Impact of Negative Arrivals and Multiple Working Vacation on Dual Supplier Inventory Model with Finite Lifetimes

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Abstract

In this paper we analyzed an inventory model with two-suppliers, finite life times, multiple working vacations and customers who arrive according to RCE process. Perishable and replenishment rates of two-suppliers are exponentially distributed. The server takes exponential working vacations when the queue is empty. Arrival process follows Poisson distribution and the probability for an ordinary customer is p and for negative customer is q. Limiting distribution of the assumed model is obtained. Numerical results are presented for cost function and various system performance parameters. The impact of two-suppliers on the optimal reorder points will be useful in developing strategies for handling various perishable inventory problems with replenishment rates.

Keywords: (s, S) policy, Two-supply inventory, Lead time, RCE process, Multiple working vacations, Matrix analytic method.

1 Introduction

In an (s, S) inventory policy, the quantity $Q(= S - s)$ is placed if inventory falls to s, so that the maximum inventory level is S. This policy has been widely discussed for almost a century. However in inventory models with more than one supplier we can improve the quality of service, develop strong relationship with the customers, reduce loss of sales due to stock shortages, enhanced profits, etc. In two-supply (s, S) inventory policy, two orders of quantities Q_1 and Q_2 are placed whenever inventory level drops to r and s respectively. For literature on inventory models with two supplies one can refer Yang and Wei-Chung [12] and Vijayashree and Uthaykumar [8].

The life time of inventory items is indefinitely long in many classic inventory models like, vegetables, food items, medical products, etc., which become unusable after a certain period. That means there exists a real - life inventory system which consists of products having a finite lifetime.

These type of products are called as perishable products and the corresponding inventory system can be considered as a perishable inventory system. Yonguri et. al. [13] studied an inventory models for perishable items with and without backlogging. "A deterministic perishable inventory model with time dependent demands is developed by Sushil and Ravendra [7]. Dinesh and Roberto [1] discussed a perishable inventory model with style goals."

"Sivakumar and Arivarignan [6] introduced negative customers in inventories with finite queues". "For more literature on this concept, one may refer Manual et al. [3, 4]."

In the working vacation (WV) model, the server without stopping the service completely, he continuous with lower service rate. "A continuous review inventory system with a single and multiple server vacations is given by Jayaraman et al. [2]. Periyasamy [5] discussed an inventory system with finite life times, retrial demands and multiple server vacations." For more literature on working vacations on may refer the papers by Vijaya Laxmi and Soujanya, [9, 10] and [11].

In the present paper, an inventory policy with two supply chains for replacement in which one having a lesser lead time is considered. Demands occur according to Poisson distribution. The arriving person may join the system with possibility p or remove one customer from the queue with probability q. The perishable, service rates in busy and WV are exponentially distributed. Limiting distributions are found. Several system performance parameters of the assumed model are presented. Also the analysis of the cost function is also carried out using direct search method.

2 Description of the model

In two supply inventory model, when inventory shrinks to $r(>\frac{S}{2})$, a quantity $Q_1(=S-r)$ is placed from the first supplier and is replenished with an exponential rate η_1. If it falls to $s(<Q_1)$ a quantity $Q_2(=S-s>s+1)$ is placed from another supplier and is replenished with an exponential rate $\eta_2(\eta_2>\eta_1)$. Arrival process follows Poisson distribution with rate λ. The arrived customer joins the queue with probability p and removes an existing customer from the end with probability $q(=1-p)$. Consider exponential Vacation time, service times during regular period and WV with rates β, μ_b and μ_v, respectively. We assume that, the server stays idle in any period at zero inventory level. Perishable rate follows exponential distribution with rate γ.

Let $N(t)$ be the length of the queue, $L(t)$ be the quantity of inventory and $\zeta(t)$ be the state of the server, which is defined as

$$\zeta(t)=\begin{cases}0, & server\ is\ active;\\ 1, & server\ is\ in\ WV.\end{cases}$$

It is clear that the Markov process $\{(N(t),\zeta(t),L(t)|t\geq 0\}$ is a state space model with

$$E=\{\{(i,j,k)|i\geq 0, j=1,2,0\leq k\leq S\}$$

Describe the order sets as

$$<i,j>=\begin{cases}((<i,j,k>))_{j=1,k=1,2,\dots,S}\\ ((<i,j,k>))_{j=2,k=0,1,\dots,S}.\end{cases}$$

$$<i>=((<i,j>))_{j=1,2.}$$

Then E can be denoted as $(<0>,<1>,\dots)$. Therefore, the transition rate matrix P is

$$P=\begin{array}{c|ccccc} & <0> & <1> & <2> & <3> & \cdots\\ <0> & A_0 & C & 0 & 0 & \dots\\ <1> & B & A & C & 0 & \dots\\ <2> & 0 & B & A & C & \dots\\ <3> & 0 & 0 & B & A & \dots\\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots\end{array}$$

where

$$A_0=\begin{array}{c|cc} & 1 & 2\\ 1 & [A_0]^{11} & [A_0]^{12}\\ 2 & [A_0]^{21} & [A_0]^{22}\end{array},$$

$$[A_0]^{11} = \begin{cases} -(p\lambda + \eta_1 + \eta_2 + l\gamma), & m = l, l = 1,2,\dots,s; \\ -(p\lambda + \eta_1 + l\gamma), & m = l, l = s+1, \dots, r; \\ -(p\lambda + l\gamma), & m = l, l = r+1, \dots, S; \\ l\gamma, & m = l-1, l = 2, \dots, S; \\ \eta_1, & m = l + Q_1, l = 1, \dots, r; \\ \eta_2, & m = l + Q_2, l = 1, \dots, s; \\ 0, & otherwise. \end{cases},$$

$$[A_0]^{12} = \begin{cases} \gamma, & m = l-1, l = 1; \\ 0, & otherwise. \end{cases}, [A_0]^{21} = \begin{cases} \beta, & m = l, l = 1,2,\dots,S; \\ 0, & otherwise. \end{cases},$$

$$[A_0]^{22} = \begin{cases} -(p\lambda + \eta_1 + \eta_2), & m = l, l = 0; \\ -(p\lambda + \eta_1 + \eta_2 + \beta + l\gamma), & m = l, l = 1,2,\dots,s; \\ -(p\lambda + \eta_1 + \beta + l\gamma), & m = l, l = s+1, \dots, r; \\ -(p\lambda + \beta + l\gamma), & m = l, l = r+1, \dots, S; \\ l\gamma, & m = l-1, l = 1, \dots, S; \\ \eta_1, & m = l + Q_1, l = 0, \dots, r; \\ \eta_2, & m = l + Q_2, l = 0, \dots, s; \\ 0, & otherwise. \end{cases},$$

$$C = 121[C_0]^{11}[C_0]^{12}2[C_0]^{21}[C_0]^{22}, [C_0]^{12} = [C_0]^{21} = 0$$

$$[C_0]^{11} = \begin{cases} p\lambda, & m = l, l = 1,\dots,S; \\ 0, & otherwise. \end{cases}, [C_0]^{22} = \begin{cases} p\lambda, & m = l, l = 0,\dots,S; \\ 0, & otherwise. \end{cases},$$

$$B = 121[B]^{11}[B]^{12}2[B]^{21}[B]^{22}, [B]^{21} = 0, [B]^{12} = \begin{cases} \mu_b, & m = l-1, l = 1; \\ 0, & otherwise. \end{cases},$$

$$[B]^{11} = \begin{cases} \mu_b, & m = l-1, l = 2,\dots, S; \\ q\lambda, & m = l, l = 1,\dots,S; \\ 0, & otherwise. \end{cases}, [B]^{22} = \begin{cases} \mu_v, & m = l-1, l = 1,\dots S; \\ q\lambda, & m = l, l = 0,\dots,S; \\ 0, & otherwise. \end{cases},$$

$$A = 121[A_1]^{11}[A_1]^{12}2[A_1]^{21}[A_1]^{22},$$

$$[A]^{11} = \begin{cases} -(p\lambda + q\lambda + \eta_1 + \eta_2 + l\gamma + \mu_b), & m = l, l = 1,2,\dots,s; \\ -(p\lambda + +q\lambda + \eta_1 + l\gamma + \mu_b), & m = l, l = s+1, \dots, r; \\ -(p\lambda + q\lambda + l\gamma + \mu_b), & m = l, l = r+1, \dots, S; \\ l\gamma, & m = l-1, l = 2, \dots, S; \\ \eta_1, & m = l + Q_1, l = 1, \dots, r; \\ \eta_2, & m = l + Q_2, l = 1, \dots, s; \\ 0, & otherwise. \end{cases},$$

$$[A]^{12} = \begin{cases} \gamma, & m = l-1, l = 1; \\ 0, & otherwise. \end{cases}, [A]^{21} = \begin{cases} \beta, & m = l, l = 1,2,\dots,S; \\ 0, & otherwise. \end{cases},$$

$$[A]^{22} = \begin{cases} -(p\lambda + q\lambda + \eta_1 + \eta_2 + \mu_v), & m = l, l = 0; \\ -(p\lambda + q\lambda + \eta_1 + \eta_2 + \mu_v + \beta + l\gamma), & m = l, l = 1,2,\dots,s; \\ -(p\lambda + q\lambda + \eta_1 + \beta + l\gamma + \mu_v), & m = l, l = s+1, \dots, r; \\ -(p\lambda + q\lambda + \beta + l\gamma + \mu_v), & m = l, l = r+1, \dots, S; \\ l\gamma, & m = l-1, l = 1, \dots, S; \\ \eta_1, & m = l + Q_1, l = 0, \dots, r; \\ \eta_2, & m = l + Q_2, l = 0, \dots, s; \\ 0, & otherwise. \end{cases},$$

3 Analysis of the Model

Initially the stability condition of the defined model is determined and then the limiting probabilities are derived in this section.

3.1 Stability condition

For the stability condition, consider the matrix $G = A + B + C$ as

$$G = 121[G]^{11}[G]^{12}2[G]^{21}[G]^{22},$$

$$[G]^{11} = \begin{cases} -(\eta_1 + \eta_2 + \mu_b + l\gamma), & m = l, l = 1; \\ -(\eta_1 + \eta_2 + l\gamma), & m = l, l = 2, \dots, s; \\ -(\eta_1 + l\gamma), & m = l, l = s+1, \dots, r; \\ -l\gamma, & m = l, l = r+1, \dots, S; \\ l\gamma, & m = l-1, l = 2, \dots, S; \\ \eta_1, & m = l + Q_1, l = 1, \dots, r; \\ \eta_2, & m = l + Q_2, l = 1, \dots, s; \\ 0, & otherwise. \end{cases},$$

$$[G]^{12} = \begin{cases} \gamma, & m = l-1, l = 1; \\ 0, & otherwise. \end{cases}, [G]^{21} = \begin{cases} \beta, & m = l, l = 1,2, \dots, S; \\ 0, & otherwise. \end{cases},$$

$$[G]^{22} = \begin{cases} -(p\lambda + q\lambda + \eta_1 + \eta_2 + \mu_v), & m = l, l = 0; \\ -(p\lambda + q\lambda + \eta_1 + \eta_2 + \mu_v + \beta + l\gamma), & m = l, l = 1,2, \dots, s; \\ -(p\lambda + q\lambda + \eta_1 + \beta + l\gamma + \mu_v), & m = l, l = s+1, \dots, r; \\ -(p\lambda + q\lambda + \beta + l\gamma + \mu_v), & m = l, l = r+1, \dots, S; \\ l\gamma, & m = l-1, l = 1, \dots, S; \\ \eta_1, & m = l + Q_1, l = 0, \dots, r; \\ \eta_2, & m = l + Q_2, l = 0, \dots, s; \\ 0, & otherwise. \end{cases},$$

Let Π be the limiting distribution of G, i.e., $\Pi G = 0$ and $\Pi e = 1$, where $\Pi = (\Pi_1, \Pi_2)$. From $\Pi G = 0$, we get

$$\Pi_1[G]^{11} + \Pi_2[G]^{21} = 0$$
$$\Pi_1[G]^{12} + \Pi_2[G]^{22} = 0$$

On solving the above two equations, one can get $\Pi_2 = -\Pi_1[G]^{12}[G]^{22^{-1}}$. Using Π_2 value is $\Pi e = 1$, we get

$$\Pi_1 = [1 - [G]^{12}[G]^{22^{-1}}]^{-1}$$
$$\Pi_2 = -[1 - [G]^{12}[G]^{22^{-1}}]^{-1}[G]^{12}[G]^{22^{-1}}$$

3.2 Computation of Steady State Vectors

The limiting distribution for the defined model is

$$\pi_i^{(j,k)} = \lim_{t\to\infty} Pr[N(t) = i, \zeta(t) = j, L(t) = k | N(0), \zeta(0), L(0)]$$

where $\pi_i^{(j,k)}$ is the probability of i^{th} demand at j^{th} state with k inventories. These probabilities are shortly represented as π_i. "The limiting distribution is given by $\pi_i = \pi_0 R^i, i \geq 1$, where R is the minimal non-negative solution of the matrix-quadratic equation $R^2B + RA + C = 0$."

For finding π_0 and π_1, we have from $\pi P = 0$,

$$\pi_1 = -\pi_0 C(A + RB)^{-1} = \pi_0 w,$$

where

$$w = -C(A + RB)^{-1}.$$

Further, $\pi_0 A_0 + \pi_1 B = 0, i.e., \pi_0(A_0 + wB) = 0$.

First take π_0 as the limiting distribution of $A_0 + wB$. Then π_i, for $i \geq 1$ can be found using

$\pi_1 = \pi_0 w, \pi_i = \pi_1 R^{i-1}, i \geq 2$. Therefore the limiting distribution of the system is obtained as follows.

$$(\pi_0 + \pi_1 + \pi_2 + \cdots)e = \pi_0(1 + w(I - R)^{-1})e.$$

4 System performance measures

1. Percentage of server busy period is
$$P_b = \sum_{i=1}^{\infty} \sum_{k=1}^{S} \pi_i^{(1,k)} * 100.$$

2. Percentage of server working vacation period is
$$P_v = \sum_{i=1}^{\infty} \sum_{k=0}^{S} i\pi_i^{(2,k)} * 100.$$

3. **Average inventory level:** The average inventory level (E_{IL}) is defined as
$$E_{IL} = \sum_{i=0}^{\infty} \sum_{k=1}^{S} k\pi_i^{(1,k)} + \sum_{i=1}^{\infty} \sum_{k=0}^{S} \pi_i^{(2,k)}.$$

4. **Average service rate:** The average service rate is defined as
$$E_{SR} = \sum_{i=0}^{\infty} \sum_{k=1}^{S} [\mu_b \pi_i^{(1,k)} + \mu_v \pi_i^{(2,k)}].$$

5. **Average server vacation period:** Average server vacation period (E_{SV}) is
$$E_{SV} = \sum_{i=0}^{\infty} \sum_{k=0}^{S} \beta\pi_i^{(2,k)}.$$

6. **Average negative arrivals:** Average negative arrivals (E_{NA}) is defined as
$$E_{NA} = \sum_{i=0}^{\infty} \sum_{k=1}^{S} q\lambda[\pi_i^{(1,k)} + \pi_i^{(2,k)}].$$

7. **Average arrival rate:** The average arrival rate (E_{AR}) is defined as
$$E_{AR} = \sum_{i=1}^{\infty} \sum_{k=1}^{S} p\lambda[\pi_i^{(1,k)} + \pi_i^{(2,k)}].$$

8. **Average replenishment rate from 1^{st} supplier:** The average replenishment rate from 1^{st} supplier (E_{RR_1}) is
$$E_{RR_1} = \sum_{i=0}^{\infty} \sum_{k=1}^{r} \eta_1\pi_i^{(1,k)} + \sum_{i=0}^{\infty} \sum_{k=0}^{r} \eta_1\pi_i^{(2,k)}.$$

9. **Average replenishment rate from 2^{nd} supplier:** The average replenishment rate from 2^{nd} supplier (E_{RR_2}) is
$$E_{RR_2} = \sum_{i=0}^{\infty} \sum_{k=1}^{s} \eta_2\pi_i^{(1,k)} + \sum_{i=0}^{\infty} \sum_{k=0}^{s} \eta_2\pi_i^{(2,k)}$$

10. **Average lifetime:** The average lifetime (E_{FR}) is defined as
$$E_{FR} = \sum_{i=0}^{\infty} \sum_{k=1}^{S} k\gamma[\pi_i^{(1,k)} + \pi_i^{(2,k)}].$$

4.1 Cost analysis

Let
C_{S_1} = Setup cost from the 1^{st} supplier,
C_{S_2} = Setup cost from the 2^{nd} supplier,
C_H = Holding cost,
C_F = Failure cost,
C_N = Loss due to negative arrivals,
C_V = Fixed cost when the server is on vacation,
C_A = Fixed cost for arrivals,
C_{ST} = service cost.
Therefore, the total average cost is defined as

$$TC(s,r) = \begin{cases} C_{S_1}E_{RR_1} + C_{S_2}E_{RR_2} + C_H E_{IL} + C_F E_{PR} + \\ C_N E_{NA} + C_V E_{SV} + C_A E_{AR} + C_{ST} E_{ST}. \end{cases}$$

5 Numerical analysis

For this section, let us fix the parameters as $S = 14, p = 0.6, q = 0.4, \lambda = 2.3, \mu_b = 1.2, \mu_v = 0.8, \eta_1 = 0.4, \eta_2 = 4.7 \gamma = 0.2, \beta = 1.2$.

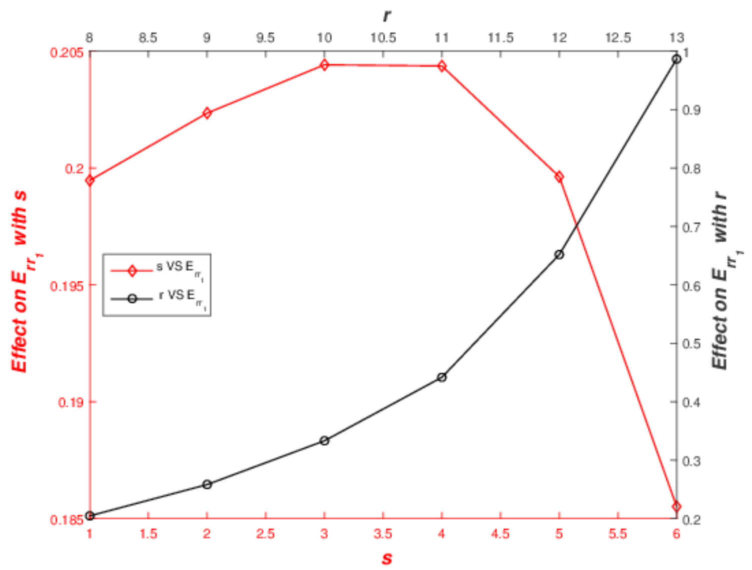


Figure 1: Effect of (s,r) on E_{RR_1}

Figure 1 presents the effect of reordering points (s,r) on the Average replenishment rate of the first supplier (E_{RR_1}). Since E_{RR_1} is effected with r, it rises as r rises and drops as s rises.

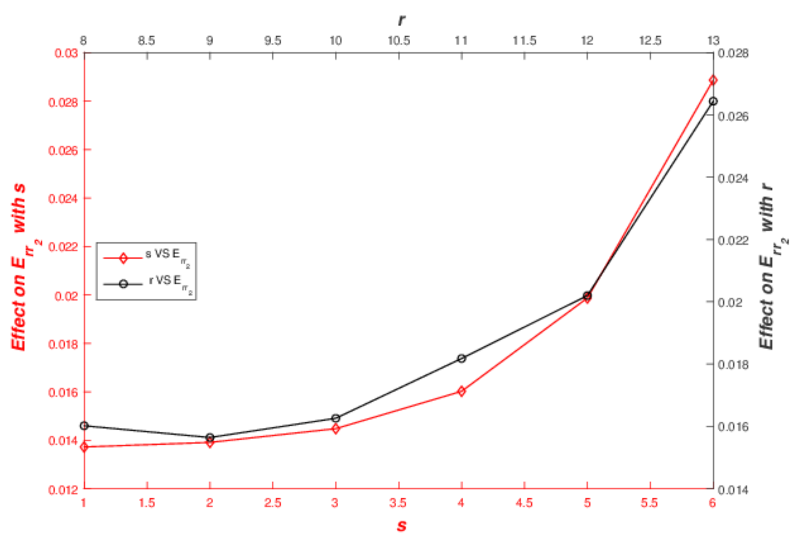
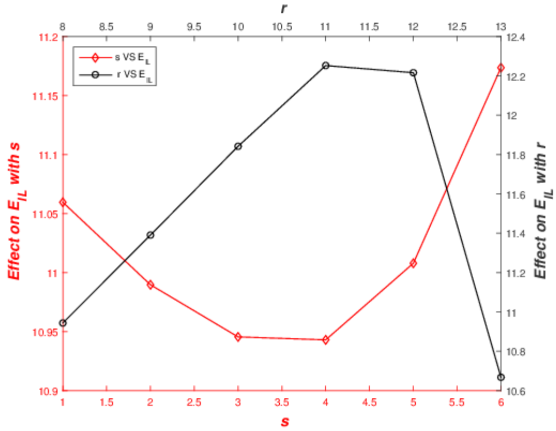


Figure 2: Effect of (s,r) on E_{RR_2}

Figure 2 presents the effect of reordering points (s, r) on the Average replenishment rate of the second supplier (E_{RR_2}). Even though E_{RR_2} is effected with s, the second replenishment order is placed after the first replenishment order is done. Due to this E_{RR_2} increases with the increase in both s and r as show in Figure 2



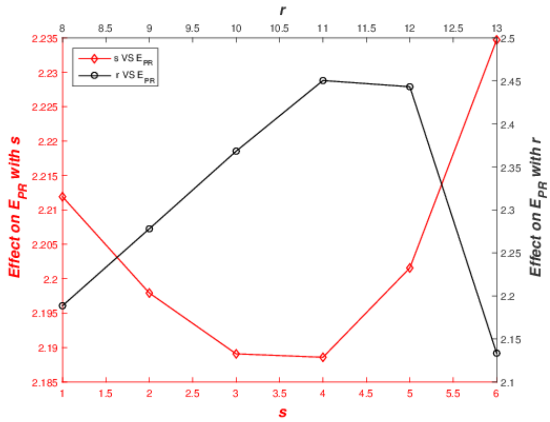


Figure 3: Effect of (s, r) on the E_{IL} Figure 4: Effect of (s, r) on the E_{PR}

The effect (s, r) on E_{IL} and E_{PR} are shown in Figure 3 and Figure 4 respectively. According to our assumption $s < S - r$, first order is placed if inventory falls to r. Also $\eta_2 > \eta_1$, replenishment time of the first supplier is greater than that of the second supplier. Due to this E_{IL} and E_{PR} increases up to $r = 11(rvsE_{IL}$ and $rvsE_{PR})$ and from $s = 4(svsE_{IL}$ and $svsE_{PR})$ which is evident from figures 3 and 4.

Table 1: Values of s^*, r^* and $TC(s^*, r^*)$

Case 1	(s^*, r^*)	(4,8)	(3,9)	(3,10)	(5,11)	(4,12)
	$TC(s^*, r^*)$	684.852	690.974	696.967	703.395	705.422
Case 2	(s^*, r^*)	(4,8)	(3,9)	(3,10)	(5,11)	(4,12)
	$TC(s^*, r^*)$	686.896	693.546	700.301	707.848	711.938
Case 3	(s^*, r^*)	(4,8)	(3,9)	(3,10)	(5,11)	(4,12)
	$TC(s^*, r^*)$	684.948	691.060	697.054	703.518	705.543
Case 4	(s^*, r^*)	(4,8)	(3,9)	(3,10)	(5,11)	(4,12)
	$TC(s^*, r^*)$	903.71	918.745	933.036	947.758	949.734
Case 5	(s^*, r^*)	(4,8)	(3,9)	(3,10)	(5,11)	(4,12)
	$TC(s^*, r^*)$	700.172	706.918	713.492	720.500	722.524
Case 6	(s^*, r^*)	(4,8)	(3,9)	(3,10)	(5,11)	(4,12)
	$TC(s^*, r^*)$	586.104	592.269	598.219	604.647	606.674
Case 7	(s^*, r^*)	(4,8)	(3,9)	(3,10)	(5,11)	(4,12)
	$TC(s^*, r^*)$	684.575	690.697	696.690	703.118	705.114
Case 8	(s^*, r^*)	(4,8)	(3,9)	(3,10)	(5,11)	(4,12)
	$TC(s^*, r^*)$	388.608	394.730	400.723	407.151	409.178
Case 9	(s^*, r^*)	(4,8)	(3,9)	(3,10)	(5,11)	(4,12)
	$TC(s^*, r^*)$	678.421	684.543	690.536	696.964	698.991

Table 1 gives s^* and r^* that minimize $TC(s, r)$, for different numerical examples which are defined as the following cases.

1. $C_{S_1} = 10, C_{S_2} = 14, C_H = 10, C_F = 13, C_N = 150, C_V = 100, C_A = 250, C_{ST} = 20$
2. $C_{S_1} = 20, C_{S_2} = 14, C_H = 10, C_F = 13, C_N = 150, C_V = 100, C_A = 250, C_{ST} = 20$

3. $C_{S_1} = 10, C_{S_2} = 20, C_H = 10, C_F = 13, C_N = 150, C_V = 100, C_A = 250, C_{ST} = 20$
4. $C_{S_1} = 10, C_{S_2} = 14, C_H = 30, C_F = 13, C_N = 150, C_V = 100, C_A = 250, C_{ST} = 20$
5. $C_{S_1} = 10, C_{S_2} = 14, C_H = 10, C_F = 20, C_N = 150, C_V = 100, C_A = 250, C_{ST} = 20$
6. $C_{S_1} = 10, C_{S_2} = 14, C_H = 10, C_F = 13, C_N = 50, C_V = 100, C_A = 250, C_{ST} = 20$
7. $C_{S_1} = 10, C_{S_2} = 14, C_H = 10, C_F = 13, C_N = 150, C_V = 50, C_A = 250, C_{ST} = 20$
8. $C_{S_1} = 10, C_{S_2} = 14, C_H = 10, C_F = 13, C_N = 150, C_V = 100, C_A = 50, C_{ST} = 20$
9. $C_{S_1} = 10, C_{S_2} = 14, C_H = 10, C_F = 13, C_N = 150, C_V = 100, C_A = 250, C_{ST} = 15$

Table 1 gives s^* and r^* that minimize $TC(s,r)$. One can observe that the optimum reorder points in all the cases is $r = 8$ for the first supplier and $s = 3$ for the second supplier. We know that the average inventory level is more when compared to other performance measures discussed in Section 4. However the total cost function increases with increase in holding cost which is evident form Case 4. Cost values are minimum in Case 8 due to decrease in the fixed cost per unit arrival. Also, cost value rises with the increase in the reordering point of the first supplier.

Conclusion

In this paper, some investigations are done on the impact of two suppliers on an inventory model having negative arrivals, finite lifetimes and multiple working vacations. The main motive of this paper is to place an order for inventory using two suppliers instead of a single supplier. The limiting distribution of the assumed model is derived. Various system performance parameters are discussed and analyzed assumed cost function to obtain s^* and r^*. Later, one can extend this paper with multi supply inventory models.

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Declaration of Conflicting Interests

The Authors declare that there is no conflict of interest.

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