Costs of Maintenance Service Policy: a New Approach on Constant Stress Partially Accelerated Life Test for Generalized Inverted Exponential Distribution

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Abstract

In this paper, we describe how to analyze and propose the accelerated life test plans for the development of the excellence and reliability of the product. We focus on estimating the costs of maintenance service policy because it has a very significant position to assist any manufacturing organization for sale and available its equipment and maintenance cost-effective. The constant-stress partially accelerated life test is assumed when the lifetime of test units follows Generalized Inverted Exponential distribution under the progressive censoring scheme. The maximum likelihood estimates, Fisher Information matrix, and the asymptotic variance and covariance matrix are obtained. The confidence intervals of the estimators are also obtained. Furthermore, a simulation study is conducted to check the accuracy of the findings.

Keywords: Life Testing, Constant-stress, Maintenance service policy, Progressive censoring, Generalized Inverted Exponential distribution, Simulation Study.

I. Introduction

In current scenario due to rapid and frequent technological changes the demand of manufacturing designs has been improving day by day due to which it is quit challenging and complex to obtain information about the lifetime of items or products under normal usage when the product of high reliability is tested because some commonly used life tests provide no or very few failures at the end of the test. So in such situation accelerated life testing (ALT) may be applied as one of the solution in which the product or material is tested under higher than usual used conditions to obtain the information quickly on the life distribution or performance of a product. These conditions are referred as stresses may be in the form of temperature, voltage, force etc. Generally there are three types of life test methods in accelerated life testing design – First is constant stress ALT, second is step stress ALT and third is progressive stress ALT. In the present research we are focusing only on constant stress accelerated life testing in which we may have fixed stress levels applied for different groups of tested items. It refers that every item is subjected to only one stress level until the item fails or the test is stopped for other reasons. ALT can be divided into two categories: complete (all failure data are available) or censored (some of the failure data are missing).

The data obtained from ALT cannot be extrapolated to use condition because in accelerated life testing (ALT), the mathematical model relating to the lifetime of an item and stress is known or can be assumed.

But, in some cases these relationships are not known and cannot be assumed, So a partially accelerated life test (PALT) can be used in such cases in which the test items are run at both normal and higher than normal stress conditions. The constant stress partially accelerated life test (CSPALT) and step stress partially accelerated life test (SSPALT) are the two commonly used methods in PALT. The products cann't be tested at either usual or higher than usual condition until and unless the test is terminated in CSPALT. On the other hand in SSPALT as an approach to accelerate failures which increases the load applied to the products in a specified discrete sequence. A sample of test items is first to run at use condition and if it does not fail for a specified time, then it is run at accelerated condition until prespecified numbers of failures are obtained or a prespecified time has reached.

In many cases when life data are analyzed, an experiment can be out of control due to many reasons like components of a system may break accidentally and all the units in the sample may not fail. This type of data is called censored or incomplete data. Due to different types of censoring, censored data can be divided into Type I censored (or time censored) data and Type II censored (or failure-censored) data. These two censoring schemes do not allow for units to be removed from the test at points other than the final termination point. Although, the removal of items or components from the test during testing is possible in the progressive type censoring scheme. In such types of situation, the multiple censoring schemes are the best choice for an engineer or reliability Weibull distribution with constant stress under the type-I censoring scheme. Anwar and Islam [4] analyzed the constant stress PALT plan for Gompertz distribution under type I censoring.

Zhang and Fang [5] dealt with an estimation of acceleration factor when the lifetime of units follows Exponential distribution under CSPALT based on type-I censored data. A new approach of constructing the exact lower and upper confidence limits is proposed by them for the acceleration factor. Sadia and Islam [6] presented a study on CSPALT plans when the lifetime of units follows Rayleigh distribution based on type-II censored data. Shi and Shi [7] dealt with a study on CSPALT using the masked series system when the lifetime of components follows Complementary Exponential distribution based on progressive type-II censoring. Ismail [8] discussed a study on CSPALT for Weibull distribution based on a hybrid censoring scheme. He makes a statistical inference by using two methods; maximum likelihood and percentile bootstrap method. Nassar and Elharoun [9] presented an inference on CSPALT for Exponentiated Weibull distribution in the case of multiply censored data. Hassan et al. [10] showed a study on CSPALT for inverted Weibull distribution in the multiply censoring scheme. Cheng and Wang [11] estimated parameters under multiply censoring scheme when the lifetime of items follows Burr XII distribution. Currently, Alam et al. [12] tackled CSPALT based on a multiply censoring scheme when the lifetime of failure units follows the Exponentiated Exponential failure model. Currently, Alam et al. [13] presented a study on ALT when the lifetime of test units follows Burr type-X for Type-II censoring and Progressive censoring, respectively. Alam et al. [14] also presented a study on maintenance service policy under SSPALT when the lifetime of test units follows the Power function failure model with progressive censoring.

The current study based on maintenance service policy problem under CSPALT for Progressive censoring when the lifetime of test units follows Generalized Inverted Exponential distribution.
The information (lifetime data) is censored when the accurate failure time of an item is unknown. Many types of censoring schemes are available, such as left, right, interval, Type-I, Type-II, hybrid, progressive, progressive Type-I, and progressive Type-II censoring, etc. We consider only the progressive Type-II censoring scheme in this paper. The Type-I and Type-II censoring schemes are mainly common and popular schemes in lifetime theory. The only major drawback in both Type-I and Type-II censoring schemes is that the experimenter cannot withdraw live items during the experiment. A newly censoring scheme which is a generalization of classical Type-II censoring scheme comes in light. It gives permission to draw item or items during the experiment.

For literature about this scheme, the authors refer to the book by Balakrishnan and Aggrawalla [15], and an article by Balakrishnan [16]. The progressive Type-II censoring is explained as follows:
The lifetime of n units are $X_1, X_2, ..., X_n$, and these test units are put on the testing. Also, suppose that $X_i, i = 1,2,..., n$ are independent and identically distributed (i.i.d) with cumulative distribution function $F(x)$ and probability distribution function $f(x)$. Before the experiment, an integer m $(m < n)$ is resolved, and the progressive Type-II censoring scheme $(R_1, R_2, ..., R_m, R_i > 0)$ and $n = m + \sum_{i=1}^{m} R_i$ is specified. Now, ith failure is observed, and after the failure, R_i functioning items are randomly removed from the test during the lifetime testing experiment. $X_{i:m:n}, i = 1,2,...n$ and m are the totally observed lifetimes, which are observed samples for the progressively Type-II censoring scheme. $x_{1:m:n} < x_{2:m:n} < ... < x_{m:m:n}$ are the observed values of the progressively Type-II right censored samples.

The paper is organized as follows; The model description, test procedure, and basic assumptions for CSPALT are given in section 2. The point estimation, interval estimation, Fisher information matrix, and confidence intervals are presented in section 3.. The estimating costs of maintenance service policy under Generalized Inverted Exponential distribution are presented in section 4. A simulation study using Monte-Carlo technique is proposed in section 5. Finally, the conclusions are made in last sections.

II. Model Description and Test Procedure

I. Model Description

In life testing theory, the one parameter (negative) Exponential distribution plays an important role because of its simplicity and it prefers to any other one parameter distribution. A generalized case of this distribution is presented by Gupta and Kundu [17] and known as Generalized Exponential distribution. A shape parameter is introduced by him. Lin et al. [18] introduced another extension of Exponential distribution, and this extension is known as Inverted Exponential distribution. They obtained the maximum likelihood estimator, confidence limits and also presented a comparison of this distribution with that of inverted Gaussian and Log-normal distributions using a maintenance data set. Bayes estimators of the parameter and risk functions under special loss functions are obtained by Dey [19]. A new distribution is presented by Abouammoh and Alshingiti [20] which is known as Generalized Inverted Exponential Distribution (GIED). Nadarajah and Kotz [21] noted that this distribution is original from the Exponentiated Frechet distribution. Due to the convenient structure of the distribution function, the GIED can be used in different applications, for example, in accelerated life testing, horse racing, queue theory, modeling wind speeds, etc.
The probability density function (pdf) for GIED is given as

$$f(t, \mu, \eta) = \frac{\eta\mu}{t^2} e^{-\mu/t}(1 - e^{-\mu/t})^{\eta-1}, \ t \geq 0, \ \mu, \eta > 0 \tag{1}$$

where, η and μ are shape and scale parameters, respectively.
The curve of the above equation (1) is shown in figure 1.

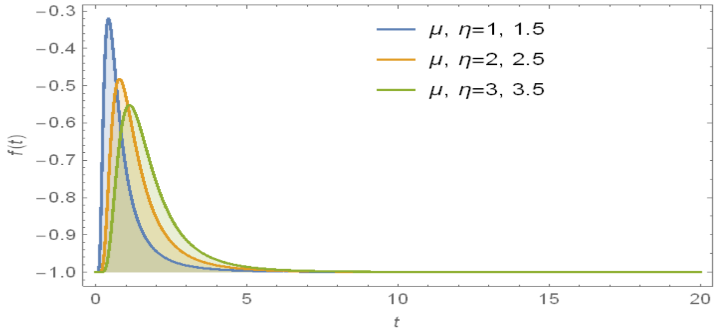

Figure1. *Probability density function curve of GIED*

The cumulative density function (cdf) for GIED is given as

$$F(t,\mu,\eta)=1-(1-e^{-\mu/t})^{\eta},\ t\geq 0\ ,\mu,\eta>0 \tag{2}$$

The curve of the above equation (2) is shown in figure 2.

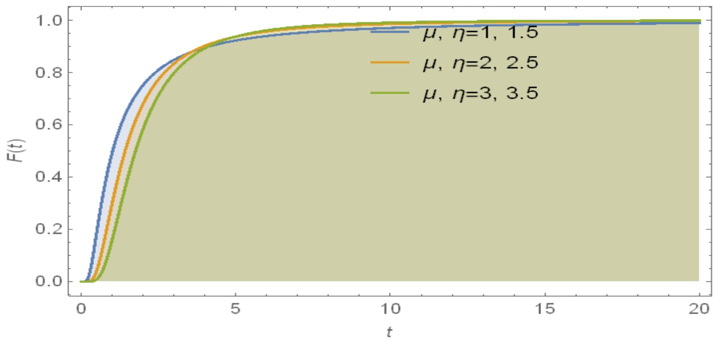

Figure2. *Cumulative density function curve of GIED*

The reliability function for GIED is given as

$$R(t,\mu,\eta)=(1-e^{-\mu/t})^{\eta}$$

The curve of the above expression is shown in figure 3.

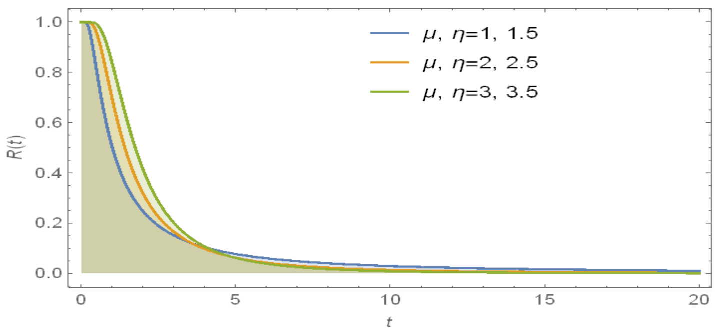

Figure3. *Reliability function curve of GIED*

The hazard function for GIED is given as

$$H(t,\mu,\eta)=\frac{\eta\mu}{t^{2}}e^{-\mu/t}(1-e^{-\mu/t})^{-1},$$

The curve of the above expression is shown in figure 4.

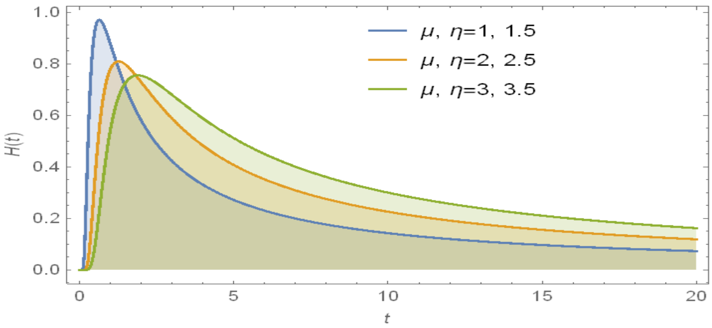

Figure4. *Hazard function curve of GIED*

Abouammoh and Alshingiti [20] and Nadarajah and Kotz [21] studied many interesting and useful properties of GIED. The hazard function of GIED depends on the shape parameter, and it can be increasing, or decreasing but not constant. If the shape parameter is greater than 4, this distribution has a unmoral and right-skewed density function Moreover, this distribution provides a better fit than Gamma, Weibull, Generalized Exponential, and Inverted Exponential distributions. The reliability estimation in the context of this distribution with progressively Type-II censoring scheme is studied by Krishna and Kumar [22].

II. Test Procedure

The test procedure of CSPALT based on progressive Type-II censored data assuming the lifetime item has GIED is described as follows

The pdf under normal condition is given as follows

$$f_1(t_i) = \frac{\eta\mu}{t^2} e^{-\mu/t_i}(1-e^{-\mu/t_i})^{\eta-1},\ t_i \geq 0,\ \mu,\eta > 0, i = 1,2,\dots,m_1 \tag{3}$$

The cdf under normal condition is given as follows

$$F_1(t_i) = 1-(1-e^{-\mu/t_i})^{\eta},\ t_i \geq 0\ , \mu,\eta > 0 \tag{4}$$

where, t_i is the observed lifetime of an item i, that is tested at normal condition.

The pdf and cdf of the lifetime $Y = \beta^{-1}T$, under accelerated condition are given in following equations, (5) and (6)

$$f_2(y_j) = \frac{\beta\eta\mu}{(\beta y_j)^2} e^{-\mu/\beta y_j}(1-e^{-\mu/\beta y_j})^{\eta-1},\ y_j \geq 0,\ \mu,\eta > 0 \tag{5}$$

$$F_2(y_j) = (1-e^{-\mu/\beta y_j})^{\eta},\ y_j \geq 0,\ \mu,\eta > 0, j = 1,2,\dots,m_2 \tag{6}$$

where, y_j is the observed lifetime of an item j, that is tested at the accelerated condition and $(\beta > 1)$ is the acceleration factor.

III. Basic Assumptions

The necessary assumptions for CSPALT are given as

- The lifetimes of items T_i $i = 1,2,\dots,m_1$ are independent and identically distributed (i.i.d.) random variable with pdf provided in equation (3), which is allocated to normal condition.
- The lifetimes of items T_i $i = 1,2,\dots,m_1$ are independent and identically distributed (i.i.d.) random variable with pdf provided in equation (3), which is allocated to normal condition.
- T_i and Y_j are mutually independent also.

- m_1 and m_2 are the total number of items at normal and accelerated conditions, respectively.

$m = m_1 + m_2$ =Total number of items.

III. Estimation Procedure

The point and interval estimation are presented in this section.

I. Point Estimation

Let $X_1, X_2, ..., X_n$ are the lifetime of n independent units which put on test. These units are independently and identically distributed (i.i.d.) as GIED distribution with probability density function, which is presented in equation (1). The m completely ordered lifetimes are denoted by

$$x_{1:m:n} < x_{2:m:n} < ... < x_{J:m:n} < x_{n_1} < x_{J+1:m:n} < ... < x_{m:m:n}$$

Here, J denoted the number of failed units in normal conditions.
Hence, the likelihood function for GIED with progressively Type-II censored data under CS-PALT is given as:

$$L(x_i, \mu, \eta, \beta) = \prod_{i=1}^{J} f_1(x_i)[1 - F_1(x_i)]^{R_i} \times \prod_{i=J+1}^{m} f_2(x_i)[1 - F_2(x_i)]^{R_i} \tag{7}$$

After putting values from equations (3), (4), (5) and (6), we get the following log likelihood function, which is given as

$$\begin{aligned} \ln L(x_i, \mu, \eta, \beta) = & \sum_{i=1}^{J} \ln(\mu\eta) + \sum_{i=1}^{J} \ln(x_i^{-2}) + (\eta - 1)\left[\sum_{i=1}^{J} \ln(1 - e^{-\mu/x_i}) + \sum_{i=J+1}^{m} \ln(1 - e^{-\mu/\beta x_i})\right] + \\ & - \mu\left[\sum_{i=1}^{J} x_i^{-1} + \sum_{i=J+1}^{m} (\beta x_i)^{-1}\right] + \sum_{i=J+1}^{m} \ln(\beta\mu\eta) + \sum_{i=J+1}^{m} \ln(\beta x_i)^{-2} \\ & + \eta\left[\sum_{i=1}^{J} R_i \ln(1 - e^{-\mu/x_i}) + \sum_{i=J+1}^{m} R_i \ln(1 - e^{-\mu/\beta x_i})\right] \end{aligned} \tag{8}$$

where, $\ln L = L(x_i, \mu, \eta, \beta) = l$

The Maximum likelihood (ML) estimates of η, μ , and acceleration factor β are obtained from the following non-linear equations (9), (10) and (11).

$$\begin{aligned} \frac{\partial l}{\partial \mu} = & \sum_{i=1}^{J} \frac{1}{\mu} + (\eta - 1)\left[\sum_{i=1}^{J} \frac{e^{-\mu/x_i}}{x_i(1 - e^{-\mu/x_i})} + \sum_{i=J+1}^{m} \frac{e^{-\mu/\beta x_i}}{\beta x_i(1 - e^{-\mu/\beta x_i})}\right] - \left[\sum_{i=1}^{J} x_i^{-1} + \sum_{i=J+1}^{m} (\beta x_i)^{-1}\right] \\ & + \sum_{i=J+1}^{m} \frac{1}{\mu} + \sum_{i=J+1}^{m} \ln(\beta x_i)^{-2} + \eta\left[\sum_{i=1}^{J} R_i \frac{e^{-\mu/x_i}}{x_i(1 - e^{-\mu/x_i})} + \sum_{i=J+1}^{m} R_i \frac{e^{-\mu/\beta x_i}}{\beta x_i(1 - e^{-\mu/\beta x_i})}\right] \end{aligned} \tag{9}$$

$$\begin{aligned} \frac{\partial l}{\partial \eta} = & \left[\sum_{i=1}^{J} \ln(1 - e^{-\mu/x_i}) + \sum_{i=J+1}^{m} \ln(1 - e^{-\mu/\beta x_i})\right] + \sum_{i=1}^{J} \frac{1}{\eta} \\ & + \sum_{i=J+1}^{m} \frac{1}{\eta} + \sum_{i=J+1}^{m} \ln(\beta x_i)^{-2} + \left[\sum_{i=1}^{J} R_i n(1 - e^{-\mu/x_i}) + \sum_{i=J+1}^{m} R_i \ln(1 - e^{-\mu/\beta x_i})\right] \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\partial l}{\partial \beta} = & -2\sum_{i=J+1}^{m} \beta^{-1} + (\eta - 1)\mu\beta^{-2} \sum_{i=J+1}^{m} x_i^{-1} \frac{e^{-\mu/\beta x_i}}{(1 - e^{-\mu/\beta x_i})} + \eta\mu\beta^{-2} \sum_{i=J+1}^{m} R_i x_i^{-1} \frac{e^{-\mu/\beta x_i}}{(1 - e^{-\mu/\beta x_i})} \\ & + \mu \sum_{i=J+1}^{m} \beta^{-2} x_i^{-1} + \sum_{i=J+1}^{m} \beta^{-1} \end{aligned} \tag{11}$$

The solution of the above three non-linear equations is impossible manually. So an iterative technique (Newton-Raphson method) is applied to solve these equations.

II. Interval Estimation

The Fisher information matrix under progressive Type-II censoring scheme is given as

$$I = \begin{bmatrix} -\frac{\partial^2 l}{\partial \mu^2} & -\frac{\partial^2 l}{\partial \mu \partial \eta} & -\frac{\partial^2 l}{\partial \mu \partial \beta} \\ -\frac{\partial^2 l}{\partial \eta \partial \mu} & -\frac{\partial^2 l}{\partial \eta^2} & -\frac{\partial^2 l}{\partial \eta \partial \beta} \\ -\frac{\partial^2 l}{\partial \beta \partial \mu} & -\frac{\partial^2 l}{\partial \beta \partial \eta} & -\frac{\partial^2 l}{\partial \beta^2} \end{bmatrix} \quad (12)$$

$$-\frac{\partial^2 l}{\partial \mu^2} = \sum_{i=1}^{J} \frac{1}{\mu^2} + (\eta - 1)\left[\sum_{i=1}^{J} \frac{e^{-\mu/x_i}}{(1-e^{-\mu/x_i})x_i^2}\left(1+\frac{e^{-\mu/x_i}}{1-e^{-\mu/x_i}}\right) + \sum_{i=J+1}^{m} \frac{e^{-\mu/\beta x_i}}{(\beta x_i)^2(1-e^{-\mu/\beta x_i})}\left(1+\frac{e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\right)\right]$$

$$+\sum_{i=J+1}^{m} \frac{1}{\mu^2} - \sum_{i=J+1}^{m} \beta^{-2} x_i^{-1} + \eta \left[\begin{array}{l} \sum_{i=1}^{J} R_i \frac{e^{-\mu/x_i}}{(1-e^{-\mu/x_i})x_i^2}\left(1+\frac{e^{-\mu/x_i}}{1-e^{-\mu/x_i}}\right) + \\ \sum_{i=J+1}^{m} R_i \frac{e^{-\mu/\beta x_i}}{(\beta x_i)^2(1-e^{-\mu/\beta x_i})}\left(1+\frac{e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\right) \end{array}\right]$$

$$-\frac{\partial^2 l}{\partial \eta^2} = \sum_{i=1}^{J} \frac{1}{\mu^2} + \sum_{i=J+1}^{m} \frac{1}{\mu^2}$$

$$-\frac{\partial^2 l}{\partial \beta^2} = -2\sum_{i=J+1}^{m} \beta^{-2} - (\eta-1)\sum_{i=J+1}^{m}\left[\frac{\mu x_i^{-1}\beta^{-2}e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\left(\frac{\mu}{x_i\beta^2}+\frac{\mu x_i^{-1}\beta^{-2}e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\right) - \frac{2\beta^{-3}\mu x_i^{-1}e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\right]$$

$$-\eta\sum_{i=J+1}^{m} R_i\left[\frac{\mu x_i^{-1}\beta^{-2}e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\left(\frac{\mu}{x_i\beta^2}+\frac{\mu x_i^{-1}\beta^{-2}e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\right) - \frac{2\beta^{-3}\mu x_i^{-1}e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\right]$$

$$+2\mu\sum_{i=J+1}^{m} \beta^{-3}x_i^{-1} + \sum_{i=J+1}^{m} \beta^{-2}$$

$$-\frac{\partial^2 l}{\partial \mu \partial \eta} = -\left[\sum_{i=1}^{J} \frac{e^{-\mu/x_i}}{x_i(1-e^{-\mu/x_i})} + \sum_{i=J+1}^{m} \frac{e^{-\mu/\beta x_i}}{\beta x_i(1-\beta x)}\right] + -\left[\sum_{i=1}^{J} R_i \frac{e^{-\mu/x_i}}{x_i(1-e^{-\mu/x_i})} + \sum_{i=J+1}^{m} R_i \frac{e^{-\mu/\beta x_i}}{\beta x_i(1-\beta x)}\right]$$

$$-\frac{\partial^2 l}{\partial \mu \partial \beta} = -(\eta-1)\left[\sum_{i=J+1}^{m} \frac{e^{-\mu/\beta x_i}}{\beta x_i(1-e^{-\mu/\beta x_i})}\left(\frac{\mu}{\beta^2 x_i} - \beta^{-1} + \frac{\mu x_i^{-1}\beta^{-2}e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\right)\right]$$

$$-\eta\left[\sum_{i=1}^{J} R_i \frac{e^{-\mu/\beta x_i}}{\beta x_i(1-e^{-\mu/\beta x_i})}\left(\frac{\mu}{\beta^2 x_i} - \beta^{-1} + \frac{\mu x_i^{-1}\beta^{-2}e^{-\mu/\beta x_i}}{1-e^{-\mu/\beta x_i}}\right)\right] - \sum_{i=J+1}^{m} \beta^{-2}x_i^{-1}$$

$$-\frac{\partial^2 l}{\partial \eta \partial \beta} = \sum_{i=J+1}^{m} \frac{e^{-\mu/\beta x_i}\mu x_i^{-1}\beta^{-2}}{1-e^{-\mu/\beta x_i}} + \sum_{i=J+1}^{m} R_i \frac{e^{-\mu/\beta x_i}\mu x_i^{-1}\beta^{-2}}{1-e^{-\mu/\beta x_i}}$$

Now, the variance-covariance matrix under progressive Type-II censoring scheme is the inverse of the Fisher Information matrix and it is given as

$$\Sigma = I^{-1} \quad (14)$$

The ML estimates of distribution parameters and β are asymptotically normally distributed and consistent in large samples.

So, the two-sided approximate $100(1-\alpha)\%$ confidence limits for distribution parameters and β are obtained in the following way:

$$\left.\begin{array}{c} L_{\hat{\eta}} = \hat{\eta} - z_{\gamma/2}\sigma(\hat{\eta}) and U_{\hat{\eta}} = \hat{\eta} + z_{\gamma/2}\sigma(\hat{\eta}) \\ L_{\hat{\mu}} = \hat{\mu} - z_{\gamma/2}\sigma(\hat{\mu}) and U_{\hat{\mu}} = \hat{\mu} + z_{\gamma/2}\sigma(\hat{\mu}) \\ L_{\hat{\beta}} = \hat{\beta} - z_{\gamma/2}\sigma(\hat{\beta}) and U_{\hat{\beta}} = \hat{\beta} + z_{\gamma/2}\sigma(\hat{\beta}) \end{array}\right\} \quad (15)$$

Where $z_{\gamma/2}$ is the $\left[100(1-\gamma)/2\right]^{th}$ standard normal percentile. $\sigma(*)$is the standard deviation for the ML estimates $\hat{\eta}, \hat{\theta}$and$\hat{\beta}$, it is calculating by taking the square root of the first diagonal element of the I^{-1}.

IV. Estimating Costs of Maintenance Service Policy under GIED

There are numbers of authors has explored the problem of maintenance service policy instance, some are as follows- Yiwei et al. [29] studied a cost-driven predictive maintenance policy for structural airframe maintenance. Maintenance policy is formally derived based on the trade-off between probabilities of occurrence of unscheduled and scheduled maintenance. Yiwei et al. [30] proposed predictive airframe maintenance strategies using model-based prognostics. According to them two predictive maintenance strategies based on the developed prognostic model and applied to fatigue damage propagation in fuselage panels. In the research of Lie et al. [31] a preventive maintenance policy is also proposed for the single-unit system failures which have sudden shocks and internal deterioration. The emphasize of the study was to minimize the expected cost per unit time defining the optimal preventive replacement interval, inspection interval, and the number of inspections. Another study was done by Sukhwa et al. [32] For designing and optimizing maintenance service policy. In another study Fabrian and Luis [33] has suggested a method to definite maintenance intervals to those of similar systems under development, and this method has been applied in an aircraft manufacturing company using the current operation database. Michail et al. [34] did one research and developed an aircraft maintenance planning optimization tool and its application to an aircraft component. In another important research by Shey-Huei et al. [35] has suggested the optimal preventive maintenance policy for multi-state systems.

The maintenance service policy ends when the arrangement period reaches time (usage level (H)). The system's renewal is not involved. The preventive and corrective maintenances are under this policy. At a constant interval of time (τ), the system should go for periodically preventive maintenance under this policy. At each failure within successive preventive maintenances, the system should go for minimally repaired. A complicated repairable system with a long life is perfect for this type of service arrangement.

The important Assumptions of the Maintenance Service Policy are:

- The successive failure and random actions are mutually independent.
- The successive failures are said to be known on the parameters of distributions.
- Only minimal repairs are conducted when the repairs were completed in maintenance.
- The Servicing activity held responsible to restores life to a bit.
- The repairs times are minor to compare to the item's life.
- The age renovation is stable even after each preventive maintenance.
- The unit amount of minimal repairs has a constant average between the unit amount of preventive maintenances and preventive maintenances.

The expected cost of maintenance service policy is the sum of the total sum of expected costs, all minimal repairs, and the expected costs of all planned preventive maintenance over the policy's period.

We can get the expected cost of maintenance service per unit time by dividing the expected total cost by the duration of service policy.

According to Rahman [30], the expected cost of maintenance service policy can be defined in the following steps

- Taking the equivalent length of the preventive maintenance period (τ), the expected cost of minimal repairs among preventive maintenance for GIED is given as
$$E(CT_{mr}) = CT_{mr}\left[\sum_{f=0}^{N-1}\int_{f\tau}^{(f+1)\tau} H(t-p\kappa)dt\right] = CT_{mr}\left[\sum_{f=0}^{N-1}\eta\ln(1-e^{-\mu/\tau})\right] \tag{16}$$
- The expected cost of preventive maintenance is given as
$$E(CT_{pm}) = N * CT_{pm} \tag{17}$$

Here, the arrangement is periodically maintained at Nth preventive maintenance.

- The total expected cost per unit time $CT(\tau, N)$ for GIED is given as

$$E(CT(\tau,N)) = \frac{E(CT_{mr}) + E(CT_{pm})}{H} = \frac{CT_{mr}\left[\sum_{f=0}^{N-1}\eta\ln(1-e^{-\mu/\tau})\right] + E(CT_{pm})}{H} \tag{18}$$

where $H = N \times \tau$

V. Simulation Study and Results

In this segment, we carry out a simulation study to check the performance of the estimators having GIED distribution using progressive Type-II censoring scheme. This study is prepared by Monte Carlo Simulation technique by R-Software. To test out the performance of estimators, the means square error (MSEs) and absolute relative bias (RAB) are estimated. The key steps for the study are

(i) The total sample m is divided into two parts, m_1 and m_2.
where $m_1 = m\pi$ and $m_2 = m(1-\pi)$

- Generate random samples of size $m_1\,(t_{1,1} < t_{2,2} < ... < t_{m_1,1})$ and $m_2\,(t_{2,1} < t_{2,2} < ... < t_{m_2,2})$ under normal and accelerated conditions, respectively, from GIED distribution by the inverse CDF method.
- Generate 1000 random samples of sizes 35, 70, and 105 and specify the following values.
Case (I) $(\mu = 0.9, \eta = 0.9, \beta = 2.2)$, Case (II) $(\mu = 0.7, \eta = 0.7, \beta = 2.5)$
Case (III) $(\mu = 0.6, \eta = 1.2, \beta = 2.2)$, Case (IV) $(\mu = 0.5, \eta = 0.9, \beta = 2.5)$
- The distribution parameters and acceleration factor are achieved for each sample and all set of parameters.
- By equation (15), for confidence levels $\alpha = 95\%, 99\%$, the two-sided confidence limits are obtained for parameters μ, η and β.
- The Newton Raphson technique is used to solve all non-linear equations.
- The above steps are replicated 1000 times with different values of parameters.
- From equations (16-18), the expected cost of maintenance service policy is estimated for preventive maintenance, total costs, minimal repairs, and expected cost rate, and the length of the maintenance service policy (H) is chosen as three years.
- At the usual cost $(CT_{pm} = 800)$, preventive maintenance be every four months $(\tau = 0.30)$. If there are failures linking two consecutive preventive maintenance, the minimal repairs will be completed at an average cost $(CT_{mr} = 650)$. Finally, the expected cost of preventive maintenance is 23360, $E(C_{pm}) = 23360$.

Table 1: The Biases and MSEs with different size of samples for progressive Type-II censoring

m	Parameters	Case I ($\mu = 0.9, \eta = 0.9, \beta = 2.2$)			Case II ($\mu = 0.7, \eta = 0.7, \beta = 2.5$)		
		Estimates	RAB	MSE	Estimates	RAB	MSE
35	μ	1.227	0.908	1.192	1.368	1.402	2.002
	η	0.409	0.504	1.063	0.208	0.608	1.155
	β	1.872	0.394	1.531	1.887	2.094	2.360
70	μ	1.098	0.611	0.969	1.109	1.318	1.559
	η	0.502	0.394	0.744	1.009	0.373	1.024
	β	1.998	0.576	0.902	2.665	1.670	2.006
105	μ	2.082	0.299	0.033	1.401	1.703	1.133
	η	0.280	0.155	0.214	0.218	0.099	0.715
	β	2.498	0.221	0.604	1.977	1.137	0.883

Table 2: The Biases and MSEs with different size of samples for progressive Type-II censoring

m	Parameters	Case III ($\mu = 0.6, \eta = 1.2, \beta = 2.2$)			Case IV ($\mu = 0.5, \eta = 0.9, \beta = 2.5$)		
		Estimates	RAB	MSE	Estimates	RAB	MSE
35	μ	1.809	1.767	2.092	2.001	0.969	2.004
	η	2.498	1.091	1.869	2.550	1.308	1.908
	β	1.005	1.351	1.531	1.990	1.782	2.400
70	μ	1.676	1.029	1.760	2.413	0.308	1.610
	η	2.012	0.762	1.444	1.610	0.810	1.042
	β	2.001	0.433	1.202	1.910	1.063	1.204
105	μ	1.302	0.650	1.033	2.915	0.344	0.772
	η	1.643	1.190	0.914	1.709	0.142	0.724
	β	0.985	0.125	8.104	2.809	0.771	1.771

Table 3: At Confidence Level $\alpha = 95\%, 99\%$, the Confidence Limits of Estimates at Various Size of Samples

m	Parameters	Case I : ($\mu = 0.9, \eta = 0.9, \beta = 2.2$)				σ	Case I I: ($\mu = 0.7, \eta = 0.7, \beta = 2.5$)				σ
		CI, $z = 1.96$		CI, $z = 2.58$			CI, $z = 1.96$		CI, $z = 2.58$		
		Lower Bound	Upper Bound	Lower Bound	Upper Bound		Lower Bound	Upper Bound	Lower Bound	Upper Bound	
35	μ	1.25	2.34	1.13	1.81	0.18	0.83	2.19	0.78	1.69	0.22
	η	1.42	2.51	0.97	1.60	0.11	1.40	2.54	1.09	1.66	0.48
	β	0.96	2.11	0.92	1.51	0.72	1.07	2.15	0.60	1.52	0.26
70	μ	1.04	2.05	0.85	1.34	0.10	1.15	2.03	1.16	1.62	0.19
	η	0.91	1.83	0.69	1.26	0.38	0.99	1.80	0.83	1.40	0.28
	β	0.99	1.54	0.93	1.29	0.59	0.81	1.47	0.65	1.45	0.40
105	μ	0.85	1.66	0.82	1.18	0.09	0.70	1.35	0.90	1.43	0.19
	η	0.69	1.32	0.77	1.10	0.28	0.77	1.23	0.81	1.16	0.36
	β	0.66	0.96	0.84	1.06	0.33	0.54	0.82	0.49	0.77	0.61

Table 4: At Confidence Level $\alpha = 95\%, 99\%$, the Confidence Limits of Estimates at Various Size of Samples

m	Parameters	Case III: $(\mu = 0.6, \eta = 1.2, \beta = 2.2)$				σ	Case I V: $(\mu = 0.5, \eta = 0.9, \beta = 2.5)$				σ
		CI, $z = 1.96$		CI, $z = 2.58$			CI, $z = 1.96$		CI, $z = 2.58$		
		Lower Bound	Upper Bound	Lower Bound	Upper Bound		Lower Bound	Upper Bound	Lower Bound	Upper Bound	
35	μ	1.04	2.11	0.91	1.90	0.09	0.78	2.16	0.97	1.91	0.11
	η	1.69	2.65	1.11	1.71	0.19	1.29	2.11	1.11	1.80	0.38
	β	1.73	2.43	0.85	1.63	0.32	1.09	1.88	0.78	1.63	0.31
70	μ	0.79	1.65	1.00	1.44	0.15	1.25	1.93	1.05	1.49	0.10
	η	0.71	1.56	0.61	1.03	0.40	0.96	1.52	0.72	1.29	0.20
	β	1.29	1.44	0.77	1.33	0.22	0.72	1.36	0.59	0.90	0.32
105	μ	0.55	1.09	0.65	0.90	0.14	0.49	0.85	0.62	0.87	0.16
	η	0.69	1.12	0.72	0.94	0.21	0.78	1.53	0.45	0.73	0.32
	β	0.79	0.99	0.85	1.26	0.28	0.76	0.99	0.66	1.16	0.22

Table 5: Expected cost rate, total cost, minimal repair time, and its confidence level for GIED

m	Minimal repair cost			Total cost			Cost rate		
	$E(CT_{mr})$	Lower Bound	Upper Bound	$E(CT_{total})$	Lower Bound	Upper Bound	$E(CT(\tau, N))$	Lower Bound	Upper Bound
Case –I $(\mu = 0.9, \eta = 0.9, \beta = 2.2)$									
35	274558.9	3872.8	8054.4	98192.3	3912.4	4562.3	96832.1	3421.6	49821.5
70	221852.0	5732.1	9023.7	84632.7	4132.5	7099.4	71093.7	5320.2	82313.4
105	19277.0	81432.4	9793.5	80981.4	2987.8	3983.3	65421.2	4542.7	69874.2
Case-II $(\mu = 0.7, \eta = 0.7, \beta = 2.5)$									
35	89198.3	45020.7	61345.8	62176.6	2970.9	36876.9	32790.6	2076.9	45786.9
70	71612.0	9112.4	11935.0	64830.3	4765.8	7876.8	25595.4	4765.2	7176.2
105	45423.8	4023.7	5839.5	59763.9	9762.3	10965.3	20954.2	4859.5	77654.5
Case-III $(\mu = 0.6, \eta = 1.2, \beta = 2.2)$									
35	53909.9	23754.7	3321.9	39654.9	3432.8	4876.8	16593.6	6543.9	9654.9
70	39976.4	44876.6	6654.4	35937.5	4325.8	7354.9	15987.8	6543.5	106543.9
105	41287.4	8565.5	106549.4	30654.1	5435.4	6543.1	29043.6	3876.3	5765.8

VI. Conclusion

This study proposed a partially accelerated life test plan under constant stress and estimating costs of maintenance service policy using the progressive Type-II censoring scheme for the Generalized Inverted Exponential distribution. The following assumptions are:

- As the sample size increases, the values of MSEs and RABs reduce and confidence intervals become narrower or the confidence interval size decreases. Thus, the MLEs have cheering statistical outcomes. We can also observe that the numerical outcomes and theoretical conclusions support each other, and our suppositions are also satisfied. (see, Table 1,2,3 and 4).

- The model parameters and costs of maintenance service policy have direct relationship for the Generalized Inverted Exponential distribution. (see Table 5)
- Also, maintenance service and sample sizes have inverse relationship. (see Table 5)

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