Reliability Prediction of Distributed System with Homogeneity in Software and Server using Joint Probability Distribution via Copula Approach

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Abstract

The present paper intends to design and development reliability models for the analysis of distributed hardware-software system. For the determination of reliability and system performance, the study analyzed a distributed system consisting of a single host with two heterogeneous software running on the host and two identical servers configured as series-parallel system. Both client (host), software and server's failure time are to be exponentially distributed while repairs follow two forms of distributions that are general and Gumbel-Hougaard family copula. The system is analyzed using supplementary variable technique with implications of Laplace transforms. The results are presented in tables and graphs. Some important measures of reliability such as availability of system, reliability of the system, MTTF and cost analysis have been discussed. Some particular cases have also been derived and examined to see the practical effect of the model.

Keywords: Distributed System, Clients, Availability, Software, MTTF, Cost Analysis

1 Introduction

Many commercial systems like, military systems, aircraft systems, institutions and industrial setting, are composed of a number of communicating devices that allows distribution, exchange and dissemination of information to various parts, units, sections or department. These are coined as distributed systems, with components running on different processors or in different processes. A distributed system is system consisting of hardware and software devices configured as network. Distributed system is a collection of computers in which each client in the cluster can assist in the execution of various functions. The devices that constitute distributed system are linked through a computer network and distribution middleware. These devices assist the system in providing powerful services, guarantees of performance, fault tolerance, and security. Each distributed system has programs running on many different computers connected via a network, have become very complicated and very difficult to get right. Reliability can be seen as the ability of a system to perform its intended function under stipulated conditions for a specified period of time. To improve distributed system reliability, many researchers have proposed different types of studies/mathematical models and proclaimed better performance by their operations. For instance, Wu (2014) discussed the modelling of distributed files systems for practical performance. Vijayalakshmi (2015) analyzed the dependability of homogeneous distributed software hardware systems. Olabiyisi et al. (2011) presented a survey on performance evaluation models for distributed system architecture. Dhulavvagol et al. (2020) discussed the performance analysis of distributed processing system using shard selection techniques on elastic search. Joarder et al. (2016) discussed the incremental repartitioning of shared-nothing distributed databases for scalable OLTP applications. Munoz-Escoi and Juan-Marin (2018) dealt with synchrony in dynamic distributed systems. Ahmed and Wu (2013) presented a survey on reliability of distributed system. Sari and Akkaya (2015) studied fault tolerance mechanism in distributed systems. Mishra and Tripathi (2014) discussed some problems, challenges and issues of distributed software system. Kovalev et al. (2015) presented model on reliability analysis of distributed computer system with architecture client-server.

Inherent in the establishment of reliability requirements in the need to estimate or predict reliability in advance of manufacturing the product. This prediction is a continuing process which takes place at several stages of progression from design through usage. The prediction of system performances is based on system architecture, components arrangements, system configuration, operative environment, ability of handler, regular repair with appropriate repair policies and uses of protection devices to minimize failure effects. Many researchers designed the various types of systems and evaluate their performances employing different types of failure possibilities and maintenance policies. To cite some, them Singh et al. (2010) studied reliability measures for a system which consisting three units at super priority, priority and ordinary unit under preemptive resume repair policy employing supplementary variable approach. Singh et al. (2013) evaluated reliability measures Availability, MTTF and cost benefit analysis for a system consisting two subsystems with controllers in series configuration under k-out-of-n: G/ policy using supplementary variable and copula approach. and A. Kumar and M. Ram (2015) studied reliability measures including sensitivity analysis of a coal handling unit for thermal power plant which consisting two subsystems in series configuration using supplementary variable techniques. M. A, El-Damcese et al (2016) studied reliability and sensitivity analysis of a k-out-of-n: G, warm standby parallel repairable system with replacement at common cause failure using Markov model by taking three different case for analytical results computations A, Kuldeep Nagiya et al. (2017) studied a tree topology network environmental analysis under reliability approach using Markov process and supplementary variable which convert Morkov process to non Morkov process. Ram Niwas and Harish Garg, (2017) analyzed reliability metric including profit function of an industrial system grounded on cost free warranty scheme.

The redundancy improves the system performance of the repairable systems. A frequently used type of redundancy is (k-out-of-n, $k \leq n$) system introduces by Birnbaum et al. (1961). A (k-out-of-n:G) which is equivalent to (k+1-out-of-n:F) have analyzed by many researchers which has applied in most of all the systems including industrial, networking systems, mechanical systems, manufacturing systems power plants and transmission and communication systems. Researchers, Rawal et al (2013), Jyoti Gulati et al (2016), Monika Gahlot et al. (2018) have analyzed the performances of the (k-out-of-n:G/F) types of repairable systems by taking different types of failures and two types of repair using copula linguistic approach and concluded that copula repair policy is better over general repair policy. Abdul, K, Lado and Singh (2019) have analyzed a system comprising two subsystems in series configuration with different types of failures and copula repair approach. Afshin Yaghoubi et al (2020) deliberated a closed form of steady state availability of cold standby repairable k-out-of-n: G system using Markov method.

In the architecture of a distributive network system we observed the three subsystems connected in series arrangement namely Clients, Software and Servers. In this model we have consider clients as subsystem 1, Software's subsystem 2 and servers as the subsystem 3. The table 1 presents the states status of the model. In the present study subsystem 1 consists two software, subsystem 2 consists of a client while subsystem 3 comprises of two servers. System failure can occur due the failure of client or the two servers or two software.

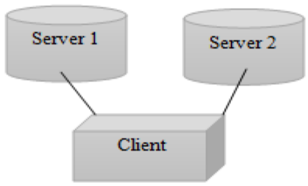


Figure 1: Proposed system

⟶ Replicated data

⟶ User traffic

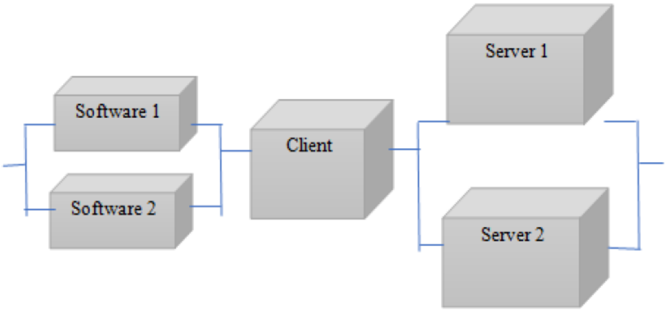
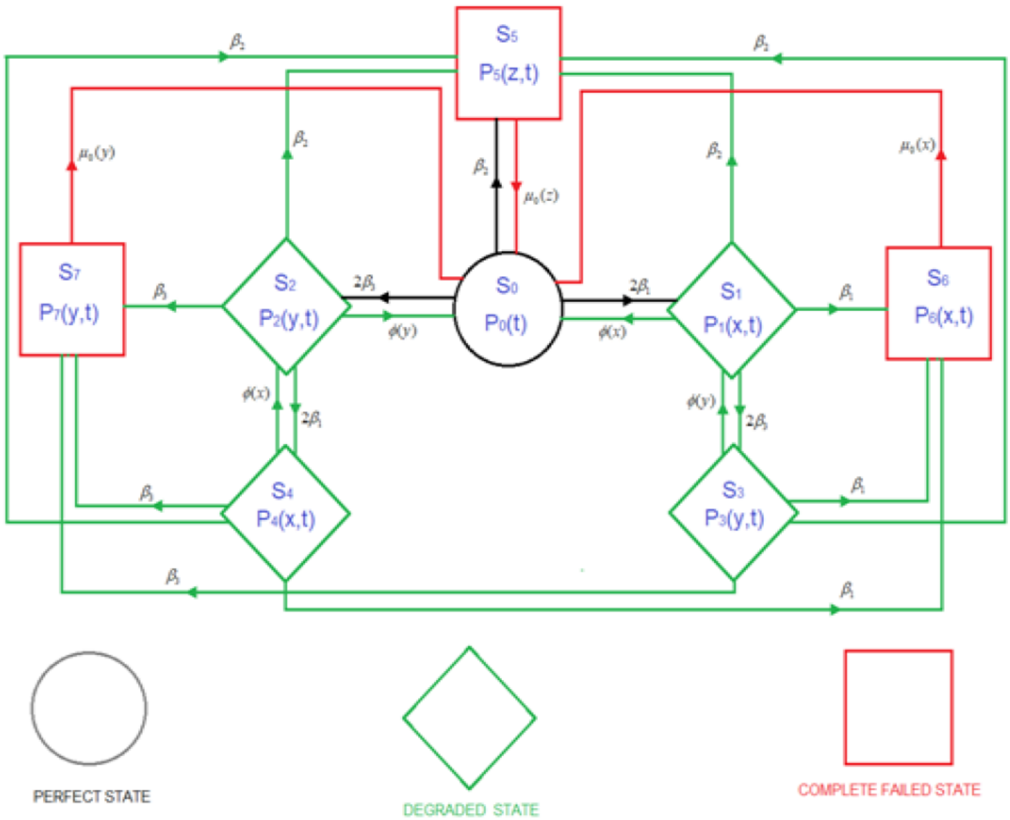


Figure 2: Reliability block diagram of the proposed system

State Structure diagram and State description Table



State Structure diagram of the Model

Table 1: States of the system table.

	Subsystem 1	Subsystem 2		Subsystem 3		
State	Client	Software 1	Software 2	Server 1	Server 2	System status
S_0	Functional	Functional	Functional	Functional	Functional	Operative
S_1	Functional	Functional	Functional	Failed	Functional	Operative
S_2	Functional	Functional	Failed	Functional	Functional	Operative
S_3	Functional	Failed	Functional	Failed	Functional	Operative
S_4	Functional	Functional	Failed	Functional	Functional	Operative
S_5	Failed	Idle	Idle	Idle	Idle	Down
S_6	Idle	Idle	Idle	Failed	Failed	Down
S_7	Idle	Failed	Failed	Idle	Idle	Down

2 Notations, Assumptions

3.1 Notations

- t: Time variable on a time scale.
- s: Laplace transform variable for all expressions.
- $\beta 1/\beta 2/\beta 3$: Failure rate of Server/ Client /Software.
- $\phi(x)\phi(y)$: Repair rate of Server/ Software
- $\mu_0(x)/\mu_0(y)/\mu_0(z)$:Repair rates for complete failed states of Server/ Client /Software
- $p_i(t)$: The probability that the system is in Si state at instants for =0 to 10
- $\overline{P}(s)$: Laplace transformation of state transition probability
- $P_i(x,t)$: The probability that a system is in state Si for i=1 . . . , the system under repair and elapse repair time is (x, t) with repair variable x and time variable t.
- $P_i(y,t)$: The probability that a system is in state Si for i=1. . . , the system under repair and elapse repair time is (y, t) with repair variable y and time variable t.
- P_i (z, t): The probability that a system is in state Si for i=1. . . , the system under repair and elapse repair time is (z, t) with repair variable z and time variable t.
- $E_p(t)$: Expected profit during the time interval [0, t)
- K_1, K_2: Revenue and service cost per unit time, respectively.
- $\mu_0(x)$: The expression of joint probability (failed state S_i to good state S_0) according to Gumbel-Hougaard family copula definition

$$\mu(x) = c_\theta(u_1, u_2(x)) = exp(x^\theta + \{\log\phi(x)^\theta\}^{\frac{1}{\theta}})1 \le \theta \le \infty$$

3.2 Assumption: The undermentioned assumptions are dealt for study of the model.

1. In the initial stage system is good working state with all components
2. The Client is using two similar software's.
3. Both Software's are identical to each other and independent to each other.

4. Servers are identical and independent to each other in working context.
5. Each software failed independent of the other.
6. Each server failed independent of the other
7. Servers works simultaneously and independently.
8. In the degraded mode with minor failure general repair is employed to maintained of servers and software's and clients.
9. The complete failed state in the system are maintained using copula repair distribution especially Gumbel Hougaard copula distribution.
10. It assumed that during repair not part of system breakdown/ damage.

Table 1: States of the system

3 Formulation of Mathematical Model

By probability of considerations and continuity arguments, we can obtain the following set of difference differential equations governing the present mathematical model.

$$\left[\frac{\partial}{\partial t}+2\beta_1+\beta_2+2\beta_3\right]P_0(t)=\int_0^\infty \phi(x)P_1(x,t)dx+\int_0^\infty \phi(y)P_2(y,t)dy \\ +\int_0^\infty \mu_0(y)P_7(y,t)dy+\int_0^\infty \mu_0(x)P_6(x,t)dx+\int_0^\infty \mu_0(z)P_5(z,t)dz \tag{1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_1+2\beta_3+\beta_2+\phi(x)\right]P_1(x,t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\beta_1+\beta_2+\beta_3+\phi(y)\right]P_2(y,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta_1+\beta_2+\beta_3+\phi(y)\right]P_3(y,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\beta_1+\beta_2+\beta_3+\phi(x)\right]P_4(x,t)=0 \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\mu(z)\right]P_5(z,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu(x)\right]P_6(x,t)=0 \tag{7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu(y)\right]P_7(y,t)=0 \tag{8}$$

Boundary Conditions: During the operational mode the repair facility is not available than the relation of two consecutive state transition probabilities can be obtain with help of boundary conditions. i.e. $p_{i+1}(0,t)=\sum_i \lambda_i p_i(0,t)$ where j represents the any state. From state transition diagram one can easily have the following relations;

$$p_1(0,t)=2\beta_1 p_0(t), p_2(0,t)=2\beta_3 p_0(t), p_3(0,t)=\beta_1\beta_3 p_0(t), \\ p_4(0,t)=4\beta_1\beta_3 p_0(t), p_5(0,t)=\beta_2[1+2\beta_1+2\beta_3+8\beta_1\beta\beta_3]p_0(t), \\ p_6(0,t)=\beta_1[2\beta_1+8\beta\beta_1\beta_3]p_0(t), p_7(0,t)=\beta_3[2\beta_3+8\beta\beta_1\beta_3]p_0(t) \tag{9}$$

Initials Conditions

$$p_0(0)=1 andotherstateprobabilitiesarezeroatt=0 \tag{10}$$

4 Solution of the Model

Taking Laplace transformation of equations (1)-(9) and using equation (10), we obtain.

$$[s+2\beta_1+\beta_2+2\beta_3]\overline{P}_0(s)=\int_0^\infty \phi_1(x)\overline{p_1}(x,s)dx+\int_0^\infty \phi_1(y)\overline{p_2}(y,s)dy+ \int_0^\infty \mu_0(z)\overline{p_5}(z,s)dz+\int_0^\infty \mu_0(y)\overline{p_7}(y,s)dy+\int_0^\infty \mu_0(x)\overline{p_6}(x,s)dx \tag{11}$$

$$\left[s+\frac{\partial}{\partial x}+\beta_1+2\beta_3+\beta_2+\phi(x)\right]\overline{p_1}(x,s)=0 \tag{12}$$

$$\left[s+\frac{\partial}{\partial y}+\beta_2+2\beta_1+\beta_3+\phi(y)\right]\overline{p_2}(y,s)=0 \tag{13}$$

$$\left[s+\frac{\partial}{\partial y}+\beta_1+\beta_2+\beta_3+\phi(y)\right]\overline{p_3}(y,s)=0 \tag{14}$$

$$\left[s+\frac{\partial}{\partial x}+\beta_1+\beta_2+\beta_3+\phi(x)\right]\overline{p_4}(x,s)=0 \tag{15}$$

$$\left[s+\frac{\partial}{\partial z}+\mu_0(z)\right]\overline{p_5}(z,s)=0 \tag{16}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{p_6}(x,s)=0 \tag{17}$$

$$\left[s+\frac{\partial}{\partial y}+\mu_0(y)\right]\overline{p_7}(y,s)=0 \tag{18}$$

$$\begin{gathered}\overline{p_1}(0,s)=2\beta_1\overline{P_0}(s),\overline{p_2}(0,s)=2\beta_3\overline{P_0}(s),\overline{p_4}(0,s)=4\beta_1\beta_3\overline{P_0}(s),\\ \overline{P_5}(0,s)=\beta_2[1+2\beta_1+2\beta_3+8\beta_1\beta_3]\overline{P_0}(s),\\ \overline{P_6}(0,s)=\beta_1[2\beta_1+8\beta_1\beta_3]\overline{P_0}(s),\overline{P_7}(0,s)=\beta_3[2\beta_3+8\beta_1\beta_3]\overline{P_0}(s)\end{gathered} \tag{19}$$

Solving (11)-(18) with the help of (19) one may get

$$\overline{P_0}(s)=\frac{1}{D(s)} \tag{20}$$

$$\overline{P_1}(s)=\frac{2\beta_2}{D(s)}\frac{(1-S_\phi(s+\beta_1+2\beta_3+\beta_2)}{(s+\beta_1+2\beta_3+\beta_2)} \tag{21}$$

$$\overline{P_2}(s)=\frac{2\beta_3}{D(s)}\frac{(1-S_\phi(s+2\beta_1+\beta_2+\beta_3)}{(s+2\beta_1+\beta_2+\beta_3)} \tag{22}$$

$$\overline{P_3}(s)=\frac{4\beta_1\beta_3}{D(s)}\frac{(1-S_\phi(s+\beta_1+\beta_2+\beta_3)}{(s+\beta_1+\beta_2+\beta_3)} \tag{23}$$

$$\overline{P_4}(s)=\frac{4\beta_1\beta_3}{D(s)}\frac{(1-S_\phi(s+\beta_1+\beta_2+\beta_3)}{(s+\beta_1+\beta_2+\beta_3)} \tag{24}$$

$$\overline{P_5}(s)=\frac{\beta_2[1+2\beta_1+2\beta_3+8\beta_1\beta_3]}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \tag{25}$$

$$\overline{P_6}(s)=\frac{\beta_1[2\beta_1+8\beta_1\beta_3]}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \tag{26}$$

$$\overline{P_7}(s)=\frac{\beta_3[2\beta_3+8\beta_1\beta_3]}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \tag{27}$$

$$\begin{aligned}D(s)&=s+2\beta_1+\beta_2+2\beta_3-\{2\beta_1\overline{S_\phi}(s+\beta_1+\beta_2+2\beta_3)\\&+\beta_2(1+2\beta_1+2\beta_3+8\beta_1\beta_3)S_{\mu_0}(s)+2\beta_3\overline{S_\phi}(s+2\beta_1+\beta_2+\beta_3)+\\&\beta_3(2\beta_3+8\beta_1\beta_3)S_{\mu_0}(s)+\beta_1(2\beta_1+8\beta_1\beta_3)S_{\mu_0}(s)\}\end{aligned} \tag{28}$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

$$\overline{P}_{up}(s) = \overline{P_0}(s)P_1(s) + P_2(s) + P_3(s) + P_4(s)$$
$$= \frac{1}{D(s)}\begin{bmatrix} 1 + 2\beta_1 \frac{(1-S_\phi(s+\beta_1+2\beta_3+\beta_2)}{(s+\beta_1+2\beta_3+\beta_2)} + 2\beta_3 \frac{(1-S_\phi(s+2\beta_1+\beta_2+\beta_3)}{(s+2\beta_1+\beta_2+\beta_3)} \\ +4\beta_1\beta_3 \frac{(1-S_\phi(s+\beta_1+\beta_2+\beta_3)}{(s+\beta_1+\beta_2+\beta_3)} + 4\beta_1\beta_3 \frac{(1-S_\phi(s+\beta_1+\beta_2+\beta_3)}{(s+\beta_1+\beta_2+\beta_3)} \end{bmatrix} \quad (29)$$

$$\overline{P}_{failed}(s) = \overline{P_5} + \overline{P_6} + \overline{P_7} \quad (30)$$

5 Analytical Study of the model

5.1 Availability of the system for copula repair

When repair follows two types of repair i.e., exponential and general distribution. Setting, the repairs,

$$\overline{S}_{exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{\frac{1}{\theta}}}(s) = \frac{exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{\frac{1}{\theta}}}{s+exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{\frac{1}{\theta}}}, \overline{S}_\phi(s) = \frac{\phi}{s+\phi}$$

in equation (29) and fixing the values of failure rates as,

, $(\beta_1 = 0.04, \beta_2 = 0.04, \beta_3 = 0.04)$, $(\beta_1 = 0.05, \beta_2 = 0.05, \beta_3 = 0.05)$, $(\beta_1 = 0.06, \beta_2 = 0.06, \beta_3 = 0.06)$, $(\beta_1 = 0.07, \beta_2 = 0.07, \beta_3 = 0.07)$ and $\phi = 1, \theta = 1, x = 1$, then taking inverse Laplace transform, one can obtain, the expressions (a, b, c, d) respectively;

- $P_{up}(t) = 0.02105544525e^{(-2777881421t)} - 0.01014421182e^{(-1.293633161t)} + 1.005687563e^{(-0.006785417978t)} - 0.01659879629e^{(-1.120000000t)}$
- $P_{up}(t) = 0.02834810927e^{(-2.800124701t)} - 0.01787925501e^{(-1.359319656t)} + 1.010420311e^{(-0.008855643005t)} - 0.02088916502e^{(-1.150000000t)}$
- $P_{up}(t) = -0.02520769946e^{(-1.180000000t)} + 0.03636839750e^{(-2.825889314t)} - 0.02675858604e^{(-1.421952360t)} + 1.015597888e^{(-0.01045832627t)}$
- $P_{up}(t) = 0.04499933873e^{(-2.855437178t)} - 0.3642027421e^{(-1.481497573t)} + 1.020957837e^{(-0.01136524989t)} - 0.02953980143e^{(-1.210000000t)}$

For, t= 0, 10, 20, 30, 40, 50, 60, 70, 80, 90.. units of time, one may get different values of $P_{up}(t)$ as shown in Table1.

Table 2: Table 1. Availability variation for copula repair

Time(t)	Availability (a) β_j=0.04, j=1, 2,3	Availability (b) β_j=0.05, j=1, 2,3	Availability (c) β_j=0.06, j=1, 2,3	Availability (d) β_j=0.07, j=1, 2,3
0	1.0000	1.0000	1.0000	1.0000
10	0.9397	0.9247	0.9147	0.9112
20	0.8780	0.8464	0.8239	0.8133
30	0.8204	0.7746	0.7420	0.7259
40	0.7666	0.7090	0.6684	0.6479
50	0.7163	0.6489	0.6020	0.5783
60	0.6693	0.5939	0.5422	0.5162
70	0.6254	0.5436	0.4884	0.4607
80	0.5844	0.4975	0.4399	0.4112
90	0.5460	0.4553	0.3962	0.3670
100	0.5102	0.4167	0.3568	0.3276

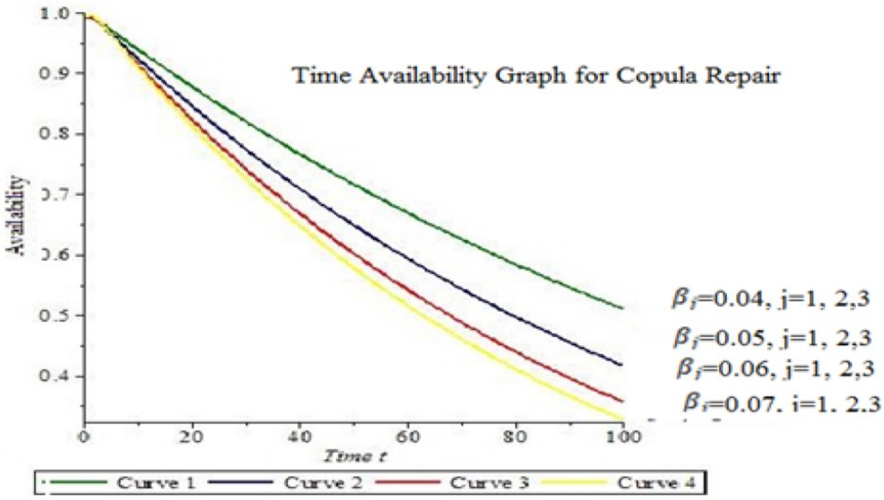


Figure 3: Time v/s. Availability graph for (Copula Repair)

5.2 Availability analysis when the system follows General repair:

If the system follows general repair than the availability of the system can be analyzed by putting $\mu = \Phi$ in equation (61). For the same set of failure rates in Sec (5.1) and taking inverse Laplace transform of resulting expressions one may get availability expressions respect to general repair rates in (a, b, c, d)

- $P_{up}(t) = 0.01149680663e^{(-1.166408188t)} + 0.0120314111e^{(-1.011271752t)} + 0.9853993116e^{(-0002320060097t)} - 10.008927529467e^{(-1.060000000t)}$
- $P_{up}(t) = 0.01347456469e^{(1.326621009t)} + 0.02937122091e^{(-1.026793715t)} + 0.9757788560e^{(-0.006585276132t)} - 0.01862464183e^{(-1.210000000t)}$
- $P_{up}(t) = -0.02918859381e^{(-1..80000000t)} + 0.01027889213e^{(-1.482860719t)} + 0.052669449984e^{(-1.047183349t)} + 0.9662152017e^{(-0.009955931640t)}$
- $P_{up}(t) = 0.004003254503e^{(-1.636498085t)} + 0.08225309992e^{(-1.072910924t)} + 0.9544778674e^{(-0.01059099182t)} - 0.0473422178e^{(-1.240000000t)}$

For different values of the time t from 0, 10. 20, 30, . . . ,100 in interval [0,100] one can obtain the table 2.

Table 3: Table 2. Availability variation for general repair

Time(t)	Availability (a) β_j=0.04, j=1, 2,3	Availability (b) β_j=0.05, j=1, 2,3	Availability (c) β_j=0.06, j=1, 2,3	Availability (d) β_j=0.07, j=1, 2,3
0	1.0000	1.0000	1.0000	1.0000
10	0.9628	0.9135	0.8746	0.8585
20	0.9407	0.8553	0.7917	0.7722
30	0.9191	0.8008	0.7167	0.6946
40	0.8980	0.7498	0.6488	0.6248
50	0.8774	0.7020	0.5873	0.5620
60	0.8573	0.6572	0.5316	0.5055
70	0.8376	0.6153	0.4812	0.4547
80	0.8184	0.5761	0.4356	0.4090
90	0.7997	0.5394	0.3943	0.3679
100	0.7813	0.5050	0.3570	0.3309

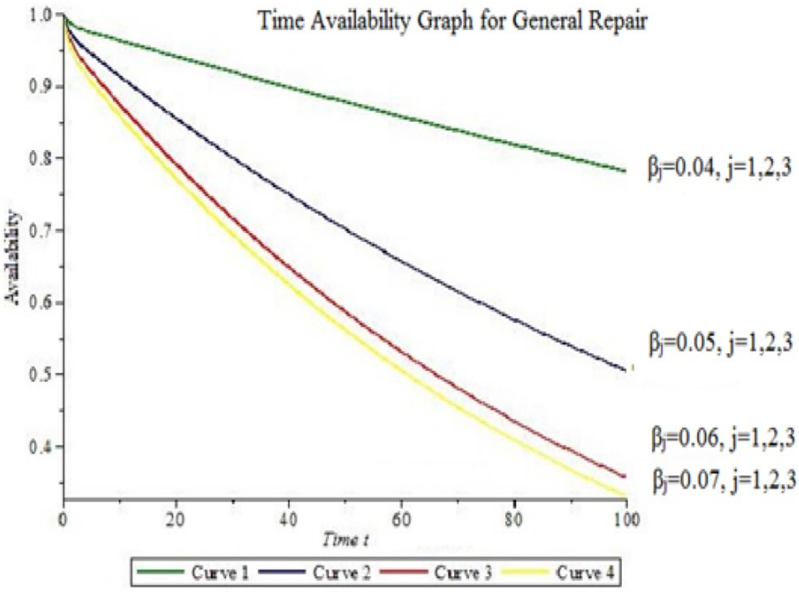


Figure 4: Availability for General Repair

Reliability Analysis:

The system performance of a non-repairable system is known as reliability. Therefor treating all repair of the system to zero in (29) and the inverse Laplace transform of resulting expression give us reliability of system. For the same set of parametric values as in section (7.1) one can obtain the expression (a, b, c, d) as under.

- $R_1(t) = 2.120000000e^{(0.200000000t)} + 2.080000000e^{(1.160000000t)} + 1.040000000e^{(0.1200000000t)}$
- $R_2(t) = -2.150000000e^{(-0.2500000000t)} + 2.100000000e^{(0.2000000000t)} + 1.050000000e^{(-0.1500000000t)}$
- $R_3(t) = 2.120000000e^{(-0.2400000000t)} + 1.060000000e^{(-0.1800000000t)} - 2.180000000e^{(-0.3000000000t)}$
- $R_4() = 1.070000000e^{(-0.2100000000t)} + 2.140000000e^{(-0.2800000000t)} - 2.210000000e^{(-0.3500000000t)}$

The graphical presentation of reliability R(t) variation is shown in figure 5.

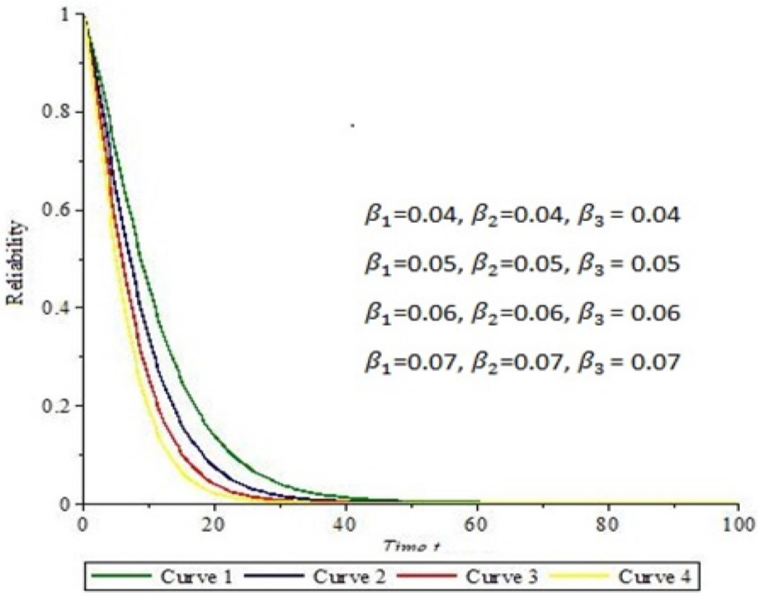


Figure 5: Time v/s Reliability Graph for set of values ($\beta 1, \beta 2, \& \beta 3$)

5.3 Mean Time to Failure (MTTF)

The MTTF is a very important measure of system performance which control failure effect on the system. It deals which unit is more important to get best performance of the system. Mathematically this can be obtained by setting, all repair to zero in equation (29) and then $\lim_{s\to 0}\overline{P}_{up}(s)$ we get expression of MTTF of the system as:

$$MTTF = \lim_{s\to 0}\overline{P}_{up}(s) = \frac{1}{2\beta_1+\beta_2+\beta_3}\left[1+\frac{2\beta_1}{\beta_1+\beta_2+2\beta_3}+\frac{2\beta_3}{2\beta_1+\beta_2+\beta_3}+\frac{8\beta_1\beta_1}{\beta_1+\beta_2+\beta_3}\right] \quad (31)$$

Setting, $\beta_2 = 0.04\beta_3 = 0.04$ and varying β_1 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (31) one may obtain MTTF of the system. Table 2 whose column 2 demonstrates variation of MTTF. with respect to β_1 .

Setting $\beta_1 = 0.04\beta_3 = 0.04$ and varying β_2 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (31) one may obtain MTTF of the system. Table 2 whose column 3 reveals variation of MTTF. with respect to β_2 .

Setting $\beta_1 = 0.04\beta_2 = 0.04$, and varying β_3 as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in (31) one may obtain MTTF of the system. Table 2 whose column 4 establishes variation of MTTF. with respect to β_3 .

Table 4: Table 3.Variation of MTTF with failure rates

Variation in failure rate	MTTF β_1	MTTF β_2	MTTF β_1
0.01	19.89	13.30	18.25
0.02	16.80	14.40	16.22
0.03	14.71	13.72	14.56
0.04	13.17	13.17	13.17
0.05	11.98	12.68	12.00
0.06	11.02	12.24	11.00
0.07	10.23	11.84	10.14
0.08	9.56	11.47	9.40
0.09	8.98	11.15	8.75
0.01	8.47	10.78	8.17

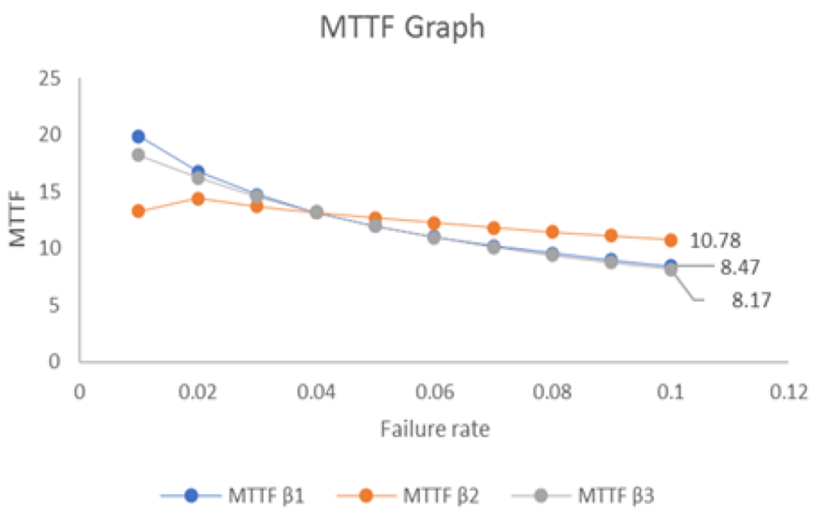


Figure 6: Failure Rate vs. M.T.T.F.

5.4 Cost Analysis

Cost analysis when the repair follow Gumbel Hougaard family Copula distribution

Let the failure and rates of system be, β_1=0.04, β_2=0.04, β_3= 0.04, ϕ = 1, θ = 1, x = 1, and the service facility be always available, then expected profit during the interval [0, t) can be given as the formula, $E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$

Where K_1 and K_2 are revenue service cost per unit time. Hence the expected profit by the operation of the system for time in interval [0, t) when the system repair follow copula repair is given as;

$$E_p(t) = K_1(-0.007579677e^{-2.777881420t}) + (-0.007841644e^{-1.2936331t} \\ -148.213060e^{-.0067854179t} + 0.014820535e^{-1.12000000t} + 148.20) - K_2 t \tag{32}$$

Setting K_1 = 1and K_2 = 0.6, 0.5, 0.4, 0.3, and 0.2 respectively and varying t =0, 10, 20, 30, 40, 50, 60, 70, 80, 90,100 units of time one get Table.

Table 5: Expected profit computation with time variation

Time t	K_2=0.6	K_2=0.5	K_2=0.4	K_2=0.3	K_2=0.2
0	0	0	0	0	0
10	3.71	4.71	5.71	6.71	7.71
20	6.80	8.80	10.80	12.80	14.80
30	9.30	12.28	15.28	18.28	21.28
40	11.22	15.22	19.22	23.22	27.22
50	12.63	17.63	22.63	27.63	32.63
60	13.56	19.56	26.56	31.56	37.56
70	14.03	21.03	28.03	35.03	40.03
80	14.07	22.07	30.07	38.07	26.07
90	13.72	22.72	31.72	40.72	49.72
100	13.00	23.00	33.00	43.00	53.00

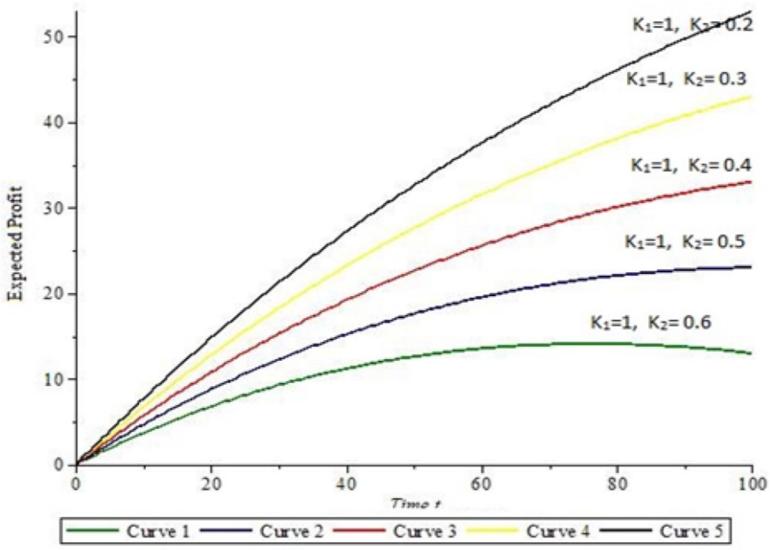


Figure 7: Time vs. Expected Profit

Cost analysis for General Repair: When the repair only follow general repair than expected profit in interval [0,t) can be given by taking $\mu_0 = \theta$. For the same values of failure rates as in copula repair we obtained the expected profit as;

$$E_p(t) = K_1(-0.0101570566e^{-1.3266210t} - 0.0286047923e^{-1.026793715t} \\ -148.1758451e^{-0.0065852769t} + 0.0166291445e^{-1.12000000t} + 148.20) - K_2(t) \tag{33}$$

SettingK_1 = 1 and K_2 = 0.6, 0.5, 0.4, 0.3, and 0.2 respectively and varying t =0, 10, 20, 30, 40, 50, 60, 70, 80, 90,100 units of time one get Table.

Table 6: Table 5: Expected profit computation with time variation

Time t	K_2=0.6	K_2=0.5	K_2=0.4	K_2=0.3	K_2=0.2
0	0	0	0	0	0
10	3.47	4.47	5.47	6.47	7.47
20	6.31	8.31	10.31	12.31	14.31
30	8.59	11.59	14.59	17.58	20.59
40	10.34	14.34	18.34	22.34	26.34
50	11.59	16.59	21.59	26.59	31.59
60	12.39	18.39	24.39	3039	36.39
70	12.75	19.75	26.75	33.75	40.74
80	12.71	20.71	28.71	36.71	44.71
90	12.28	21.28	30.28	39.28	48.28
100	11.50	21.50	31.50	41.50	51.50

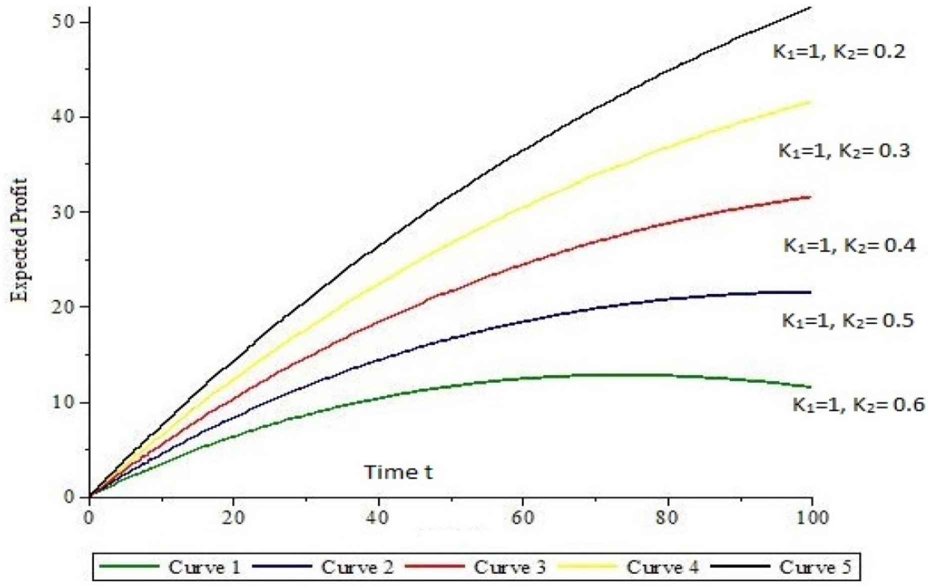


Figure 8: Expected profit as function of time t

6 Result Interpretation and Conclusion

In this paper we have analyzed the system performance based of different types of failures in the associated elements of distributed system via two types of repair employing copula repair approach and general repair. Table1 and Fig.1 provides information availability variation with respect of time. The figure 1 explain that when the failure rates increases than availability deceases. Availability when the failure rates are fixed at $\beta_1 = \beta_2 = \beta_3 = 0.04$, then availability is higher than the values of failure rates $\beta_1 = \beta_2 = \beta_3 = 0.07$. Table 2 and the corresponding Figure 2 presents the availability of system when the repair follows general distribution. By comparing the results from table1 and table 2 we conclude that availability of the system is better when the repair follows copula distribution.

Figure3 presents variation of reliability as non-repairable system. The reliability of system decreases when the failure rates of the subsystems increases. From the Reliability graph it is clear that rate of decrement in reliability values is much high than availability values. One can understand need of regular repair for repairable system to achieving excellent performance. The Figure 4 yields the mean-time-to-failure (MTTF) of the system with respect to variation in failure rates $\beta_1, \beta_2 and \beta_3$ respectively.

When revenue cost per unit time K_1 is fixed at 1, service costs K_2 = 0.6, 0.5. 0.4. 0.3 and 0.2 the Tables 4 & 5, Figures 5 & 6 presents expected profit incurred by operation of the system. The incurred profit is high when the system repair follows copula distribution. In the both cases it can perceive that as service cost decrease profit increases. Therefor one can conclude that copula repair must be recommended over general repair.

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