A Two Non-Identical Unit Parallel System Subject to Two Types of Failure and Correlated Life Times

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Abstract

The paper deals with the reliability and cost-benefit analysis of a two non-identical unit system with two types of failure. The units are named as unit-1 and unit-2 and they are arranged in a parallel configuration. Unit-1 can fail due to hardware or due to human error failure whereas unit-2 fails due to normal cause. A single repairman is considered with the system for all types of failure in the units and unit-1 gets priority in repair over the unit-2. The repair time distributions of unit-1 are taken as general with different c.d.fs and the repair time distribution of unit-2 is taken as exponential. Failure time distribution of unit-1 due to human error is taken exponential. Whereas the random variable denoting the failure time of unit-1 due to hardware failure and random variable denoting the failure time of unit-2 are assumed to be correlated random variables having their joint distribution as bivariate exponential (B.V.E.).

Keywords: Transition probabilities, mean sojourn time, bi-variate exponential distribution, regenerative point, reliability, MTSF, availability, expected busy period of repairman, net expected profit.

I. Introduction

Reliability is an important concept in the planning design and operation stages of various complex systems. Gupta et al. (2014) analysed a two non-identical unit parallel system with two independent repairmen-skilled and ordinary. A failed unit is first attended by skilled repairman to perform first phase repair and then it goes for second phase repair by ordinary repairman. Both types of repair discipline are FCFS. Chaudhary et al. (2015) analysed a two non-identical unit parallel system model assuming that an administrative delay occurs in getting the repairman available with the system whenever needed. Upon failure of a unit, the other operating unit shares the load of failed unit. Chopra and Ram (2017) analysed a two non-identical unit parallel system with two types of failures: common cause failure and partial failure. The repairman is not always available with the system to repair a failed unit. Whenever a unit fails, the repairman is called to come at the system and he takes some significant time to reach at the system. This time is known as waiting time of repairman during which the failed unit waits for repair. Chandra et al. (2020) performed the reliability and cost benefit analysis of the two identical and non-identical unit parallel system models by using Semi – Markov Process in regenerative point. A study of comparison is made between the reliability characteristics for both the system models under study. In these papers, the authors did not consider the concept of human error failure. In all the above system models the authors have considered single cause of failure in a unit i.e. normal cause.

Mahmoud and Moshref (2010) analysed a two-unit cold standby system by considering two cause of failure in a unit namely-Due to hardware and Due to human error. It has also been assumed that an operating unit goes for preventive maintenance (PM) to increase the system effectiveness. All the distributions of random variables involved in the system model are taken to follow arbitrary distributions. Kumar and Malik (2011) carried out the profit analysis of a computer system model with software and hardware failure subject to maximum operation time (MOT) and maximum repair time (MRT). An operating unit goes for preventive maintenance (PM) after completing MOT, if it is not failed during this time. Further if a failed unit is not repaired during MRT, it is replaced by new one. The priority to software replacement is given over hardware repair. Singh et al. (2016) analysed a two-unit warm standby system with two types of repairman. The first type of repairman, usually called regular repairman who is always remains available with the system to attend a failed unit. If he might not be able to do some complex repairs within some tolerable (patience) time, an expert repairman is called from the outside to complete the repair of the failed unit and he takes some significant time to reach at the system. Further an operating unit may fail either due to hardware or due to human error. In all the above system models the common assumptions considered is that the failure and repair times of the units are taken to uncorrelated random variables.

Gupta and co-workers [2008,2018] analysed two unit parallel and standby system models under different sets of assumptions by taking the failure and repair times as correlated random variables having their joint distribution as bivariate exponential. They have considered only single type of failure in an operating unit. Some authors including [1999, 2013] analysed two-unit parallel system models by taking the joint distribution of life times of the units working in parallel as bivariate exponential. They have also considered the single type of failure in an operating unit. **The objective of the present paper is to study a two non-identical unit parallel system subject to two causes of failure in an operating unit-Due to hardware and Due to human error**. Human failure is defined as a failure to perform a prescribed task which could result in damage to the equipment and property. There exist a number of causes for human error; e.g., lack of good job environments, poor training or skill of the operating personnel and so on. On the other hand, hardware failure occurs due to flaws in design, poor quality control and poor maintenance.

The life time of the units working in parallel form are taken to be correlated random variables having their joint distribution as bivariate exponential with different parameters as the form of joint p.d.f. given below.

$$f(x_1, x_2) = \alpha_1\alpha_2(1-r)e^{-\alpha_1 x_1-\alpha_2 x_2}\, I_0\left(2\sqrt{\alpha_1\alpha_2 r x_1 x_2}\right); \quad x_1, x_2, \alpha_1, \alpha_2 > 0; \quad 0 \le r < 1$$

where, $$I_0(z) = \sum_{k=0}^{\infty}\frac{(z/2)^{2k}}{(k!)^2}$$

is the modified Bessel function of type-I and order zero.

By using regenerative point technique, the following measures of system effectiveness are obtained-

i. Transient-state and steady-state transition probabilities.
ii. Mean sojourn time in various regenerative states.
iii. Reliability and mean time to system failure (MTSF).
iv. Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
v. The expected busy period of repairman in time interval (0, t).
vi. Net expected profit earned by the system in time interval (0, t) and in steady-state.

II. System Description and Assumptions

1. The system comprises of two non-identical units-unit-1 and unit-2. Initially, both the units are operative in parallel configuration.
2. Each unit has two modes-Normal (N) and Total failure (F).
3. Unit-1 can fail either due to hardware or human error. Whereas unit-2 can fail only due to its normal cause.
4. The system failure occurs when both the units are totally failed.
5. A single repairman is always available to repair the failed unit-1 either due to hardware or human error and the failed unit-2. The unit-1 gets priority in repair over the unit-2.
6. Failure time of unit-1 due to human error is taken exponential distribution whereas the failure time of unit-1 due to hardware and failure time of unit-2 due to normal cause are assumed to be correlated random variables having their joint distribution as bivariate exponential (B.V.E.) with density function as follows-

$$f(x_1, x_2) = \alpha_1\alpha_2(1-r)e^{-\alpha_1 x_1 - \alpha_2 x_2} I_0\left(2\sqrt{\alpha_1\alpha_2 r x_1 x_2}\right); \quad x_1, x_2, \alpha_1, \alpha_2 > 0; \quad 0 \le r < 1$$

where, $I_0(z) = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$

7. The repair time distribution of unit-1 failed either due to hardware or due to human error are taken as general with different c.d.fs whereas the repair time distribution of unit-2 failed due to normal cause is taken as exponential.
8. A repaired unit always works as good as new.

III. Notations and States of the System

We define the following symbols for generating the various states of the system-

N_{o1}^1, N_{o2}^2	:	Unit-1 and Unit-2 in normal (N) mode and operative.
F_{r1}^1	:	Unit-1 is in failure (F) mode and repair which is failed due to hardware failure.
F_{r2}^1	:	Unit-1 is in failure (F) mode and repair which is failed due to human error.
F_r^2, F_{wr}^2	:	Unit-2 is in failure (F) mode and under repair/waits for repair.

Considering the above symbols in view of assumptions stated in section-2, the possible states of the system are shown in the transition diagram represented by **Figure. 1**. It is to be noted that the epochs of transitions into the state S_4 from S_1, S_5 from S_2 are non-regenerative, whereas all the other entrance epochs into the states of the systems are regenerative.

The other notations used are defined as follows:

E	:	Set of regenerative states.
$X_i\left(i=1,2\right)$	:	Random variables representing the failure time of uni1-1 in N-mode and unit-2 respectively for i=1,2.
$f\left(x_1,x_2\right)$	:	Joint p.d.f. of $\left(x_1,x_2\right)$. $f(x_1,x_2)=\alpha_1\alpha_2\left(1-r\right)e^{-\alpha_1x_1-\alpha_2x_2}\,I_0\left(2\sqrt{\alpha_1\alpha_2rx_1x_2}\right)$; $x_1,x_2,\alpha_1,\alpha_2>0$; $0\le r<1$ where, $I_0\left(z\right)=\sum_{k=0}^{\infty}\frac{(z/2)^{2k}}{(k!)^2}$
$g_i\left(x\right)$	:	Marginal p.d.f. of $X_i=x$ $=\alpha_i\left(1-r_i\right)e^{-\alpha_i(1-r)x}$
$k_1\left(x_1\middle\|X_2=x_2\right)$	:	Conditional p.d.f. of $X_1\|X_2=x$. $=\alpha_1e^{-\left(\alpha_1x_1+\alpha_2rx\right)}I_0\left(2\sqrt{\alpha_1\alpha_2rxx_1}\right)$
$k_2\left(x_2\middle\|X_1=x\right)$	:	Conditional p.d.f. of $X_2\|X_1=x$. $=\alpha_2e^{-\left(\alpha_2x_2+\alpha_1rx\right)}I_0\left(2\sqrt{\alpha_1\alpha_2rxx_2}\right)$
$K_i\left(\cdot\|x\right)$	:	Conditional c.d.f. of $X_i\|X_j=x,\ i\ne j$; $i,j=1,2$.
λ	:	Constant failure rate of unit-1 due to Human error.
β	:	Constant repair rate of unit-2 due to normal cause.
$G_1\left(\cdot\right),G_2\left(\cdot\right)$	:	c.d.f. of repair time of unit-1 failed due to hardware failure and unit-1 failed due to human error.
$q_{ij}(\cdot),q_{ij}^{(k)}\left(\cdot\right)$	:	p.d.f. of transition time from state S_i to S_j and S_i to S_j via S_k.
$p_{ij},p_{ij}^{(k)}$	:	Steady-state transition probabilities from state S_i to S_j and S_i to S_j via S_k.
$p_{ij\|x},p_{ij\|x}^{(k)}$	:	Steady-state transition probabilities from state S_i to S_j and S_i to S_j via S_k when it is known that the unit has worked for time x before its failure.
*	:	†Symbol for Laplace Transform i.e. $q_{ij}^*\left(s\right)=\int e^{-st}q_{ij}\left(t\right)dt$

~ : Symbol for Laplace Stieltjes Transform i.e. $\tilde{Q}_{ij}(s) = \int e^{-st} dQ_{ij}(t)$

© : Symbol for ordinary convolution i.e.

$$A(t) © B(t) = \int_0^t A(u) B(t-u) du$$

†The limits of integration are 0 to ∞ whenever they are not mentioned.

TRANSITION DIAGRAM

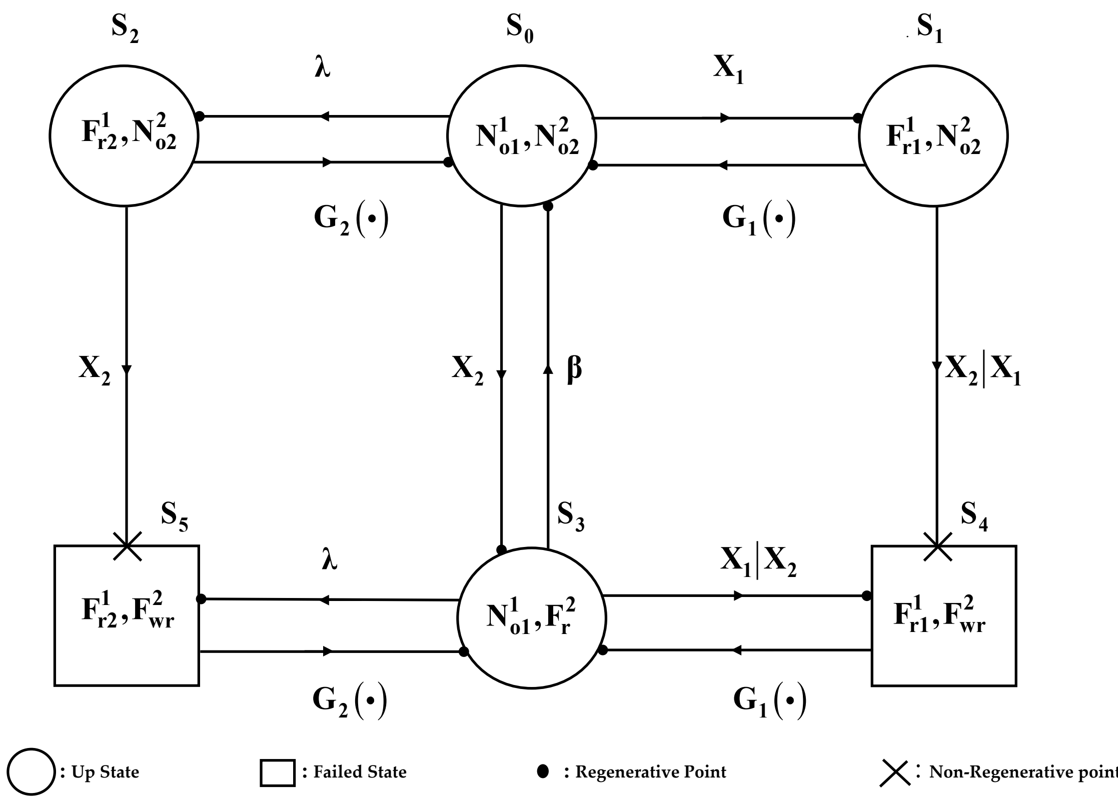

Figure. 1

IV. Transition Probabilities and Sojourn Times

Let $X(t)$ be the state of the system at epoch t, then $\{X(t); t \geq 0\}$ constitutes a continuous parametric Markov-Chain with state space $E = \{S_0 \text{ to } S_5\}$. The various measures of system effectiveness are obtained in terms of steady-state transition probabilities and mean sojourn times in various states. First we obtain the direct conditional and unconditional transition probabilities in terms of

$$\alpha_1' = \frac{\alpha_1}{\alpha_1 + \lambda + \beta}, \qquad \alpha_2' = \frac{\alpha_2}{\alpha_2 + \theta_1}$$

as follows-

$$p_{01} = \int \alpha_1 (1-r) e^{-\{\lambda+\alpha_1(1-r)+\alpha_2(1-r)\}t} dt = \frac{\alpha_1(1-r)}{\lambda+\alpha_1(1-r)+\alpha_2(1-r)}$$

Similarly,

$$p_{02} = \frac{\lambda}{\lambda+\alpha_1(1-r)+\alpha_2(1-r)}, \qquad p_{03} = \frac{\alpha_2(1-r)}{\lambda+\alpha_1(1-r)+\alpha_2(1-r)}$$

$$p_{20} = \tilde{G}_2\left[\alpha_2(1-r)\right], \qquad p_{23}^{(5)} = 1-\tilde{G}_2\left[\alpha_2(1-r)\right]$$

$$p_{43} = \int dG_1(t) = 1, \qquad p_{53} = \int dG_2(t) = 1$$

$$p_{10|X} = \int dG_1(u)\bar{K}_2(u|x)$$

Similarly,

$$p_{13|X}^{(4)} = \int \bar{G}_1(u) dK_2(u|x)$$

$$p_{30|X} = \int \beta e^{-(\lambda+\beta)u}\left(\int_u^\infty \alpha_1 e^{-(\alpha_1 y+\alpha_2 rx)} \sum_{j=0}^{\infty} \frac{(\alpha_1\alpha_2 rxy)^j}{(j!)^2} dy\right) du = \frac{\beta}{\lambda+\beta}\left[1-\alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}\right]$$

$$p_{34|X} = \alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}, \qquad p_{35|X} = \frac{\lambda}{\lambda+\beta}\left[1-\alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}\right]$$

The unconditional transition probabilities with correlation coefficient from some of the above conditional transition probabilities can be obtained as follows:

$$p_{10} = \int p_{10|X} g_1(x) dx = \int p_{10|x}\{\alpha_1(1-r)e^{-\alpha_1(1-r)x}\} dx$$

Similarly,

$$p_{13}^{(4)} = \int p_{13|x}^{(4)}\{\alpha_1(1-r)e^{-\alpha_1(1-r)x}\} dx, \qquad p_{30} = \frac{\beta}{\lambda+\beta}\left\{1-\frac{\alpha_1'(1-r)}{(1-r\alpha_1')}\right\}$$

$$p_{34} = \frac{\alpha_1'(1-r)}{(1-r\alpha_1')}, \qquad p_{35} = \frac{\lambda}{\lambda+\beta}\left\{1-\frac{\alpha_1'(1-r)}{(1-r\alpha_1')}\right\}$$

It can be easily verified that,

$$p_{01}+p_{02}+p_{03} = 1, \qquad p_{10}+p_{13}^{(4)} = 1, \qquad p_{20}+p_{23}^{(5)} = 1$$
$$p_{30}+p_{34}+p_{35} = 1, \qquad p_{43} = p_{53} = 1 \tag{1-5}$$

V. Mean Sojourn Times

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transiting into any other state. If random variable U_i denotes the sojourn time in state S_i then,

$$\psi_i = \int P\left[U_i > t\right] dt$$

Therefore, its values for various regenerative states are as follows-

$$\psi_0 = \int e^{-\{\lambda+\alpha_1(1-r)+\alpha_2(1-r)\}t} dt = \frac{1}{\lambda+\alpha_1(1-r)+\alpha_2(1-r)} \tag{6}$$

$$\psi_{1|X} = \int \bar{G}_1(t)\bar{K}_2(t|x)dt = \int \bar{G}_1(t)\left(\int_t^{\infty} \alpha_2 e^{-(\alpha_2 u+\alpha_1 rx)} \sum_{j=0}^{\infty} \frac{(\alpha_1\alpha_2 rxu)^j}{(j!)^2} du\right)dt$$

So that,

$$\psi_1 = \int \psi_{1|x} g_1(x)dx = \int \psi_{1|x}\alpha_1(1-r)e^{-\alpha_1(1-r)x}dx \tag{7}$$

$$\psi_2 = \int \bar{G}_2(t)e^{-\alpha_2(1-r)t}dt \tag{8}$$

$$\psi_{3|X} = \frac{1}{\beta+\lambda}\left\{1-\alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}\right\}$$

So that,

$$\psi_3 = \frac{1}{\beta+\lambda}\left[1-\frac{\alpha_1'(1-r)}{(1-r\alpha_1')}\right] \tag{9}$$

$$\psi_4 = \int \bar{G}_1(t)dt \tag{10}$$

$$\psi_5 = \int \bar{G}_2(t)dt \tag{11}$$

VI. Analysis of Characteristics

a) Reliability and MTSF

Let $R_i(t)$ be the probability that the system operates during (0, t) given that at t=0 system starts from $S_i \in E$. To obtain it we assume the failed states S_4 and S_5 as absorbing. By simple probabilistic arguments, the value of $R_0(t)$ in terms of its Laplace Transform (L.T.) is given by

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^* + q_{03}^* Z_3^*}{1-q_{01}^* q_{10}^* - q_{02}^* q_{20}^* - q_{03}^* q_{30}^*} \tag{12}$$

We have omitted the argument's from $q_{ij}^*(s)$ and $Z_i^*(s)$ for brevity. $Z_i^*(s); i = 0,1,2,3$ are the L. T. of

$$Z_0(t) = e^{-\{\lambda+\alpha_1(1-r)+\alpha_2(1-r)\}t}, \qquad Z_1(t) = \bar{G}_1(t)\int \bar{K}_2(t|x)g_1(x)dx$$

$$Z_2(t) = e^{-\alpha_2(1-r)t}\bar{G}_2(t), \qquad Z_3(t) = e^{-(\lambda+\beta)t}\int \bar{K}_1(t|x)g_2(x)dx$$

Taking the Inverse Laplace Transform of (12), one can get the reliability of the system when system initially starts from state S_0.

The MTSF is given by,

$$E(T_0) = \int R_0(t)dt = \lim_{s\to 0} R_0^*(s) = \frac{\psi_0 + p_{01}\psi_1 + p_{02}\psi_2 + p_{03}\psi_3}{1-p_{01}p_{10} - p_{02}p_{20} - p_{03}p_{30}} \tag{13}$$

b) Availability Analysis

Let $A_i(t)$ be the probability that the system is up at epoch t, when initially it starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of Laplace transform, one can obtain the value of $A_0(t)$ in terms of its Laplace transforms i.e. $A_0^*(s)$ given as follows-

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{14}$$

where,

$$N_1(s) = \left[1 - q_{34}^* q_{43}^* - q_{35}^* q_{53}^*\right]\left(Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*\right) + \left\lfloor q_{01}^* q_{13}^{(4)*} + q_{02}^* q_{23}^{(5)*} + q_{03}^* \right\rfloor Z_3^*$$

and

$$D_1(s) = \left[1 - q_{34}^* q_{43}^* - q_{35}^* q_{53}^*\right]\left(1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*\right) - q_{30}^* \left\lfloor q_{01}^* q_{13}^{(4)*} + q_{02}^* q_{23}^{(5)*} + q_{03}^* \right\rfloor \tag{15}$$

where, $Z_i(t)$, i=0,1,2,3 are same as given in section VI(a).

The steady-state availability of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) \tag{16}$$

We observe that

$$D_1(0) = 0$$

Therefore, by using L. Hospital's rule the steady state availability is given by

$$A_0 = \lim_{s\to 0} \frac{N_1(s)}{D_1'(s)} = \frac{N_1}{D_1'} \tag{17}$$

where,

$$N_1 = p_{30}\left(\psi_0 + p_{01}\psi_1 + p_{02}\psi_2\right) + \left(1 - p_{01}p_{10} - p_{02}p_{20}\right)\psi_3$$

and

$$D_1' = p_{30}\left\lfloor \psi_0 + p_{01}\left(\psi_1 + p_{14}\psi_4\right) + p_{02}\left(\psi_2 + p_{25}\psi_5\right)\right\rfloor + \left(1 - p_{01}p_{10} - p_{02}p_{20}\right)\left[\psi_3 + p_{34}\psi_4 + p_{35}\psi_5\right] \tag{18}$$

The expected up time of the system in interval (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0(u)\,du$$

So that, $$\mu^*_{up}(s) = \frac{A_0^*(s)}{s} \tag{19}$$

c) Busy Period Analysis

Let $B_i^1(t)$, $B_i^2(t)$ and $B_i^3(t)$ be the respective probabilities that the repairman is busy in the repair of unit-1 failed due to hardware, unit-1 failed due to human error and unit-2 failed due to normal cause at epoch t, when initially the system starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of L. T., one can obtain the values of above three probabilities in terms of their L. T. i.e. $B_i^{1*}(s)$, $B_i^{2*}(s)$ and $B_i^{3*}(s)$ as follows-

$$B_i^{1*}(s) = \frac{N_2(s)}{D_1(s)}, \qquad B_i^{2*}(s) = \frac{N_3(s)}{D_1(s)} \quad \text{and} \quad B_i^{3*}(s) = \frac{N_4(s)}{D_1(s)} \tag{20-22}$$

where,

$$N_2(s) = q_{01}^*\left[1 - q_{34}^* q_{43}^* - q_{35}^* q_{53}^*\right]\left(Z_1^* + q_{14}^* Z_4^*\right) + q_{34}^*\left\lfloor q_{01}^* q_{13}^{(4)*} + q_{02}^* q_{23}^{(5)*} + q_{03}^* \right\rfloor Z_4^*$$

$$N_3(s) = q_{02}^*\left[1 - q_{34}^* q_{43}^* - q_{35}^* q_{53}^*\right]\left(Z_2^* + q_{25}^* Z_5^*\right) + q_{35}^*\left(q_{01}^* q_{13}^{(4)*} + q_{02}^* q_{23}^{(5)*} + q_{03}^*\right) Z_5^*$$

$$N_4(s) = \left\lfloor q_{01}^* q_{13}^{(4)*} + q_{02}^* q_{23}^{(5)*} + q_{03}^* \right\rfloor Z_3^*$$

and $D_1(s)$ is same as defined by the expression (15) of section VI(b).

Also Z_4^* and Z_5^* are the L. T. of

$$Z_4(t)=\bar{G}_1(t), \quad Z_5(t)=\bar{G}_2(t)$$

The steady state results for the above three probabilities are given by-

$$B_o^1=\lim_{s\to 0} sB_0^*(s)=N_2/D_1', \qquad B_o^2=N_3/D_1' \qquad \text{and} \qquad B_0^3=N_4/D_1' \tag{23-25}$$

$$N_2=p_{30}p_{01}(\psi_1+p_{14}\psi_4)+\lfloor p_{34}(1-p_{01}p_{10}-p_{02}p_{20})\rfloor\psi_4$$

$$N_3=p_{30}p_{02}(\psi_2+p_{25}\psi_5)+\lfloor p_{35}(1-p_{01}p_{10}-p_{02}p_{20})\rfloor\psi_5$$

$$N_4=[1-p_{01}p_{10}-p_{02}p_{20}]\psi_3$$

and D_1' is same as given in the expression (18) of section VI(b).

The expected busy period in repair of unit-1 failed due to hardware, unit-1 failed due to human error and unit-2 failed due to normal cause during time interval (0, t) are respectively given by-

$$\mu_b^1(t)=\int_0^t B_0^1(u)du, \qquad \mu_b^2(t)=\int_0^t B_0^2(u)du \qquad \text{and} \qquad \mu_b^3(t)=\int_0^t B_0^3(u)du$$

So that,

$$\mu_b^{1*}(s)=B_0^{1*}(s)/s \qquad \mu_b^{2*}(s)=B_0^{2*}(s)/s \qquad \text{and} \qquad \mu_b^{3*}(s)=B_0^{3*}(s)/s \tag{26-28}$$

d) Profit Function Analysis

The net expected total cost incurred in time interval (0, t) is given by

$P(t)=$ Expected total revenue in (0, t) - Expected cost of repair in (0, t)

$$=K_0\mu_{up}(t)-K_1\mu_b^1(t)-K_2\mu_b^2(t)-K_2\mu_b^3(t) \tag{29}$$

Where, K_0 is the revenue per- unit up time by the system during its operation. K_1, K_2 and K_3 are the amounts paid to the repairman per-unit of time when he is busy in repair of unit-1 failed due to hardware, unit-1 failed due to human error and unit-2 failed due to normal cause respectively.

The expected total profit incurred in unit interval of time is $P=K_0A_0-K_1B_0^1-K_2B_0^2-K_2B_0^3$

VII. Particular Case

When the repair time of unit-1 failed due to hardware and human error also follow exponential with p.d.fs as follows-

$$g_1(t)=\theta_1 e^{-\theta_1 t}, \qquad g_2(t)=\theta_2 e^{-\theta_2 t}$$

The Laplace Transform of above density function are as given below-

$$g_1^*(s)=\tilde{G}_1(s)=\frac{\theta_1}{s+\theta_1}, \qquad g_2^*(s)=\tilde{G}_2(s)=\frac{\theta_2}{s+\theta_2}$$

Here, $\tilde{G}_1(s)$ and $\tilde{G}_2(s)$ are the Laplace-Stieltjes Transforms of the c.d.fs $G_1(t)$ and $G_2(t)$ corresponding to the p.d.fs $g_1(t)$ and $g_2(t)$.

In view of above, the changed values of transition probabilities and mean sojourn times.

$$p_{10}=1-\frac{\alpha_2'(1-r)}{(1-r\alpha_2')}, \qquad p_{13}^{(4)}=\frac{\alpha_2'(1-r)}{(1-r\alpha_2')}, \qquad p_{20}=\frac{\theta_2}{\alpha_2(1-r)+\theta_2}$$

$$p_{23}^{(5)}=\frac{\alpha_2(1-r)}{\alpha_2(1-r)+\theta_2}, \qquad \psi_1=\frac{1}{\alpha_2(1-r)+\theta_1}, \qquad \psi_2=\frac{1}{\alpha_2(1-r)+\theta_2}$$

VIII. Graphical Study of Behaviour and Conclusions

For a more clear view of the behaviour of system characteristics with respect to the various parameters involved, we plot curves for **MTSF** and **profit function** in **Fig. 2** and **Fig. 3** w.r.t. $\boldsymbol{\alpha_1}$ for three different values of correlation coefficient $\mathbf{r}$ =**0.25, 0.35** and **0.45** and two different values of repair parameter $\boldsymbol{\theta_1}$=**0.7** and **0.9** while the other parameters are kept fixed as $\boldsymbol{\lambda = 0.09}$, $\boldsymbol{\alpha_2 = 0.045}$, $\boldsymbol{\beta = 0.8}$, $\boldsymbol{\theta_2 = 0.7}$ .From the curves of **Fig. 2,** we observe that **MTSF** increases uniformly as the values of $\mathbf{r}$ and $\boldsymbol{\theta_1}$ increase and it decreases with the increase in $\boldsymbol{\alpha_1}$. Further, to achieve **MTSF** at least **94** units we conclude from smooth curves that the value of $\boldsymbol{\alpha_1}$ must be less than **0.118, 0.190 and 0.332** respectively for $\mathbf{r = 0.25}$ **, 0.35 , 0.45** when $\boldsymbol{\theta_1 = 0.9}$. Whereas from dotted curves we conclude that the value of $\boldsymbol{\alpha_1}$ must be less than **0.100, 0.171, 0.294** for $\mathbf{r = 0.25}$ **, 0.35** and **0.45** when $\boldsymbol{\theta_1 = 0.7}$.

Similarly, **Fig. 3** reveals the variations in **profit (P)** w.r.t. α_1 for varying values of $\mathbf{r}$ and $\boldsymbol{\theta_1}$, when the values of other parameters are kept fixed as $\boldsymbol{\lambda = 0.09}$, $\boldsymbol{\alpha_2 = 0.045}$, $\boldsymbol{\beta = 0.8}$, $\boldsymbol{\theta_2 = 0.7}$, $\mathbf{K_0 = 160}$, $\mathbf{K_1 = 400}$, $\mathbf{K_2 = 250}$ and $\mathbf{K_3 = 350}$. Here also the same trends in respect of $\boldsymbol{\alpha_1}$, $\mathbf{r}$ and $\boldsymbol{\theta_1}$ are observed as in case of **MTSF**. Moreover, we conclude from the smooth curves that the system is profitable only if $\boldsymbol{\alpha_1}$ is less than **0.581, 0.700** and **0.850** respectively for $\mathbf{r = 0.25}$ **, 0.35 , 0.45** when $\boldsymbol{\theta_1 = 0.9}$. From dotted curves, we conclude that the system is profitable only if $\boldsymbol{\alpha_1}$ is less than **0.520, 0.612** and **0.759** respectively for $\mathbf{r = 0.25}$ **, 0.35** and **0.45** when $\boldsymbol{\theta_1 = 0.7}$.

Behaviour of MTSF w.r.t. $\boldsymbol{\alpha_1}$ for different values of $\mathbf{r}$ and $\boldsymbol{\theta_1}$

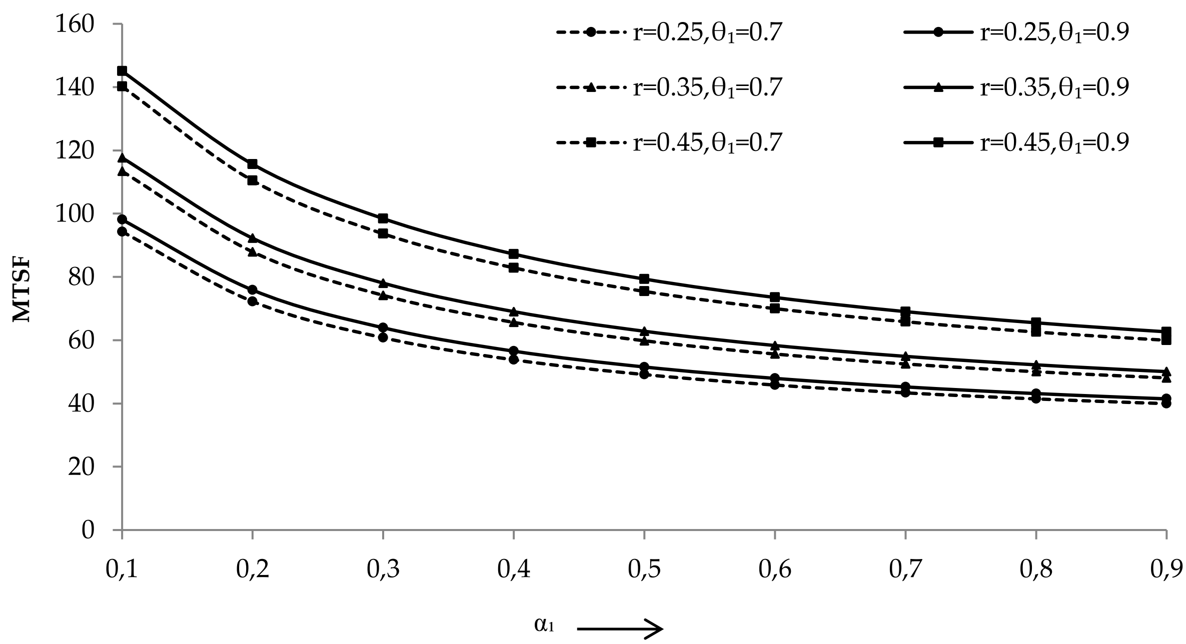

Figure.2

Behaviour of PROFIT (P) w.r.t. $\boldsymbol{\alpha_1}$ for different values of $\mathbf{r}$ and $\boldsymbol{\theta_1}$

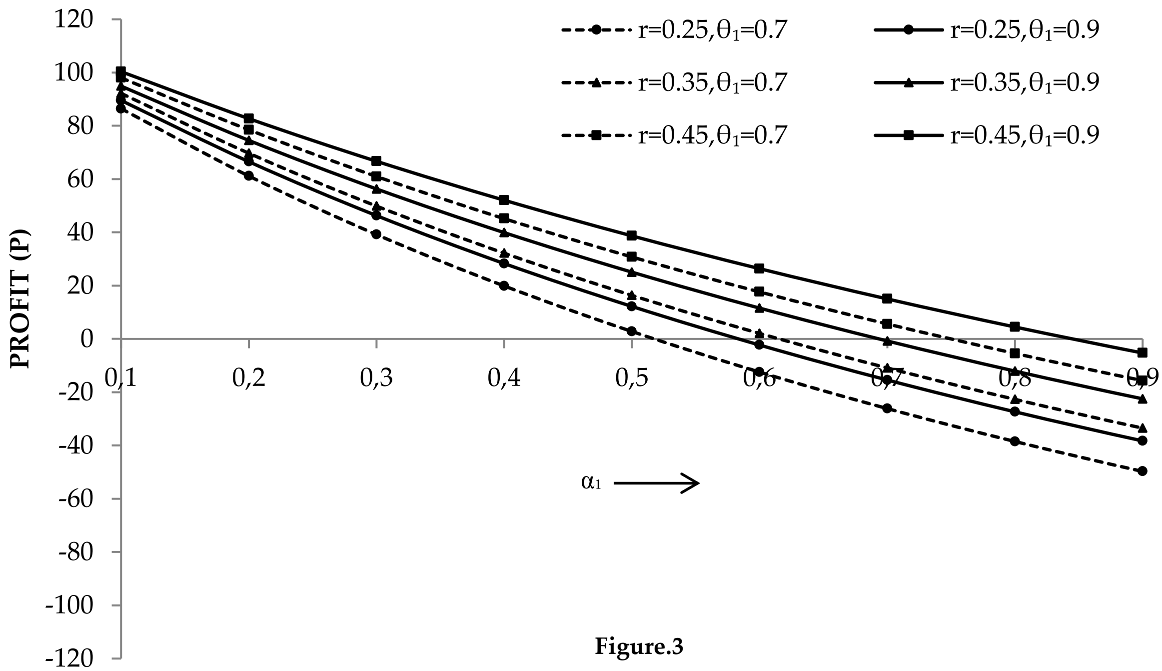

Figure.3

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References

[1] Chandra, A., Naithani, A., Gupta, S., & Jaggi, C. K. (2020). Reliability and cost analysis comparison between two-unit parallel systems with non-identical and identical consumable units. *Journal of Critical Reviews*, *7*(7), 695-702.

[2] Chaudhary, A., Jaiswal, S., & Gupta, R. (2015). A two unit parallel load sharing system with administrative delay in repair. *Journal of Reliability and Statistical Studies*, *8*(1), 1-13.

[3] Chopra, G., & Mangey, R. (2017). Stochastic analysis of two non-identical unit parallel system incorporating waiting time. *International Journal of Quality and Reliability Management*, 34(6), 817-832.

[4] Gupta, R., & Vaishali (2018). A two non-identical unit parallel system with correlated failure and repair times of repair. *International Journal of Agricultural and Statistical Sciences*, Vol.14, 2, pp.721-730.

[5] Gupta, R., Kishan, R., & Kumar, P. (1999). A two-non-identical-unit parallel system with correlated lifetimes. *International Journal of Systems Science*, *30*(10), 1123-1129.

[6] Gupta, R., Kujal, S., & Mahi, M. (2014). A two dissimilar unit parallel system with two phase repair by skilled and ordinary repairmen. *International Journal of System Assurance Engineering and Management*, *5*(4), 554-561.

[7] Gupta, R., Mahi, M., & Sharma, V. (2008). A two component two unit standby system with correlated failure and repair times. *Journal of Statistics and Management Systems*, *11*(1), 77-90.

[8] Gupta, R., Sharma, P. K., & Shivakar (2013). A two-unit active redundant system with two physical conditions of repairman and correlated life times. *Journal of Ravishankar University*, b-24- 26, 40-51.

[9] Kumar, A., & Malik, S. C. (2011). Profit analysis of a computer system with priority to software replacement over hardware repair subject to maximum operation and repair times. *International Journal of Engg. Science and Technology (IJEST)*, *3*(10), 7452-7468.

[10] Mahmoud, M. A. W., & Moshref, M. E. (2010). On a two-unit cold standby system considering hardware, human error failures and preventive maintenance. *Mathematical and Computer Modelling*, *51*(5-6), 736-745.

[11] Singh, P., Kumar, P., & Kumar, A. (2016). Reliability analysis of two-unit warm standby system subject to hardware and human error failures. *International Journal of Computer Science and Information Technologies*, *7*(3), 1296-1309.