Designing of Special Type Double Sampling Plan for Life Tests Based on Percentiles Using Exponentiated Frechet Distribution

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Abstract

Acceptance Sampling performs a major role in industry for monitoring process and assessing complains for reducing the cost and inspection time. A truncated life test may be conducted to determine the smallest sample size to ensure certain percentile life time of an items. The operating characteristic function values are obtained according to different quality levels and the minimum ratios of the true average life to the specified average life at the specified producer's risk are derived. Useful tables are provided for the proposed sampling plan.

Keywords: Special Type Double Sampling Plan, Exponentiated Frechet Distribution, Percentile

I. Introduction

In numerous statistical quality control (SQC) and reliability studies, acceptance sampling plans are executed to achieve the satisfactory inferential idea about a product. The acceptance sampling plans assosciated with accepting or rejecting a large-sized lot of products or items on the basis of the quality of products in a sample which are taken from lot. An acceptance sampling is a specified plan that determines the smallest sample size to be used for testing. If the quality characteristics of product follows a lifetime of an item, then it is termed as acceptance sampling technique for life test experiments or Reliability Sampling Plan. A common process in life testing is to terminate the life test by a predetermined time t_0 and mention the number of failures. One of the mission of these experiments is to set a lower confidence limit on the mean life. It is then to establish a specified mean life with a given probability of at least p* which provides protection to consumers.

Many studies has been carried out for the designing of single sampling plan and double sampling plans for truncated life tests under different statistical distributions for life time. Truncated life tests in the exponential distribution was first evaluated by Epstein (1954). Further, various authors developed truncated life test sampling plan using various distributions, such as Goode and Kao

(1961) using Weibull distribution, Gupta and Groll (1961) using gamma distribution, Kantam and Rosaiah (1998) using half logistic distribution, Kantam et al., (2001) using log-logistic distribution, Rosaiah and Kantam (2005) using inverse Rayleigh distribution, Balakrishnan et al., (2007) using generalized Birnbaum-Saunders distribution, Rosaiah et al., (2006) using exponentited log-logistic distribution etc.

Furthermore Lio et al (2009) have studied acceptance sampling plans from truncated life tests based on the assumptions of Brinbaum-Saunders distribution for percentiles and they also proposed that the acceptance sampling plan based on mean may not satisfy the demand of engineering on specific percentile of strength or the breaking stress. This describes the material strength of the products is declined significantly and may not meet consumer’s satisfactory level. Percentiles gives more knowledge regarding the life time distribution than the mean life does. When the life time distribution is symmetric, the 50th percentile or the median is similar to the mean life. Hence, developing acceptance sampling plans based on percentiles of life time distributions can be used as a generalization of the developing acceptance sampling plans based on the mean life of items or products.

Rao and Kantam (2010) developed Acceptance Sampling plans for truncated life tests based on the log-logistic distribution for percentiles. Rao *et al*. (2012) also developed the Acceptance Sampling plans for percentiles based on the Inverse Rayleigh Distribution. Rao *et al*.(2014) further proposed the Acceptance Sampling plans based on the percentiles of Exponentiated half log logistic distribution. The exponentiated Fréchet distribution has been used in various fields of research applications. Acceptance sampling plan based on truncated life tests for the exponentiated Fréchet distribution (EFD) using average life was discussed by Al-Nassar and Al-Omari (2013). Gui and Aslam (2015) discussed acceptance sampling plans based on truncated life tests for weighted exponential distribution. Govindaraju (1984) has developed the Special Type Double Sampling Plan. The main aim of this paper is to develop the Special Type Double Sampling Plan for percentiles the lifetime distribution follows Exponentiated Frechet Distribution.

II. Exponentiated Frechet Distribution (EFD)

The Exponentiated Frechet Distribution was introduced and studied quite extensively by Nadarajah and Kotz .The EFD is mentioned as the inverse of exponentiated Weibull distribution. The probability density function (pdf) and cumulative distribution function (cdf) of EFD are given as

$$f(t;\sigma,\lambda,\theta)=\sigma^{\lambda}\lambda\theta t^{-(1+\lambda)}\left[1-e^{-(\sigma/t)^{\lambda}}\right]^{\theta-1};\ t>0,\sigma,\lambda>0,\theta>0 \qquad (1)$$

$$F(t:\sigma,\lambda,\theta)=1-\left[1-e^{-(\sigma/t)^{\lambda}}\right]^{\theta};\quad t>0,\sigma,\lambda>0,\theta>0 \qquad (2)$$

where $\sigma>0$ is a scale parameter, $\lambda>0$ and $\theta>0$ are the parameters. When $\theta=1$ it is the particular case for standard Frechet Distribution.

The moment generating function of the Exponentiated Frechet Distribution is given by

$$M_x(t)=\sigma^{r}\theta\Gamma(\theta+1)\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{r=0}^{\infty}(-1)^{\theta-i-1}\frac{t^{r}(\theta-i-j)^{\frac{r}{\lambda}-1}\Gamma[(1-\frac{r}{\lambda}),x(\theta-i-j)]}{\Gamma(i+1)\Gamma(\theta-i+1)\Gamma(r+1)} \qquad (3)$$

Hazard Function

The hazard function for any distribution is given by

$$h(t)=\frac{f(t)}{1-F(t)} \qquad (4)$$

Thus the hazard function for Exponentiated Frechet Distribution is given by

$$h(t) = \frac{\sigma^{\lambda} \lambda \theta t^{-(1+\lambda)} [1 - e^{-(\sigma/t)^{\lambda}}]^{\theta - 1}}{[1 - e^{-(\sigma/t)^{\lambda}}]^{\theta}} \quad (5)$$

Percentile Estimator

The percentile estimator of any distribution is given by

$$P(t \le t_q) = q \quad (6)$$

The 100 q-th percentile of the EFD is as follows

$$t_q = \sigma \eta_q \quad (7)$$

t_q and q are directly proportional. Let

$$\eta_q = (-\ln(1 - (1-q)^{1/\alpha}))^{-(1/\lambda)} \quad (8)$$

Replacing the scale parameter $\sigma = t_q / \eta_q$

Hence, for fixed values $\lambda = \lambda_0$ and $\theta = \theta_0$, the quantile t_q is a function of the scale parameter σ, that is, $t_q \ge t_q^0 \Leftrightarrow \sigma \ge \sigma_0$

Where
$$\sigma_0 = \frac{t_q^0}{(-\ln(1 - (1-q)^{1/\alpha_0})^{-(1/\lambda_0)}}$$

σ_0 also depends on α_0 and λ_0, to build up acceptance sampling plans for the EFD ascertain $t_q \ge t_q^0$, equivalently that σ exceeds σ_0.

Replacing the scale parameter by $\sigma_0 = t_q^0 / \eta_q$, one can obtain the cumulative distribution function of Exponentiated Frechet Distribution is

$$F(t) = 1 - [1 - e^{-(t\eta_q / t_q)^{-\lambda}}]^{\theta} \quad (9)$$

Let $\delta = t / t_q$
$$F(t) = 1 - [1 - e^{-(\eta_q \delta_q^0 / (t_q / t_q^0))^{-\lambda}}]^{\theta} \quad (10)$$

Taking partial derivative with respect to δ

$$\partial F(t)/\partial \delta = \theta \lambda \eta_q (t_q^0 / t_q) [1 - e^{-(\eta_q \delta_q^0 / (t_q / t_q^0))^{-\lambda}}]^{\theta - 1} e^{-(\eta_q \delta_q^0 / (t_q / t_q^0))^{-\lambda}}$$

III. Designing of the Special Type Double Sampling under Life Time Distributions

Assume that a life test is conducted and will be terminated at time t_0. A probability P^* to reject a bad lot is used to protect consumers. A bad lot means that the true *100qth* percentile is below the supposed 100qth percentile that is, $t_q < t_q^0$. The lot is confirmed as a good one if the lifetime data hold the null hypothesis $H_0 : t_q \ge t_q^0$ against the alternative

$H_1 : t_q < t_q^0$. The consumer's risk $1 - p^*$ is used as the significance level for this hypothesis testing and p^* is the consumer's confidence level.

I. Operating Procedure

The operating procedure of special type double sampling plan for the truncated life test has the following steps:

* Take a random sample of size n_1 from the lot and put on the test for pre-assigned experimental time t_0 and observe the number of defectives d_1.
  If $d_1 > 1$ reject the lot.
* If d_1 = 0, draw a second random sample of size n_2 and put them on the test for time t_0 and observe the number of defectives d_2.
  If $d_2 \le 1$ accept the lot, Otherwise reject the lot.

In a special Type Double Sampling Plan the decision of acceptance is made only after inspecting the second sample. This aspect differs from usual double sampling plan in which decision of acceptance can be made even before the inspection of the second sample.

In Special Type Double Sampling Plan $n_1 and n_2$ denotes the sample size. For the proposed sampling plan the probability of acceptance is given by

$$p_a(p) = (1-p)^{n1+n2} + n_2 p (1-p)^{n-1}$$

The minimum sample size $n_1 and n_2$ can be calculated using the equation.

$$(1-p)^{n1+n2} + n_2 p (1-p)^{n-1} \le 1 - p^*$$

where, p is the probability that an item fails before t0, which is given by

$$F(t) = 1 - [1 - e^{-(\eta_q \delta_q^0 / (t_q / t_q^0))^{-\lambda}}]^{\theta}$$

Determination of the minimum sample sizes for special type double sampling plan reduces to

Minimize $$ASN = n_1 + n_2 (1-p)^n$$

Subject to

$$(1-p)^{n1+n2} + n_2 p (1-p)^{n-1} \le 1 - p^*$$

where n_1 and n_2 are integers. The minimum sample sizes satisfying the condition can be obtained by search procedure

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II. Construction of Tables

Step 1: Find the value of η for θ=2 and q=0.25.

Step 2: Set the evaluated η, $\lambda = 1,2$ and t/t_q =0.09, 0.11, 0.13, 0.15, 0.17, 0.19

Step 3: Find the smallest value for $n_1 and n_2$ satisfying $p_a(p) \le 1 - p^*$, where, p^* is probability of rejecting the bad lots.

Step 4: For the $n_1 and n_2$ value obtained find the ratio $d_{0.25}$ such that $p_a(p) \ge 1 - \alpha$, where $\alpha = 0.05$, p= $F(\frac{t}{t_q^0}, \frac{1}{d_q})$ and $d_q = t_q / t_q^0$.

Example

Consider the life time distribution as Exponentiated Frechet Distribution if the experimenter is interested in showing that the true unknown 25^{th} percentile life. Let the consumer risk be $1-p^{*}=0.05$. Suppose λ =2, θ=2, $\alpha=0.05, \beta=0.10$, $t_q/t_q^0=3, \delta_q^0=0.13$ and η can be calculated as 2.713, then the minimum sample size n_1 *and* n_2 can be obtained from table 1.The values of n_1 and n_2 can be obtained as 10 and 6 respectively. The operating characteristics values for the proposed sampling plan is obtained and tabulated in table 2. For n_1=10, n_2=5, $t_q/t_q^0=5$, $t/t_q^0=0.11$ and λ =2 then the operating characteristic function obtained from table 2 is 0.914607. In table 3, the respective ratio $d_{0.25}$ values is estimated by fixing the producer's risk, α = 0.05.

Table 1: *Minimum Sample Size values of n_1 and n_2 for the 25^{th} percentile of Special Type Double Sampling Plan under the assumption of Exponentiated Frechet Distribution.*

p^{*}	λ	t_q/t_q^0	t/t_q^0					
			0.09	0.11	0.13	0.15	0.17	0.19
0.99	1	2	12,11	9,6	7,4	6,4	6,2	5,1
		3	6,5	5,3	4,3	3,3	3,2	4,1
		5	4,1	3,2	3,1	3,1	3,1	3,1
	2	2	798,698	430,384	100,59	40,16	21,8	14,1
		3	420,376	69,55	26,15	12,9	10,1	7,1
		5	50,32	16,11	7,7	6,5	4,3	4,1
0.95	1	2	9,5	7,4	5,4	5,3	4,3	3,2
		3	8,7	6,4	4,3	3,3	3,2	2,2
		5	6,5	5,4	4,4	4,3	3,2	3,1
	2	2	227,196	57,32	22,12	13,4	8,3	3,2
		3	41,26	16,8	10,6	6,6	5,4	4,1
		5	10,5	6,4	4,3	3,2	2,2	2,1
0.90	1	2	11,9	7,5	5,5	5,4	4,4	3,2
		3	10,9	9,9	9,6	8,5	5,4	4,3
		5	9,9	8,6	5,4	4,3	2,2	2,1
	2	2	169,156	47,21	18,8	10,3	7,1	5,1
		3	31,21	13,5	6,4	4,3	3,2	3,1
		5	23,11	8,4	5,1	4,1	3,1	2,2
0.75	1	2	99,97	24,18	11,5	5,4	4,3	3,3
		3	4,4	4,4	4,3	3,3	3,2	2,2
		5	5,4	4,4	4,3	3,2	3,1	2,1
	2	2	102,94	25,17	11,5	5,4	4,1	3,1
		3	105,83	20,11	8,3	4,2	3,1	2,1
		5	5,3	3,1	2,2	2,1	2,1	2,1

Table 2: *Operating Characteristic value for the 25^{th} percentile of Special Type Double Sampling Plan under the assumption of Exponentiated Frechet Distribution.*

p^*	λ	n_1	n_2	t_q / t_q^0	t / t_q^0					
					0.09	0.11	0.13	0.15	0.17	0.19
0.99	1	12	11	2	0.605243	0.772442	0.945587	0.978034	0.993716	0.995808
		6	5	3	0.764119	0.873191	0.979135	0.982082	0.982126	0.98509
		4	1	5	0.858265	0.965345	0.97215	0.979003	0.979099	0.972293
	2	40	16	2	0.662366	0.893797	0.976779	0.989225	0.991036	0.993717
		26	15	3	0.784096	0.988032	0.988048	0.988048	0.988048	0.988048
		16	11	5	0.855601	0.985653	0.993700	0.994604	0.996402	0.996405
0.95	1	9	5	2	0.691921	0.993712	0.995503	0.995506	0.996402	0.997302
		8	7	3	0.779626	0.994879	0.980334	0.988358	0.99282	0.997302
		6	5	5	0.792806	0.994607	0.996402	0.997300	0.997302	0.9982
	2	22	12	2	0.563506	0.685675	0.891024	0.994600	0.995503	0.994604
		16	8	3	0.695503	0.859640	0.996402	0.997302	0.998200	0.994604
		10	5	5	0.791028	0.914607	0.996402	0.997302	0.9982	0.998201
0.90	1	11	9	2	0.491008	0.699190	0.791917	0.992815	0.995503	0.996402
		9	6	3	0.607231	0.790231	0.880355	0.994607	0.996402	0.997302
		8	6	5	0.889190	0.912811	0.995503	0.996402	0.998200	0.998201
	2	47	21	2	0.487946	0.692818	0.995508	0.996405	0.997302	0.998200
		31	21	3	0.611458	0.778502	0.950136	0.975503	0.996402	0.997300
		23	11	5	0.790827	0.877635	0.972136	0.985503	0.996405	0.997302
0.75	1	24	18	2	0.411092	0.678502	0.790136	0.995503	0.996402	0.997300
		4	4	3	0.590827	0.777635	0.890136	0.995503	0.996405	0.997302
		5	4	5	0.696400	0.896400	0.946402	0.997300	0.997302	0.998200
	2	25	17	2	0.407400	0.582110	0.769282	0.996404	0.997302	0.998201
		20	11	3	0.595503	0.696400	0.896402	0.997302	0.997302	0.998201
		5	3	5	0.695506	0.797302	0.940982	0.998201	0.998201	0.998201

Table 3: *The ratio* $d_{0.25}$ *for accepting the lot with the producer's risk of 0.05 when* $\theta = 2$

p^*	λ	t / t_q^0					
		0.09	0.11	0.13	0.15	0.17	0.19
0.99	2	2.344003	2.029148	1.027736	1.203372	1.251749	1.814854
	1	0.409894	0.396737	0.617704	0.464991	0.470858	0.486277
	0.75	0.379255	0.366778	0.326859	0.391388	0.295660	0.310311
	0.50	0.291685	0.270410	0.287141	0.312855	0.265462	0.262976
	0.25	0.217611	0.213765	0.228062	0.251749	0.258777	0.235822
0.95	2	3.628452	2.422327	1.74862	1.311969	1.02182	1.015266
	1	0.893549	0.670501	0.617704	0.535661	0.471674	0.501266
	0.75	0.628414	0.539050	0.475576	0.427177	0.387902	0.343006
	0.50	0.440772	0.400338	0.306644	0.340266	0.319427	0.253812
	0.25	0.258447	0.294061	0.228302	0.272212	0.238937	0.232853
0.90	2	2.422327	1.74862	1.311969	1.021820	1.501266	3.030129
	1	0.670501	0.617704	0.535661	0.471674	0.536410	0.697645
	0.75	0.539050	0.475576	0.427177	0.387902	0.253812	0.522105
	0.50	0.400338	0.306644	0.340266	0.319427	0.232853	0.383096
	0.25	0.294061	0.228302	0.272212	0.210886	0.207543	0.308925
0.75	2	1.679687	1.456120	1.835661	1.02182	1.190118	1.118228
	1	0.438826	0.514024	0.535661	0.470858	0.421973	0.553591
	0.75	0.398042	0.413224	0.316928	0.324011	0.359442	0.264479
	0.50	0.320421	0.335890	0.275242	0.319427	0.224150	0.238487
	0.25	0.245847	0.283336	0.238053	0.265462	0.206561	0.214106

IV. Conclusion

This paper indicates the Special Type Double Sampling Plan based on the percentiles of Exponentiated Frechet Distribution (EFD), when the life test is truncated for a pre-specified time. Special Type Double Sampling plan requires less Average Sample Number than the Single Sampling Plan results in the reduction of inspection cost. For these plans, the smallest sample sizes, which assured the median life time, is examined under the given consumer's risk and producer's risk. The industrial practitioner and the experimenter has to adopt the proposed plan to save the cost and time of the experiment. Extensive tables have been provided for the industrial use according to various parameters and percentile values.

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