Inverse Ishita Distribution: Properties and Applications

Kamlesh Kumar Shukla
•
Department of Community Medicine, NIIMS, Noida
International University, Greater Noida, India
Email: kkshukla22@gmail.com

Abstract

In this paper, new lifetime distribution has been proposed which is an inverse of Ishita distribution Shanker and Shukla (2017). Its statistical and mathematical properties including Reneyi entropy and Stress-strength reliability measure have been discussed. Maximum likelihood estimation method has been used to estimate its parameter. Simulation study has also been carried out to check the behavior of maximum likelihood estimator. Finally, proposed distribution has been applied on lifetime datasets and compares its superiority over other inverse lifetime distributions.

Keywords: Inverse distribution, Stress-strength Reliability, Maximum likelihood estimation

I. Introduction

Shanker and Shukla (2017) proposed a one parameter lifetime distribution with the following probability density function (pdf) and cumulative distribution function (cdf) respectively:

$$f_1(y;\theta)=\frac{\theta^3}{\theta^3+2}\left(\theta+y^2\right)e^{-\theta x} \quad ;y>0,\ \theta>0 \tag{1.1}$$

$$F_1(y;\theta)=1-\left[1+\frac{\theta y(\theta y+2)}{\left(\theta^3+2\right)}\right]e^{-\theta y} \ ;y>0,\theta>0 \tag{1.2}$$

This distribution is known as the Ishita distribution. The mathematical and statistical properties including its parameter estimation can be shown in Shanker and Shukla (2017). It's important applications from biological and engineering data have been described in their paper, and its superiority have been discussed over other one parameter life time classical distributions such as Lindley, Exponential distribution. Shukla (2019) has discussed and compared specially one parameter lifetime distribution including Lindley and Ishita distribution from Biological, Engineering, Agricultural and demographic data[2]. Lindley distribution proposed by Lindley (1958) and its pdf and cdf are defined respectively by

$$f_2(y;\theta)=\frac{\theta^2}{\theta+1}(1+y)e^{-\theta y} \quad ;y>0,\ \theta>0 \tag{1.3}$$

$$F_2(y;\theta)=1-\left[1+\frac{\theta y}{(\theta+1)}\right]e^{-\theta y} \ ;y>0,\theta>0 \tag{1.4}$$

Detailed about its mathematical and statistical properties including hazard rate and application from lifetime data have been discussed by Gitany et al (2008). The inverse Lindley distribution has been proposed by Sharma et al (2015) which is an inverse of Lindley distribution (ILD). It can be shown its important statistical properties, stress and strength reliability measure including its stress and strength parameter estimation method on real lifetime data in their paper. Its pdf and cdf are given respectively by

$$f_3\left(x;\theta\right)=\frac{\theta^2}{(\theta+1)x^3}\left(1+x\right)e^{-x/\theta}\quad ;x>0,\ \theta>0 \tag{1.5}$$

$$F_3\left(x;\theta\right)=1+\frac{\theta/x}{\left(\theta+1\right)}e^{-x/\theta}\ ;x>0,\theta>0 \tag{1.6}$$

In another study, Keller and Kamath (1982) introduced Inverse exponential distribution (IED) and it has been studied and discussed as a lifetime model. Its pdf and cdf are given respectively by

$$f_4\left(x;\theta\right)=\frac{\theta}{x^2}e^{-x/\theta}\quad ;x>0,\ \theta>0 \tag{1.7}$$

$$F_4\left(x;\theta\right)=e^{-x/\theta}\ ;x>0,\theta>0 \tag{1.8}$$

The main motivation of this paper is to introduce a new life time distribution are:

(i) It is observed that Ishita distribution is more flexible distribution than Lindley and exponential distribution especially for biological data.

(ii) Inverse of Ishita distribution would also be more flexible and gives good fit over Inverse Lindley distribution as well as exponential distribution.

The study has been divided into twelve sections, introduction of proposed study are discussed in the first section. Inverse Ishita distribution has been defined in the second section. Survival and hazards rate function are discussed in third section. Moment has been derived in fourth section. In the fifth section, stochastic ordering has been discussed. Reneyi Entropy measure has been discussed in sixth section. Order statistics of proposed distribution has been discussed in seventh section. Stress-Strength reliability measure has been derived in the eighth section. Maximum likelihood estimation method has been derived for estimation parameter of proposed distribution in ninth section. Simulation study for proposed distribution has been carried out in the tenth section. In the eleventh section, application of proposed distribution on real lifetime data has been presented. Conclusions have been given in the last section.

II. Inverse Ishita distribution

If a random variable Y has an Ishita distribution, the variable $X=\frac{1}{Y}$ will have an Inverse Ishita distribution (IID) of equation (1.1). A random variable X is said to have an Inverse Ishita distribution with scale parameter θ and its pdf and cdf are defined respectively by;

$$f_5\left(x;\theta\right)=\frac{\theta^3}{(\theta^3+2)x^4}\left(1+\theta x^2\right)e^{-\theta/x}\quad ;x>0,\ \theta>0 \tag{2.1}$$

$$F_5\left(x;\theta\right)=1+\frac{\frac{\theta}{x}\left(\frac{\theta}{x}+2\right)}{\left(\theta^3+2\right)}e^{-\theta/x}\ ;x>0,\theta>0 \tag{2.2}$$

The behavior of proposed distribution for varying value of θ has been presented in figure1. It is observed from figure1 that pdf of IID is decreasing as increased value of parameter θ

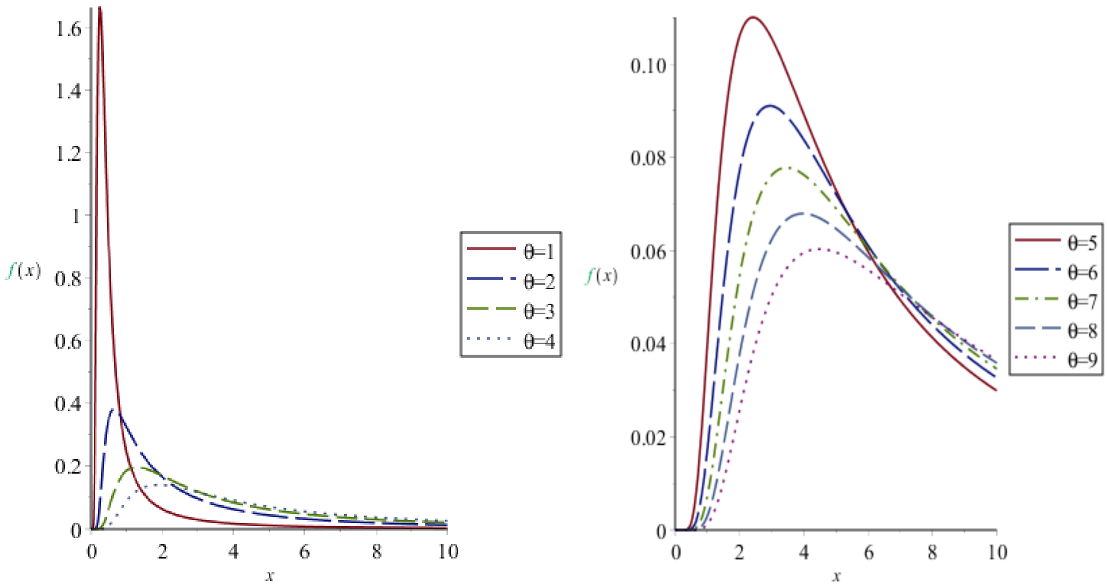

Figure1. *pdf of IID for varying value of* θ

III. Survival and Hazards function

Survival function $S\left(x;\theta\right)$ of IID can be defined as

$$S\left(y;\theta\right)=1-F_5(y;\theta)$$

$$S\left(x;\theta\right)=1-\left(1+\frac{\frac{\theta}{x}\left(\frac{\theta}{x}+2\right)}{\left(\theta^3+2\right)}\right)e^{-\theta/x}\ ;x>0,\theta>0$$

And hazard function $h\left(x;\theta\right)$ of IID can be defined as

$$h\left(x;\theta\right)=\frac{f_5(x;\theta)}{S(x;\theta)}$$

$$h\left(x;\theta\right)=\frac{\theta^3\left(1+\theta x^2\right)e^{-\theta/x}}{\left[x^2(\theta^3+2)-\left(x^2(\theta^3+2)+2\theta x+\theta^2\right)e^{-\theta/x}\right]}$$

The nature of survival and hazard function of IID for varying value of parameter θ are presented in figure2&3 respectively. Hazard function of IID distribution is also uni-model in x, and achieves its maxi-mum value at x_0. The turning point (x_0) of hazard function can be obtained as the solution of the following equation.

$$x_0(\theta^3+2)e^{\frac{\theta}{x_0}}\left(\frac{x_0^{\,2}\theta^2}{2}-\theta x_0^{\,3}+\frac{\theta}{2}-2x_0\right)+\theta x_0^{\,4}(\theta^3+2)+\theta^2 x_0^{\,3}$$
$$+2x_0^{\,2}(\theta^3+2)+3x_0\theta+\theta^2=0$$

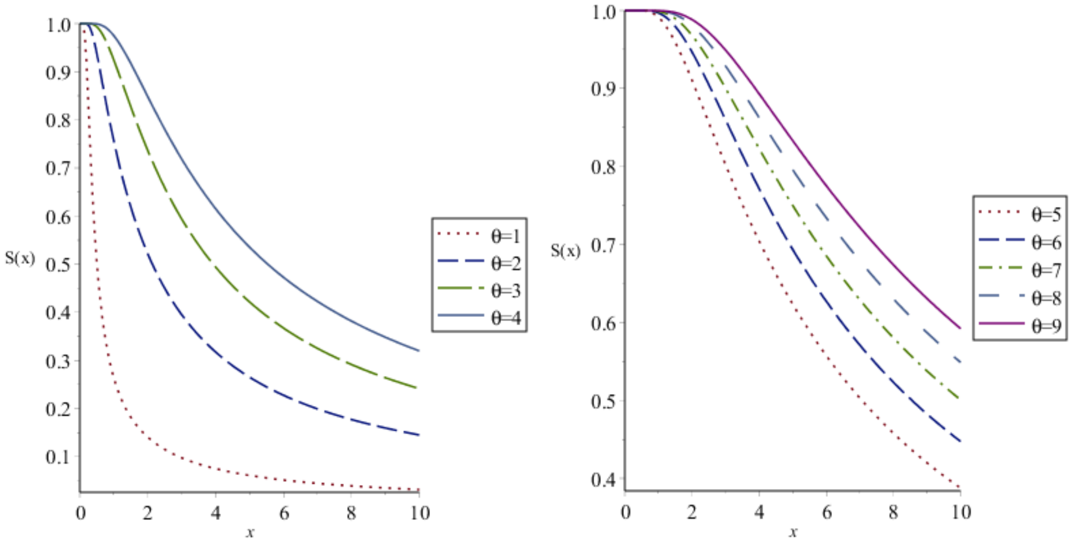

Figure2. *Survival function of IID for varying value of* θ

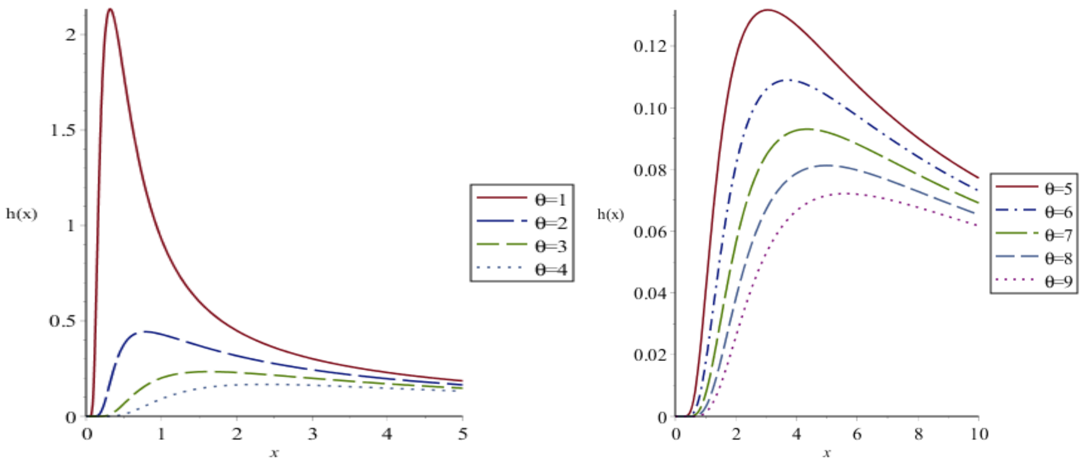

Figure3. *Hazard function of IID for varying value of* θ

IV. Moments

Moments of a distribution are used to study the most important characteristics of the distribution including mean, variance, skewness, kurtosis, etc. The r th moment about origin μ_r' of IID can be expressed in explicit expression in terms of complete gamma functions.

Theorem1: Suppose Y_i follows IID (θ). Then the r th moment about origin μ_r' of IID is

$$\mu_r'=\frac{\gamma(3-r)}{\theta^{-r}\left[(\theta^3+2)\right]};r\le 2 \tag{4.1}$$

Considering (2.1), we have

$$\mu_r' = \int_0^\infty x^r \cdot \frac{\theta^3}{(\theta^3+2)x^4}\left(\theta x^2+1\right)e^{-\theta/x}dx$$
$$= \frac{\theta^3}{(\theta^3+2)}\left[\theta\int_0^\infty x^{r-2}e^{-\theta/x}dx + \int_0^\infty x^{r-4}e^{-\theta/x}dx\right]$$
$$= \frac{\theta^3}{(\theta^3+2)}\left[\theta\int_0^\infty x^{r-1-1}e^{-\theta/x}dx + \int_0^\infty x^{r-3-1}e^{-\theta/x}dx\right] \tag{4.2}$$

Using inverse gamma function $\int_0^\infty y^{n-\alpha-1}e^{-\theta/y}dy = \frac{\gamma(\alpha-n)}{\theta^{\alpha-n}}$, (4.2) can be written as

$$\mu_r' = \frac{\gamma(3-r)}{\theta^{-r}\left[(\theta^3+2)\right]} \tag{4.3}$$

It will exist if $r \le 2$, therefor only mean and Variance can be calculated substituting $r = 1,2$ in (4.3)

Mean and Variance are $\frac{\theta}{(\theta^3+2)}$, $\frac{\theta^2(\theta^3+1)}{(\theta^3+2)^2}$

Moments (4.3) does not exist at $r > 2$, therefore only coefficient of variance and Index of dispersion can be calculated of IID, which are $\sqrt{(\theta^3+1)}$ and (θ^3+1) respectively. Natures of mean and variance of IID are presented in figure5:

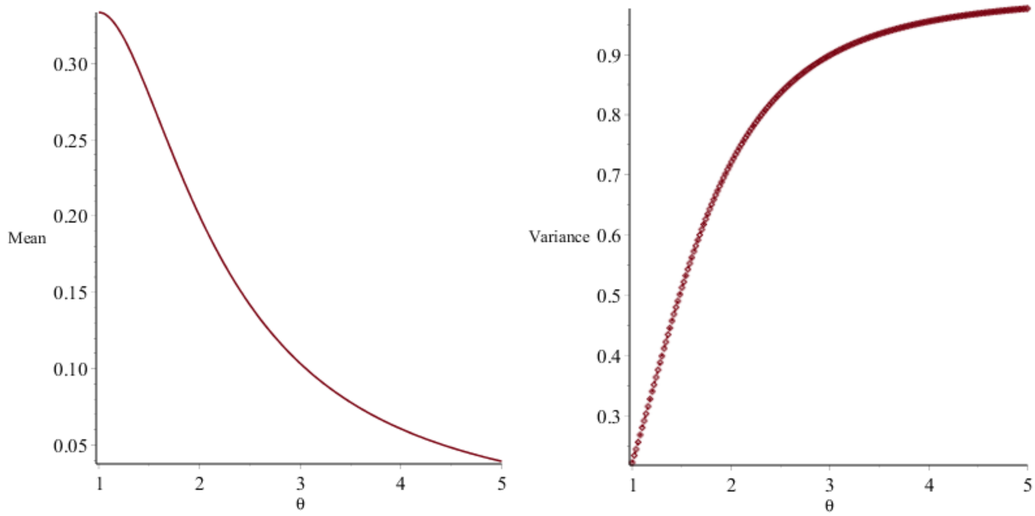

Figure4. *Mean Variance of IID for varying value of* θ

V. Stochastic Ordering

Stochastic ordering of positive continuous random variables is an important tool for judging their comparative behavior. A random variable X is said to be smaller than a random variable Y in the

(i) stochastic order $\left(X \le_{st} Y\right)$ if $F_X\left(x\right) \ge F_Y\left(x\right)$ for all x

(ii) hazard rate order $\left(X \le_{hr} Y\right)$ if $h_X\left(x\right) \ge h_Y\left(x\right)$ for all x

(iii) mean residual life order $\left(X \leq_{mrl} Y\right)$ if $m_X\left(x\right) \leq m_Y\left(x\right)$ for all x

(iv) likelihood ratio order $\left(X \leq_{lr} Y\right)$ if $\frac{f_X\left(x\right)}{f_Y\left(x\right)}$ decreases in x.

The following results due to Shaked and Shanthikumar(1994) are well known for establishing stochastic ordering of distributions

$$X \leq_{lr} Y \Rightarrow X \leq_{hr} Y \Rightarrow X \leq_{mrl} Y \tag{5.1}$$
$$\Downarrow$$
$$X \leq_{st} Y$$

The IID is ordered with respect to the strongest 'likelihood ratio ordering' as shown in the following theorem:

Theorem2: Let X and Y ~ IID$\left(\theta_1\right)$ and$\left(\theta_2\right)$ respectively. If $\theta_1 > \theta_2$ then $X \leq_{lr} Y$ and hence $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$.

Proof: We have

$$\frac{f_X(x)}{f_Y(x)} = \frac{\theta_1^3\left(\theta_2^3+2\right)}{\theta_2^3\left(\theta_1^3+2\right)}\left(\frac{\theta_1 x^2+1}{\theta_2 x^2+1}\right)e^{-(\theta_1-\theta_2)/x} \quad ; x>0$$

Now

$$\log\frac{f_X(x)}{f_Y(x)} = \log\left[\frac{\theta_1^3\left(\theta_2^3+2\right)}{\theta_2^3\left(\theta_1^3+2\right)}\right]+\log\left(\frac{\theta_1 x^2+1}{\theta_2 x^2+1}\right)-\left(\theta_1-\theta_2\right)/x.$$

.

This gives $$\frac{d}{dx}\log\frac{f_X(x)}{f_Y(x)} = \frac{2x(\theta_1-\theta_2)}{(\theta_2 x^2+1)^2}+\frac{\left(\theta_1-\theta_2\right)}{x^2}$$

$$\frac{d}{dx}\log\frac{f_X(x)}{f_Y(x)} = \frac{(2x+1)(\theta_1-\theta_2)}{x^2(\theta_2 x^2+1)^2} > 0, for \theta_1 > \theta_2$$

Thus if $\theta_1 > \theta_2$, $\frac{d}{dx}\log\frac{f_X(x)}{f_Y(x)} > 0$. This means that $X \leq_{lr} Y$ and hence $X \leq_{hr} Y$, $X \leq_{mrl} Y$ and $X \leq_{st} Y$.

VI. Order Statistics

Let $X_1, X_2, ..., X_n$ be a random sample of size n from IID (2.1). Let $X_{(1)} < X_{(2)} < ... < X_{(n)}$ denote the corresponding order statistics. The p.d.f. and the c.d.f. of the kth order statistic, say $Y = X_{(k)}$ are given by

$$f_Y\left(y\right) = \frac{n!}{\left(k-1\right)!\left(n-k\right)!}F^{k-1}\left(y\right)\left\{1-F\left(y\right)\right\}^{n-k} f\left(y\right)$$

$$= \frac{n!}{\left(k-1\right)!\left(n-k\right)!}\sum_{l=0}^{n-k}\binom{n-k}{l}\left(-1\right)^l F^{k+l-1}\left(y\right)f\left(y\right)$$

and

$$F_Y(y)=\sum_{j=k}^{n}\binom{n}{j}F^j(y)\{1-F(y)\}^{n-j}$$

$$=\sum_{j=k}^{n}\sum_{l=0}^{n-j}\binom{n}{j}\binom{n-j}{l}(-1)^l F^{j+l}(y),$$

respectively, for $k=1,2,3,...,n$.

Thus, the pdf and the cdf of kth order statistics of IID are obtained as

$$f_Y(y)=\frac{n!\theta^3\left(1+\theta x^2\right)e^{-\theta/x}}{\left(\theta^3+2\right)(k-1)!(n-k)!}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\times\left[1+\frac{\theta/x\left(\theta/x+2\right)}{\theta^3+2}e^{-\theta/x}\right]^{k+l-1}$$

and

$$F_Y(y)=\sum_{j=k}^{n}\sum_{l=0}^{n-j}\binom{n}{j}\binom{n-j}{l}(-1)^l\left[1+\frac{\theta/x\left(\theta/x+2\right)}{\theta^3+2}e^{-\theta/x}\right]^{j+l}$$

VII. Renyi Entropy

An entropy of a random variable X is a measure of variation of uncertainty. A popular entropy measure is Renyi (1961). If X is a continuous random variable having probability density function $f(.)$, then Renyi entropy is defined as

$$T_R(\gamma)=\frac{1}{1-\gamma}\log\left\{\int f^{\gamma}(x)\,dx\right\}$$

where $\gamma>0$ and $\gamma\neq1$.

Thus, the Renyi entropy of IID(2.1) can be obtained as

$$T_R(\gamma)=\frac{1}{1-\gamma}\log\left[\int_0^{\infty}\frac{\theta^{3\gamma}}{\left(\theta^3+2\right)^{\gamma}}\left(1+\theta x^2\right)^{\gamma}e^{-\theta\gamma/x}dx\right]$$

$$=\frac{1}{1-\gamma}\log\left[\int_0^{\infty}\frac{\theta^{3\gamma}}{\left(\theta^3+2\right)^{\gamma}}\left(1+\theta x^2\right)^{j}e^{-\theta\gamma/x}dx\right]$$

$$=\frac{1}{1-\gamma}\log\left[\int_0^{\infty}\frac{\theta^{3\gamma}}{\left(\theta^3+2\right)^{\gamma}}\sum_{j=0}^{\infty}\binom{\gamma}{j}\left(\theta x^2\right)^{j}e^{-\theta\gamma/x}dx\right]$$

$$=\frac{1}{1-\gamma}\log\left[\sum_{j=0}^{\infty}\binom{\gamma}{j}\frac{\theta^{3\gamma+j}}{\left(\theta^3+2\right)^{j}}\int_0^{\infty}e^{-\theta\gamma/x}\,x^{2j+2-1-1}dx\right]$$

$$= \frac{1}{1-\gamma} \log \left[\sum_{j=0}^{\infty} \binom{\gamma}{j} \frac{\theta^{3\gamma+j}}{\left(\theta^3+2\right)^{\gamma}} \frac{\Gamma(-2j-1)}{(\theta\gamma)^{-2j-1}} \right]$$

VIII. Stress-strength Reliability

The stress- strength reliability describes the life of a component which has random strength X that is subjected to a random stress Y. When the stress applied to it exceeds the strength, the component fails instantly and the component will function satisfactorily till $X > Y$. Therefore, $R = P(Y < X)$ is a measure of component reliability and in statistical literature it is known as stress-strength parameter.

Let X and Y be independent strength and stress random variables having IID(2.1) with parameter (θ_1) and (θ_2) respectively. Then the stress-strength reliability R can be obtained as

$$R = P(Y < X) = \int_0^{\infty} P(Y < X \mid X = x) f_X(x)\, dx$$

$$= \int_0^{\infty} f(x;\theta_1)\; F(x;\theta_2)\, dx$$

$$= \int_0^{\infty} \frac{\theta_1^3}{(\theta_1^3+2)x^4}\left(1+\theta_1 x^2\right) e^{-\theta_1/x} \left(1 + \frac{\frac{\theta_2}{x}\left(\frac{\theta_2}{x}+2\right)}{\left(\theta_2^3+2\right)} \right) e^{-\theta_2/x} dx$$

Using inverse gamma function, it can be written as

$$R = \frac{\theta_1^3 \begin{bmatrix} 2\theta_1\theta_2(\theta_1+\theta_2)^3 + 2\theta_1(\theta_1+\theta_2)^2(\theta_2^3+\theta_2^2+2) \\ 12\theta_2(\theta_1+\theta_2) + 24(\theta_2^3+\theta_2^2+2) \end{bmatrix}}{\left(\theta_1^3+2\right)\left(\theta_2^3+2\right)\left(\theta_1+\theta_2\right)^5}.$$

IX. Maximum Likelihood Estimation Method

Let $(x_1, x_2, x_3, \ldots, x_n)$ be a random sample of size n from (2.1). The likelihood function, L of IID is given by

$$L = \left(\frac{\theta^3}{(\theta^3+2)} \right)^n \prod_{i=1}^{n} \frac{\left(\theta x_i^2+1\right)}{x^4} e^{-\sum_i \theta/x}$$

The log likelihood function is thus obtained as

$$\ln L = n \ln\left(\frac{\theta^3}{(\theta^3+2)} \right) + \sum_{i=1}^{n} \ln(\theta x_i^2+1) - \sum_{i=1}^{n} \ln(x_i^4) - \theta \sum_{i=1}^{n} (1/x_i)$$

the maximum likelihood estimate $\hat{\theta}$ of parameter θ is the solution of the log-likelihood equation $\frac{\partial \log L}{\partial \theta} = 0$. It is obvious that $\frac{\partial \log L}{\partial \theta} = 0$ will not be in closed form and hence some numerical optimization technique can be used e the equation for θ. In this paper the nonlinear method available in R software has been used to find the MLE of the parameter θ.

X. Simulation Study

In this section, simulation study of proposed distribution has been carried out using acceptance and rejection method. To conduct simulation study, following steps have been follows: generate 1000 samples of size n = 20, 40, 60, 80 from the IID (2, 3, 4, and 5) compute the Bias and MSE using MLE$\left(\theta\right)$ and proposed value of parameter$\left(\theta\right)$

Table1. *Average bias error (ABE) and Mean square error (MSE)of parameter*

n	Parameter $\left(\theta\right)$	θ=2		θ=3		θ=4		θ=5	
		Bias	MSE	ABE	MSE	ABE	MSE	ABE	MSE
20	2	-0.02383	0.011362	-0.00135	3.63E-05	0.01573	0.00495	0.024822	0.01232
	3	-0.07383	0.109032	-0.05135	0.052731	-0.0342	0.02347	-0.02518	0.01267
	4	-0.12383	0.306701	-0.10135	0.205427	-0.0842	0.14200	-0.07518	0.11303
	5	-0.17383	0.604371	-0.15135	0.458122	-0.1342	0.36052	-0.12518	0.31339
40	2	-0.01157	0.005358	0.000535	1.15E-05	0.00947	0.00358	0.015616	0.00975
	3	-0.03657	0.053506	-0.02446	0.023940	-0.0155	0.00964	-0.00938	0.00352
	4	-0.06157	0.151654	-0.04946	0.097869	-0.0405	0.06570	-0.03438	0.04728
	5	-0.08657	0.299802	-0.07446	0.22179	-0.0655	0.17176	-0.05938	0.14105
60	2	-0.00932	0.005216	0.000379	8.62E-06	0.00682	0.00279	0.01061	0.0067
	3	-0.02599	0.04053	-0.01628	0.015917	-0.0098	0.00581	-0.00604	0.00219
	4	-0.04266	0.109177	-0.03295	0.06515	-0.0265	0.04216	-0.02271	0.03095
	5	-0.05932	0.211157	-0.04965	0.147734	-0.0431	0.11185	-0.03938	0.09305
80	2	0.007964	0.005074	0.000437	1.53E-05	0.00585	0.00274	0.008270	0.00547
	3	-0.00454	0.001646	-0.01206	0.011639	-0.0066	0.00353	-0.00422	0.00143
	4	-0.01704	0.023218	-0.02456	0.048264	-0.0191	0.02932	-0.01672	0.02238
	5	-0.02954	0.06979	-0.03706	0.109888	-0.0316	0.08012	-0.02922	0.06834

From the above table it is observed that most average bias errors are negative and it is decreasing as increased value of sample size.

XI. Application on real data

In this section, proposed distribution has been applied on two real datasets and compares with one parameter Inverse Lindley distribution (ILD) and Inverse Exponential distribution (IED).

Goodness of fit has been decided using Akaike information criteria (AIC), Bayesian Information and criteria (BIC) values respectively, which are calculated for each distribution and also compared. As we know that best goodness of fit of the distribution can be decided on the basis of minimum value of AIC and BIC. Comparison of distributions is shown in table 2 as well as their fitted plots are presented in figure5&6. Table 2 shows that AIC & BIC of IID, ILD and IED (three distributions) have been calculated and compared, and it is observed that IID has minimum value of AIC and BIC in comparison to ILD and IED. It can be say that it is good model and can be considered for life time data. From the figure 5&6, it can be seen that expected value of IID is much closed to probability of observed value on both the data sets in comparison to ILD and IED.

Data set 1: This data set represents remission times (in months) of a random sample of 128 bladder cancer patients reported in Lee and Wang (2003)

0.08,2.09,3.48,4.87,6.94,8.66,13.11,23.63,0.20,2.23,3.52,4.98,6.97,9.02,13.29,0.40,2.26,3.57,5.06,7.09,9.22, 13.80,25.74,0.50,2.46,3.64,5.09,7.26,9.47,14.24,25.82,0.51,2.54,3.70,5.17,7.28,9.74,14.76,6.1,0.81,2.62,3.82 ,5.32,7.32,10.06,14.77,32.15,2.64,3.88,5.32,7.39,10.34,14.83,34.26,0.90,2.69,4.18,5.34,7.59 10.66,15.96,36.66,1.05,2.69,4.23,5.41,7.62,10.75,16.62,43.01,1.19,2.75,4.26,

5.41,7.63,17.12,46.12,1.26,2.83,4.33,5.49,7.66,11.25,17.14,79.05,1.35,2.87,5.62,7.87,11.64,17.36,1.40,3.02, 4.34,5.71,7.93,11.79,18.10,1.46,4.40,5.85,8.26
11.98,19.13,1.76,3.25,4.50,6.25,8.37,12.02,2.02,3.31,4.51,6.54,8.53,12.03,20.28,2.02,3.36,6.76,12.07,21.73, 2.07,3.36,6.93,8.65,12.63,22.69

Data Set 2: This data set is given by Linhart and Zucchini (1986)which represents the failure times of the air conditioning system of an airplane:
23, 261 ,87 ,7, 120, 14 ,62, 47 ,225, 71 ,246 ,21, 42 ,20, 5, 12, 120 ,11, 3, 14, 71 ,11, 14, 11, 16, 90, 1, 16 ,52, 95

Table 2: *MLE's, Standard Errors, - 2ln L, AIC and BIC of the fitted distributions for data set*

Data	Distribution	Parameter	$-2\ln L$	AIC	BIC
1	**IID**	$\theta = 2.9017$	**913.28**	**915.28**	**916.72**
	ILD	$\theta = 3.0857$	931.72	933.72	935.15
	IED	$\theta = 2.4799$	915.65	917.65	919.08
2	**IID**	$\theta = 11.1891$	**317.99**	**319.99**	**321.33**
	ILD	$\theta = 12.0387$	318.53	320.53	921.96
	IED	$\theta = 11.1798$	918.12	320.12	321.56

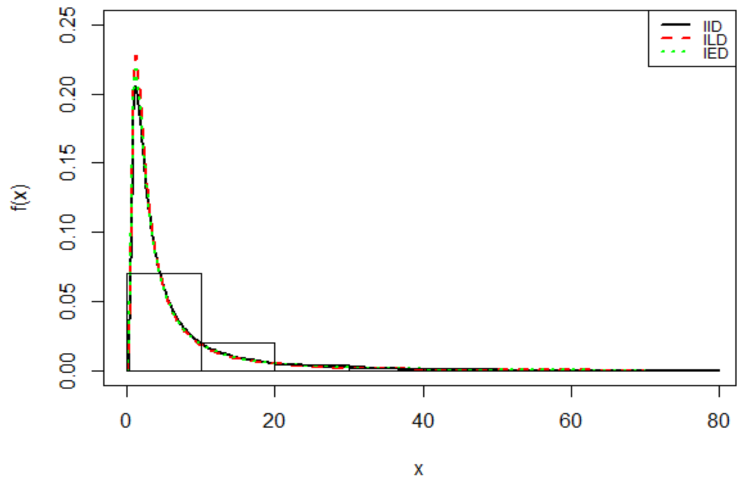

Figure 5. *Fitted plot of distributions for dataset-1*

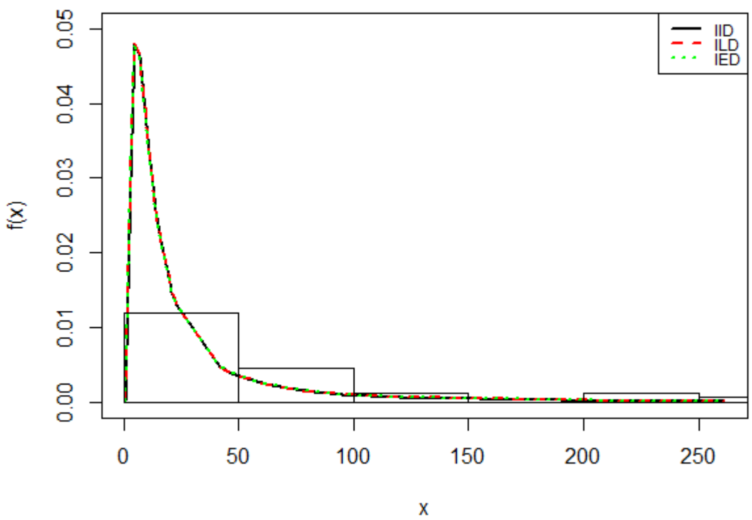

Figure6. *Fitted plot of distributions for dataset-2*

XII. Conclusions

A new lifetime distribution has been proposed. Statistical and mathematical properties including Renyi Entropy and Stress-Strength reliability have been derived. Simulation study of proposed distribution has been carried out to know the behavior of MLE estimate of parameter. Maximum Likelihood Estimation (MLE) Method has been used to estimate its parameter. The Proposed distribution has been applied on two real data sets and compare with two one parameter ILD and IED. It is observed from the table2 that IID gives better fit over both distributions on both the data sets.

Acknowledgement

Author is thankful to the chief editor as well as reviewer for the valuable comments to improve the quality of paper.

References

[1] Ghitany, M.E., Atieh, B. and Nadarajah, S. (2008): Lidley distribution and its Application, Mathematics Computing and Simulation, 78, 493 – 506.

[2] Keller, A. Z., & Kamath, A. R. (1982). Reliability analysis of CNC machine tools. *Reliability Engineering*, 3, 449–473. doi:10.1016/0143-8174(82)90036-1

[3] Lee , ET and Wang JW. Statistical methods for survival data analysis, 3rd edition, John Wiley and Sons, New York, NY, USA, 2003.

[4] Lindley, D. V. (1958), Fiducial distributions and Bayes' theorem, Journal of the Royal Statistical Society, Series B, 20, 102- 107.

[5] Linhart, H. and Zucchini, W. Model Selection, John Wiley, New York, 1986.

[6] Renyi, A. (1961): On measures of entropy and information, in proceedings of the 4th Berkeley symposium on Mathematical Statistics and Probability, 1, 547 – 561, Berkeley, university of California press.

[7] Shaked, M. and Shanthikumar, J.G.(1994): Stochastic Orders and Their Applications, Academic Press, New York.

[8] Sharma, V. K., Singh, S. K., Singh, U., and Agiwal, V. (2015), The inverse Lindley distribution: a Stress-strength reliability model with application to head and neck cancer data, Journal of Industrial and Production Engineering, 2015 Vol. 32, No. 3, 162–173, ttp://dx.doi.org/10.1080/21681015.2015.1025901

[9] Shukla, K. K. (2019), A comparative study of one parameter distribution Biom & Biostats International J. 8(3),111-123.

[10] Shanker, R. and Shukla, K. K. (2017), "Ishita Distribution and its application to model lifetime data", Biometrics & Biostatistics International Journal,5(2),1-9