On a Reliability of Tree-Like Transportation Networks

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Abstract

The degree of reliability of the transportation tree-like networks is proposed to be estimated by the index of operational reliability, which is the relative volume of the product not delivered to the point for some time due to the failures of its elements. A method is proposed for calculating this index using a characteristic feature of the structure of transportation network – a tree-like structure and assumes the time invariance of failure and repair flows of its elements. On such structure, the Y-shaped structure-forming fragment is distinguished, the assessment of the reliability of which (in the accepted understanding) is carried out analytically using the concept of the state space. Each of Y-shaped fragment is virtually replaced by one fictitious element, the destruction parameter of which is calculated from the condition of equality of the volumes of the product undelivered to the network output during such a replacement. The calculation of the operational index is reduced to a step-by-step recurrent procedure using the results obtained in the previous step.

Key words: transportation network, product, operational reliability, Y-shaped fragment, failure-repair process, virtual equivalence.

It is difficult to point presently a field of production activity in which, to greater or lesser extent, transportation networks would not be used. With all their diversity, under the transportation network, in the general case, one can understand the aggregate of transport links (mains) along which the necessary movement of a certain “product” in space is carried out.

Transportation networks are classified according to various criteria, in particular, according to their topology. This article discusses a class of tree-like networks designed to transport a product entering at some network inlets to its only outlet, i.e. performing, figuratively speaking, an “aggregation” function (Fig. 1). Examples of such networks, under varying degrees of idealization, are: oil or gas transportation systems from production sites to the main pipeline; sewer networks of large cities; a network of approach lines to the gravity hump of a railway sorting yard of the station; assembly conveyors of various flow-line productions and much more. One of the properties of a transportation network, like any technical object, is the reliability of its functioning. In the “classical” reliability theory, an object is usually estimated using a set of quantitative indexes such as “survival function”, “mean operating time between failures”, and others [1]. For most technical objects, such indexes most often correspond to the everyday insights about the reliability of the object’s functioning and are easily interpreted physically.

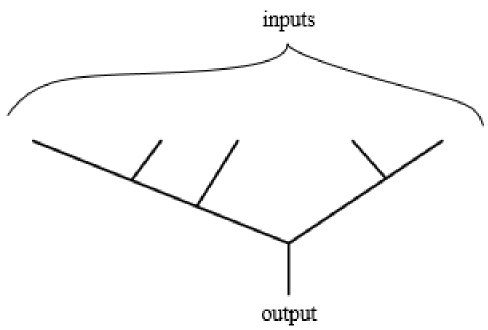

Fig. 1. Tree-like transportation network.

When it comes to transportation networks, these indexes sometimes lead to ambiguity in their interpretation and require additional explanations and clarifications [2-4]. Moreover, when analyzing the operation of transportation networks, it often turns out that such generally different concepts as "reliability" and "operational efficiency" are so interrelated that it is very difficult to separate them from each other. Therefore, the choice of a single quantitative index, to a certain extent, uniting these two concepts, as well as the development of an engineering methodology for its calculation, seems to be actual and important task. One of the possible solution to this problem may be the approach proposed in this article.

For the uniqueness of further understanding, we will specially stipulate the terminology and assumptions accepted in this work:

- topologically, the network is a simply connected labeled graph with several entrance nodes (inputs) and one outgoing (output);
- each transport link of the network will be considered as an "element";
- regardless of the physical character, what the network is intended to transport will be called "product";
- the intensity of the product flow arriving into *i*-th inlet of the network will be called "product flow rate";
- the movement of the product along each element of the network is unidirectional;
- in the process of functioning, any element of the network can be one of two possible states: "in operation" or "under repair";
- transition of any element from working state to repair and back occurs at random moment of continuous time;
- flows of failures and repairs of the *i*-th element are stationary [5] with parameters λ_i and μ_i, respectively.

Consider the simplest Y-shaped transportation network (Fig. 2a)); to Fig. 2b) we will return later.

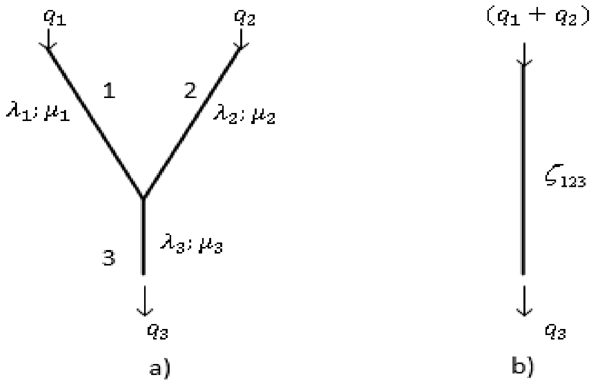

Fig. 2. Y-shaped transportation network a) and its equivalent b).

The network consists of three elements (1, 2 and 3); into inputs 1 and 2 come product value per time unit q_1 and q_2. The purpose of the network is to transport the incoming product to the output of 3nd element (q_3). In the process of functioning, the network elements can fail with rates λ_1, λ_2 and λ_3, respectively, be repaired (with rates μ_1, μ_2 and μ_3), and re-enter in operation. As a result, a part of the product ΔQ that arrived at the network inlets during the time *T* is not delivered to its output. Let us evaluate ΔQ, assuming that all listed technological parameters and parameters of the failure-restoration process are given.

We depict the graph of states for Y-shaped network shown in Fig. 2a). For this purpose, let us number and describe all possible states of the network (there are 8 of them) and assign each of them its stationary possibility p_i ($i = 0 \div 7$), namely:

0: (all three elements in operation) - p_0;

1: (1st element under repair, 2nd and 3rd in operation) - p_1;

2: (2nd element under repair, 1st and 3rd in operation) - p_2;

3: (3rd element under repair, 1st and 2nd in operation) - p_3;

4: (1st and 2nd elements under repair, 3rd in operation) - p_4;

5: (1st and 3rd elements under repair, 2nd in operation) - p_5;

6: (2nd and 3rd elements under repair, 1st in operation) - p_6;

7: (all three elements are under repair) - p_7.

Then, the state space graph for Y-shaped network will be shown in the following form (Fig. 3).

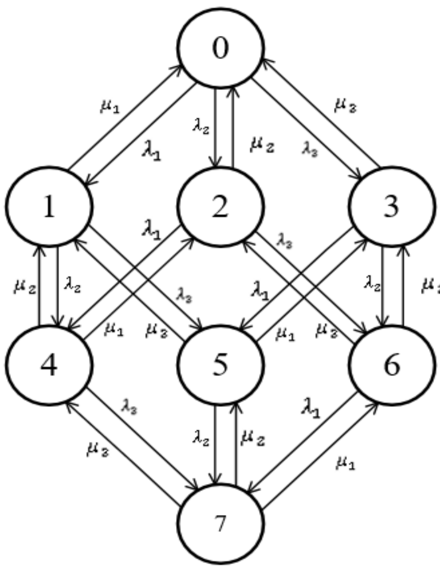

Fig. 3. State space graph for Y-shaped network.

Determination of the probability values p_i is realized in line with a common procedure [5] and reduces to solving of the matrix equation:

$$A \cdot \tilde{P} = \tilde{B}, \quad (1)$$

where (under preceding notations and taking account of the normalization condition $\sum_{i=0}^{7} p_i = 1$) A - is the square matrix of size $[8 \times 8]$ having the form:

$$A = \begin{bmatrix} (\lambda_1+\lambda_2+\lambda_3) & -\mu_1 & -\mu_2 & -\mu_3 & 0 & 0 & 0 & 0 \\ -\lambda_1 & (\lambda_2+\lambda_3+\mu_1) & 0 & 0 & -\mu_2 & -\mu_3 & 0 & 0 \\ -\lambda_2 & 0 & (\lambda_1+\lambda_3+\mu_2) & 0 & -\mu_1 & 0 & -\mu_3 & 0 \\ -\lambda_3 & 0 & 0 & (\lambda_1+\lambda_2+\mu_3) & 0 & -\mu_1 & -\mu_2 & 0 \\ 0 & -\lambda_2 & -\lambda_1 & 0 & (\lambda_3+\mu_1+\mu_2) & 0 & 0 & -\mu_3 \\ 0 & -\lambda_3 & 0 & -\lambda_1 & 0 & (\lambda_2+\mu_1+\mu_3) & 0 & -\mu_2 \\ 0 & 0 & -\lambda_3 & -\lambda & 0 & 0 & (\lambda_1+\mu_2+\mu_3) & -\mu_1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix},$$

B - is the row-matrix of free members [0 0 0 0 0 0 0 1], P - is the row-matrix of required stationary probabilities [p_0 p_1 p_2 p_3 p_4 p_5 p_6 p_7], and "~" – is the transposition symbol.

With every state i can associate a certain product volume ΔQ_i undelivered to the network output at time T due to the network elements failure. This volume is easily determined from Fig. 2a) and equals to: for the state $0 \rightarrow \Delta Q_0 = 0$; $1 \rightarrow \Delta Q_1 = q_1 T$; $2 \rightarrow \Delta Q_2 = q_2 T$; $3 \div 7 \rightarrow \Delta Q_3 \div \Delta Q_7 = (q_1 + q_2)T$.

The undelivered product volume ΔQ is calculated as expectation of the random variable ΔQ_i:

$$\Delta Q = \sum_{i=0}^{7} p_i \cdot \Delta Q_i \,. \tag{2}$$

We introduce a new index of operational reliability.

As a quantitative measure (index) of the operational reliability of Y-shaped transportation network (Fig. 2a)) γ *we will be considering the relative part of the product undelivered to the output due to the failure of network elements*, i.e. [6]:

$$\gamma = \frac{\Delta Q}{Q} = \frac{\sum_{i=0}^{7} p_i \cdot \Delta Q_i}{Q}, \tag{3}$$

where Q is the product volume incoming at the network inputs during the time T ; in the case under consideration $Q = (q_1 + q_2)T$.

As can be seen from (3) the value γ is normalized and can vary from 0 to 1. The value $\gamma = 0$ corresponds to a reliable, and $\gamma = 1$ to an unreliable network. Intermediate values γ characterize the degree 0f reliability of this object (in considered sense).

The index γ is more informative than generally accepted indexes of the "classical" reliability theory ("survival function", etc.) since its value depends not only on the technological parameters of the transportation network ("product flow rate" for inputs), but also from the location of the failed element in the object structure.

For this class of tasks such an index is applicable to characterize a reliability of not only three-element network considered above, but also networks containing any (usually n) number of elements. However, methods for calculating it for practical cases, when n can reach ten or even hundreds, causes significant difficulties: the number of possible network states is 2^n [5], and, for large n, the calculation of their stationary probabilities becomes very resource-expendable even for modern computing technologies. It is possible to use topological calculation methods [17-19, etc.], which directly from the state graph of a complex system allow us to formalize the reliability indicators.

However, for the considered class of problems of analyzing systems with network structures and a specific reliability indicator, the use of known topological methods is also largely resource intensive.

Overcome these difficulties allows an approach proposed further. This approach proceeds from the fact that the probabilities of simultaneous failure of two or more elements of Y-shaped network shown in Fig. 2a) are extremely low. It is important to note that this assumption is applies only to each element of Y-shaped fragment. The state graph of such system is shown in Fig. 4.

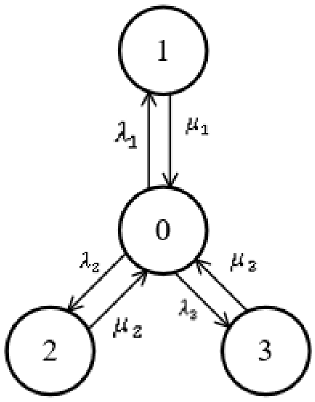

Fig. 4. State graph for Y-shaped network corresponding to the simplifying assumption.

The state numbers accepted in Fig. 4, their contents and notations of the assigned possibilities p_i ($i = 0 \div 3$) are explained below:

0: (all three elements in operation) - p_0;

1: (1st element under repair, 2nd and 3rd in operation) - p_1;

2: (2nd element under repair, 1st and 3rd in operation) - p_2;

3: (3rd element under repair, 1st and 2nd in operation) - p_3.

In this case the solution of equation (3) simplifies and leads to following expressions for stationary possibilities [7]:

$$p_0 = \frac{1}{1+\zeta_1+\zeta_2+\zeta_3}; \tag{4}$$

$$p_1 = \frac{\zeta_1}{1+\zeta_1+\zeta_2+\zeta_3}; \tag{5}$$

$$p_2 = \frac{\zeta_2}{1+\zeta_1+\zeta_2+\zeta_3}; \tag{6}$$

$$з_2 = \frac{\zeta_3}{1+\zeta_1+\zeta_2+\zeta_3}, \tag{7}$$

when dimensionless parameters are additionally introduced: $\zeta_1 = \lambda_1 / \mu_1$; $\zeta_2 = \lambda_2 / \mu_2$; $\zeta_3 = \lambda_3 / \mu_3$.

Each i-th state corresponds to the volume of the product undelivered to the network output that will be now (see Fig. 2a)): $0 \rightarrow \Delta Q_0 = 0$; $1 \rightarrow \Delta Q_1 = q_1 T$; $2 \rightarrow \Delta Q_2 = q_2 T$; $3 \rightarrow \Delta Q_3 = (q_1 + q_2)T$, and the expectation ΔQ is calculated as:

$$\Delta Q = \sum_{i=0}^{3} p_i \cdot \Delta Q_i = \frac{(\zeta_1+\zeta_3)q_1 + (\zeta_2+\zeta_3)q_2}{1+\zeta_1+\zeta_2+\zeta_3} \cdot T. \tag{8}$$

The results of performed numerical calculations given in [3] allow us to speak that a calculation error is relative and is acceptable when replacing the exact formula (2) with approximate (8) amounting to the tenth parts of a percent.

Let us carry out the equivalency operation symbolically shown in Fig. 2, i.e. let us replace the Y-shaped network with one element, choosing the parameter ζ_{123} that determines its "failure-repair" process from the condition of equality of expectations ΔQ corresponding to Fig. 2a) and Fig. 2b). Applying the technique described in [8], we get:

$$\zeta_{123} = \frac{(\zeta_1 + \zeta_3)q_1 + (\zeta_2 + \zeta_3)q_2}{(1+\zeta_2)q_1 + (1+\zeta_1)q_2}. \tag{9}$$

Considering that ζ_1 and ζ_2 are usually negligible in comparison with unity, we have with a high degree of accuracy finally:

$$\zeta_{123} = \frac{(\zeta_1 + \zeta_3)q_1 + (\zeta_2 + \zeta_3)q_2}{q_1 + q_2}. \tag{10}$$

It can be shown [3] that the value ζ_{123} calculated by (10) numerically coincides with the quantitative measure of the reliability of Y-shaped network (see (3)) proposed in this article. Thus, the system in Fig. 2a) can be formally replaced by one equivalent fictitious element (Fig. 2b)), the only parameter ζ_{123} of which is unambiguously expressed through the parameters of the original Y-shaped network.

The operation of equivalency can be applied not only to Y-shaped network, but also to any three-like transportation structure having an arbitrary number of elements. This possibility follows from the topological feature of such networks.

Consider again the network shown in Fig. 1. It is easy to see that this tree-like transportation network is a certain connection (composition) of Y-shaped fragments. In this sense, such fragment of the network can be considered as a structure-forming fragment. Taking into account that each such fragment can be virtually replaced by one equivalent fictitious element, the following procedure is proposed for quantifying the reliability (in sense considered here) of the network as a whole.

Determination of the operational reliability index is reduced to a recurrent step-by-step procedure for equivalent Y-shaped network fragments, at each stage of which the results of calculations at the previous step are used as input data. Each such step, starting from the inputs, leads to a new (virtual) network, in respect of which the procedure is repeated. The equivalence process ends when the original network is represented by only one fictitious Y-shaped fragment, the reliability index of which is determined in an elementary way (by analogy with formula (10).

It is most convenient to demonstrate the application of this algorithm with a specific numerical example. As such an example, consider the network shown in Fig. 1, enumerating its elements and designating the parameters used in further calculations (Fig. 5a)).

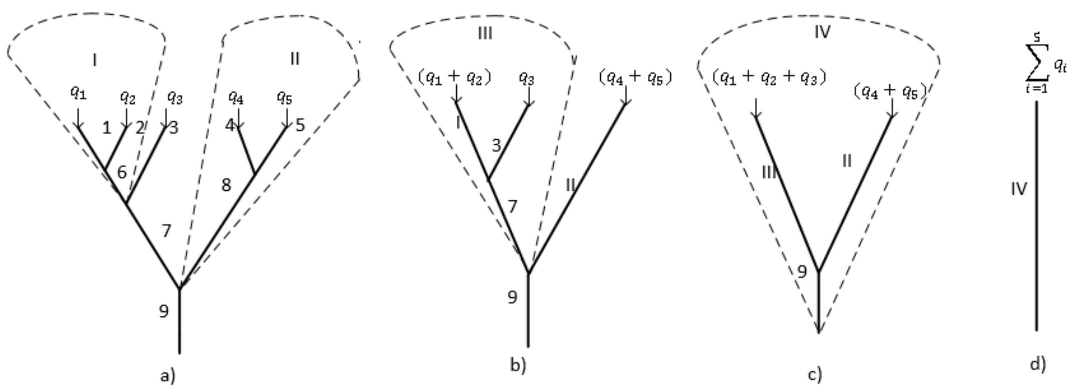

Fig. 5. Transportation network a) and its virtual transformations b), c), d).

We assume the values of the failure (λ_i) and repair (μ_i) rates for all elements of the original network proceeding only from convenience of considerations in calculations carrying out. Besides, we take into account that they mutual relations are typical for real objects [3]. These data are tabulated in Table 1.

Table 1. Input data for the numerical example.

Element number; i	1	2	3	4	5	6	7	8	9
λ_i ; proper units	0,5	0,7	0,8	0,2	0,9	0,4	0,2	0,3	0,1
μ_i; proper units	365	365	365	365	365	365	300	300	180
$\zeta_i \times 10^3$	1,37	1,92	2,19	0,54	2,47	1,09	0,67	1,0	0,56

In lower line of Table 1 the values of dimensionless parameter ζ_i characterizing the "failure-repair" process of *i*-th element of the considered network are given. In addition, we assign the product flow rates at network inputs (in proper units): $q_1 = 1{,}5$; $q_2 = 1{,}5$; $q_3 = 2{,}0$; $q_4 = 2{,}0$; $q_5 = 3{,}0$.

We proceed to the step-by-step procedure.

Step 1. From Fig. 5a) it can be seen that from the network elements adjacent to the inputs, two Y-shaped fragments can be distinguished: I, including elements 1, 2 and 6, and II, including elements 4, 5 and 8. In Fig. 5a) these fragments are conventionally enclosed in contours bounded by dashed lines. Let us carry out the equivalence of these fragments calculating, respectively, ζ_I and ζ_{II} according to the formula (10):

$$\zeta_I = \frac{(\zeta_1 + \zeta_6)q_1 + (\zeta_2 + \zeta_6)q_2}{q_1 + q_2} = 2{,}735 \cdot 10^{-3}, \quad (11)$$

$$\zeta_{II} = \frac{(\zeta_4 + \zeta_8)q_4 + (\zeta_5 + \zeta_8)q_8}{q_4 + q_5} = 2{,}698 \cdot 10^{-3} \quad (12)$$

This makes it possible to replace the original network with its virtual analogue (Fig. 5b)).

Step 2. On the network thus obtained, we carry out again the equivalence operation (contour III), and calculate ζ_{III} using the results (11) and (12):

$$\zeta_{III} = \frac{(\zeta_I + \zeta_7)(q_1 + q_2) + (\zeta_3 + \zeta_7)q_3}{q_1 + q_2 + q_3} = 3{,}187 \cdot 10^{-3}. \quad (13)$$

We turn to the network shown in Fig. 5c).

Step 3. Similarly, we equivalent the system of elements III, II and 9 (contour IV). We calculate ζ_{IV} :

$$\zeta_{IV} = \frac{(\zeta_{III} + \zeta_9)(q_1 + q_2 + q_3) + (\zeta_{II} + \zeta_9)(q_4 + q_5)}{q_1 + q_2 + q_3 + q_4 + q_5} = 3{,}502 \cdot 10^{-3}. \quad (14)$$

Step 4. We pass to one virtual element (Fig. 5d)) which replaces the entire original network.

However, as explained above, the value ζ_{IV} numerically coincides with quantitative measure of reliability (operational reliability index) γ for the original network. Physically, for this example, the result obtained indicates that, on average, about 0,35 % of the product volume entering the network inputs is not delivered to the output due to the unreliability of its elements.

Thus, the proposed method for determining the index of the operational reliability of the transportation network is solved in just four steps of the procedure of sequential virtual transformations (equivalence) of the object, at each of which simple intermediate calculations are performed. Let us note, by the way, that the solution of this problem by drawing up the Kolmogorov equations would lead to the need to determine the stationary probabilities of network states from a system of 512 interconnected algebraic equations.

In conclusion we note that the operational index γ can be useful in solving many practical problems, such as, for example, assessing a technical condition of the network at the current time, drawing up plans for medium- and long-term actions for the sequence of repairs and renovations of the network, developing alternate versions for its expansion and development (if necessary), and others. At the same time, considering the informational richness of index γ, at least some of this kind of tasks can be posed and solved as optimization ones [9].

Finally, one more remark. The beginning of the article, as a limitation of the method under discussion, the assumption about the time invariance of failure and repair flows of all transportation network elements is indicated. Meanwhile, there are cases when for some elements this assumption clearly contradicts reality, and, thus, makes the application of the developed method incorrect. This difficulty can be circumvented by approximate “stationarization” non-stationary flows of events. The methodology of such stationarization has be developed and published both for special cases: seasonally changing [10-13] or linearly increasing in time failure rate [14], and for the case when network element “ages” according to an arbitrary but well-known law [15-16].

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