A New Ranking in Hexagonal Fuzzy number by Centroid of Centroids and Application in Fuzzy Critical Path

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Abstract

This paper intends to introduce a different ranking approach for obtaining the critical path of the fuzzy project network. In the network, each activity time duration is viewed by the fuzzy hexagonal number. This study proposes an advanced ranking approach by applying the centroid of the Hexagonal fuzzy number. The Hexagon is separated into two right angles and one polygon. By applying the right angle and polygon centroid formula, we can calculate the centroid of each plane and calculate the centroid of the centroid. It also focuses on the arithmetic operations in Hexagonal fuzzy numbers. The developed strategy has been described by a numerical illustration and is correlated with a few of the existing ranking approaches.

Keywords: Fuzzy critical path, fuzzy triangular number, ranking function, centroid, the centroid of centroid, hexagonal fuzzy number.

I. Introduction

Construction is an essential planning tool and organizes the implementation of a specific project. The network diagram plays a critical aspect in the completion period of the formative project. The Critical Path Method (CPM) is a successful approach to scheduling and controlling large management and construction projects. The Critical Path Method was developed at the beginning of the 1960s; with the support of the critical path, the decision-maker will follow an acceptable technique of maximizing the project period and the possible tools to achieve the project's earliest completion and quality.

The fuzzy set theory can always play significant role in dealing with the complexity of the activity's durations in a project network in this type of problem.

In 1965, Zadeh [8] recommended the fuzzy set concept to represent undefined terms. Jain [10, 11] recommended a ranking approach applying the notion of maximizing the fuzzy number of the order set in 1976.

Yager [12, 13] also suggested some ranking functions, where the hypothesis of normality or convexity is not assumed. In1981, FPERT [20] was suggested by Stefan Chanas and Kamburowski for project completion time estimation. They did not suggest FPERT as an alternative approach to probabilistic methods because there is no statistical correlation between them. Therefore, there was a need for developing the concept of ranking fuzzy numbers. Subsequently, Lee and Li [5] suggested fuzzy ranking depending on two distinctive factors: mean and distribution of fuzzy numbers in 1981. Cheng suggested a later coefficient of variance (CV index) in 1988 to enhance Lee and Li's concept [3]. In 2006, Abbasbandy et. al [17] introduced a ranking approach based on sign distance. Asady et. al [2] suggested a new ranking fuzzy number strategy by minimization of distance. There are some limitations to Asady's strategy. So, Abbasbandy and Hajjari [18] suggested the magnitude of fuzzy numbers in 2009 to enhance Asady's strategy. In 2013, Rajarajeswari and Sahaya Sudha [14, 15] suggested a new ranking function in linear fuzzy hexagonal numbers and applied it to the fuzzy linear programming problem. In 2015, Thamaraiselvi and Santhi [21] resolved the fuzzy transportation problem by applying the magnitude of Hexagonal fuzzy numbers. In 2016, Sudha and Revathi [1] proposed a new ranking on Hexagonal fuzzy numbers and utilized it to a fuzzy linear programming problem. Selvakumari and Sowmiya [19] proposed a methodology in 2017 for locating fuzzy critical paths utilizing Pascal's triangle graded mean integration when the period of every activity is expressed as a Linear Hexagonal fuzzy number. In 2017, Elumalai et al; introduced [6] to solve the fuzzy transportation problem by applying the Robust ranking approach. Rajendran et.al; proposed a new ranking in generalized hexagonal fuzzy numbers in 2018, and results compared with the magnitude of a fuzzy hexagonal number. In 2020, Leela-Apiradee et. al; introduced [9] Hexagonal fuzzy number cardinality, which is used to understand a technique for categorizing Hexagonal fuzzy numbers, and proposed a ranking approach Hexagonal fuzzy numbers, especially on their possible mean values. In 2020, Avishek et.al.[4] introduced a new ranking and defuzzification idea to transform a fuzzy hexagonal number into a crisp number determining its significance for solving decision-making problems. In 2020, Thirupathi et al. [22] Introduced a new raking approach depending on the fuzzy hexagonal number utilizing the centroid formula of triangle and rectangle and considering the distance from the origin to centroid centroided. It is considered as a ranking in Hexagonal fuzzy numbers.

2. Basic Definitions

2.1 Fuzzy Set [8]

"A fuzzy set $\tilde{A}$ is defined on the universal set of real numbers; $\tilde{R}$ is a fuzzy number with the following its membership function.

(i) $\mu_{\tilde{A}}(x)$:$\tilde{R}\rightarrow$ [0,1] is continuous
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, p] \cup [s, \infty]$
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[p, q]$ and strictly decreasing on $[r, s]$
(iv) $\mu_{\tilde{A}}(x)$=1 for all $x \in [q, r]$ where $p \leq q \leq r \leq s$."

2.2 Fuzzy Number [8]

"A Fuzzy set $\tilde{A}$ of the real line $\tilde{R}$ with membership function $\mu_{\tilde{A}}(x)$: $\tilde{R}\rightarrow$ [0,1] is called the fuzzy number, if

(i) $\tilde{A}$ must be a normal and convex fuzzy set.
(ii) The support of $\tilde{A}$ is finite."

2.3 Generalized Fuzzy Number [13]

"A fuzzy set $\tilde{A}$ is known as a generalized fuzzy number on a universal set of real numbers if its membership function has the following conditions:

(i) $\mu_{\tilde{A}}(x)$: $\tilde{R} \rightarrow [0,1]$ is continuous
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [-\infty, p] \cup [s, \infty]$
(iii) $\mu_{\tilde{A}}(x)$is strictly increasing on $[p, q]$ and strictly decreasing on $[r, s]$
(iv) $\mu_{\tilde{A}}(x) = \omega$ for all $x \in [q, r]$ where 0<ω≤1."

2.4 Trapezoidal Fuzzy Number [14]

"A fuzzy number $\tilde{A}$ is a Trapezoidal fuzzy number denoted by (p, q, r, s), and its membership function is given below. Where $p \leq q \leq r \leq s$.

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-p}{p-q}, for\, p \leq x \leq q \\ 1, \quad for\, q \leq x \leq r \\ \frac{s-x}{s-r}, \ for\, r \leq x \leq s \\ 0, \quad otherwiise" \end{cases} \tag{1}$$

2.5 Generalized Trapezoidal Fuzzy Number [14]

"Generalized Fuzzy number $\tilde{A} = (p, q, r, s, \omega)$is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \omega\left(\frac{x-p}{p-q}\right), for\, p \leq x \leq q \\ \omega, \quad for\, q \leq x \leq r \\ \omega\left(\frac{s-x}{s-r}\right), \ for\, r \leq x \leq s \\ 0, \quad otherwise" \end{cases} \tag{2}$$

Generalized Trapezoidal fuzzy number diagram represented in Figure 1.

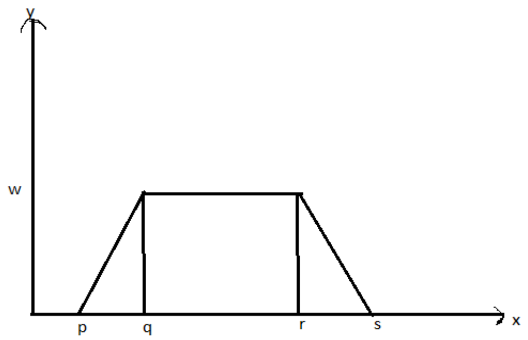

Figure 1: Graphical representation of GTFN

2.6 Hexagonal Fuzzy Number [14]

"A Fuzzy number $\tilde{A}_H$ is a Hexagonal fuzzy number represented by $\tilde{A}_H = (p, q, r, s, t, u)$where a, b, c, d, e, f are real numbers and its membership function $\mu_{\tilde{A}}(x)$is given below

$$\mu_{\tilde{A}_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-p}{q-p}\right) \ , \ for\ p \leq x \leq q \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-q}{r-q}\right), \ for\ q \leq x \leq r \\ 1\ , \quad for\ r \leq x \leq s \\ 1 - \frac{1}{2}\left(\frac{x-s}{t-s}\right), \ for\ s \leq x \leq t \\ \frac{1}{2}\left(\frac{u-x}{u-t}\right), \quad for\ t \leq x \leq u \\ 0\ , \quad otherwise" \end{cases}$$

2.7 Generalized Hexagonal Fuzzy Number [16]

"A generalized Hexagonal Fuzzy number represented by $\tilde{A}_H = (p, q, r, s, t, u, \omega)$ where a, b, c, d, e, f are real numbers and its membership function $\mu_{\tilde{A}_H}(x)$ is given by

$$\mu_{\tilde{A}_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-p}{q-p}\right), & for\ p \le x \le q \\ \frac{1}{2} + \frac{\omega}{2}\left(\frac{x-q}{r-q}\right), & for\ q \le x \le r \\ \omega, & for\ r \le x \le s \\ 1 - \frac{\omega}{2}\left(\frac{x-s}{t-s}\right), & for\ s \le x \le t \\ \frac{1}{2}\left(\frac{u-x}{u-t}\right), & for\ t \le x \le u \\ 0, & otherwise" \end{cases}$$

Generalized Hexagonal Fuzzy Number diagram represented in Figure 2.

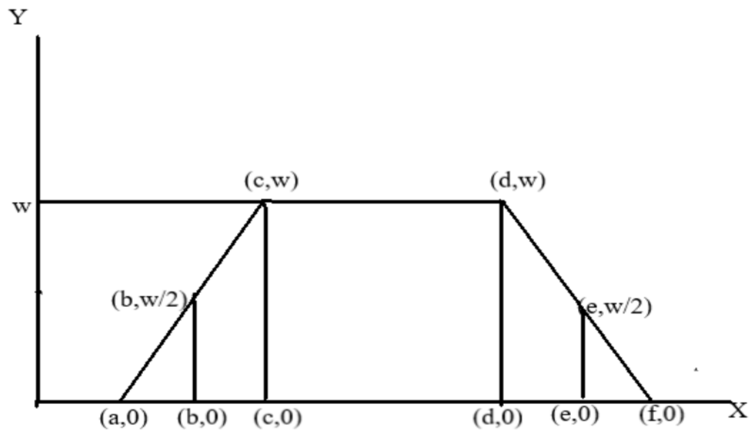

Figure2: *Generalized Hexagonal Fuzzy Number*

2.8 Ordering of Hexagonal Fuzzy Number [16]

"Let $\tilde{A}_H = (p_1, p_2, p_3, p_4, p_5, p_6)$ and $\tilde{B}_H = (q_1, q_2, q_3, q_4, q_5, q_6)$ be in fuzzy real number be the set of real Hexagonal fuzzy numbers

(i) $\tilde{A}_H \simeq \tilde{B}_H$ iff $p_i = q_i,\ i = 1,2,3,4,5,6$
(ii) $\tilde{A}_H \le \tilde{B}_H$ iff $p_i \le q_i,\ i = 1,2,3,4,5,6$
(iii) $\tilde{A}_H \ge \tilde{B}_H$ iff $p_i \ge q_i, i = 1,2,3,4,5,6$"

2.9 Ranking of Hexagonal Fuzzy Number [16]

An effective approach for comparing fuzzy numbers is to use a ranking function $\mathcal{R}: F(R) \longrightarrow R$, where $F(R)$ is a collection of fuzzy numbers that maps each fuzzy number into a real number, where a natural number order exists. For any two Hexagonal fuzzy numbers $\tilde{P}_H = (p_1, p_2, p_3, p_4, p_5, p_6)$ and $\tilde{Q}_H = (q_1, q_2, q_3, q_4, q_5, q_6)$ have the following comparison.

(i) $\tilde{P}_H = \tilde{Q}_H \Leftrightarrow R(\tilde{P}_H) = R(\tilde{Q}_H)$
(ii) $\tilde{P}_H \le \tilde{Q}_H \Leftrightarrow R(\tilde{P}_H) \le R(\tilde{Q}_H)$
(iii) $\tilde{P}_H \ge \mathrm{Q} \Leftrightarrow R(\tilde{P}_H) \ge R(\tilde{Q}_H)$

3. Proposal of a new ranking in Linear Hexagonal Fuzzy Number

We suggest a successful method for calculating the rank of Hexagonal fuzzy numbers. The Proposal ranking in the Hexagonal fuzzy number diagram is represented in Figure 3.

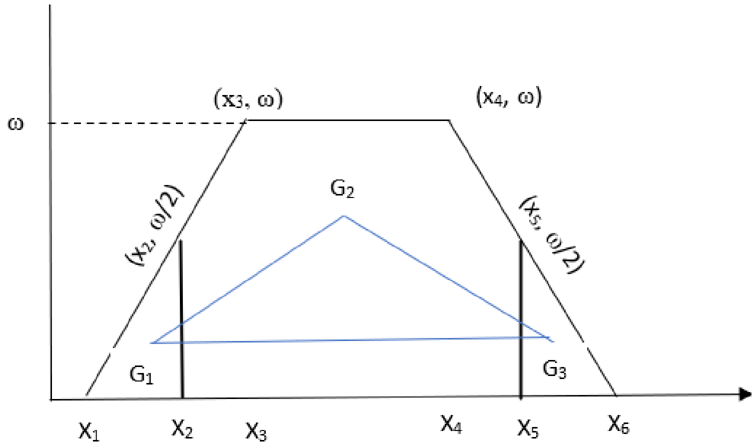

Figure3: *Proposed Ranking Method*

In Figure 3, the hexagonal is split into two right angles and one polygon. By applying the centroid formula of right angle and polygon, calculate the centroid of triangles and polygon, respectively. The circumcentre of the centroids of the fuzzy hexagonal number is taken into a balancing point of Hexagon in Figure 3. The circumcentre of the centroids of this three-plane figure is taken as the ranking of generalized Hexagonal fuzzy numbers. Let G_1, G_2, and G_3 be the centroid of the three plane figures.

G_1 specify the centroid of the right angle with vertices $(x_1, 0), \left(x_2, \frac{\omega}{2}\right), (x_2, 0)$.
G_2 specify the centroid of the triangle with vertices
$(x_2, 0), \left(x_2, \frac{\omega}{2}\right), (x_3, \omega), (x_4, \omega), \left(x_5, \frac{\omega}{2}\right), (x_5, 0)$
G_3 specify the centroid of the right angle with vertices $(x_5, 0), \left(x_5, \frac{\omega}{2}\right), (x_6, 0)$
The centroid of these three planes is;

$$G_1 = \left(\frac{x_1 + 2x_2}{3}, \frac{\omega}{6}\right), G_2 = \left(\frac{2x_2 + x_3 + x_4 + 2x_5}{6}, \frac{\omega}{2}\right), G_3 = \left(\frac{2x_5 + x_6}{3}, \frac{\omega}{6}\right) \text{ respectively.}$$

The circumcentre of G1, G_2, and G_3 is

$$G_{\tilde{A}_H} = \left(\frac{2x_1 + 6x_2 + x_3 + x_4 + 6x_5 + 2x_6}{18}, \frac{5\omega}{18}\right)$$

Therefore, the generalized Hexagonal fuzzy number $\tilde{A}_H = (x_1, x_2, x_3, x_4, x_5, x_6, \omega)$ new ranking function is:

$$R(\tilde{A}_H) = (\bar{x}_0 \bar{y}_0) = \left(\frac{2x_1 + 6x_2 + x_3 + x_4 + 6x_5 + 2x_6}{18}\right) * \frac{5\omega}{18}$$

Here, we utilize 4 sets of Hexagonal fuzzy numbers. These are opted from ref [16] to analyze the suggested method with convinced current ranking methods. The sets and the outcome obtained by the suggested method are given in Table1.

Table1: *Ranking order obtained results by the suggested method*

Sets	Hexagonal Fuzzy Numbers	$\mathcal{R}(\tilde{F}_H)$	Conclusion
Example 1			
Set-1	$\tilde{A}$(1,2,3,4,5,6;0.8)	0.7777	$\tilde{D} < \tilde{C} < \tilde{B} < \tilde{A}$
	$\tilde{B}$(-1,0,2,4,5,6;0.8)	0.5679	
	$\tilde{C}$ (-2, -1,0,2,4,6;0.8)	0.3456	
	$\tilde{D}$(-2, -1,0,1,2,3;0.8)	0.1111	
Example2			
Set-2	$\tilde{A}$ (1,0,0.2,0.3,0.4,0.5;0.6)	0.017	$\tilde{C}$<$\tilde{D}$<$\tilde{B}$<$\tilde{A}$
	$\tilde{B}$(-1, -0.5,0,0.4,0.5,1;0.6)	0.003	
	$\tilde{C}$ (-1, -0.6,0.3,0.2,0.5,1;0.6)	-0.006	
	$\tilde{D}$(-1, -0.2, -0.1,0.2,0.3,1;0.6)	0.006	
Example 3			
Set-3	$\tilde{A}$(0.1,0.2,0.4,0.6,0.7,0.9;1)	0.12963	$\tilde{B}$ >$\tilde{A}$
	$\tilde{B}$(0.2,0.4,0.6,0.7,0.8,0.9;1)	0.16512	
Example 4			
Set-4	$\tilde{A}$(0.2,0.3,0.5,0.6,0.7,0.9;0.7)	0.1	Â >$\hat{B}$
	$\tilde{B}$(0.1,0.2,0.4,0.5,0.6,0.9;0.7)	0.08	

The Proposal ranking method is compared with some existing methods represented in Table 2.

Table2: *The comparison of different ranking methods*

Ranking method	Set1	Set2	Set3	Set4
Avishek method [4]	$\tilde{D} < \tilde{C} < \tilde{B} < \tilde{A}$	$\tilde{C}$<$\tilde{D}$<$\tilde{B}$<$\tilde{A}$	$\tilde{B}$>$\tilde{A}$	$\tilde{A}$> $\tilde{B}$
Nagoor method [7]	$\tilde{D} < \tilde{C} < \tilde{B} < \tilde{A}$	$\tilde{C}$<$\tilde{D}$<$\tilde{B}$<$\tilde{A}$	$\tilde{B}$>$\tilde{A}$	$\tilde{A}$>$\tilde{B}$
Rajendran method [16]	$\tilde{D} < \tilde{C} < \tilde{B} < \tilde{A}$	$\tilde{C}$<$\tilde{D}$<$\tilde{B}$<$\tilde{A}$	$\tilde{B}$>$\tilde{A}$	$\tilde{A}$>$\tilde{B}$
Proposal method	$\tilde{D} < \tilde{C} < \tilde{B} < \tilde{A}$	Ĉ <$\hat{D}$<$\hat{B}$ < Â	$\tilde{B}$>$\tilde{A}$	$\tilde{A}$>$\tilde{B}$

4 Fuzzy Critical by a new ranking in Hexagonal fuzzy number

4.1 Analytical Example

This section, come out with numerical application of the proposal fuzzy set CPM-based methodology on an activity network. We consider a network with a set of fuzzy events $\tilde{A}$= {1, 2, 3, 4, 5, 6, 7} and the fuzzy activity time represented as a Hexagonal fuzzy number for each activity in table3. (All durations in days) Furthermore, the project network diagram is represented in Figure 4.

Table 3: *Fuzzy project network information*

Activity	Hexagonal fuzzy numbers
1-2	(3,7,11,15,19,24)
1-3	(3,5,7,9,10,12)
2-4	(11,14,17,21,25,30)
3-4	(3,5,7,9,10,12)
2-5	(5,7,10,13,17,21)
3-6	(7,9,11,14,18,22)
4-7	(7,9,11,14,18,22)
5-7	(2,3,4,6,7,9
6-7	(5,7,8,11,14,17)

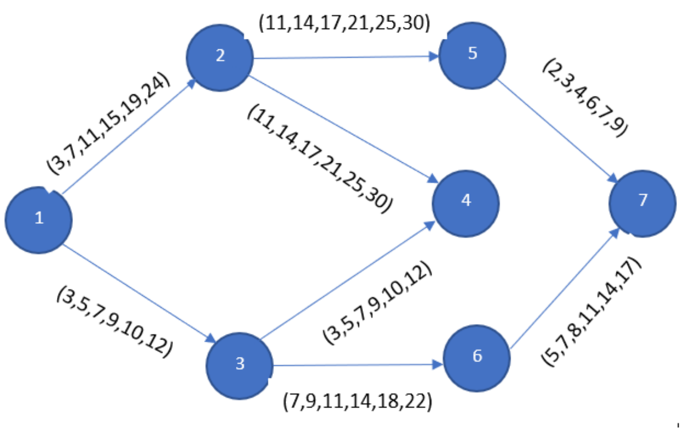

Figure 4: *Fuzzy project network*

Hexagonal fuzzy number transformed into an activity duration by proposal method. This activity duration taken as the time between the nodes and fuzzy critical path is calculating by applying the traditional method. The expected time between activities represented in Table4 and the related diagram is represented in Figure5.

Table 4: Expected time between activities

Activity	Hexagonal Fuzzy Numbers	Expected time
1-2	(3,7,11,15,19,24)	3.64
1-3	(3,5,7,9,10,12)	2.09
2-4	(11,14,17,21,25,30)	5.46
3-4	(3,5,7,9,10,12)	2.09
2-5	(5,7,10,13,17,21)	3.37
3-6	(7,9,11,14,18,22)	3.78
4-7	(7,9,11,14,18,22)	3.78
5-7	(2,3,4,6,7,9	1.41
6-7	(5,7,8,11,14,17)	2.91

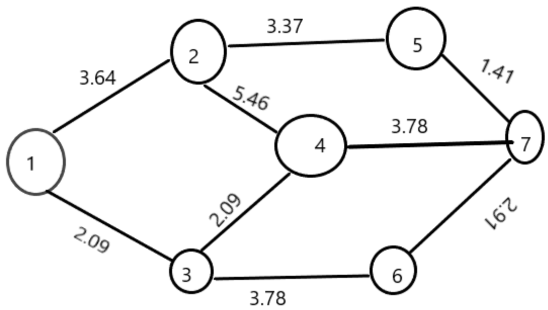

Figure 6.5. *Expected time between activities*

The possible paths of the project network's total duration time are represented in Table5.

Table 5. *Possible paths of project network total duration time*

Path	Expected Time
1-2-5-7	8.42
1-2-4-7	12.88
1-3-4-7	7.96
1-3-6-7	8.78

In table 5, the maximum value is 12.88.
Therefore, the project completion duration is 12.88, and the critical path is 1-2-4-7.

5 Comparison with Existing methods

Existing Method1[14]

In 2013, Rajarajeswari and Sahaya Sudha suggested a revised ranking in Hexagonal fuzzy numbers. Their new ranking function in Hexagonal fuzzy number is;

$$\mathcal{R}(\tilde{A}_H) = \frac{2f_1 + 3f_2 + 4f_3 + 4f_4 + 3f_5 + 2f_6}{18} \times \frac{5}{18}$$

Existing Method2 [21]

In 2015, Thamaraiselvi et. al. suggested the Magnitude of Hexagonal fuzzy numbers. The magnitude of Hexagonal fuzzy number is;

$$Mag(\tilde{A}_H) = \frac{2f_1 + 3f_2 + 4f_3 + 4f_4 + 3f_5 + 2f_6}{18}$$

Existing Method3 [1]

In 2016, Sahaya Sudha and Revathi introduced an improved ranking in generalized Hexagonal fuzzy numbers. Their new ranking function is;

$$\mathcal{R}(\tilde{A}_H) = \frac{2f_1 + 4f_2 + 9f_3 + 9f_4 + 4f_5 + 2f_6}{6} \times \frac{11\omega}{6}$$

Existing Method4 [16]

In 2017, Rajendran et.al., suggested a revised ranking in generalized Hexagonal fuzzy numbers. In their method, the new ranking function is;

$$\mathcal{R}(\tilde{A}_H) = \frac{2f_1 + 3f_2 + 4f_3 + 4f_4 + 3f_5 + 2f_6}{18} \times \frac{5\omega}{18}$$

Existing Method5 [22]

In 2020, Thirupathi et.al., suggested a revised ranking function in generalized Hexagonal fuzzy numbers. The revised ranking function is;

$$\mathcal{R}(\tilde{A}_H) = \frac{2f_1 + 4f_2 + 3f_3 + 3f_4 + 4f_5 + 2f_6}{18} \times \frac{13\omega}{36}$$

Existing Method6 [4]

In 2020, Avishek et.al., suggested a ranking in generalized Hexagonal fuzzy numbers. Their developed ranking function in Hexagonal fuzzy number is;

$$\mathcal{R}(\tilde{A}_H) = \frac{4f_1 + 10f_2 + 16f_3 + 16f_4 + 10f_5 + 4f_6}{12} \times \frac{5\omega}{6}$$

Table 6 represents project completion time estimates using the developed model and existing methodologies. Figure 5 presented the fuzzy critical path and project completion time of some recent methods as well as the proposal method.

Table 6: *Fuzzy Critical path compare with existing methods*

Method	Critical path	Project completion time
Rajendran et.al method	1-2-4-7	12.01
Magnitude of HFN	1-2-4-7	45.89
Rajarajeswari and Sahaya Sudha	1-2-4-7	12.73
Thirupathi et.al	1-2-4-7	16.62
Avishek et.al	1-2-4-7	128.74
Proposal method	1-2-4-7	12.88

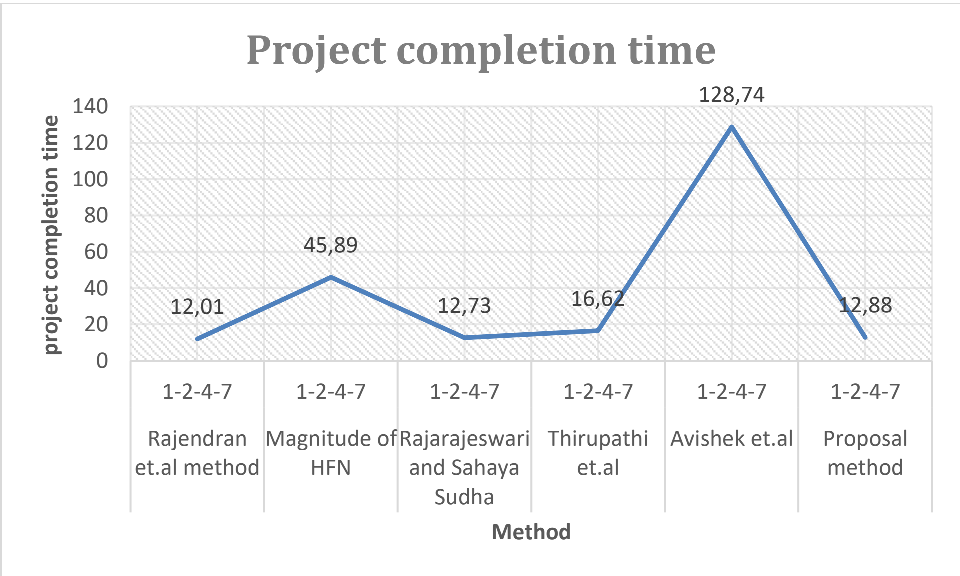

Figure 5: *Fuzzy critical path compared with existing methods graph*

4. Conclusion

In this paper, activity durations in the network represented by Hexagonal fuzzy numbers and suggested an advanced ranking function by Hexagonal fuzzy numbers with the centroid of centroid method. The new ranking function has been applied to calculate the critical path for the fuzzy project network. Numerous experiments have been conducted, and the results are correlated with some of the available methods. The attained results are similar to Rajendran, Rajarajeswari, and Tirupati methods, and completion time is less when compared to the magnitude of hexagonal fuzzy numbers and Avishek et al. method despite having the same critical path in all methods. The Avishek method gives the highest value in the comparison results and is not correlated with any existing method. So, the proposed method is better than the Avishek method. Moreover, the investigators can use the present concept on Hexagonal fuzzy numbers in numerous domains such as Engineering problems, Transportation problems, Neural networks, Cloud computing, image processing, mobile computing, etc. Further attention proceeded by constructing a new ranking function by various kinds of fuzzy numbers with project networks.

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