Probabilistic analysis of a multi-state warm standby k-out-of-n: G system in a series configuration using copula linguists

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Abstract

This paper discusses the reliability analysis of repairable complex system comprising of two subsystems in series configuration together with the controllers. The two subsystems, consisting of three undistinguishable units in a parallel arrangement and functioning under 1-out-of-3: G operational policy. Controllers control both the subsystems and can be unstable, and the malfunction result in the controller prevents system operation. The system may have an unforeseeable catastrophic failure due to which the system may not perform its function once the situation arises. The failure rate of the units is constant, and the exponential distribution is assumed to obey. The two forms of repair namely general repair and Goumbel-Hougard copula repair are used to restore the existing failed units of the system. The supplementary variable technique with Laplace transformation is used to evaluate the output of the system. Using Stochastic theory, differential equations are derived to obtain essential features of reliability such as availability of the system, reliability of the system, MTTF, and profit analysis. Graphs were drawn to highlight the behavior of the results. Tables and figures display the findings and suggest that copula repair is a more efficient repair policy for the improved performance of repairable systems. It brings a different aspect to the research world to adopt multi-dimensional repair in the form of the copula. Besides, the findings of the model are useful for system engineers and maintenance managers.

Keywords: k-out-of-n: G/F system configuration; availability; reliability; MTTF; Controller; catastrophic failure; Gumbel-Hougaard family copula distribution.

I. Introduction

In the design of complex engineering systems, specifically in the manufacturing sector, the research community is lacking in the prospect of developing new frameworks. The architecture of the model must be such that it can execute the task effectively and meet high levels of availability and reliability. Every enhancement in system reliability is always associated with the cost of the system; the improvement in reliability is defensible to the degree that the cost of the system unapproachability exceeds the cost of the basic service offered. Reliability may be increased by the procurement and installation of new paraphernalia or the repair of existing facilities. In addition to the financial component of retaining the status of every industry, customer loyalty is often a crucial prerequisite.

The system reliability and its measures has a crucial role to play in preserving status and customer satisfaction. There are many mechanisms to increase system efficiency and redundancy to boost system efficiency and benefit gained. Any equivalent or non-identical components that are retained in standby mode assist system operations as required after the main unit/component has failed. As per the available reliability theory literature, in particular, three types of standby units viz. cold standby, mild standby, and hot standby have been tested by many scholars in the past. Moreover, redundancy is very cost-effective in ensuring a certain degree of efficiency of the system. Therefore, to increase the stability and efficiency of the k-out-of-n system configuration in which at least k components out of n have to run for the system to be operational play a critical role. To explore some examples of such type of configured structure, a telecommunications system with four transmitters can be modeled as a 2-out-of-4: G system. An extensive bus with six tires four is enabled to perform tasks for time is a 4-out-of-6: G system. Overwhelmingly k-out-of-n system plays a crucial role in system reliability theory for the proper operation of the system. The k-out-of-n-type warm standby method has found various applications in the field of reliability, including redundant system inspection, network architecture, power generation, and transmission networks, etc.

Extensive attempts have been made over the last decades by many scholars, including Kullstam (1981), Zhao (1994), Coit (2001), Park and Pham (2012), Wu and Guan (2005), Xing et al. (2012), and Ram et al. (2013) to establish strategies for solving k-out-of-n types of systems and computing availability, MTBF, and MTTR and other probabilistic measures for repairable systems. They have researched the performance of complex repairable systems employing k-out-of-n: G/F, operational schemes. Following the performance assessment of complex repairable systems, Zuo and Tian (2006) measured the performance of a series-parallel system under varying operating policy conditions. Malinowski (2016) has established a network of inflow points, transit-only nodes, and outflow points. In their network, arcs were regulated, and components were repairable with constant failure and repair rates. The efficiency of this network's performance is determined by the ratio of the total demand met at all the outflow points to the total demand needed at these points. Levitin et al. (2013) looked at mixed-designed series-parallel systems through reliability measures assuming random failure propagation time. The exact reliability formula for consecutive repairable k-out-of-n-type operative systems was showed by Liang et al. (2010). Sharma and Kumar (2017) measured availability and other efficiency measurements of the successive k-out-of- n machining system using standby with multiple working vacations. Eryilmaz (2007, 2009, 2010) has developed formulas for consecutive k-out-of-n: F system using lifetime distribution, reliability, and properties of the k-out-of-n system with arbitrarily dependent components and mixture representations for the protection of successive- k systems. A system with (M+N) units under k-out-of-(M+N): G scheme in which the M units were inactive warm standby mode has been analyzed by Zhang (2006). Kumar and Gupta (2007) evaluated the reliability characteristics of a 1-out-of-2 warm standby system comprising of the main unit with a supporting unit, including a repair facility. Cha et al. (2014) suggested a competing risk model reliability analysis by considering two types of failures partial failure and complete failure as a catastrophic failure phenomenon. They compared deterioration with catastrophic failure and showed that a catastrophic failure is more troubling as the system may not accomplish its function after a catastrophic failure happens. Levitin and Dai (2012) analyzed a multi-state sliding window system with multiple failures mode. Every element can have separate states for minor failed, major failures, and complete failed states. The reliability of the whole system also relies on component reliability, which is assembled in the system.

The controller is a device that controls the output variables and operating conditions imposed with the given dynamical systems. Several researchers around the globe have published their findings on the reliability of complex repairable systems using controllers. It can be used in engineering systems particularly in electronics to control a circuit, in computers as a peripheral unit, in software design to create an interface between models and views, game controllers, etc.

Controllers can also be used in other systems such as linguistics (control the verb), aviation (control the air traffic), biomedical, economic, and socio-economic systems. Digital computers are an integral part of complex engineering systems to control the variables like as to control centrifugal force for controlling speed, to control the furnace temperature, thermostat controller to control room temperature, etc. Ogata (2009) introduced and explained the idea of controllers for modern engineering systems. Authors such as Singh et al. (2013) investigated a system consisting of two subsystems in a series configuration with controllers in which the first subsystem functions under k-out-of-n: G, policy and the second subsystem has three similar units in parallel arrangements. The study under different failure rates and two forms of repairs was carried out. Computation of availability projected that multi-repair would result in better execution of the system. In all the research papers listed above, all the authors discussed several failures and a single method of repair. They fail to note if we have more than one form of repair between two adjacent states that could be possible in a variety of complex systems. If this is feasible, we can test the reliability characteristics using Goumbel-Hougard Copula repair distribution for a completely failed condition. Copulas allow one to isolate the dependency structure in a distribution where two or more variable quantities are involved. Copulas have been introduced by Nelson (2006). To quote some similar work posed by some authors including Ibrahim et al. (2017), Jia et al. (2017), Kumar et al. (2017), and Singh et al. (2020b) examined the reliability measurements of systems comprising subsystems in series configurations and k-out-of-n: G/ F policy with implications of a joint probability distribution. Monika et al. (2019) tested a complex repairable system via a switch and human failure and copula approach. Singh et al. (2020a) investigated a repairable network system of three server-based computer labs under a 2-out-of-3: G scheme. Raghav et al. (2020) studied a dynamic system with two subsystems in a series configuration with imperfect switching devices with copula linguistic approach implications and concluded that copula repair predicts better performance over the general repair. Rawal et al. (2013) analyzed a model of the internet data center including a redundant server with the main mail server trickling different types of failure and two types of repair employing copula distribution. Confirming the various operating choices in the system, some critical analysis was carried out to determine the various reliability features of the system. A repairable warm standby k-out-of-n: G and 2-out-of-4: G systems in series under catastrophic failure and a switching device was recently studied by Poonia et al. (2020) using copula repair. This model was built by taking n-k+1 states into account in the first subsystem in such a way that it formed a finite series during solution unlike as done in the past. Via this article, the scientific community is advised by the authors to carry out multi-dimension repairs in the form of copulas, since they have excellent results over the general repair.

2. Model description and notations

2.1 System description

Refer to the literature discussed in the introduction, none of the authors studied any system consisting of the k-out-of-n: G form of operating strategy with controllers under catastrophic failure. In order to close the difference, we examined the reliability of a repairable warm standby system in series configurations with two subsystems (namely subsystem-1 & 2). Each subsystem is having three similar units in a parallel configuration and follow 1-out-of-3: good working strategy. Units in both subsystems are connected to the controller for the proper functionality of the system, which could be unstable at the time of need and the switching time is instantaneous. Also, the system could face unexpected catastrophic failures during service. There are four types of possible states for the system operation: perfect state, minor failed state, major failed state, and completely failed states. The failure rates of the functional and standby units of each subsystem are constant in nature, but they follow exponential distributions. The repair system is fitted with two distributions general and

Goumbel-Hougard copula distribution. The rate of repair of each device in subsystem-1 and subsystem-2 is regarded as the same but different from each subsystem.

The article is formulated in the following way: Through Section-1, we studied the related work that can be retrieved in various articles. Section-2 of the manuscript deliberates system description, assumptions, and state description. Section-3 consists of system configuration and transition diagram. Section-4 presents mathematical modeling using differential equations. The empirical results for the different output measures of the system are simulated by considering a few specific cases listed in section-5. With the help of graphs, the concluding remarks on our findings with interpretations are provided in Section 6. MAPLE (software) is used to obtain both explicit expressions and numerical evaluations for reliability physiognomies.

2.2. Assumptions

In this article, we consider the following assumptions:

1. Subsystem-1 / subsystem-2 operates effectively until one or more units, are in good working order i.e. "1-out-of-3: G " policy.
2. Both the subsystems have a control unit that is unstable in the system, and the controller's function is as long as the controller fails, the whole system fails immediately."
3. An unforeseeable catastrophic failure of the system could occur at any time (t).
4. The system has four states: Good, minor partially failed, major partially failed, and utterly failed.
5. If the unit has been restored, it is again operational in both the subsystems. No failure was reported due to machine repair.
6. The repairman is available full time and ready to restore minor and major faults.
7. A repair person is available to full time and may repair partially or fully failed units.
8. Partially failed states are restored by employing general repair, while the Gumbel-Hougaard copula can be activated to reinstate the system in case of a complete failure.

2.3. Notations

s , t Laplace transform / Time scale variable

λ_1 / μ_1 Failure rate of each unit in subsystem-1/subsystem-2.

$\lambda_{c_1} / \lambda_{c_2}$ The failure rate of controllers in subsystem-1/subsystem-2.

λ_{c_T} Failure rate related to the catastrophic failure mode.

$\phi_1(x)/\psi_1(y)$ Repair rate of one unit in subsystem-1/subsystem-2.

$\phi_2(x)/\psi_2(y)$ Repair rate of two units in subsystem-1/subsystem-2.

$P_0(t)$ The state transition probability that the system is in S_i state at an instant for $i=0$.

$\bar{P}(s)$ Laplace transformation of the state transition probability $P(t)$.

$P_i(x,t)$ The probability that the system is in state S_i $i=1$ to 8 and the system is under repair with elapsed repair time is x, t x repaired variable and t is time variable.

$E_p(t)$ Expected profit in the interval $\left[0,t\right)$.

K_1, K_2 Revenue generated and service cost per unit time respectively.

$\mu_0(x)$ An expression of the joint probability from failed state S_i to good state S_0 according to Gumbel-Hougaard family copula is given as;

$$\mu_0(x) = C_\theta\{u_1(x), u_2(x)\} = \exp\left[x^\theta + \{\log\phi(x)\}^\theta\right]^{1/\theta} \quad \text{where } u_1(x) = \phi(x), \quad u_2(x) = e^x$$

Here θ is the parameter $1 < \theta < \infty$.

3. System configuration, transition diagram, and state description

The system configuration is shown in Fig 1 (a) while the state transition diagram in Fig 1 (b). The state description of the model highlights that initially all the units in both the subsystems are functioning perfectly and it in a state of S_0. After one unit has failed in either subsystem, it switches to S_1 or S_4 which are regarded as minor partially failed states. If two units have failed in any subsystem, they will be passed to S_2 or S_5 that are the major partially failed states. In both cases, to restore the system we use general repair. States S_3, S_6, S_7, and S_8 are complete failed states due to failure of all the three units in subsystem-1 or2, or due to failure controllers or catastrophic failure. In these complete failed states, a multidimensional repair in the form of the copula is used to restore the system.

Table 1 State Description

State	Description
S_0	This is a perfect state and all units of subsystem-1 and subsystem-2 are in good working condition.
S_1, S_4	The indicated state is deteriorated and deemed to be a minor failed state but is in operational mode after the failure of anyone unit in subsystem-1/2. The remaining two units are well-functioning. The system is being restored through general repair.
S_2, S_5	The indicated state is deteriorated and deemed to be a major failed state but is in operational mode after the failure of any two units in subsystem-1/2. The remaining unit is well-functioning. The system is being restored through general repair.
S_3, S_6 S_7, S_8	The states suggest that the system is in complete failure mode and is being revived using the copula distribution of the Gumbel-Hougaard family.

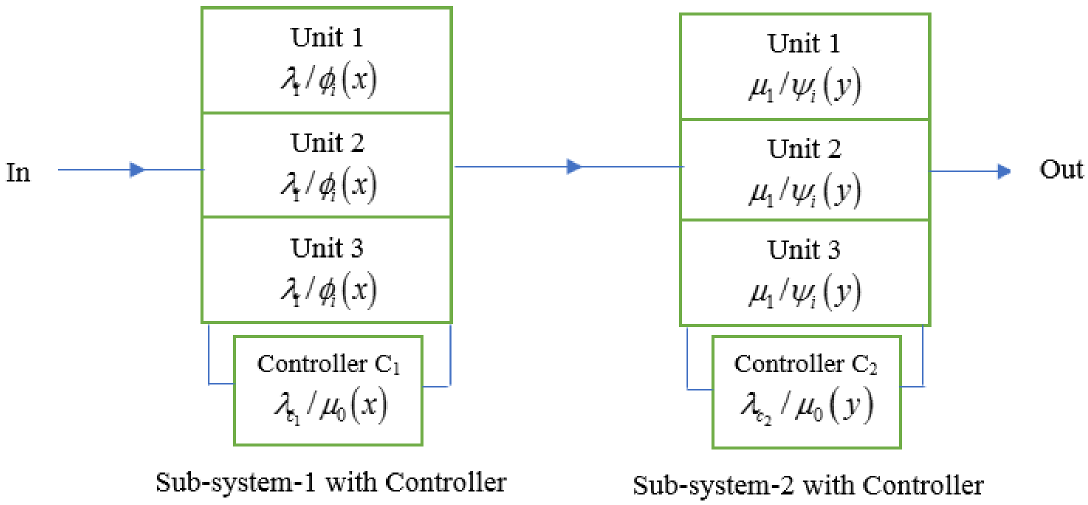

Figure 1 (a) System configuration

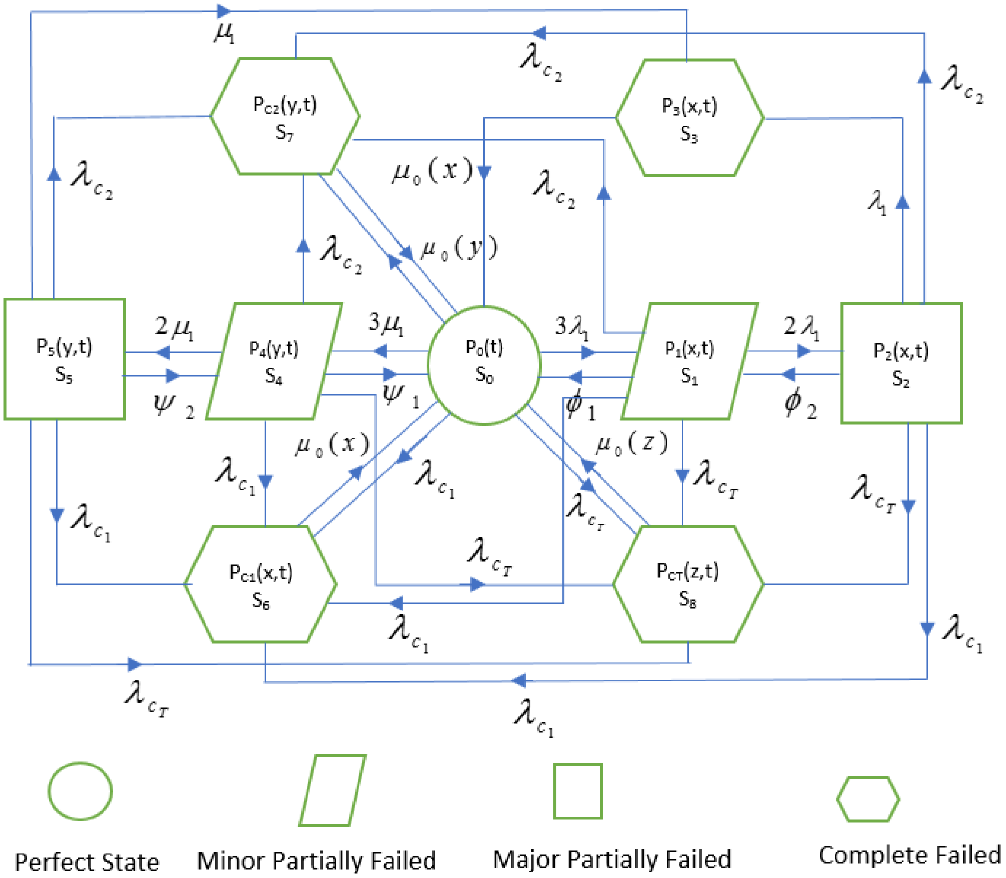

Figure 1 (b) State transition diagram of the model

4. Preparation of mathematical model

Based on stochastic theory arguments, one can easily develop the set of differentials equations associated with the existing mathematical model for the above-mentioned state transition diagram.

$$\left[\frac{\partial}{\partial t}+3\lambda_1+3\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right]P_0(t)=\left[\int_0^\infty \phi_1(x)P_1(x,t)dx+\int_0^\infty \psi_1(y)P_4(y,t)dy\right.$$
$$+\int_0^\infty \mu_0(x)P_3(x,t)dx+\int_0^\infty \mu_0(x)P_{c_1}(x,t)dx$$
$$\left.+\int_0^\infty \mu_0(y)P_{c_2}(y,t)dy+\int_0^\infty \mu_0(z)P_{c_T}(z,t)dz\right] \quad (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\phi_1(x)\right]P_1(x,t)=0 \quad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\phi_2(x)\right]P_2(x,t)=0 \quad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_3(x,t)=0 \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\psi_1(y)\right]P_4(y,t)=0 \quad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\psi_2(y)\right]P_5(y,t)=0 \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_{c_1}(x,t)=0 \quad (7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right]P_{c_2}(y,t)=0 \quad (8)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\mu_0(z)\right]P_{c_T}(z,t)=0 \quad (9)$$

Boundary conditions

$$P_1(0,t)=3\lambda_1 P_0(t) \quad (10)$$

$$P_2(0,t)=2\lambda_1 P_1(0,t)=6\lambda_1^2 P_0(t) \quad (11)$$

$$P_4(0,t)=3\mu_1 P_0(t) \quad (12)$$

$$P_5(0,t)=2\mu_1 P_4(0,t)=6\mu_1^2 P_0(t) \quad (13)$$

$$P_3(0,t)=\lambda_1 P_2(0,t)+\mu_1 P_5(0,t)=6\left(\lambda_1^3+\mu_1^3\right)P_0(t) \quad (14)$$

$$P_{c_1}(0,t)=\lambda_{c_1}\left[P_0(t)+P_1(0,t)+P_2(0,t)+P_4(0,t)+P_5(0,t)\right] \quad (15)$$

$$P_{c_2}(0,t)=\lambda_{c_2}\left[P_0(t)+P_1(0,t)+P_2(0,t)+P_4(0,t)+P_5(0,t)\right] \quad (16)$$

$$P_{c_T}(0,t)=\lambda_{c_T}\left[P_0(t)+P_1(0,t)+P_2(0,t)+P_4(0,t)+P_5(0,t)\right] \quad (17)$$

Initial conditions

$P_0(0)=1$, and other state probabilities are zero at $t=0$ (18)

Solution of the model
Taking Laplace transformation of equations (1) to (17) and using equation (18), we obtain

$$\left[s+3\lambda_1+3\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right]\bar{P}_0(s)=1+\int_0^\infty \phi_1(x)\bar{P}_1(x,s)dx+\int_0^\infty \psi_1(y)\bar{P}_4(y,s)dy$$
$$+\int_0^\infty \mu_0(x)\bar{P}_3(x,s)dx+\int_0^\infty \mu_0(x)\bar{P}_{c_1}(x,s)dx$$
$$+\int_0^\infty \mu_0(y)\bar{P}_{c_2}(y,s)dy+\int_0^\infty \mu_0(z)\bar{P}_{c_T}(z,s)dz] \quad (19)$$

$$\left[s+\frac{\partial}{\partial x}+2\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\phi_1(x)\right]\bar{P}_1(x,s)=0 \quad (20)$$

$$\left[s+\frac{\partial}{\partial x}+\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\phi_2(x)\right]\bar{P}_2(x,s)=0 \quad (21)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\bar{P}_3(x,s)=0 \quad (22)$$

$$\left[s+\frac{\partial}{\partial y}+2\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\psi_1(y)\right]\bar{P}_4(y,s)=0 \quad (23)$$

$$\left[s+\frac{\partial}{\partial y}+\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\psi_2(y)\right]\bar{P}_5(y,s)=0 \quad (24)$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\bar{P}_{c_1}(x,s)=0 \quad (25)$$

$$\left[s+\frac{\partial}{\partial y}+\mu_0(y)\right]\bar{P}_{c_2}(y,s)=0 \quad (26)$$

$$\left[s+\frac{\partial}{\partial z}+\mu_0(z)\right]\bar{P}_{c_T}(z,s)=0 \tag{27}$$

Boundary conditions

$$\bar{P}_1(0,s)=3\lambda_1\bar{P}_0(s) \tag{28}$$

$$\bar{P}_2(0,s)=2\lambda_1\bar{P}_1(0,s)=6\lambda_1^2\bar{P}_0(s) \tag{29}$$

$$\bar{P}_4(0,s)=3\mu_1\bar{P}_0(s) \tag{30}$$

$$\bar{P}_5(0,s)=2\mu_1\bar{P}_4(0,s)=6\mu_1^2\bar{P}_0(s) \tag{31}$$

$$\bar{P}_3(0,s)=\lambda_1\bar{P}_2(0,s)+\mu_1\bar{P}_5(0,s)=6\left(\lambda_1^3+\mu_1^3\right)\bar{P}_0(s) \tag{32}$$

$$\bar{P}_{c_1}(0,s)=\lambda_{c_1}\left[1+3(\lambda_1+\mu_1)+6\left(\lambda_1^2+\mu_1^2\right)\right]\bar{P}_0(s) \tag{33}$$

$$\bar{P}_{c_2}(0,s)=\lambda_{c_2}\left[1+3(\lambda_1+\mu_1)+6\left(\lambda_1^2+\mu_1^2\right)\right]\bar{P}_0(s) \tag{34}$$

$$\bar{P}_{c_T}(0,s)=\lambda_{c_T}\left[1+3(\lambda_1+\mu_1)+6\left(\lambda_1^2+\mu_1^2\right)\right]\bar{P}_0(s) \tag{35}$$

Now solving all the equations with the boundary conditions, one may get

$$\bar{P}_0(s)=\frac{1}{D(s)} \tag{36}$$

$$\bar{P}_1(s)=\frac{3\lambda_1}{D(s)}\frac{1}{\left(s+2\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right)} \tag{37}$$

$$\bar{P}_2(s)=\frac{6\lambda_1^2}{D(s)}\frac{1}{\left(s+\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right)} \tag{38}$$

$$\bar{P}_3(s)=\frac{6\left(\lambda_1^3+\mu_1^3\right)}{D(s)}\frac{1}{s} \tag{39}$$

$$\bar{P}_4(s)=\frac{3\mu_1}{D(s)}\frac{1}{\left(s+2\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right)} \tag{40}$$

$$\bar{P}_5(s)=\frac{6\mu_1^2}{D(s)}\frac{1}{\left(s+\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right)} \tag{41}$$

$$\bar{P}_{c_1}(s)=\frac{\lambda_{c_1}}{D(s)}\left[1+6\left(\lambda_1^2+\mu_1^2\right)+3\lambda_1+3\mu_1\right]\frac{1}{s}=\frac{\lambda_{c_1}}{D(s)}\frac{U}{s} \tag{42}$$

$$\bar{P}_{c_2}(s)=\frac{\lambda_{c_2}}{D(s)}\left[1+6\left(\lambda_1^2+\mu_1^2\right)+3\lambda_1+3\mu_1\right]\frac{1}{s}=\frac{\lambda_{c_2}}{D(s)}\frac{U}{s} \tag{43}$$

$$\bar{P}_{c_T}(s)=\frac{\lambda_{c_T}}{D(s)}\left[1+6\left(\lambda_1^2+\mu_1^2\right)+3\lambda_1+3\mu_1\right]\frac{1}{s}=\frac{\lambda_{c_T}}{D(s)}\frac{U}{s} \tag{44}$$

where $D(s)=s+3\lambda_1+3\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}-3\lambda_1P-3\mu_1R-6\left(\lambda_1^3+\mu_1^3\right)T-\left(\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right)UT$

$$P=\bar{S}_{\phi_1}\left(s+2\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right)=\frac{\phi_1}{s+2\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\phi_1}$$

$$Q=\bar{S}_{\phi_2}\left(s+\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right)=\frac{\phi_2}{s+\lambda_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\phi_2}$$

$$R=\bar{S}_{\psi_1}\left(s+2\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right)=\frac{\psi_1}{s+2\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\psi_1}$$

$$S=\bar{S}_{\psi_2}\left(s+\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}\right)=\frac{\psi_2}{s+\mu_1+\lambda_{c_1}+\lambda_{c_2}+\lambda_{c_T}+\psi_2}$$

$T = \bar{S}_{\mu_0}(s) = \frac{\mu_0}{s+\mu_0}$ and $U = 1+6\left(\lambda_1^2+\mu_1^2\right)+3\lambda_1+3\mu_1$

Sum of Laplace transformations of the state transitions, where the system is in operational mode and failed state at any time, is as follows

$$\bar{P}_{up}(s) = \bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_4(s)+\bar{P}_5(s) \quad (45)$$

$$\bar{P}_{down}(s) = 1-\bar{P}_{up}(s) \quad (46)$$

5. Evaluation of Reliability Characteristics

5.1 Availability of the system

If the system performs with lowered efficiency i.e. it is in partial failure mode then the system is restored through general distribution, but in case of complete failure, repair follows a multivariate distribution namely Gumbel-Hougaard copula distribution, which uses the followings path.

$$\bar{S}_{\mu_0}(s) = \bar{S}_{\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}}(s) = \frac{\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}}{s+\exp\left[x^\theta+\{\log\phi(x)\}^\theta\right]^{1/\theta}} \text{ and } \bar{S}_\phi(s) = \frac{\phi}{s+\phi}.$$

We consider both general distribution and copula distribution while evaluating $\bar{P}_{up}(s)$. Taking the values of failure rates as $\lambda_1 = 0.02, \mu_1 = 0.03, \lambda_{c1} = 0.021, \lambda_{c2} = 0.022, \lambda_{cT} = 0.025$, $\theta = 1$ $x = 1$ $\phi_i = \psi_i = 1(i = 1,2)$ in equations (45). computing inverse Laplace transform, with Maple 17 software one can obtain the following availability expression of the system. Here we have considered the following particular cases:

(a) When both the subsystems have switching device, we obtain,

$$\begin{aligned} P_{up}(t) = {} & 0.030148e^{-2.8040t}+0.024319e^{-1.2900t}-0.003139e^{-1.1309t}-0.011268e^{-1.0955t} \\ & -0.021207e^{-1.0481t}-0.029779e^{-1.0383t}+1.007386e^{-0.0093t}-0.001840e^{-1.0880t} \\ & +0.005382e^{-1.0980t} \end{aligned} \quad (47)$$

(b) When subsystem-2 does not have a switching device i.e. $\lambda_{s_2} = 0$, we obtain,

$$\begin{aligned} P_{up}(t) = {} & -0.001895e^{-1.0660t}+0.005182e^{-1.0760t}+0.020739e^{-2.7765t}+0.027661e^{-1.2728t} \\ & -0.003152e^{-1.1089t}-0.011093e^{-1.0734t}-0.021381e^{-1.0261t}-0.030907e^{-1.0162t} \\ & +1.014845e^{-0.0104t} \end{aligned} \quad (48)$$

(c) When both subsystems 1 and 2 do not have a switching device i.e. $\lambda_{s_1} = \lambda_{s_2} = 0$, we obtain,

$$\begin{aligned} P_{up}(t) = {} & -0.001948e^{-1.0450t}+0.005009e^{-1.0550t}+0.011482e^{-2.7501t}+0.031176e^{-1.2564t} \\ & -0.003163e^{-1.0879t}-0.010942e^{-1.0522t}-0.021539e^{-1.0051t}-0.032022e^{-0.9951t} \\ & +1.021947e^{-0.0114t} \end{aligned} \quad (49)$$

Similar expressions can be obtained for availability under general repair by taking $\mu_0 = 1$. Put the values of t as $t = 0, 5, 10, 15, 20, 25, 30, 35, 40, 45$ and 50 units. The variation in availability under general repair and copula repair can be seen in table-2 and corresponding figure-2.

Table 2 Variation in availability for various t under copula and general repair.

Time (t)	Copula Repair			General Repair		
	(a)	(b)	(c)	(a)	(b)	(c)
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9610	0.9628	0.9648	0.9217	0.9357	0.9498
10	0.9173	0.9141	0.9116	0.8813	0.8895	0.8981
15	0.8754	0.8676	0.8610	0.8426	0.8456	0.8490
20	0.8354	0.8234	0.8132	0.8057	0.8038	0.8027
25	0.7972	0.7815	0.7680	0.7704	0.7640	0.7588
30	0.7607	0.7417	0.7254	0.7367	0.7263	0.7174
35	0.7260	0.7040	0.6851	0.7044	0.6903	0.6782
40	0.6928	0.6681	0.6471	0.6735	0.6562	0.6411
45	0.6611	0.6341	0.6112	0.6440	0.6238	0.6061
50	0.6309	0.6018	0.5772	0.6158	0.5929	0.5730

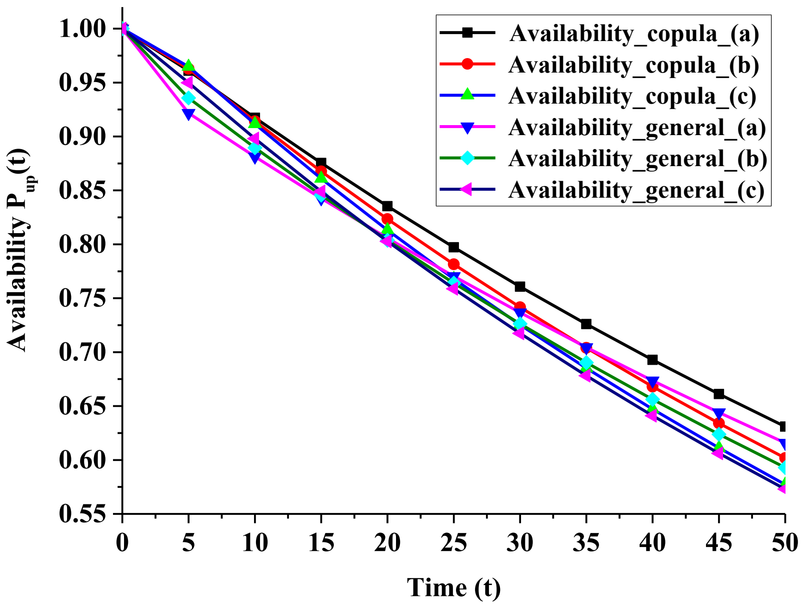

Figure 2 Variation in availability for various t under copula and general repair.

5.2 Reliability of the system

Reliability is the probabilistic measure of a non-repairable system. Taking all repair rates to zero and obtain the inverse Laplace transform in (45), we get the reliability of the system after taking the failure rates as $\lambda_1 = 0.02, \mu_1 = 0.03, \lambda_{c1} = 0.021, \lambda_{c2} = 0.022, \lambda_{cT} = 0.025$. Now consider the same cases as availability, we have

(a) When both the subsystems have switching device, we obtain,

$$R_i(t) = 0.049568e^{-0.0880t} + 12.572026e^{-0.1280t} + 0.141705e^{-1.1309t} + 2.158779e^{-0.1080t} - 4.384342\ 10^{-41} e^{-1.4681t}\left(3.1754\ 10^{41}\cosh(1.3332t) + 3.1887\ 10^{41}\cosh(1.3332t)\right) \quad (50)$$

Similar expressions for the reliability of the system can be obtained in the other two cases. For different values of time-variable $t = 0, 5, 10, 15, 20, 25, 30, 35, 40, 45$ and 50 units of time, one may get different values of reliability $R(t)$ for all the three cases as shown in table-3 and figure-3.

Table 3 Variation in reliability corresponding to the different cases

Time (t)	(a)	(b)	(c)
0	1.0000	1.0000	1.0000
5	0.6798	0.7589	0.8429
10	0.4189	0.5220	0.6440
15	0.2466	0.3431	0.4701
20	0.1419	0.2203	0.3354
25	0.0807	0.1399	0.2365
30	0.0456	0.0883	0.1659
35	0.0257	0.0557	0.1161
40	0.0145	0.0351	0.0813
45	0.0082	0.0221	0.0570
50	0.0046	0.0140	0.0401

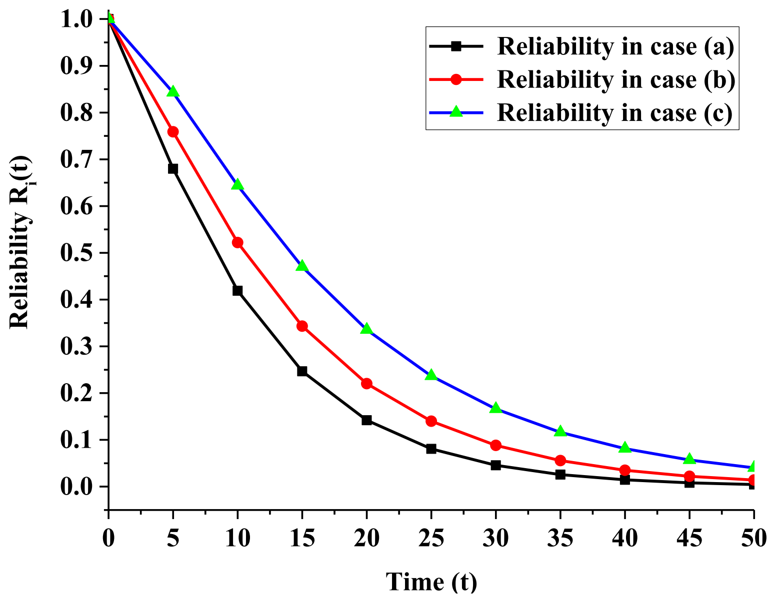

Figure 3 Reliability for various time (t)

5.3 Mean Time to Failure

Taking all repair rate to zero and the limit as s tends to zero in (45) for the exponential distribution; we can obtain the MTTF as:

$$MTTF = \frac{1}{\lambda}\left[1 + \frac{3\lambda_1}{2\lambda_1 + \mu} + \frac{6\lambda_1^2}{\lambda_1 + \mu} + \frac{3\mu_1}{2\mu_1 + \mu} + \frac{6\mu_1^2}{\mu_1 + \mu}\right] \tag{51}$$

where $\lambda = 3\lambda_1 + 3\mu_1 + \lambda_{c_1} + \lambda_{c_2} + \lambda_{c_T}$ and $\mu = \lambda_{c_1} + \lambda_{c_2} + \lambda_{c_T}$

Now taking the values of different parameters a $\lambda_1 = 0.02, \mu_1 = 0.03, \lambda_{c_1} = 0.021, \lambda_{c_2} = 0.022$ and $\lambda_{c_T} = 0.025$ and varying $\lambda_1, \mu_1, \lambda_{c_1}, \lambda_{c_2}$ and λ_{c_T} one by one respectively as $0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10$ in (56), the variation of MTTF, with respect to failure rates can be obtained as given in table-4 and figure-4.

Table 4 Computation of MTTF corresponding to the failure rates

Failure Rate	**MTTF**				
	λ_1	μ_1	λ_{c_1}	λ_{c_1}	λ_{c_T}
0.01	11.2065	12.2242	11.9856	12.1127	12.5101
0.02	10.7387	11.5194	10.8417	10.9466	11.2732
0.03	10.1469	10.7387	9.8897	9.9776	10.2506
0.04	9.5608	10.0101	9.0855	9.1602	9.3916
0.05	9.0201	9.3626	8.3997	8.4619	8.6605
0.06	8.5337	8.7955	7.8031	7.8589	8.0309
0.07	8.1002	8.3002	7.2842	7.3331	7.4836
0.08	7.7143	7.8666	6.8277	6.8709	7.0035
0.09	7.3704	7.4852	6.4231	6.4615	6.5793
0.10	7.0628	7.1480	6.0623	6.0966	6.2018

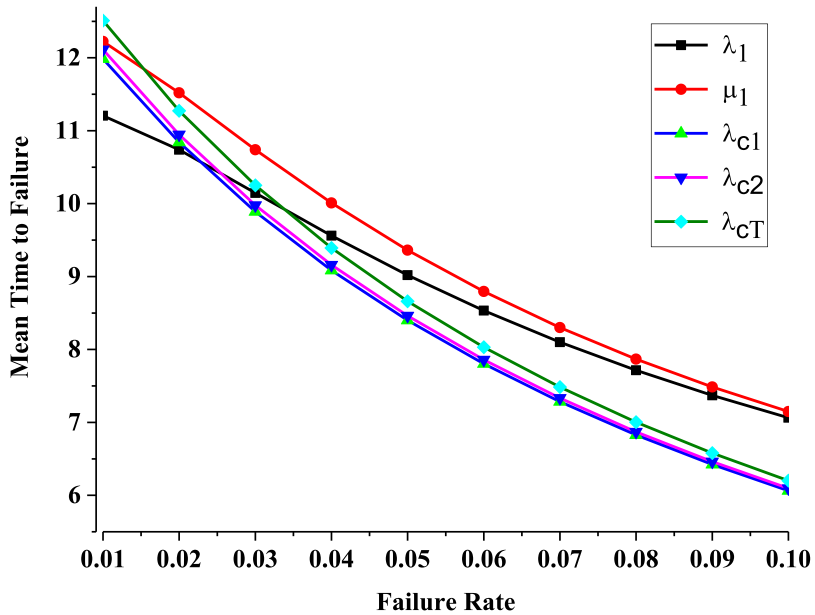

Figure 4 MTTF as a function of failure rates

5.4 Sensitivity Analysis of the system

The model's sensitivity analysis shows how the variance in the mathematical model's output can be attributed to various causes of uncertainty in its input or input variation by considering other inputs as constants. Sensitivity can be attained by taking the partial differentiation of the mean time to failure with respect to the failure rates of the system. Setting the parameters as $\lambda_1 = 0.02$, $\mu_1 = 0.03, \lambda_{c1} = 0.021, \lambda_{c2} = 0.022, \lambda_{cT} = 0.025$ in the partial differentiation of equation (51) obtained using maple, we will get the sensitivity of the system as shown in table-5 and figure-5.

Table 5 Computation of sensitivity with regard to the failure rates

Failure Rate	$\frac{\partial(MTTF)}{\partial\lambda_1}$	$\frac{\partial(MTTF)}{\partial\mu_1}$	$\frac{\partial(MTTF)}{\partial\lambda_{C1}}$	$\frac{\partial(MTTF)}{\partial\lambda_{C2}}$	$\frac{\partial(MTTF)}{\partial\lambda_{CT}}$
0.01	-31.0467	-56.2651	-125.8712	-128.4334	-136.6005
0.02	-56.4646	-77.9323	-103.9328	-105.8611	-111.9739
0.03	-59.9943	-76.3944	-87.2011	-88.6883	-93.3809
0.04	-56.6471	-68.9347	-74.1542	-75.3247	-79.0038
0.05	-51.3805	-60.6203	-63.7889	-64.7261	-67.6625
0.06	-45.9257	-52.9484	-55.4214	-56.1832	-58.5629
0.07	-40.8758	-46.2792	-48.5727	-49.1999	-51.1544
0.08	-36.3955	-40.6038	-42.8988	-43.4211	-45.0452
0.09	-32.4899	-35.8047	-38.1477	-38.5870	-39.9506
0.10	-29.1073	-31.7451	-34.1312	-34.5041	-35.6596

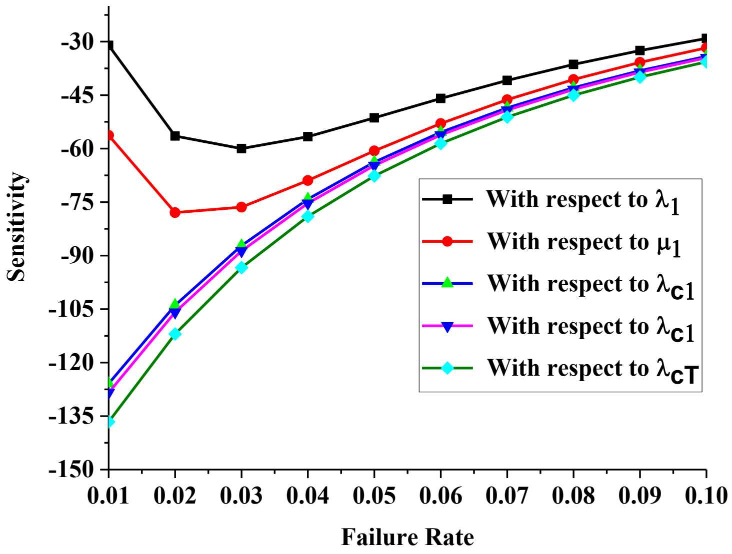

Figure 5 Sensitivity with respect to failure rates

5.5 Cost Analysis of the system

Incurred profit as the system follows copula repair and general repair has been calculated by assuming the same failure and repair rate as per section 5.1. Let us assume the service facility to be open at all times, then the estimated profit to be realized in the interval $\left[0, t\right)$ is

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{52}$$

Where K_1 and K_2 are the revenue generation and service cost in unit time, respectively? Thus

$$\begin{aligned} E_p(t) = K_1 \{ & -0.010751e^{-2.8040t} - 0.018850e^{-1.2900t} + 0.020233e^{-1.0481t} - 107.640872e^{-0.0093t} \\ & + 0.002776e^{-1.130912t} + 0.010286e^{-1.0954t} + 0.028680e^{-1.0383t} - 0.004901e^{-1.0980t} \\ & + 0.001691e^{-1.0880t} + 107.611709 \} - K_2 t \end{aligned} \tag{53}$$

A similar expression can be obtained in case of general repair. Let $K_1 = 1$ $K_2 = 0.1, 0.2, 0.3, 0.4$ and 0.5 us varying $t = 0, 5, 10, 15, 20, 25, 30, 35, 40, 45$ and 50 units of time in Eq. (52), the expected profit under copula repair and general repair can be seen in table-6 and 7 and corresponding diagrams -6 and 7.

Table-6: Expected profit computation in copula repair policy

Time (t)	K_2=0.1	K_2=0.2	K_2=0.3	K_2=0.4	K_2=0.5
0	0.00	0.00	0.00	0.00	0.00
5	4.39	3.89	3.39	2.89	2.39
10	8.58	7.58	6.58	5.58	4.58
15	12.56	11.06	9.56	8.06	6.56
20	16.34	14.34	12.34	10.34	8.34
25	19.92	17.42	14.92	12.42	9.92
30	23.32	20.32	17.32	14.32	11.32
35	26.53	23.03	19.53	16.03	12.53
40	29.58	25.58	21.58	17.58	13.58
45	32.46	27.96	23.46	18.96	14.46
50	35.19	30.19	25.19	20.19	15.19

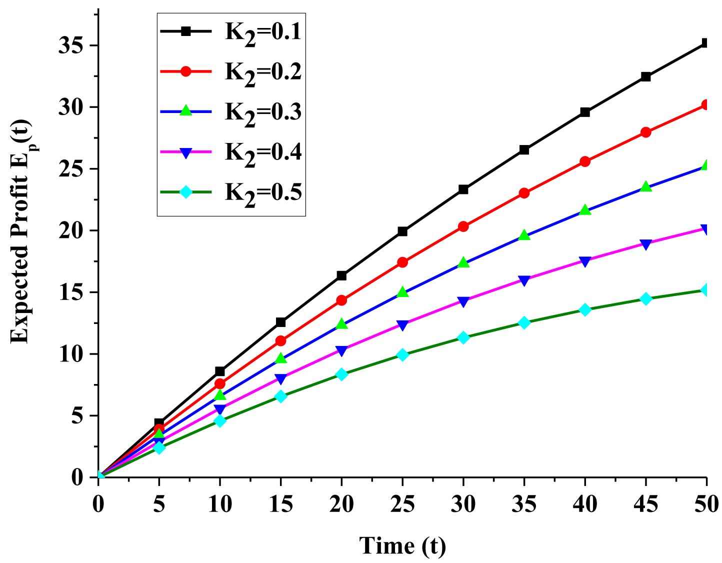

Figure-6: Computation of expected profit in copula repair policy

Table-7: Expected profit computation in general repair policy

Time (t)	K_2=0.1	K_2=0.2	K_2=0.3	K_2=0.4	K_2=0.5
0	0.00	0.00	0.00	0.00	0.00
5	4.23	3.73	3.23	2.73	2.23
10	8.24	7.24	6.24	5.24	4.24
15	12.05	10.55	9.05	7.55	6.05
20	15.67	13.67	11.67	9.67	7.67
25	19.11	16.61	14.11	11.61	9.11
30	22.38	19.38	16.38	13.38	10.38
35	25.48	21.98	18.48	14.98	11.48
40	28.42	24.42	20.42	16.42	12.42
45	31.22	26.72	22.22	17.72	13.22
50	33.86	28.86	23.86	18.86	13.86

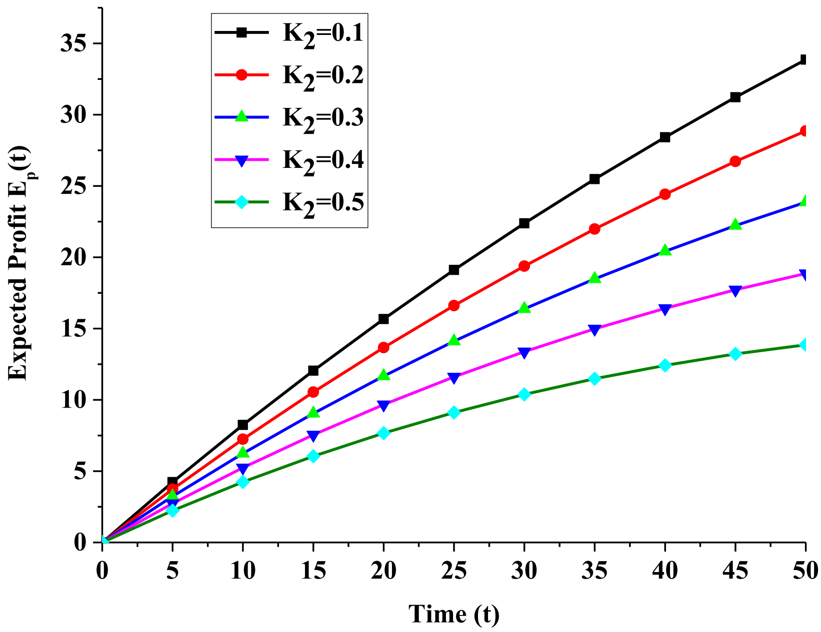

Figure-7: Computation of expected profit in copula repair policy

6. Result discussion and conclusion

This paper analyzes the probabilistic measures of a repairable system consisting of two subsystems in series arrangement with controllers under catastrophic failure. Each subsystem consists of three replica units in a parallel configuration and operates under 1-out-of-3: G strategy. A study of the model with the support of supplementary variables confirms that copula repair is a better and more effective repair policy. The following decisions can be made based on the analysis carried out in this paper:

Table-2 and figure-2 include the variation in the availability of the system in three possible situations under copula repair and general repair when failure rates are set at different time-related values. It can be easily shown that the availability decreases as time t increases in all situations, but it is better in the case of copula repair with controllers. The availability is low with general maintenance and without controllers. Moreover, not only the availability highlights the need for multivariate repair in the form of copulas but also the necessity of controllers.

Table-3 and figure-3 show evidence for the reliability of the system at various time values. The graph revealed a steep decrease in reliability from the top to the bottom in a succinct time in all three situations, depending on the failure rate of units. Furthermore, it can be found that the corresponding values of availability are higher than the reliability, which underlines the need for systematic repair for all dynamic systems for healthier outcomes.

The MTTF of the system concerning variation in $\lambda_1, \mu_1, \lambda_{s_1}, \lambda_{s_2}$, and λ_{c_T} indicated in table-4 and corresponding figure-4. It can be seen that the MTTF of the system reduces with rising values of all the parameters. The MTTF was observed to be the largest in the case of μ_1. Thus, MTTF of the system in all possible scenarios is decreasing as failure rates $\lambda_1, \mu_1, \lambda_{s_1}, \lambda_{s_2}$, and λ_{c_T} increase from 0.01 to 0.10.

Careful observations in table-5 and accompanying figure-5 demonstrate the sensitivity of the system and it is very important to note that sensitivity improves with a rise in failure rate values.

A critical analysis from table 6 (under copula repair) and 7 (under general repair) and figures 6 and 7 indicate that the estimated profit increases as the service cost K_2 decreases, while revenue cost per unit time is set at K_1=1. The estimated predicted profit is maximum for K_2= 0.1 while the minimum profit for K_2=0.5. One may observe that as service cost reduces, benefit swells with the variation of time. In comparison, copula repair is a more efficient repair approach for greater performance of repairable systems, since earnings are higher in the case of copula repair.

The model developed in this paper was found to be highly advantageous in proper maintenance analysis, decision, and evaluation of performance. As far as future studies are concerned, we may increase the number of units in both the subsystems. Furthermore, the optimum reliability and availability of the system can be determined.

Conflicts of Interest: Authors hereby state that there are no conflicts of interest is in this manuscript. This is soul work of authors which has not submitted in any other journal. The manuscript follows ethically guidelines of submissions for the Journal.

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