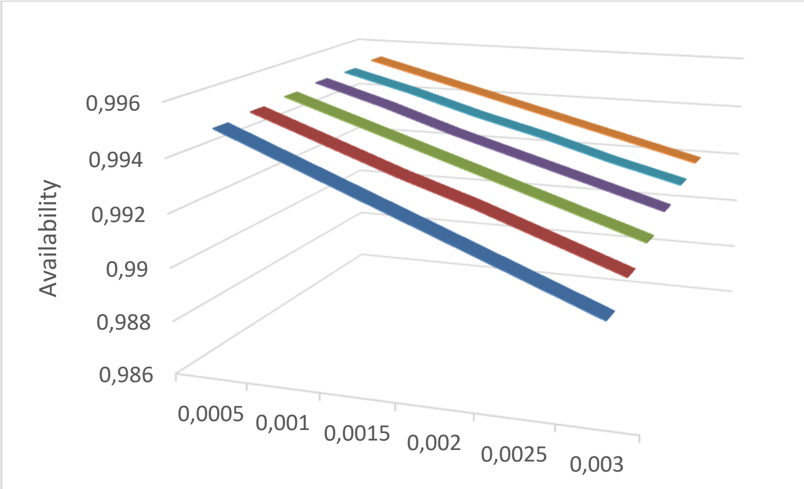


Figure 8a: Availability against λ_1 for $\mu_1 \in [0.45:0.05:0.7]$

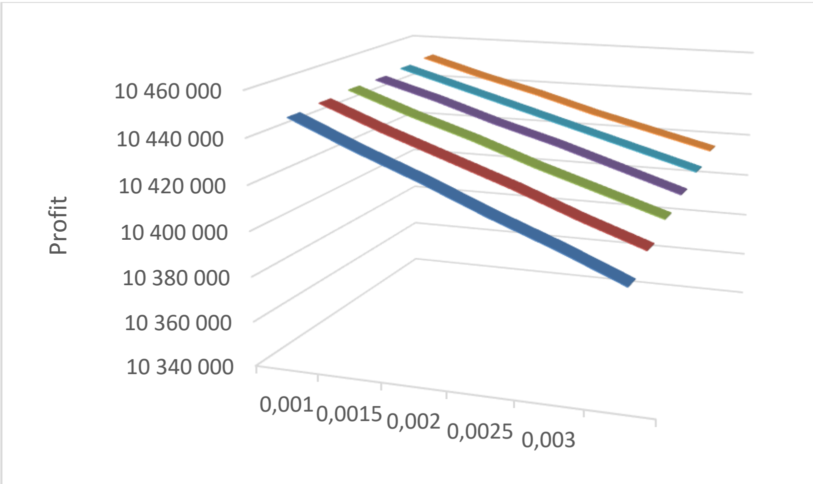


Figure 8b: Profit against λ_1 for $\mu_1 \in [0.45:0.05:0.7]$

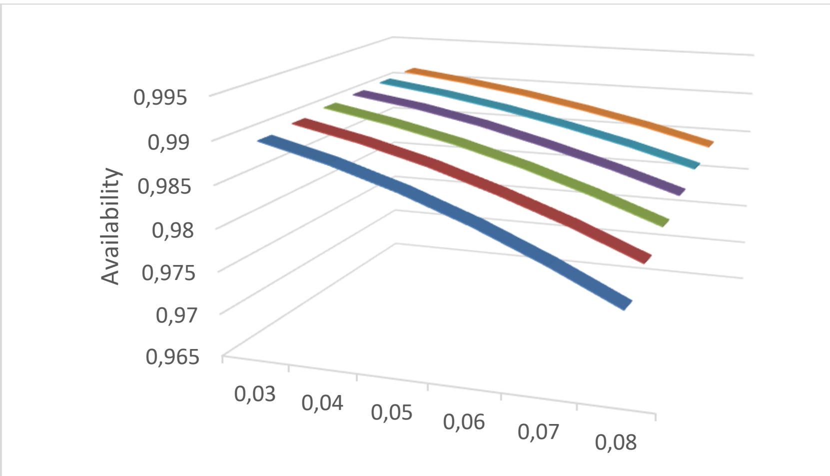


Figure 9a: Availability against λ_2 for $\mu_2 \in [0.5:0.05:0.8]$

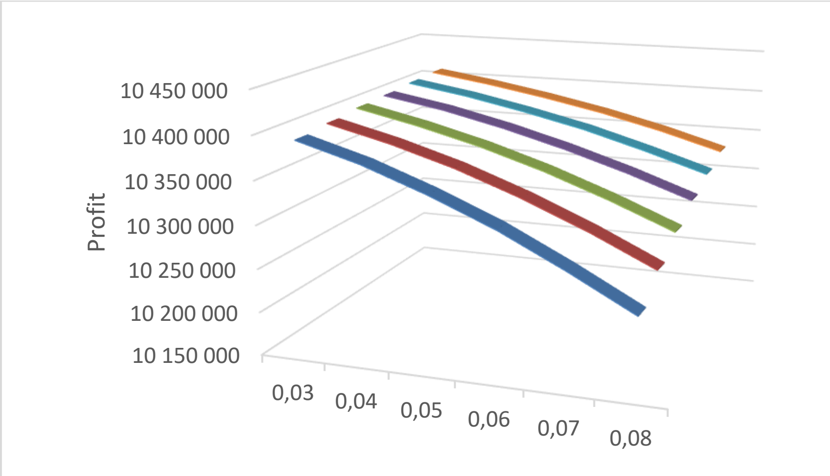


Figure 9b: Profit against λ_2 for $\mu_2 \in [0.5:0.05:0.8]$

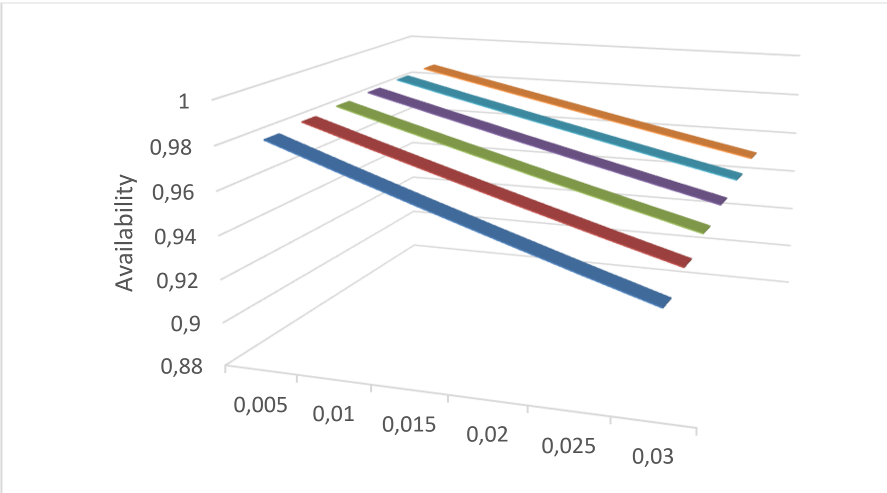


Figure 10a: Availability against λ_3 for $\mu_3 \in [0.4:0.05:0.65]$

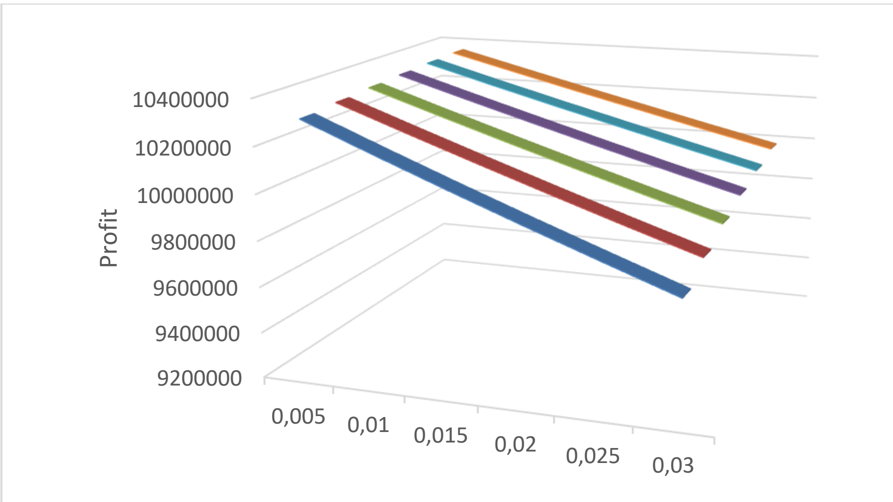


Figure 10b: Profit against λ_3 for $\mu_3 \in [0.4:0.05:0.65]$

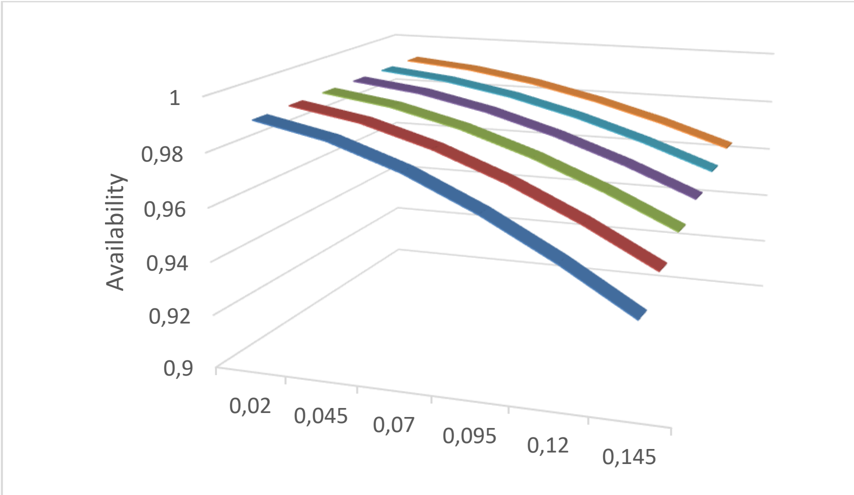


Figure 11a: Availability against λ_4 for $\mu_4 \in [0.5:0.05:0.75]$

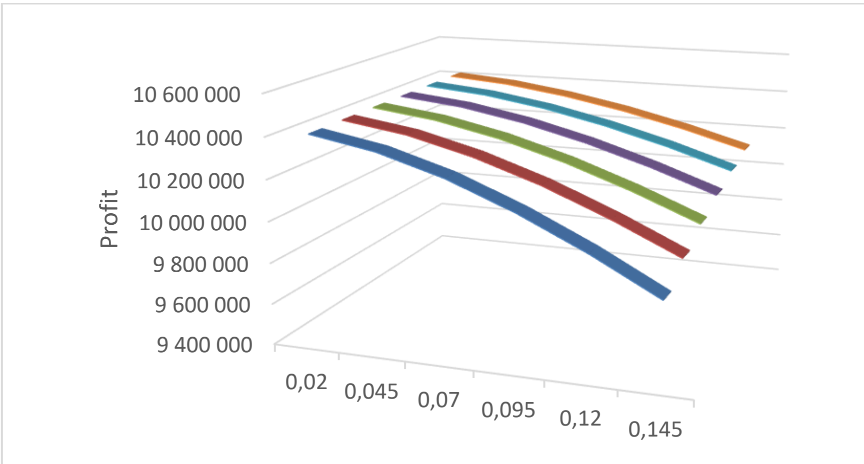


Figure 11b: Profit against λ_4 for $\mu_4 \in [0.5:0.05:0.75]$

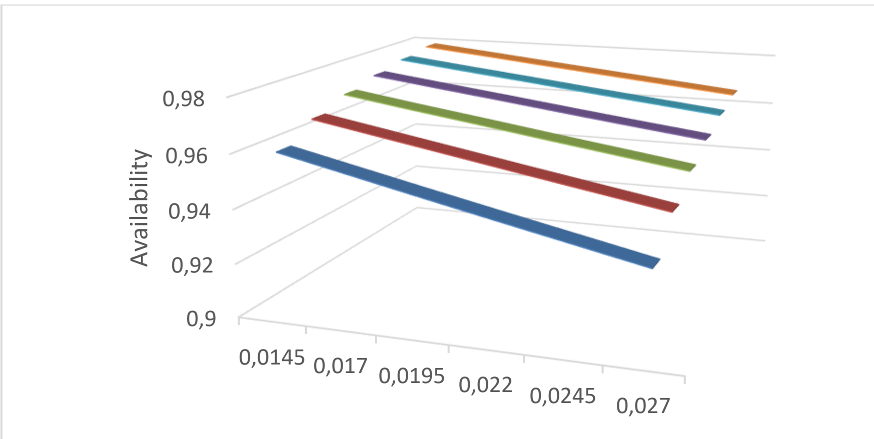


Figure 12a: Availability against λ_5 for $\mu_5 \in [0.0145:0.0025:0.027]$

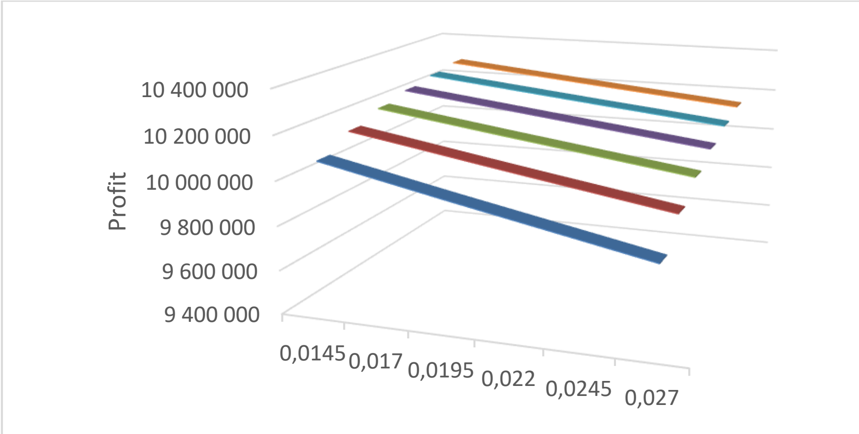


Figure 12b: Profit against λ_5 for $\mu_5 \in [0.0145 : 0.0025 : 0.027]$

Table 2 and Figures 8a and 8b present the impact of failure and repair rates of subsystem A against the availability and profit for different values of parameters λ_1 and μ_1. It is evident from Table 2 and Figure 8a and 8b that availability and profit shows increasing pattern with respect to repair rate μ_1 and decreasing pattern with respect to failure rate λ_1. It is clear that availability and profit are higher with the higher value of μ_1 **and lower with higher value of** λ_1.

Table 3 and Figures 9a and 9b display the effect of failure and repair rates of subsystem B against the profit for different values of parameters λ_2 and μ_2. It is evident from Table 3 and Figure 9a and 9b that the availability and profit shows increasing pattern with respect to repair rate μ_2 and decreasing pattern with respect to failure rate λ_2. It is clear that availability and profit are higher with the higher value of μ_2 **and lower with higher value of** λ_2.

Results from Table 4 and Figures 10a and 10b present the impact of failure and repair rates of subsystem C against availability and profit for different values of parameters μ_3 and λ_3. It is evident from Table 4 and Figure 5a that the profit shows increasing pattern with respect to repair rate μ_3 and decreasing pattern with respect to failure rate λ_3. It is clear that availability and profit are higher with the higher value of μ_3 **and lower with higher value of** λ_3.

It is evident from Table 5 and Figures 11a and 11b that availability and profit increases and decreases with increase in the values of parameters μ_4 and λ_4. It is evident from Table 5 and Figure 11a and 11b that availability and profit shows increasing pattern with respect to repair rate μ_4 and decreasing pattern with respect to failure rate λ_4. It is clear that availability and profit are higher with the higher value of μ_4 **and lower with higher value of** λ_4.

Table 6 and Figures 12a and 12b present the impact of failure and repair rates of subsystem E against the availability and profit for different values of parameters μ_5 and λ_5. It is evident from Table 6 and Figure 12a and 12b that availability and profit shows increasing pattern with respect to repair rate μ_5 and decreasing pattern with respect to failure rate λ_5. It is clear that profit is higher with the higher value of μ_5 **and lower with higher value of** λ_5.

V. Conclusion

In this paper, we constructed a series-parallel system configuration consisting of five subsystems to study the cost analysis of the system. Explicit expressions for steady-state availability, busy period and profit function for the system are derived. In this research work, mathematical models of availability and profit are developed and validated for each of the subsystem operation in a sachet water system. Numerical results presented have shown the effect of both failure and repair rates on profit. From the analysis, it is evident that profit can be enhancing through:

- ✓ Proper maintenance planting to avoid the occurrence of catastrophic failure.
- ✓ Maintaining the system availability at the highest order.
- ✓ Adding more fault tolerant redundant units/subsystem

Mathematical models of the system are developed in the form of availability, busy period of repairman due to minor and major failure and as well as profit function. Availability and Profit generated are presented in the Tables 2 to 6. The effects of failure and repair rates of all the subsystems are presented in the form of profit matrices. It is evident from the availability and profit matrices that as failure/repair rates increases, availability and profit tend to decrease/increase.

With modifications and assumptions, the model in this paper will plant management to avoid an incorrect reliability assessment and consequent erroneous decision making, which may lead to unnecessary expenditures. The present work can extend to incorporate failure dependency, condition monitoring to enable management in determining the optimal maintenance/ replacement time.

On the basis of the surface plots, tables and figures, it is evident that the availability and profit can be enhanced through higher values of repair rates together with lower values of failure rates. Thus, higher system availability and revenue can be achieved through repair of early failure of units, individual subsystem replacement, and proper maintenance planning to avoid the occurrence of catastrophic failure, and by adding fault tolerant units/subsystems. The present work can be extended further for a system to containing multi-subsystems with multi units and solve using human reliability analysis techniques

References

[1] Aggarwal A. K, Kumar S, Singh V, Garg TK, 2014. Markov modelling and reliability analysis of urea synthesis system of a fertilizer plant. Journal of Industrial Engineering International, 11, 1-14, DOI 10. 1007/s40092-014-0091-5.

[2] Aggarwal, A. K.,Kumar, S. and Singh, V, 2017. Mathematical modeling and fuzzy availability analysis for serial processes in the crystallization system of a sugar plant, Journal of Industrial Engineering International, 13:47–58, DOI 10.1007/s40092-016-0166-6.

[3] Arvind K Lal, A. K., Manwinder Kaur M and Lata, S. (2013). Behavioral study of piston manufacturing plant through stochastic models, Journal of Industrial Engineering International, 9(24), 1-10. http://www.jiei-tsb.com/content/9/1/24.

[4] Fadi N. Sibai, F.N. (2014). Modelling and output power evaluation of series parallel photovoltaic modules, International Journal of Advanced Computer Science and Applications,5(1),129-136.

[5] Garg D, Kumar K, and Singh J., 2010a. Availability analysis of a cattle feed plant using matrix method. Int J Eng 3(2):201–219

[6] Garg S, Singh J and Singh D. V., 2010b. Availability analysis of crankcase manufacturing in a

two-wheeler automobile industry. Appl Math Model 34:1672–1683

[7] Garg H and Sharma S.P., 2011. Multi-objective optimization of crystallization unit in a fertilizer plant using particle swarm optimization. Int J Appl Sci Eng 9(4):261–276

[8] Gupta S, and Tewari P.C., 2011. Simulation modeling in a availability thermal power plant. J Eng Sci Technol Rev 4(2):110–117.

[9] Khanduja R, Tewari PC, Kumar D., 2012. Steady state behaviour and maintenance planning of bleaching system in a paper plant. Int J Ind Eng 7(12):39–44

[10] Kadiyan S, Garg RK, Gautam R., 2012. Reliability and availability analysis of uncaser system in a brewery plant. Int J Res Mech Eng Technol 2(2):7–11.

[11] Kumar, A., Saini, M. and Malik, S. C., 2014. Stochastic modelling of a concrete mixture plant with preventive maintenance, Application and Applied Mathematics, 9(1): 13-27.

[12] Kumar V and Mudgil V., 2014. Availability optimization of ice cream making unit of milk plant using genetic algorithm, IntManag Bus Stu, 4(3), 17-19.

[13] Kumar S, and Tewari PC., 2011. Mathematical modelling and performance optimization of CO2 cooling system of a fertilizer plant. Int J IndustEng Comp, 2, 689-698

[14] Kumar R, Sharma AK, Tewari PC., 2011. Performance modelling of furnace draft air cycle in a thermal plant. Int J EngSci Tech, 3(8), 6792-6798.

[15] Kumar, A and Lata, S., 2012. Reliability evaluation of condensate system using fuzzy Markov Model, Annals of Fuzzy Mathematics and Informatics, 4(2), 281-291.

[16] Ram, M and Manglik, M., 2016. An analysis to multi-state manufacturing system with common cause failure and waiting repair strategy, Cogent Engineering 3: 1266185, 1-20, http://dx.doi.org/10.1080/23311916.2016.1266185

[17] Tewari PC, Khaduja R, and Gupta, M., 2012 Performance enhancement for crystallization unit of a sugar plant using genetic algorithm technique. J Ind Eng Int 8(1):1–6

[18] Yusuf, I., Sani, B and Yusuf, B. (2019). Profit analysis of a series-parallel system under partial and complete failures, Journal of Applied Sciences, 19(6),565-574.