RT&A, No 1 (61)
Volume 16, March 2021

Availability and Cost Analysis of Complex Tree Topology of Computer Network with Multi-Server Using Gumbel-Hougaard Family Copula Approach

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Abstract

In relation to types and advantages of computer network topologies, in this research work, availability and cost analysis of complex computer network is considered to focus on a tree topology network that has four subsystems A,B,C and D and all the subsystems are figured in series and parallel configuration, the subsystem A and B served as computer servers together with two units each and are working 1-out-of-2: G/F policy while C and D subsystems both has three units and are working in 2-out-of-3: G/F scheme, A and B attached to subsystems C and D respectively. The system has two types of failure, degraded (partial failure) or complete failed states. The system is analyzed using supplementary variables techniques and Laplace transform. Copula family and general distribution are employed to restore the complete failed and partial failed states respectively. Computed results have been highlighted by the means of tables and graphs.

Keywords: Availability, Reliability, Sensitivity, (MTTF) Mean time to system failure, Topology, Gumbel-Hougaard family, Cost Analysis**.**

I. Introduction

The paramount of computer network is a needed requirement for improving the quality, efficiency and performance of telecommunications, manufacturing, industries, and hospital equipment. Good computer network system depends on the availability and reliability of the subsystems or units. Clients are connected in some predefined protocol called topologies, accordance of need passing information state topology configuration, where clients are connected to the main server. Among the different types of topology network is tree topology, where one central hub and multiple secondary hubs are used, failed of this central hub amount to complete failure of the system, but the subsystems remain in operational degraded state.

To improve system reliability and availability, implementation of redundant components are required, where some units are working while some remain reserves for immediate action, such operational system style is called k-out-of-n: G/F scheme. In this approach k units must work from the domain n of the system to operate, failure of more than k units results to complete failure of the system. Among many researched from different scholars in computer network topology and reliability theory model that includes, Zhang [1] analyzed on computer network reliability analysis based on intelligent cloud computing method. Saulius and Genadijus [2] investigated reliability of multi-server computer networks. Pradeep and Yogesh [3] studied software reliability growth model for three-tier client server system. Xin et al. [4] focused on reliability analysis of network service model. Potapov et al [5] studied reliability in the model of an information system with client server architecture. Kovalev et al [6] analyzed reliability analysis of distributed computer systems with client server. Fong and Hui [7] studied application of middleware in the three tier client/server database design methodology. Sumit and Anshul [8] studied on an introduction to computer networking. Yunhuai et al [9] investigated opportunity based topology control in wireless sensor network. Geon Yoon, Dae Hyun et al [10] focused on ring topology-based redundancy ethernet for industrial network. Nurul et al [11] studied the performance study of star topology in small internetworks. Ruhimat et al [12] analyzed optimal computer network based on graph topology model. Kudeep et al [13] studied tree topology network environment analysis under reliability approach, nonlinear. Nupur et al [14] focused on an approach to investigating reliability Indices for tree topology. System performance depends on system configuration and repair dynamics. Researchers have adopted different types of failure and repair, a lot of them have considered general repair while many adopted copula [15] which is now considered as the wider and better performance results compared to the general repair to cite few are Abubakar and Singh [16] have examined assessment and performance of industrial system using Gumbel Hougaard copula approach. Kabiru et al. [17] have focused on reliability assessment of complex system with two subsystems using joint distribution. Ibrahim et al. [18] have analyzed the performance analysis of multi-computer system with three subsystems in series. Muhammad et al [19] studied cost benefit analysis of tree different series parallel dynamo configurations . Pratap et al [20] have examined on the assessment of complex system with two subsystems and multi types failure and repair. Yusuf et al [21] studied performance analysis of multi computer system consisting of active parallel homogeneous.

Copula distributions that couples different types distributions, since it deals with more than one repair of the repairable systems, Gumbel Hougaard family distribution is one among different types of copula family which consider more than one repairs. The authors in the present research study have consider a tree topology system with multi-servers, the system has four subsystems named as A, B, C and D, subsystems A and B stands as servers and are working 1-out-of-2: G/F policy, respectively, while C and D subsystems both are working in 2-out-of-3: G/F scheme, A and B attached to subsystems C and D respectively. The system work in both series and parallel configuration, Gumbel Hougaard family copula distribution employed for computation and illustration. Lastly, [S_1 , S_2 , S_3, S_4 ,S_5, S_6, S_7, S_8 , S_9 ,S_{10}, S_{11} , S_{12}] represents the states of operation in degraded/partial failure while [S_{13} , S_{14}, S_{15}, and S_{16}] are completely failed states and S_0 is at perfect operational state. The degraded points have repaired by general repair and completely failed states have repaired under Gumbel Hougaard family copula. We invited supplementary variables and Laplace transformation to analyze the system. Measures in reliability among availability, reliability, and MTTF and cost analysis are all treated by the means of tables and graphs.

II. State Description, Assumption and Notations

Table 1: State Description

State	State Description
S_0	The state S_0 represents a perfect state in which both the subsystems are in good working condition.
S_1	S_1, represents a degraded state with minor partial failure in the subsystem A, due to the failure of the first server of the subsystem A.
S_2	State S_2 represents a degraded state with minor partial failure in the subsystem C, due to the failure of the first unit of the subsystem C.
S_3	state S_3 represents a degraded state with minor partial failure in the subsystem D, due to the failure of the first unit of the subsystem D.
S_4	S_4, represents a degraded state with minor partial failure in the subsystem B, due to the failure of the first server of the subsystem B.
S_5	S_5, represents a degraded state with minor failure, due to the failure of the one unit of subsystems C and D.
S_6	S_6, represents a degraded state with minor failure, due to the failure of the one unit of subsystems D and server of subsystem A.
S_7	This state accounts for a degraded state with major partial failure, due to the failure of first servers of subsystem A and B.
S_8	S_8, represents a degraded state with minor failure, due to the failure of the one unit of subsystem C and a server of subsystem B.
S_9	This state accounts for a degraded state with major partial failure, due to the failure of one units of the subsystem C and D together with a server in subsystem B.
S_{10}	S_{10}, reveals a degraded state with major partial failure, due to the failure of one unit of the subsystem D and a server in subsystems A and B.
S_{11}	S_{11}, reveals a degraded state with major partial failure, due to the failure of one unit of the subsystem C and a server in subsystems A and B.
S_{12}	S_{12}, reveals a degraded state with major partial failure, due to the failure of one unit of the subsystems C and D together with a server in subsystems A and B.
S_{13}	The state S_{13} represent a complete failed state, due to failure of subsystems C and D the system is under repair using copula.
S_{14}	The state S_{14} represent a complete failed state, due to failure of servers in subsystems A and B the system is under repair using copula.
S_{15}	The state S_{15} represent a complete failed state, due to failure of subsystems A and D the system is under repair using copula.
S_{16}	The state S_{16} represent a complete failed state, due to failure of subsystems B and C the system is under repair using copula distribution.

The state description reveals that, S_0 is a state where the system is in a perfect state where both subsystems are in good working condition. S_1, S_2, S_3, S_4, S_5, S_6 and S_8 are the states where the system is in minor partial failure but operational mode. The states S_7, S_9,S_{11}, and S_{12} are in major partial failure in which the system is working under the critical stage, and further failure in any unit in the subsystems might be a cause of complete failure. The statesS_{13}, S_{14}, S_{15}, and S_{16} are the complete failed state of the model. The minor and major failed states will be respire by employing general repair, but the complete failed state will be restored using Gumbel- Hougaard family copula distribution.

Assumption

The following assumptions are taken throughout the discussion of the model:

(i) Initially, S_0 is the state where all units in the systems are in its perfect good state.

(ii) The subsystems A and B are working as servers, with two units each, failure of one unit tends the system to a partial failure (degraded) state and its follows general distribution for repair and if more than one fails then its leads to complete failed state of the system and it restore using copula.

(iii) Both the subsystems C and D has three units and at least two units must work, if first or second units failed the system function under degraded state and it repaired by general distribution otherwise complete failed state of the entire system.

(iv) It is assumed that a repaired system works like a new and no damage appears during repair.

(v) As soon as the failed unit gets repaired, it performs its task normally.

(vi) All failure rates are constants and follow a negative exponential distribution

Table 2: Notations

t :	Time variable on time scale.
s :	A variable for Laplace transform for all expressions.
λ_A / λ_B/λ_C/λ_D:	Failure rates of units of subsystems A, B, C and D
$\varphi(x)$	Repair rates for all unit of subsystems i.e. A, B, C and D
$\mu_0(x)$, $\mu_0(y)$:	Repair rates for complete failed states.
$P_i(x,t)$:	The probability that the system is in S_i state at instant's' for i =0 to 12.
$\bar{P}_i(s)$:	Laplace transformation of state transition probability P (t).
$E_p(t)$	Expected profit during the time interval [0, t).
K_1, K_2:	Revenue and service cost per unit time in the interval [0, t) respectively.
$S_\varphi(x)$	Standard repair distribution function $S_\varphi(x) = \varphi(x)e^{-\int_0^\infty \varphi(x)}$
$L[S_\varphi(x)]$:	$\bar{s}_\varphi(x) = \int_0^\infty e^{-sx}\,\varphi(x)e^{-\int_0^\infty \varphi(x)}dx$ is the Laplace transform of $S_\varphi(x)$
$\mu_0(x) = C_\theta(u_1(x), u_2(x))$	The expression of joint probability (failed state S_i to good state S_0) according to Gumbel-Hougaard family copula is given as $C_\theta(u_1(x),u_2(x)) = \exp[x^\theta + \{\log\phi(x)\}^\theta]^{1/\theta}$, where, $u_1 = \phi(x)$, and $u_2 = e^x$, where θ as a parameter, $1\leq \theta \leq \infty$.

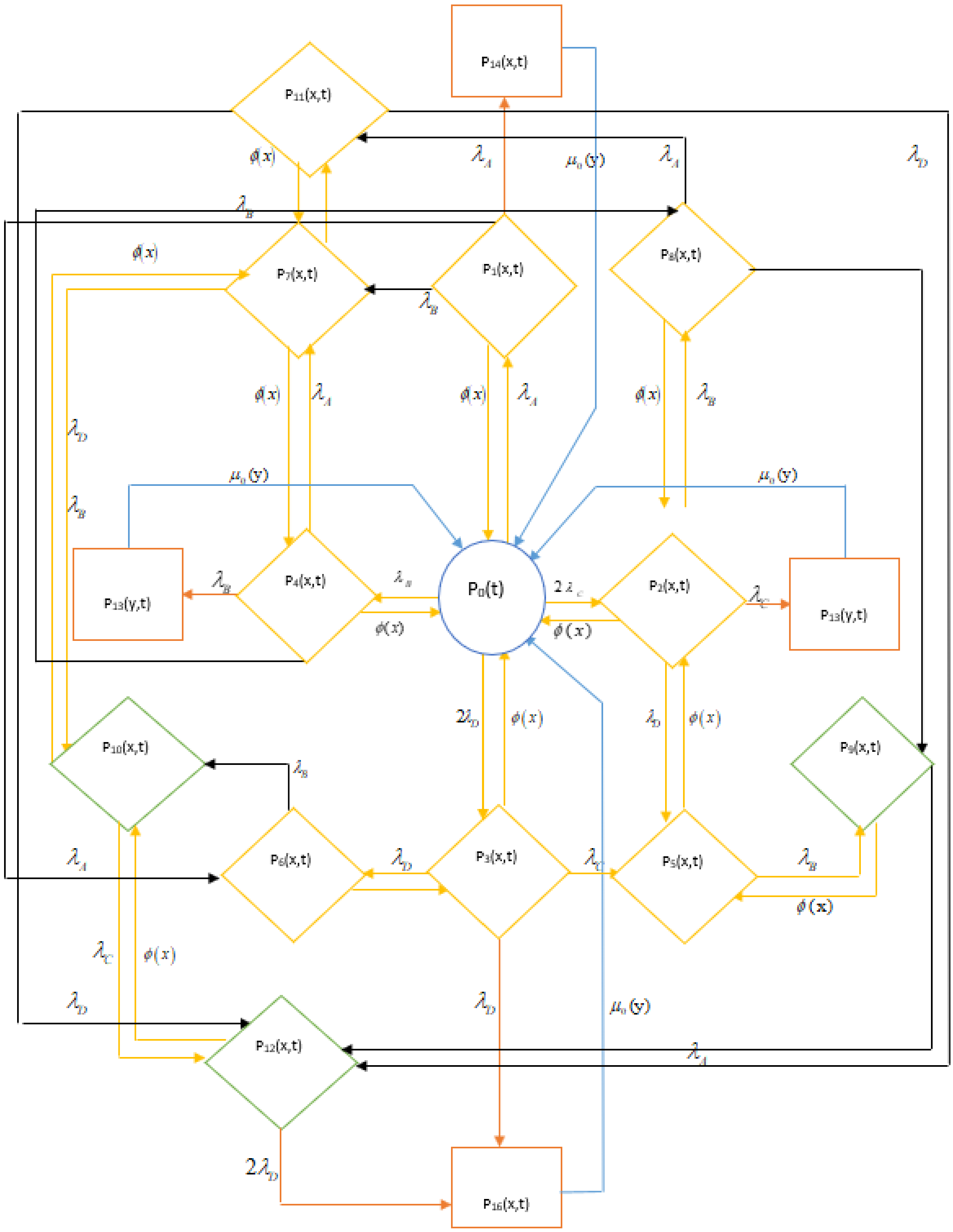

Figure 1: Transition Diagram

III. Formulation and Solution of Mathematical Model

By the probability of considerations and continuity arguments, the following sets of difference differential equations are associated with the mathematical model:

$$\left(\frac{\partial}{\partial t}+\lambda_A+\lambda_B+2\lambda_C+2\lambda_D\right)P_0(t)$$
$$=\int_0^\infty \Phi_1(x)P_1(x,t)dx+\int_0^\infty \Phi_2(x)P_2(x,t)dx+\int_0^\infty \Phi_3P_3(x,t)dx+\int_0^\infty \Phi_4(x)P_4(x,t)dx$$
$$+\int_0^\infty \mu_0P_{13}(y,t)dy+\int_0^\infty \mu_0P_{14}(y,t)dy+\int_0^\infty \mu_0P_{15}(y,t)dy$$
$$+\int_0^\infty \mu_0P_{16}(y,t)dy \qquad (1)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_A+\lambda_B+\Phi(x)\right)P_1(x,t)=0 \tag{2}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_B+\lambda_C+\lambda_D+\Phi(x)\right)P_2(x,t)=0 \tag{3}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_C+2\lambda_D+\Phi(x)\right)P_3(x,t)=0 \tag{4}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_A+2\lambda_B+\Phi(x)\right)P_4(x,t)=0 \tag{5}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_B+\Phi(x)\right)P_5(x,t)=0 \tag{6}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_B+\Phi(x)\right)P_6(x,t)=0 \tag{7}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_C+\lambda_D+\Phi(x)\right)P_7(x,t)=0 \tag{8}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_A+\lambda_D+\Phi(x)\right)P_8(x,t)=0 \tag{9}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_A+\Phi(x)\right)P_9(x,t)=0 \tag{10}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_C+\Phi(x)\right)P_{10}(x,t)=0 \tag{11}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_D+\Phi(x)\right)P_{11}(x,t)=0 \tag{12}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_D+\Phi(x)\right)P_{12}(x,t)=0 \tag{13}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)P_{13}(y,t)=0 \tag{14}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)P_{14}(y,t)=0 \tag{15}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)P_{14}(y,t)=0 \tag{16}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)P_{15}(y,t)=0 \tag{17}$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)P_{16}(y,t)=0 \tag{18}$$

Boundary conditions

$$P_1(0,t)=\lambda_AP_0(t) \tag{19}$$
$$P_2(0,t)=2\lambda_CP_0(t) \tag{20}$$
$$P_3(0,t)=2\lambda_DP_0(t) \tag{21}$$
$$P_4(0,t)=\lambda_BP_0(t) \tag{22}$$
$$P_5(0,t)=4\lambda_C\lambda_DP_0(t) \tag{23}$$
$$P_6(0,t)=(\lambda_A^2+\lambda_D^2)P_0(t) \tag{24}$$
$$P_7(0,t)=(\lambda_A\lambda_C+\lambda_A\lambda_B)P_0(t) \tag{25}$$
$$P_8(0,t)=(2\lambda_A\lambda_C+\lambda_B^2)P_0(t) \tag{26}$$
$$P_9(0,t)=(\lambda_B(2\lambda_C\lambda_D)+\lambda_A(\lambda_B\lambda_C+\lambda_B^2))P_0(t) \tag{27}$$
$$P_{10}(0,t)=(\lambda_B(\lambda_D^2+\lambda_A\lambda_B\lambda_C)+\lambda_D(\lambda_A\lambda_C+\lambda_A\lambda_B))P_0(t) \tag{28}$$
$$P_{11}(0,t)=(\lambda_A(\lambda_B^2+\lambda_B\lambda_C)+\lambda_C(\lambda_A\lambda_C+\lambda_A\lambda_B))P_0(t) \tag{29}$$
$$P_{12}(0,t)=[\lambda_A\lambda_B(2\lambda_C\lambda_D)+\lambda_A\lambda_D(\lambda_B\lambda_C+\lambda_B^2)+\lambda_C\lambda_B(\lambda_D^2+\lambda_A\lambda_B\lambda_C)$$
$$+\lambda_C\lambda_D(\lambda_A\lambda_C+\lambda_A\lambda_B)+\lambda_D\lambda_A(\lambda_B\lambda_C+\lambda_B^2)+\lambda_D\lambda_C(\lambda_A\lambda_C+\lambda_A\lambda_B)]P_0(t) \tag{30}$$

$$P_{13}(0,t) = 2\lambda_C^2 P_0(t) \quad (31)$$
$$P_{14}(0,t) = \lambda_A^2 P_0(t) \quad (32)$$
$$P_{15}(0,t) = \lambda_B^2 P_0(t) \quad (33)$$
$$P_{16}(0,t) = (4\lambda_D^2 + 2\lambda_D P_{12}(0,t))P_0(t) \quad (34)$$

Solution of the Model

By taking the Laplace transformation of equations (1) to (34) with the help of initial condition$P_0(0) = 1$, one may obtain:

$$\begin{aligned}(s + \lambda_A + \lambda_B + 2\lambda_C + 2\lambda_D) &= 1 \\ &+ \int_0^\infty \Phi_1(x)P_1(x,s)dx + \int_0^\infty \Phi_2(x)P_2(x,s)dx + \int_0^\infty \Phi_3(x)P_3(x,s)dx \\ &+ \int_0^\infty \Phi_4(x)P_4(x,s)dx + \int_0^\infty \mu_0(y)P_{12}(y,s)dy + \int_0^\infty \mu_0(y)P_{14}(y,s)dy \\ &+ \int_0^\infty \mu_0(y)P_{15}(y,s)dy + \int_0^\infty \mu_0(y)P_{16}(y,s)dy \quad (35)\end{aligned}$$

$$\left(s + \frac{\partial}{\partial x} + 2\lambda_A + \lambda_B + \Phi_1(x)\right)\bar{P}_1(x,s) = 0 \quad (36)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_B + 2\lambda_C + \lambda_D + \Phi_2(x)\right)\bar{P}_2(x,s) = 0 \quad (37)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_C + 2\lambda_D + \Phi_3(x)\right)\bar{P}_3(x,s) = 0 \quad (38)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_A + 2\lambda_B + \Phi_4(x)\right)\bar{P}_4(x,s) = 0 \quad (39)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_B + \Phi_5(x)\right)\bar{P}_5(x,s) = 0 \quad (40)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_B + \Phi_6(x)\right)\bar{P}_6(x,s) = 0 \quad (41)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_C + \lambda_D + \Phi_7(x)\right)\bar{P}_7(x,s) = 0 \quad (42)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_A + \lambda_D + \Phi_8(x)\right)\bar{P}_8(x,s) = 0 \quad (43)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_A + \Phi_9(x)\right)\bar{P}_9(x,s) = 0 \quad (44)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_C + \Phi_{10}(x)\right)\bar{P}_{10}(x,s) = 0 \quad (45)$$
$$\left(s + \frac{\partial}{\partial x} + \lambda_D + \Phi_{11}(x)\right)\bar{P}_{11}(x,s) = 0 \quad (46)$$
$$\left(s + \frac{\partial}{\partial x} + 2\lambda_D + \Phi_{12}(x)\right)\bar{P}_{12}(x,s) = 0 \quad (47)$$
$$\left(s + \frac{\partial}{\partial x} + \mu_0(x)\right)\bar{P}_{13}(x,s) = 0 \quad (48)$$
$$\left(s + \frac{\partial}{\partial y} + \mu_0(y)\right)\bar{P}_{14}(y,s) = 0 \quad (49)$$
$$\left(s + \frac{\partial}{\partial y} + \mu_0(y)\right)\bar{P}_{15}(y,s) = 0 \quad (50)$$
$$\left(s + \frac{\partial}{\partial y} + \mu_0(y)\right)\bar{P}_{16}(y,s) = 0 \quad (51)$$

The Laplace transformations of the boundary conditions are:

$$\bar{P}_1(0,s) = \lambda_A \overline{P_0}(s) \quad (52)$$
$$\bar{P}_2(0,s) = 2\lambda_C \overline{P_0}(s) \quad (53)$$
$$\bar{P}_3(0,s) = 2\lambda_D \overline{P_0}(s) \quad (54)$$
$$\bar{P}_4(0,s) = \lambda_B \overline{P_0}(s) \quad (55)$$
$$\bar{P}_5(0,s) = 4\lambda_C\lambda_D \overline{P_0}(s) \quad (56)$$
$$\bar{P}_6(0,s) = (\lambda_A^2 + \lambda_D^2\ )\bar{P}_0(s) \quad (57)$$
$$\bar{P}_7(0,s) = \ (\lambda_A\lambda_C + \lambda_A\lambda_B)\ \bar{P}_0(s) \quad (58)$$
$$\bar{P}_8(0,s) = (\lambda_B^2 + 2\lambda_B\lambda\ _C)\bar{P}_0(s) \quad (59)$$
$$\bar{P}_9(0,s) = (\lambda_B(2\lambda_C\lambda_D) + \lambda_A(\lambda_B\lambda_C + \lambda_B^2))\bar{P}_0(s) \quad (60)$$
$$\bar{P}_{10}(0,s) = (\lambda_B(\lambda_D^2 + \lambda_A\lambda_B\lambda_C) + \lambda_D(\lambda_A\lambda_C + \lambda_A\lambda_B))\bar{P}_0(s) \quad (61)$$
$$\bar{P}_{11}(0,s) = (\lambda_A(\lambda_B^2 + \lambda_B\lambda_C) + \lambda_C(\lambda_A\lambda_C + \lambda_A\lambda_B))\ \bar{P}_0(s) \quad (62)$$

$$\bar{P}_{12}(0,s) = [\lambda_A\lambda_B(2\lambda_C\lambda_D) + \lambda_A\lambda_D(\lambda_B\lambda_C + \lambda_B^2) + \lambda_C\lambda_B(\lambda_D^2 + \lambda_A\lambda_B\lambda_C) + \lambda_C\lambda_D(\lambda_A\lambda_C + \lambda_A\lambda_B) + \lambda_D\lambda_A(\lambda_B\lambda_C + \lambda_B^2) + \lambda_D\lambda_C(\lambda_A\lambda_C + \lambda_A\lambda_B)]\bar{P}_0(s) \tag{63}$$

$$\bar{P}_{13}(0,s) = 2\lambda_C^2(s)\bar{P}_0(s) \tag{64}$$

$$\bar{P}_{14}(0,s) = \lambda_A^2(s)\bar{P}_0(s) \tag{65}$$

$$\bar{P}_{15}(0,s) = \lambda_B^2(s)\bar{P}_0(s) \tag{66}$$

$$\bar{P}_{16}(0,s) = (2\lambda_D^2 + 2\lambda_D P_{12}(0,t))\bar{P}_0(s) \tag{67}$$

Now solving equations (35) to (51) with the help of equations (52) to (67), yields,

$$\bar{P}_0(s) = \frac{1}{M(s)} \tag{68}$$

$$\bar{P}_1(s) = \frac{1}{M(s)}\left\{\frac{\lambda_A}{S+2\lambda_A+\lambda_B+\Phi(x)}\right\} \tag{69}$$

$$\bar{P}_2(s) = \frac{1}{M(s)}\left\{\frac{2\lambda_C}{S+\lambda_B+2\lambda_C+\lambda_D+\Phi(x)}\right\} \tag{70}$$

$$\bar{P}_3(s) = \frac{1}{M(s)}\left\{\frac{2\lambda_D}{S+2\lambda_D+\lambda_C+\Phi(x)}\right\} \tag{71}$$

$$\bar{P}_4(s) = \frac{1}{M(s)}\left\{\frac{\lambda_B}{S+2\lambda_B+\lambda_A+\Phi(x)}\right\} \tag{72}$$

$$\bar{P}_5(s) = \frac{1}{M(s)}\left\{\frac{4\lambda_C\lambda_D}{S+\lambda_B+\Phi(x)}\right\} \tag{73}$$

$$\bar{P}_6(s) = \frac{1}{M(s)}\left\{\frac{(\lambda_A^2+\lambda_D^2)}{S+\lambda_B+\Phi(x)}\right\} \tag{74}$$

$$\bar{P}_7(s) = \frac{1}{M(s)}\left\{\frac{(\lambda_A\lambda_C+\lambda_A\lambda_B)}{S+\lambda_A+\Phi(x)}\right\} \tag{75}$$

$$\bar{P}_8(s) = \frac{1}{M(s)}\left\{\frac{(2\lambda_B^2+\lambda_B\lambda\ C)}{S+\lambda_D+\Phi(x)}\right\} \tag{76}$$

$$\bar{P}_9(s) = \frac{1}{M(s)}\left\{\frac{(\lambda_B(2\lambda_C\lambda_D)+\lambda_D(\lambda_B\lambda_C+\lambda_B^2))}{S+\lambda_A+\Phi(x)}\right\} \tag{77}$$

$$\bar{P}_{10}(s) = \frac{1}{M(s)}\left\{\frac{(\lambda_B(\lambda_D^2+\lambda_A\lambda_B\lambda_C)+\lambda_D(\lambda_A\lambda_C+\lambda_A\lambda_B))}{S+\lambda_C+\Phi(x)}\right\} \tag{78}$$

$$\bar{P}_{11}(s) = \frac{1}{M(s)}\left\{\frac{(\lambda_A(\lambda_B^2+\lambda_B\lambda_C)+\lambda_C(\lambda_A\lambda_C+\lambda_A\lambda_B))}{S+\lambda_D+\Phi(x)}\right\} \tag{79}$$

$$\bar{P}_{12}(s)\frac{1}{M(s)}\left\{\frac{\begin{array}{c}[\lambda_A\lambda_B(2\lambda_C\lambda_D)+\lambda_A\lambda_D(\lambda_B\lambda_C+\lambda_B^2)+\lambda_C\lambda_B(\lambda_D^2+\lambda_A\lambda_B\lambda_C)\\+\lambda_C\lambda_D(\lambda_A\lambda_C+\lambda_A\lambda_B)+\lambda_D\lambda_A(\lambda_B\lambda_C+\lambda_B^2)+\lambda_D\lambda_C(\lambda_A\lambda_C+\lambda_A\lambda_B)]\end{array}}{S+2\lambda_D+\Phi(x)}\right\} \tag{80}$$

The Laplace transformations of the state transition probabilities that the system is in operational mode. i.e. perfect and partially failed state $(S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}, S_{11}, S_{12})$ at any time are as follows:

$$\bar{P}_{up}(s) = \bar{P}_0(s) + \bar{P}_1(s) + \bar{P}_2(s) + \bar{P}_3(s) + \bar{P}_4(s)+\bar{P}_5(s) + \bar{P}_6(s) + \bar{P}_7(s) + \bar{P}_8(s) + \bar{P}_9(s) + \bar{P}_{10}(s) + \bar{P}_{11}(s) + \bar{P}_{12}(s) \tag{81}$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s) \tag{82}$$

Where,

$$M(s)=\left\{(s+\lambda_A+\lambda_B+2\lambda_C+2\lambda_D)-\left(\begin{array}{l}\frac{\lambda_A}{s+2\lambda_A+\lambda_B+\varphi(x)}+\frac{2\lambda_C}{s+2\lambda_B+\lambda_D+\varphi(x)}+\frac{2\lambda_D}{s+2\lambda_D+\lambda_C+\varphi(x)}+\frac{\lambda_B}{s+\lambda_A+2\lambda_B+\varphi(x)}\\+\frac{2\lambda_C{}^2\mu_0(y)}{s+\mu_0(y)}+\frac{2\lambda_B{}^2\mu_0(y)}{s+\mu_0(y)}+\frac{2\lambda_A{}^2\mu_0(y)}{s+\mu_0(y)}+\frac{(\lambda_D{}^2+2\lambda_D P_{12}(0,t))\mu_0(y)}{s+\mu_0(y)}\end{array}\right)\right\} \tag{83}$$

The $\bar{P}_{up}(s)$ and $\bar{P}_{dawn}(s)$ are the system Laplace transform of the state probabilities in operative and failed state. Then,

$$\overline{P_{up}}(s)=\sum_{i=0}^{12}\overline{P_i}(s) \quad and \quad \overline{P}_{dawn}(s)=1-\overline{P}_{up}(s)$$

$$P_{up}(t)=\left(\begin{array}{l} 1+\dfrac{\lambda_A}{s+2\lambda_A+\lambda_B+\varphi(x)}+\dfrac{2\lambda_C}{s+2\lambda_B+\lambda_D+\varphi(x)}+\dfrac{2\lambda_D}{s+2\lambda_D+\lambda_C+\varphi(x)}+\dfrac{\lambda_B}{s+\lambda_A+2\lambda_B+\varphi(x)} \\ +\dfrac{4\lambda_C\lambda_D}{s+\lambda_B+\varphi(x)}+\dfrac{\lambda_A{}^2\lambda_D{}^2}{s+\lambda_B+\varphi(x)}+\dfrac{\lambda_A\lambda_C+\lambda_A\lambda_B}{s+\lambda_A+\varphi(x)}+\dfrac{\lambda_B{}^2+\lambda_A\lambda_C}{s+\lambda_D+\varphi(x)}+\dfrac{\lambda_B(2\lambda_C\lambda_D)+\lambda_D(\lambda_B\lambda_C+\lambda_B{}^2)}{s+\lambda_D+\varphi(x)} \\ \dfrac{\lambda_B(\lambda_D{}^2+\lambda_A{}^2)+\lambda_D(\lambda_A\lambda_C+\lambda_A\lambda_B)}{s+\lambda_C+\varphi(x)}+\dfrac{2\lambda_A(\lambda_B{}^2+\lambda_B\lambda_C)+\lambda_C(\lambda_A\lambda_C+\lambda_A\lambda_B)}{s+\lambda_D+\varphi(x)} \\ \dfrac{\lambda_A\lambda_B(2\lambda_C\lambda_D)+\lambda_A\lambda_D(\lambda_B\lambda_C+\lambda_B{}^2)+\lambda_C\lambda_B(\lambda_D{}^2+\lambda_A{}^2)+2\lambda_C\lambda_D(\lambda_A\lambda_C+\lambda_A\lambda_B)+\lambda_A\lambda_D(\lambda_B\lambda_C+\lambda_B{}^2)}{s+\lambda_C+\varphi(x)} \end{array}\right) \tag{84}$$

IV. Analytical Study of the Model

I. Availability Analysis

By Setting $S_{\mu_0}(s)=\overline{S}_{\exp[x^\theta+\{\log\varphi(x)\}^\theta]^{1/\theta}}(s)=\dfrac{\exp[x^\theta+\{\log\varphi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\varphi(x)\}^\theta]^{1/\theta}}$, $\overline{S}_{\varphi_S}(s)=\dfrac{\varphi_S}{s+\varphi_S}$,

The expression of availability is obtained by taking the inverse Laplace transform of equation (84) together with the values of failure rates, λA=0.01, λB=0.02, λC=0.03, λD=0.04, at $\phi(x)=\theta=x=1$ and $\mu_0(x)=\mu_0(y)=2.781$

$$\overline{P_{up}}(t)=\left\{\begin{array}{l} -0.000007268385466e^{-1.030000000t}-0.000001279136863e^{-1.080000000t}+0.001158951568e^{-2.721478824t} \\ -0.009672831318e^{-1.210427182t}+0.0008133658410e^{-1.111354496t}+0.0001409557370e^{-1.076073851t} \\ -0.0001619227838e^{-1.052980075t}+1.012125757e^{-0.005985571315t}-0.001673078923e^{-1.020000000t} \\ -0.0004724824273e^{-1.010000000t}-0.00034152440368^{-1.040000000t} \end{array}\right\} \quad ...(85)$$

Through variation of time t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.., units, we get different values of $P_{up}(t)$ with the help of expression (85) as shown in Table 1 and figure 2.

Table 3: Variation of Availability with respect to time (t)

Time(t)	Availability
0	1
1	0.999
2	0.998
3	0.993
4	0.988
5	0.982
6	0.976
7	0.97
8	0.964
9	0.95

Figure 2: Variation of Availability with respect to time (t)

II. Reliability Analysis

Taking all repair rates to zero with the same value of failure and repair rates in equation (85), i.e $\phi(x)$ and $\mu_0(x)$ and λ_A=0.01, λ_B=0.02, λ_C=0.03, λ_D=0.04, and then taking inverse Laplace transform, we obtained the expression of reliability as:

$$R(t)=\begin{Bmatrix} 0.2000000000e^{-0.05000000000t}-1.500000000e^{-0.1200000000t}+0.8000000000e^{-0.1500000000t} \\ +0.01133333333e^{-0.01000000000t}+0.000287428571e^{-0.03000000000t}+0.0001421800000e^{-0.08000000000t} \\ +0.0203500000e^{-0.04000000000t}+2.951145391e^{-0.1000000000t}+0.05007500000e^{-0.02000000000t} \\ +0.06666666667e^{-0.07000000000t} \end{Bmatrix} \quad ...(86)$$

For different values of time t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.., units of time, we get different values of Reliability as shown in Table 2. and Figure. 3.

Table 4: Variation of reliability with respect to time (t)

Time(t)	Reliability
0	1
1	0.984
2	0.96
3	0.932
4	0.899
5	0.864
6	0.827
7	0.789
8	0.751
9	0.712

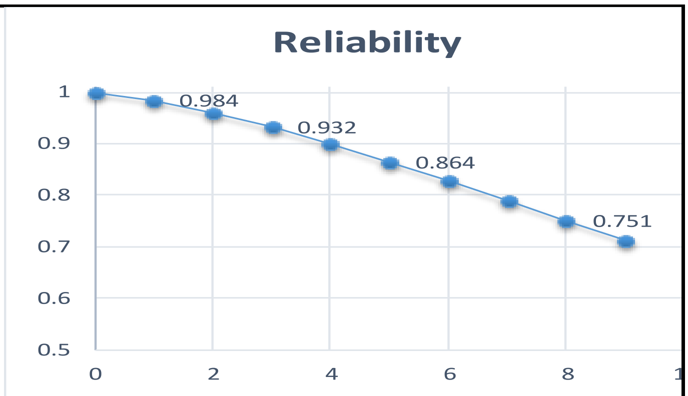

Figure 3: Variation of reliability with respect to time (t)

III. Mean Time To Failure (MTTF) Analysis:

We obtained the expression for MTTF by taking all repairs zero in equation (85), and set the limit of s tends to zero:

$$M.T.T.F.=\lim_{s\to 0}\bar{P}_{up}(s)=\frac{1}{\left\{(s+\lambda_A+\lambda_B+2\lambda_C+2\lambda_D)-\left(\frac{\lambda_A}{2\lambda_A+\lambda_B}+\frac{2\lambda_C}{s+2\lambda_B+\lambda_D}+\frac{2\lambda_D}{s+2\lambda_D+\lambda_C}+\frac{\lambda_B}{s+\lambda_A+2\lambda_B}\right)\right\}}\begin{Bmatrix} \frac{\lambda_A}{2\lambda_A+\lambda_B}+\frac{2\lambda_C}{2\lambda_B+\lambda_D}+\frac{2\lambda_D}{2\lambda_D+\lambda_C}+\frac{\lambda_B}{\lambda_A+2\lambda_B} \\ +\frac{4\lambda_C\lambda_D}{\lambda_B}+\frac{\lambda_A^2\lambda_D^2}{\lambda_B}+\frac{\lambda_A\lambda_C+\lambda_A\lambda_B}{\lambda_A}+\frac{\lambda_B^2+\lambda_A\lambda_C}{\lambda_D}+\frac{\lambda_B(2\lambda_C\lambda_D)+\lambda_D(\lambda_B\lambda_C+\lambda_B^2)}{\lambda_D} \\ \frac{\lambda_B(\lambda_D^2+\lambda_A^2)+\lambda_D(\lambda_A\lambda_C+\lambda_A\lambda_B)}{\lambda_C}+\frac{2\lambda_A(\lambda_B^2+\lambda_B\lambda_C)+\lambda_C(\lambda_A\lambda_C+\lambda_A\lambda_B)}{\lambda_D} \\ \frac{\lambda_A\lambda_B(2\lambda_C\lambda_D)+\lambda_A\lambda_D(\lambda_B\lambda_C+\lambda_B^2)+\lambda_C\lambda_B(\lambda_D^2+\lambda_A^2)+2\lambda_C\lambda_D(\lambda_A\lambda_C+\lambda_A\lambda_B)+\lambda_A\lambda_D(\lambda_B\lambda_C+\lambda_B^2)}{\lambda_C} \end{Bmatrix}$$

Setting λ_A=0.01, λ_B=0.02, λ_C=0.03, λ_D= 0.04, and varying λ_A, λ_B, λ_C and λ_D respectively as, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, in (85), we get the variation of M.T.T.F. with respect to failure rates as shown in Table.3 and corresponding Figure.4.

Table 5: Variation of MTTF with respect to failure rate

Failure Rate	MTTF λ_1	MTTF λ_2	MTTF λ_3	MTTF λ_4
0.1	38.26	48.74	59.2	104.82
0.2	33.02	34.45	48.26	122.02
0.3	30.94	29.17	43.61	132.97
0.4	29.81	27.18	41.03	141.13
0.5	29.11	26.75	39.38	148.3
0.6	28.63	27.19	38.24	154.82
0.7	28.27	28.16	37.4	160.94
0.8	28.01	29.48	36.75	166.81
0.9	27.79	31.05	36.25	172.49

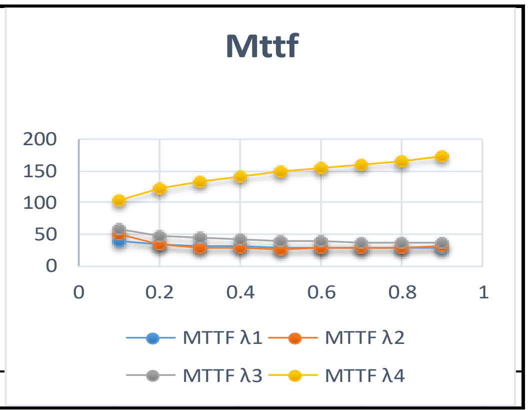

Figure 4: Variation of MTTF with respect to failure rate

IV. Sensitivity analysis of (MTTF):

The computation of sensitivity MTTF is studied through the partial differentiation of MTTF with respect to the failure rates of the system, by introducing the set of parametric variation of the failure rates λ_A=0.01, λ_B=0.02, λ_C=0.03, and λ_D=0.04 from the resulting expression, we calculated the MTTF sensitivity as shown in Table 4 and the corresponding value in Figure.5

Table 6: Sensitivity of MTTF as a function of failure rates

Failure Rate	$\frac{\partial(Mttf)}{\lambda_1}$	$\frac{\partial(Mttf)}{\lambda_2}$	$\frac{\partial(Mttf)}{\lambda_3}$	$\frac{\partial(Mttf)}{\lambda_4}$
0.1	-225	-242.6	-332.6	-599.7
0.2	-115.9	-82.62	-157.7	-384.8
0.3	-78.69	-32.95	-100.8	-278.4
0.4	-59.68	-11.24	-73.32	-213.8
0.5	-48.11	0.177	-57.36	-170.4
0.6	-40.31	6.914	-46.99	-139.2
0.7	-34.69	11.222	-39.74	-115.7
0.8	-30.45	14.144	-34.41	-97.44
0.9	-27.14	16.219	-30.31	-82.78

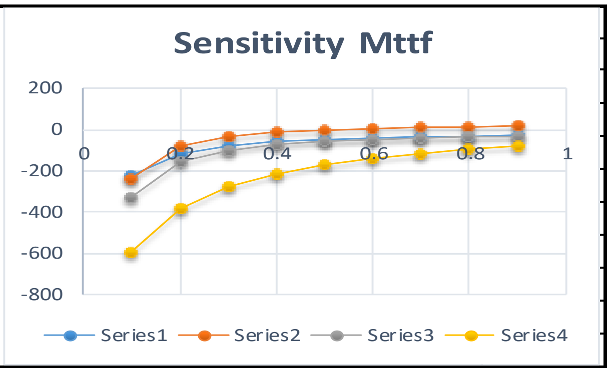

Figure 5: Sensitivity of MTTF as a function of failure rates

V. Cost Analysis

The expected profit over the time interval [0, t), can be calculate by the followeing relation $E_p(t) = K_1\int_0^t P_{up}(t)dt - K_2 t$. If the service facility of the system is always available, where k_1 is revenue generated and k_2 service cost per unit time. For the same set of the parameter of failure and repair rates in (84), we obtained the expression of cost benefit analysis.

$$E_p(t) = \left\{ \begin{array}{l} -0.000007056684918e^{-1.030000000t} + 0.00000001184385984e^{-1.080000000t} - 0.0004258536050e^{-2.721478824t} \\ +0.007991254213^{-1.210427182t} + 0.0007318689436e^{-1.111354496t} + 0.0001309907651e^{-1.076073851t} \\ +0.0001537757339e^{-1.052980075t} - 169.094266083e^{-0.005985571315t} + 0.001640273454e^{-1.020000000t} \\ +0.0004678043835e^{-1.010000000t} + 0.0003283884969e^{-1.040000000t} \end{array} \right\} \quad (86)$$

By fixing the revenue K_1= 1 and taking the values K_2= 0.6, 0.5, 0.4, 0.3, 0.2 and 0.1 respectively together with the variation of t =0, 1, 2, 3, 4, 5, 6, 7, 8, 9, Units of time, we obtained the results for expected profit as shown in Table 5 and Figure.6

Table 7: Expected Profit in [0,t) t=0,1 ,2, 3, 4…9

Time (t)	$E_P(t)$: K_2=0.6	$E_P(t)$: K_2=0.5	$E_P(t)$: K_2=0.4	$E_P(t)$: K_2=0.3	$E_P(t)$: K_2=0.2	$E_P(t)$: K_2=0.1
0	0	0	0	0	0	0
1	0.401	0.501	0.601	0.701	0.801	0.901
2	0.802	1.002	1.202	1.402	1.602	1.802
3	1.198	1.498	1.798	2.098	2.398	2.698
4	1.589	1.989	2.389	2.789	3.189	3.589
5	1.974	2.474	2.974	3.474	3.974	4.474
6	2.353	2.953	3.553	4.153	4.753	5.353
7	2.727	3.427	4.127	4.827	5.527	6.227
8	3.095	3.895	4.695	5.495	6.295	7.095
9	3.457	4.357	5.257	6.157	7.057	7.957

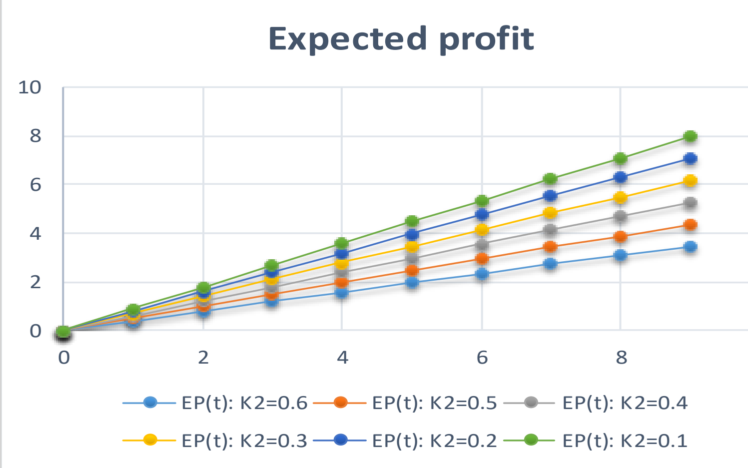

Figure 6: Expected Profit in [0,t) t=0,1 ,2, 3, 4…9

V. Discussion

The performance of the system under the assessment of reliability measures for different values of failure and repair rates. Table.1 and figure 2 shows the information of availability of the complex tree topology with respect to the variation in time when the failure rates are fixed at different values particularly, λ_A=0.01, λ_B=0.02, λ_C=0.03 and λ_D=0.04. The availability of the system decreases slowly, as the probability of failure increases, after sufficient long interval of time the system availability will tend to zero. However, one can simply predict the future behavior of the complex system at any stage for any given set of parametric values.

Table.2 and figure.3 analyzed the reliability of the system when the repair rate setup to zero. The figure shown clearly that the reliability of the system is decreasing faster compare to availability, which evidently proved that when the repairs provided the performance of the system is quite better. Table.3 and figure.4 assess the information of mean time to failure of the system (MTTF) with respect to variation of failure rates. The value change of MTTF is directly propotional to the system reliability. The computations MTTF for different values of failure rates, λ_A, $\lambda_{B,}$ $\lambda_{C,}$ and λ_D from the figure the variation in MTTF corresponding to failure rates λ_D is high compared to other failure which indicates that the system will not be affected with higher variations in values λ_D. The MTTF due to $\lambda_{A,}$ $\lambda_{B,}$ and λ_C will influence the operation of the system.

Table.4 and figure.5 shows the variation of sensitivity MTTF with respect to the values of parameters. which obtained from partial derivative of MTTF with respect to the corresponding failure rate, the variation of sensitivity MTTF corresponding to failure rates λ_D is lower compared to other failure rates.

Table.5 and figure.6 provide the information on how the profit has been generated, by fixing revenue cost per unit time K_1= 1, and varies the service costs K_2 = 0.6, 05. 0.4, 0.3, 0.2 and 0.1, if we examine critically from Figure.6 we can reveals that the expected profit increases for low service cost. Which finally shows the Networking system of tree topology system is reliable.

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