

Multi-objective Model for Daily Diet Planning

¹Mohd Arif Khan, ^{2*}Ahteshamul Haq, and ³Aquil Ahmed

•

Department of Statistics & Operations Research
Aligarh Muslim University, Aligarh-202002, Uttar Pradesh, India
[1mohdarifkhan4012@gmail.com](mailto:mohdarifkhan4012@gmail.com), [2*a.haq@myamu.ac.in](mailto:a.haq@myamu.ac.in), [3aquilstats@gmail.com](mailto:aquilstats@gmail.com)

*Corresponding author

Abstract

In this paper, we present the development of a daily diet model using fuzzy multi-objective goal programming (GP) to satisfy daily nutrient. We have designed the objective function as minimize the cost of diet, Saturated Fat and carbohydrate. This paper consists of ten consumed foodstuffs as the decision variable. The daily diet's tolerable lower and upper intake level is given for the Protein, Vit. B₆, Vit. C and Calcium. This paper aims to present a stepwise solution procedure based on fuzzy GP to obtain the compromise solution of the diet problem. Finally, a numerical example is illustrated to compare the daily diet plan with weighted GP, pre-emptive GP and fuzzy GP.

Keywords: Diet Planning, Multi-objective, Weighted goal programming, Pre-emptive Goal Programming, Fuzzy Goal Programming

I. Introduction

It is a severe problem throughout life to meet their health goals based on the daily diet. Researcher and scientists have been using different types of mathematical programming to solve this type of question. With the assist of operations research techniques, it is quite viable to discover a listing of foodstuffs in an appropriate quantity which can grant all nutrient pointers in a day. Firstly, the diet problem by using linear programming was solved by Smith [1]. Anderson and Earle [2] have done the comparative study of diet planning through linear programming and GP approach for daily nutritional requirements of Thais. Nutritionists are turning into extra conscious about the overdoses of vitamins and want for a balanced consumption of all nutrient. The essential traits of the real-world decision-making problems going through human beings at present are multidimensional and have multiple objectives which include economics, social, environmental and technical ones. Hence, it appears natural that the consideration of many objectives in the actual decision-making process requires multi-objective approaches rather than a single objective.

Linear GP is one of many techniques for dealing with the modelling, solution, and analysis of multiple and conflicting objective by reducing it to a single (or sequential) objective one. Since Charnes and Cooper [3] introduced the concept of GP. Pre-emptive GP is a particular case of GP in which the more critical (upper level) goals are optimized before lower-level goals. Once complication concerned the weighting of goals in the objective function, Ignizio [4] demonstrated the use of weighted GP in diet planning and presented the results of the problem involving the selection of foodstuffs for improvement in nutritional balance by minimizing the cost of foodstuffs. Many authors have been worked on diet planning, some of them are listed in Table 1.

Table 1. Research Review Summary

Authors	Model Objective		Techniques used					Remarks	
	Single	Multi	LP	IP	WGP	FGP	PGP		
	Eghbali et al. [5]		✓				✓		
Nath et al. [6]	✓		✓					Trial and Error Method, Nutritional model	
Eghbali et al. [7]		✓				✓		Multi-objective Fuzzy Programming, Diet Problem	
Mamat [8]		✓				✓		Diet Planning, nutritional requirements, Fuzzy Programming	
Eghbali-Zarch et al. [9]		✓				✓		Mixed Integer Linear Programming, Diet Plan, Jimenez and Epsilon-constraint Method	
Sheng and Sufahani [10]	✓			✓				Diet Planning, Integer Programming	
Ali et al. [11]	✓			✓				diet planning for boarding schools, Zero One Integer Programming	
Bhargava et al. [12]	✓					✓		weighted GP, Diet planning	
Proposed model		✓				✓	✓	✓	Diet plan, pre-empty GP, weighted GP, fuzzy programming

Decerle et al. [13] highlight the relationship between working time, quality of service and route balancing for the home health care problem by using Pareto based approach. The objectives of the model are the minimization of the total working time of the caregivers while maximizing the quality of service and minimizing the maximal working time difference among nurses and auxiliary nurses. Nguyen and Montemanni, [14] propose mixed linear programming to find the best schedule minimizing the costs due to the non-respect of patients' time windows and exceeded hours of caregivers. En-nahli et al. [15] develop a multi-objective optimization problem in which the model tries to satisfy the Home Health Care Services objectives. On the other hand, 'patients and caregivers' objectives that satisfy all patients by assigning their wished caregiver, help to get solutions taking into account the priority of a patient and the affinity patient-caregiver. Mutingi & Mbohwa [16] present a multi-agent architecture that facilitates decision making characterised with multiple objectives and the capabilities of a multi-agent system and Web services as to facilitate effective decisions for home healthcare services by using genetic algorithm. Niakan & Rahimi [17] presents a multi-objective mathematical model to address a Healthcare Inventory Routing Problem for medicinal drug distribution to healthcare facilities. The first part of objective function minimizes total inventory and transportation costs, while satisfaction is maximized by minimizing forecast error which caused by product shortage and the number of expired drugs; Greenhouse Gas emissions are also minimized. A hybridized possibilistic method is applied to cope with uncertainty, and an interactive fuzzy approach is considered to solve an auxiliary crisp multi-objective model and find optimal solutions. Othman et al. [18] composed two phases: the first one is an assignment procedure based on fuzzy logic and the second phase is based on an evolutionary method to solve the problem of medical staff scheduling which improves the performance of the scheduling system in order to help physicians to manage the organization better. Turgay & Taşkın [19] presents fuzzy GP using exponential membership function, which uses the modelling, and solving of health care system for optimal, efficient management and prioritized for the strategic

planning and resource allocation. Zhang et al. [20] examine the health-care facilities that should be located to improve the equity of accessibility, reduce the population that falls outside the coverage range, raise the total accessibility for the entire population, and decrease the cost of building new facilities and use genetic algorithm-based multi-objective optimization approach to yield a set of Pareto solutions. The multi-objective optimization approach is used to optimize the location of new health-care facilities which provides a set of different plans that compare the values of the objectives and comparing the Pareto solutions with other solutions.

In this paper, we present the development of a multi-objective daily diet model using pre-emptive and fuzzy GP to satisfy daily nutrients with an example. The objective function is designed to minimize the cost of the diet, Saturated Fat and carbohydrate. The objective of this approach is to select diets to meet specific nutritional requirements. The comparison for the daily diet plan with weighted GP, pre-emptive GP and Fuzzy GP is also shown.

II. Formulation of the Diet Model

A diet is required to propose by the dietician for the special needs of the patient. An integer number of units of the diet can be composed as ten basic foodstuffs termed as Food 1, Food 2, ..., Food 10. The nutrients that are used in the model are Saturated Fat, Carbohydrate, Protein, Vitamin C, Vitamin B₆ and Calcium. The lower and upper levels of Protein, Vitamin C, Vitamin B₆ and Calcium are used as constraints.

In the diet model, x_j ($j = 1, 2, \dots, n$) represents the different types of food items that work as a decision variable and the cost of food, Saturated Fat, and Carbohydrates for each food are C_{Dj}, S_{Fj}, C_{dj} ($j = 1, 2, \dots, n$). Then the objective function will be

$$\text{Min } Z_1 = \sum_{j=1}^{10} C_{Dj} x_j, \text{ Min } Z_2 = \sum_{j=1}^{10} S_{Fj} x_j, \text{ Min } Z_3 = \sum_{j=1}^{10} C_{dj} x_j$$

The constraints of the model satisfy the nutrients requirements. The nutrient contents of the food items in respect to the diet concerning nutrients are represented on the left-hand side of the constraints, and the right-hand side of the constraint is lower and upper demand of each nutrient (Protein, Vitamin B₆, Vitamin C, and Calcium). The upper requirement of Saturated Fat also works as a constraint. Then the constraints are as follow:

$$\begin{aligned} \sum_{j=1}^n P_j x_j &\geq P_{Min}, \sum_{j=1}^n P_j x_j \leq P_{Max} && \text{; for Protein} \\ \sum_{j=1}^n V_{Bj} x_j &\geq V_{BMin}, \sum_{j=1}^n V_{Bj} x_j \leq V_{BMax} && \text{; for Vitamin B}_6 \\ \sum_{j=1}^n V_{Cj} x_j &\geq V_{CMin}, \sum_{j=1}^n V_{Cj} x_j \leq V_{CMax} && \text{; for Vitamin C} \\ \sum_{j=1}^n C_{lj} x_j &\geq C_{lMin}, \sum_{j=1}^n C_{lj} x_j \leq C_{lMax} && \text{; for Calcium} \\ \sum_{j=1}^n S_{Fj} x_j &\leq S_{FMax} && \text{; for Saturated Fat} \\ x_j &\geq 0, x_j \leq 4, x_j \in \text{Integer } \forall j = 1, 2, \dots, n \end{aligned}$$

III. Procedure for Solving Multi-Objective Problem

Fuzzy GP is flexible and powerful techniques that can be applied to a variety of decision-making problems that have multiple objectives. Therefore, we can use this approach to obtain the optimal compromise solution for the formulated models. The stepwise solution procedure is given as follows:

Step 1: Solve the multiple objective problems by considering a single objective at a time and ignoring the others with the given set of constraints. The solution thus obtained is the idle solution. The payoff matrix constructs using idle solutions. Finally, the payoff matrix helps to construct the aspiration level to each objective function.

Step 2: The aspiration level of the objective function is set as the goal value $(g_k, k = 1, 2, 3)$.

Find $X = (x_1, x_2, \dots, x_n)$ to optimize the following fuzzy goals

$$Z_1(X) \preceq g_1, Z_2(X) \preceq g_2, Z_3(X) \preceq g_3$$

Subject to the constraint

$$\begin{aligned} \sum_{j=1}^{10} P_j x_j \geq P_{Min}, \sum_{j=1}^{10} P_j x_j \leq P_{Max}, \sum_{j=1}^{10} V_{Bj} x_j \geq V_{BMin}, \sum_{j=1}^{10} V_{Bj} x_j \leq V_{BMax} \\ \sum_{j=1}^{10} V_{Cj} x_j \geq V_{CMin}, \sum_{j=1}^{10} V_{Cj} x_j \leq V_{CMax}, \sum_{j=1}^{10} C_{lj} x_j \geq C_{lMin}, \sum_{j=1}^{10} C_{lj} x_j \leq C_{lMax} \\ \sum_{j=1}^n S_{Fj} x_j \leq S_{FMax} \quad x_j \geq 0, \quad x_j \leq 4 \quad \forall j = 1, 2, \dots, 10 \end{aligned}$$

where, $(g_1) = \text{Min}(Z_1(x))$, $(g_2) = \text{Min}(Z_2(x))$ and $(g_3) = \text{Min}(Z_3(x))$. The symbol ' \preceq ' (the type of fuzzy-min) referring to that $Z_1(X)$, $Z_2(X)$ and $Z_3(X)$ should be approximately less than or equal to the aspiration level g_1 , g_2 and g_3 up to the specified tolerance limit.

Step 3: Construct the fuzzy linear membership function the membership function of the fuzzy goal of $Z_1(X) \preceq g_1$ (i.e., fuzzy-min) as:

$$\mu_1(Z_1(X)) = \begin{cases} 1, & \text{if } Z_1(X) \leq g_1 \\ \frac{U_1 - Z_1(X)}{U_1 - g_1}, & \text{if } g_1 \leq Z_1(X) \leq U_1 \\ 0, & \text{if } Z_1(X) \geq U_1 \end{cases}$$

where, the upper tolerance limit for the fuzzy goal $Z_1(x)$ is U_1 .

Similarly, the membership function for the fuzzy goal $Z_2 \preceq g_2$ (i.e., fuzzy-min)

$$\mu_2(Z_2(X)) = \begin{cases} 1, & \text{if } Z_2(X) \leq g_2 \\ \frac{U_2 - Z_2(X)}{U_2 - g_2}, & \text{if } g_2 \leq Z_2(X) \leq U_2 \\ 0, & \text{if } Z_2(X) \geq U_2 \end{cases}$$

where, the upper tolerance limit for the fuzzy goal $Z_2(x)$ is U_2 .

Similarly, the membership function for the fuzzy goal $Z_3 \preceq g_3$ (i.e., fuzzy-min)

$$\mu_3(Z_3(X)) = \begin{cases} 1, & \text{if } Z_3(X) \leq g_3 \\ \frac{U_3 - Z_3(X)}{U_3 - g_3}, & \text{if } g_3 \leq Z_3(X) \leq U_3 \\ 0, & \text{if } Z_3(X) \geq U_3 \end{cases}$$

Where the upper tolerance limit for the fuzzy goal $Z_3(x)$ is U_3 .

Step 4: Finally, the mathematical form of all the above-given steps are summarised as:

$$\text{Max } D(\mu) = \mu_1(Z_1(X)) + \mu_2(Z_2(X)) + \mu_3(Z_3(X))$$

subject to constraint

$$\sum_{j=1}^{10} P_j x_j \geq P_{Min}, \sum_{j=1}^{10} P_j x_j \leq P_{Max}, \sum_{j=1}^{10} V_{Bj} x_j \geq V_{BMin}, \sum_{j=1}^{10} V_{Bj} x_j \leq V_{BMax}$$

$$\sum_{j=1}^{10} V_{Cj} x_j \geq V_{CMin}, \sum_{j=1}^{10} V_{Cj} x_j \leq V_{CMax}, \sum_{j=1}^{10} C_{lj} x_j \geq C_{lMin}, \sum_{j=1}^{10} C_{lj} x_j \leq C_{lMax}$$

$$\mu_1(Z_1(X)) = \frac{U_1 - Z_1(X)}{U_1 - g_1}, 0 \leq \mu_1(Z_1(X)) \leq 1, \mu_2(Z_2(X)) = \frac{U_2 - Z_2(X)}{U_2 - g_2}, 0 \leq \mu_2(Z_2(X)) \leq 1$$

$$\mu_3(Z_3(X)) = \frac{U_3 - Z_3(X)}{U_3 - g_3}, 0 \leq \mu_3(Z_3(X)) \leq 1, x_j \geq 0, x_j \leq 4, x_j \in \text{Integer } \forall j = 1, 2, \dots, n$$

$D(\mu)$ is called the fuzzy achievement function. Finally, we have a single objective problem that can be solved by using a suitable classical optimization technique.

IV. Numerical case study

A diet is required to propose by the dietician for the special needs of the patient. An integer number of units of the diet can be composed as ten basic foodstuffs termed as Food 1, Food 2, ..., Food 10. The values of protein, vitamin C, vitamin B6, saturated fat and calcium ideally fall between the bounds which are given in Table 2. The data is taken from Bhargava et al. [12].

Table 2: Nutritional and cost of the foodstuffs

Nutritions	Food Types										Daily Demand	
	1	2	3	4	5	6	7	8	9	10		
Protein (g)	3.3	25.5	2.5	11.0	27.3	3.3	1.2	1.2	2.6	0.4	40.0	15.0
Vit B ₆ (mg)	0.06	0.10	0.02	0.22	0.29	0.11	0.29	0.05	0.07	0.06	1.0	2.0
Vit C (mg)	1	0	0	0	0	44	11	7	10	6	50	100
Calcium(mg)	120	720	11	35	7	40	6	20	18	4	700	1000
Sat. Fat (g)	1.0	21.7	0.6	0.4	5.2	0.2	0.1	0.1	0.3	0.0	-	15
Carbohydrates (g)	5.0	0.1	0.0	75.7	0.0	1.1	23.2	2.6	30.9	11.8		
Cost (Rs.)	2.5	15.0	3.3	1.8	20.0	2.5	1.5	6.0	1.5	1.6		

The bounds for the three objective functions are as: $29.9 \leq Z_1 \leq 54.5$, $5.7 \leq Z_2 \leq 7.8$, and $161.3 \leq Z_3 \leq 366.7$. Using these bounds, the corresponding linear membership functions for the three objective functions are constructed as follows:

$$\mu_1(Z_1(X)) = \begin{cases} 1, & \text{if } Z_1(X) \leq 29.9 \\ \frac{54.5 - Z_1(X)}{54.5 - 29.9}, & \text{if } 29.9 \leq Z_1(X) \leq 54.5 \\ 0, & \text{if } Z_1(X) \geq 54.5 \end{cases}$$

$$\mu_2(Z_2(X)) = \begin{cases} 1, & \text{if } Z_2(X) \leq 5.7 \\ \frac{7.8 - Z_2(X)}{7.8 - 5.7}, & \text{if } 5.7 \leq Z_2(X) \leq 7.8 \\ 0, & \text{if } Z_2(X) \geq 7.8 \end{cases}$$

$$\mu_3(Z_3(X)) = \begin{cases} 1, & \text{if } Z_3(X) \leq 161.3 \\ \frac{366.7 - Z_3(X)}{366.7 - 161.3}, & \text{if } 161.3 \leq Z_3(X) \leq 366.7 \\ 0, & \text{if } Z_3(X) \geq 366.7 \end{cases}$$

Using the method defines in Section 3; we calculate the compromise solution for the model, which is given in table 3.

Table 3: Compromise solution

Objective Values	The optimal number of quantities of each Foodstuff
$Z_1=33.0, Z_2=6.9, Z_3=251.9$	$x_1=4, x_2=0, x_3=2, x_4=3, x_5=0, x_6=2, x_7=0, x_8=1, x_9=0, x_{10}=0$

This solution is accepted by the DM, which belongs to the preferred compromise solution of fuzzy acceptance rate 0.620488. The membership values with the percentile contribution of each objective are shown in Fig. 1.

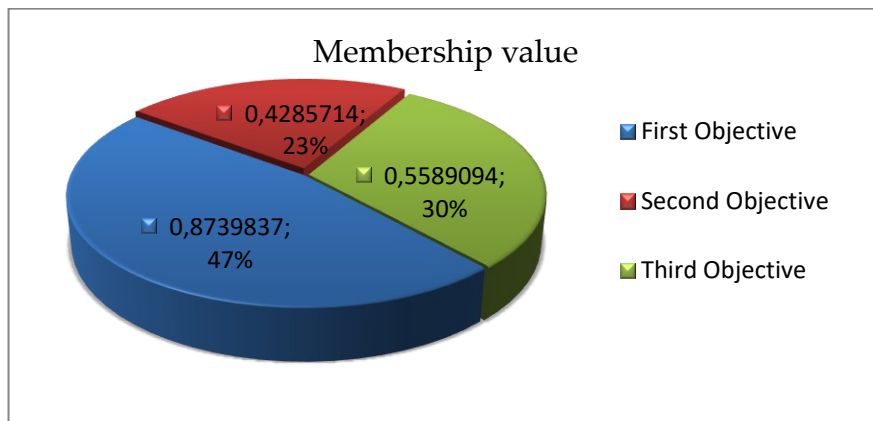


Fig. 1: Membership values of the objective with percentile contribution

Table 4: Comparison with weighted, pre-emptive and fuzzy gp

Method	Units of Foodstuff										P	Nutrition Quantity					Cost
	1	2	3	4	5	6	7	8	9	10		C_d	S_{Fl}	V_B	V_C	C_l	
Weighted GP	4	0	3	2	0	1	0	3	3	0	57.4	273.0	8.0	1.21	99	737	48.5
Pre-emptive GP	4	0	0	3	0	1	0	4	0	0	54.3	258.6	5.8	1.21	76	705	41.9
Fuzzy GP	4	0	2	3	0	2	0	1	0	0	59.0	251.9	6.9	1.21	99	707	33.0

The optimal compromise values of each foodstuffs with Weighted, Pre-emptive and Fuzzy GP are shown in Fig. 2.

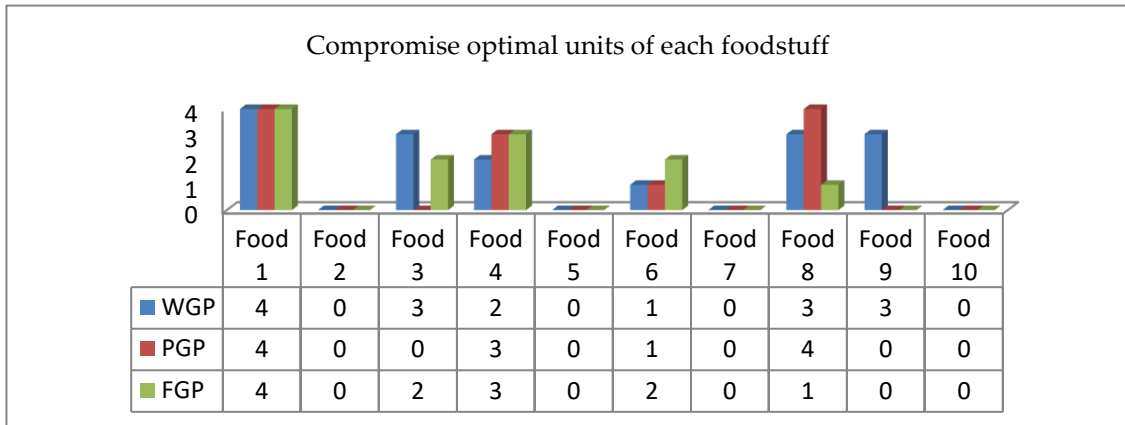


Fig 2: Compromise optimal unit for foodstuff

The comparison of membership values of each objective solved by Pre-emptive and Fuzzy GP are shown in Fig. 3.

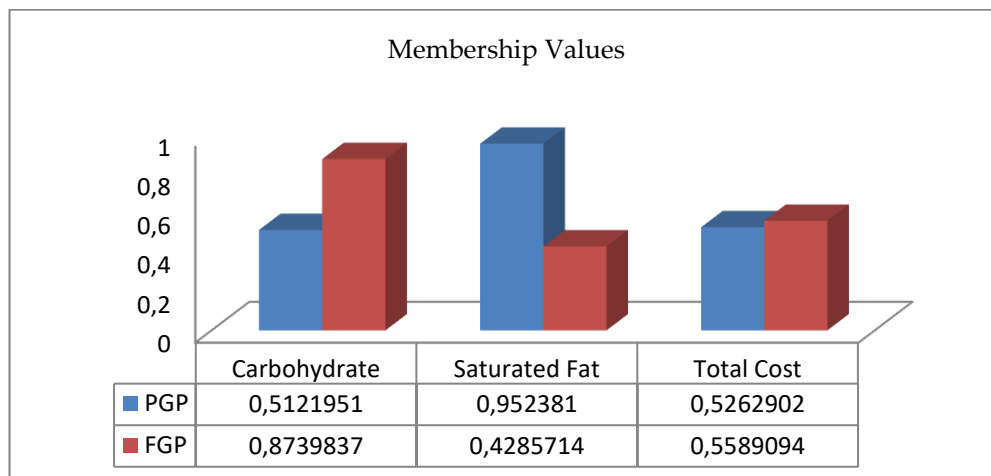


Fig 3: Graphical representation of membership values

V. Conclusion

The human body needs foods with a low content of Saturated Fat and Carbohydrate, although high in Protein, Vitamin B₆, Vitamin C, and Calcium. The multi-objective daily diet model is solved by using pre-emptive and fuzzy GP to satisfy daily nutrients through an example. We have designed the objective function as minimizing the total cost of the diet, Saturated Fat and carbohydrate. We use this approach to select diets and meet precise nutritional requirements. The comparison for the daily diet plan with weighted GP, pre-emptive GP and fuzzy GP is also shown. The finding obtained in the fuzzy programming approach has been contrasted with the weighted GP approach and the pre-emptive GP approach, and it demonstrates that the fuzzy GP approach gives a more precise and accurate solution and is a useful technique.

References

- [1] Smith, V. E. (1959). Linear programming models for the determination of palatable human diets. *Journal of Farm Economics*, 41(2), 272-283.
- [2] Anderson, A. M., & Earle, M. D. (1983). Diet planning in the third world by linear and goal programming. *Journal of the Operational Research Society*, 34(1), 9-16.
- [3] A. Charnes, & Cooper, W. W. (1961). *Management Models and Industrial Applications of linear programming*. J. Wiley.
- [4] Ignizio, J. P. (1976). *Goal Programming and Extensions* Lexington Books. Toronto: DC Health and Company.
- [5] Eghbali, H., Abdoos, E., Ashtiani, S., & Ahmadvand, M. (2013). Modelling the optimal diet problem for renal patients with fuzzy analysis of nutrients. *International Journal of Management and Fuzzy Systems*, 1(1), 7-14.
- [6] Nath, T., & Talukdar, A. (2014). Linear programming technique in fish feed formulation. *International Journal of Engineering Trends and Technology*, 11(17), 132-135.
- [7] Eghbali, H., Eghbali, M. A., & Kamyad, A. V. (2012). Optimizing human diet problem based on price and taste using multi-objective fuzzy linear programming approach. *An International Journal of Optimization and Control: Theories & Applications (IJOCTA)*, 2(2), 139-151.
- [8] Mamat, M., Zulkifli, N. F., Deraman, S. K., & Noor, N. M. M. (2012). Fuzzy linear programming approach in balance diet planning for eating disorder and disease-related lifestyle. *Applied Mathematical Sciences*, 6(103), 5109-5118.
- [9] Eghbali-Zarch, M., Tavakkoli-Moghaddam, R., Esfahanian, F., Azaron, A., & Sepehri, M. M. (2017). A New Multi-objective Optimization Model for Diet Planning of Diabetes Patients under Uncertainty. *Health Education and Health Promotion*, 5(3), 37-55.
- [10] Sheng, L. Z., & Sufahani, S. (2018, April). Optimal Diet Planning for Eczema Patient Using Integer Programming. In *Journal of Physics: Conference Series* (Vol. 995, No. 1, p. 012049). IOP Publishing.
- [11] Ali, M., Sufahani, S., & Ismail, Z. (2016). A new diet scheduling model for Malaysian school children using zero-one optimization approach. *Global Journal of Pure and Applied Mathematics*, 12(1), 413-419.
- [12] Bhargava, A. K., Bansal, D., Chandramouli, A. B., & Kumar, A. (2011). Weighted Goal Programming Model Formulation and Calculation of Diet Planning. *International Transactions in Mathematical Sciences & Computer*, 4(1).
- [13] Decerle, J., Grunder, O., El Hassani, A. H., & Barakat, O. (2019). A memetic algorithm for multi-objective optimization of the home health care problem. *Swarm and evolutionary computation*, 44, 712-727.
- [14] Nguyen, T. V. L., & Montemanni, R. (2013). Scheduling and routing in-home health care service. In *Society 40th Anniversary Workshop-FORS40* (p. 52).
- [15] En-nahli, L., Allaoui, H., & Nouaouri, I. (2015). Multi-objective modelling to human resource assignment and routing problem for home health care services. *IFAC-PapersOnLine*, 48(3), 698-703.
- [16] Mutingi, M., & Mbohwa, C. (2013, July). A home healthcare multi-agent system in a multi-objective environment. In *25th Annual Conf of the SAIIE* (pp. 1-10).
- [17] Niakan, F., & Rahimi, M. (2015). A multi-objective healthcare inventory routing problem; a fuzzy possibilistic approach. *Transportation Research Part E: Logistics and Transportation Review*, 80, 74-94.
- [18] Othman, S. B., Hammadi, S., & Quilliot, A. (2015). Multi-objective evolutionary for multi-skill health care tasks scheduling. *IFAC-PapersOnLine*, 48(3), 704-709.
- [19] Turgay, S., & Taşkın, H. (2015). Fuzzy goal programming for health-care organization. *Computers & Industrial Engineering*, 86, 14-21.

- [20] Zhang, W., Cao, K., Liu, S., & Huang, B. (2016). A multi-objective optimization approach for health-care facility location-allocation problems in highly developed cities such as Hong Kong. *Computers, Environment and Urban Systems*, 59, 220-230.