Optimal System for Five Units Serial Systems under Partial and Complete Failure

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Abstract

The present paper studies and compared some reliability characteristics of series-parallel systems containing five units each under partial and complete failure. Four different system configurations are considered in this paper. It is assumed that both the failure and repair rates of each system configuration follow exponential distribution. The steady-state availability, busy period of repairman due to partial and complete failure, profit function, mean time to failure (MTTF) have been derived, examined and compared. The system configurations are compared analytically in terms of their availability and mean time to failure (MTTF). Cost-benefit measure has been evaluated for all the system configurations. The computed results are presented in tables and figures. From the analysis, system configuration II is observed to be the optimal configuration.

Keywords: Reliability, availability, standby, partial, complete failure, configuration, cost benefit.

I. Introduction

In reliability analysis, the performance evaluation of repairable systems is a matter of great importance. Maintaining the reliability of the system is indispensable. System performance can be measure through some reliability characteristics such as availability, mean time to failure, profit and benefit-cost analysis. The system availability of some engineering systems depends on the system structure, preventive maintenance, redundancy and also on the component availability. To affirm system failure and high system performance of complex systems, it is necessary to have a system component of higher availability. Generally, increasing redundant units or using units with high availability can also enhance system performance. Performance of systems increases significantly through redundancy optimization, using components with high availability, system's structural design and maintenance through repair and preventive maintenance.

To achieve high system reliability and availability, the system must be maintained at the highest order. To achieve this end, numerous researchers have designed different types of mathematical models to study and compare their reliability, availability and mean time to failure. For instance, Singh and Abdul Kareem [8] discussed the cost assessment of complex repairable systems consisting two subsystems in series configuration using Gumbel Hougaard family copula. Berk et al [2] have discussed the reliability assessment of safety-critical sensor information. Sanusi *et al* [12] have recently studied the performance evaluation of an industrial configured as series-parallel system. Wang *et al* [16] have presented the reliability analysis of two-dissimilar unit warm standby repairable system with priority in use. Singh and Ayagi [15] discussed the study of availability of standby complex system under waiting repair and human failure using Gumbel-Hougaard family copula distribution.

Harish Garg [4] discussed the study of the multi objective non-linear programming problem for reliability (GSA) and the results have been compared with the results computed by practice swarm optimization (PSO) methodology. Malik and Tewari [10] analyzed the performance of a system and maintenance priorities decision for the water flow system of a coal-based thermal power plant. Kumar et al [1] have recently studied the reliability analysis of a redundant system with 'FCFS' repair policy subject to weather conditions. Niwas and Garg [11] presented an approach for analyzing the reliability and profit of an industrial system based on the cost-free warranty repair policy. More recently, Sanusi and Yusuf [13] have presented the study of cost analysis of 2-out-of-4 system connected to two-unit parallel supporting device for operation. Mortazavi et al. [9] have evaluated the MTBF and other reliability parameters for a 2-out-of-3: G redundant repairable systems with common cause failures considering fuzzy rates for failures and repair via a case study of a centrifugal water pumping system. Saini et al [14] have investigated microprocessor systems using RAMD approach. Yang et al [19] discussed the reliability assessment of system with inconsistent priors and multi-level data. Gahlot et al. [5] investigated the performance assessment of serial system with different types of failure and repair policy. Zhang [23] dealt with the reliability analysis of computer networks based on intelligent cloud computing methods. Zhao et al [24] have discussed the reliability analysis of aero-engine compressor rotor system considering cruise characteristics. Ibrahim et al [6] have studied the reliability assessment of complex system consisting two subsystems connected in series configuration using Gumbel-Hougaard family copula distribution. Kakkar et al [7] have examined availability analysis of two parallel unit system under the provision of maintenance. Yusuf et al [22] have analyzed some reliability characteristics of a linear consecutive 2-out-of-4system connected to 2-out-of-4 supporting device for operation.

Some research works in the field of reliability and performance analysis of systems with standby components/units have shown that optimality among the systems under considerations is not unique and it depends on the value of some parameters. Some studies such as Wang and Learn [17], Wang et al. [18], EL-Sherbeny [3] and Yen and Wang [20] did take into the effect of cost benefit such as cost/availability and cost/mean time to failure on system reliability. Wang and Learn [17], Wang et al. [18], EL-Sherbeny [3] and Yen and Wang [20] have studied cost benefit analysis of various standby systems in which the optimality among configurations in the study depends only on particular parameter using cost/MTTF and depends on the other parameter using cost/availability. The present paper is motivated by the work of Wang and Learn [17], Wang et al. [18], EL-Sherbeny [3] and Yen and Wang [20] to study reliability of four 30MW power plant systems consisting of five units each arranged in series parallel and to determine the unique optimal system among the systems under study.

The contributions of this paper are as follow:

(i) To develop the explicit expressions for availability, busy period of repairman due partial and complete failure, mean time to failure and profit function.
(ii) To perform analytical comparison between the systems in order to rank them in terms of their availability and mean time to failure.
(iii) To study and compare the four systems in terms of their profit and cost benefit.
(iv) To determine the optimal system among the systems with cold standby.

The structure of this paper is organized as follows. Section 2 gives the description of the systems considered and their reliability block diagrams. Section 3 deals with the formulation of the models. Comparison between the systems analytically in terms of their availability and mean time to failure and numerically in terms of their profit and cost benefit are presented in Section 4. Conclusions are given in Sections 5.

II. Description of the System Configurations

The present study considers $30MW$ power plant arranged in the following four series parallel configurations as shown in Figures 1-4 below: System configuration I is a series parallel system which has three $10MW$ primary units and two $10MW$ cold standby components. System configuration II is a series parallel system having two $15MW$ primary components and three $15MW$ cold standby components. System configuration III is a series parallel system with three $10MW$ primary units and two $10MW$ cold standby units. System configuration IV is a series parallel system which consists of three subsystems with two subsystems arranged in parallel and serial to the other subsystem with two $15MW$ primary components and three $10MW$ cold standby components. It is assumed that all switchover time are instantaneous and switching is perfect. It is also assumed that the switch from standby to operation is perfect. Each of the primary units fails independently of the state of the others and has an exponential failure time with parameter β_0 and is replace with cold standby unit if available while the failed unit is immediately sent for repairs and the time to repair is exponential with parameter α_0 . All failures are assumed to be repairable. System failure occurs when all units in the same subsystem have failed. A failure is partial if the system has not failed completely otherwise the failure is complete (system failure).

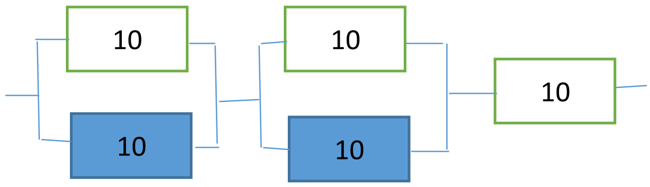

Figure 1: *Reliability block diagram of System Configuration I*

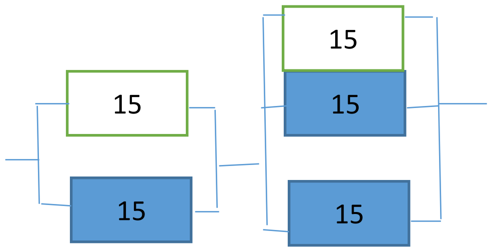

Figure 2: *Reliability block diagram of System Configuration II*

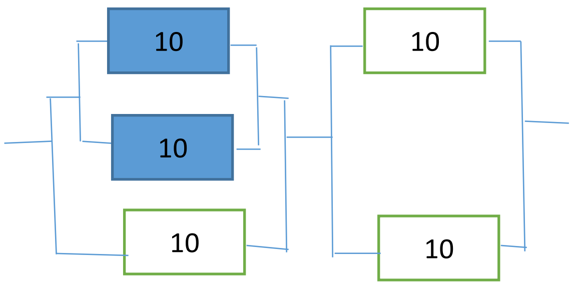

Figure 3: *Reliability block diagram of System Configuration III*

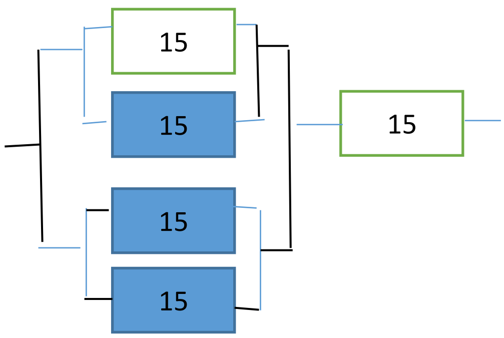

Figure 4: *Reliability block diagram of Configuration IV*

III. Reliability Models Formulation

Formulation of System Configuration I

Mean time to failure (MTTF) Analysis of System Configuration I

Let the probability that the system is in state i at time t be $h_i(t)$ and $H(t)=\left[h_1(t),h_2(t),\ldots,h_{11}(t)\right]$ be the probability row vector time t with initial conditions

$$h_k(0)=\begin{cases}1, & k=0\\ 0, & k=1,2,3,\ldots,11\end{cases}$$

The differential-difference equations derived from system configuration I are given by:

$$\left.\begin{aligned}
\frac{d}{dt}h_0(t)&=-3\beta_0h_0(t)+\alpha_0h_1(t)+\alpha_0h_2(t)+\alpha_0h_{10}(t)\\
\frac{d}{dt}h_1(t)&=-(3\beta_0+\alpha_0)h_1(t)+\beta_0h_0(t)+\alpha_0h_3(t)+\alpha_0h_5(t)+\alpha_0h_9(t)\\
\frac{d}{dt}h_2(t)&=-(3\beta_0+\alpha_0)h_2(t)+\beta_0h_0(t)+\alpha_0h_3(t)+\alpha_0h_4(t)+\alpha_0h_{11}(t)\\
\frac{d}{dt}h_3(t)&=-(3\beta_0+2\alpha_0)h_3(t)+\beta_0h_1(t)+\beta_0h_2(t)+\alpha_0h_6(t)+\alpha_0h_7(t)+\alpha_0h_8(t)\\
\frac{d}{dt}h_i(t)&=-\alpha_0h_i(t)+\beta_0h_j(t)
\end{aligned}\right\}\quad(1)$$

$i=4,5,6,\ldots,11$ and $j=0,1,2,3$

Equation (1) can be written in matrix form as:

$$H'(t)=M_1H(t)\quad(2)$$

Where

$$M_1=\begin{pmatrix}
-3\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0 & 0\\
\beta_0 & -\delta_0 & 0 & \alpha_0 & 0 & \alpha_0 & 0 & 0 & 0 & \alpha_0 & 0 & 0\\
\beta_0 & 0 & -\delta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0\\
0 & \beta_0 & \beta_0 & -\delta_1 & 0 & 0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0\\
0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0\\
0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0\\
\beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0\\
0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0
\end{pmatrix}$$

To compute the *MTTF* of system configuration I, the procedure requires deleting rows and columns of absorbing states of matrix M_1 and take the transpose to produce a new matrix, Q_1 as adopted in Wang and Kuo [17], Wang et al [16] and Wang et al [18]. The time expected to reach the absorbing state is calculated from:

$$MTTF_1 = H(0)\left(-Q_1^{-1}\right)(1,1,1,1)^T \tag{3}$$

Thus, the *MTTF* expression for system configuration I is:

$$MTTF_1 = \frac{2\alpha_0^2 + 11\alpha_0\beta_0 + 17\beta_0^2}{\beta_0\left(2\alpha_0^2 + 15\alpha_0\beta_0 + 27\beta_0^2\right)} \tag{4}$$

Where $H(0) = [1,0,0,0]$ and $Q_1 = \begin{pmatrix} -3\beta_0 & \beta_0 & \beta_0 & 0 \\ \alpha_0 & -\delta_0 & 0 & \beta_0 \\ \alpha_0 & 0 & -\delta_0 & \beta_0 \\ 0 & \alpha_0 & \alpha_0 & -\delta_1 \end{pmatrix}$

Availability and Busy period of System Configuration I

To compute the availability of system configuration I, the differential difference equation given in (2) are expressed in the form:

$$\begin{pmatrix} h_0' \\ h_1' \\ h_2' \\ h_3' \\ h_4' \\ h_5' \\ h_6' \\ h_7' \\ h_8' \\ h_9' \\ h_{10}' \\ h_{11}' \end{pmatrix} = \begin{pmatrix} -3\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0 & 0 \\ \beta_0 & -\delta_0 & 0 & \alpha_0 & 0 & \alpha_0 & 0 & 0 & 0 & \alpha_0 & 0 & 0 \\ \beta_0 & 0 & -\delta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0 \\ 0 & \beta_0 & \beta_0 & -\delta_1 & 0 & 0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \\ h_{11} \end{pmatrix}$$

The steady state availability (i.e. the sum of the probabilities of all the operational states), busy period due to partial failure and complete failure are respectively given by:

$$A_{V1}(\infty) = h_0(\infty) + h_1(\infty) + h_2(\infty) + h_3(\infty) \tag{5}$$

$$B_{h1} = h_1(\infty) + h_2(\infty) + h_3(\infty) \tag{6}$$

$$B_{h2} = h_4(\infty) + h_5(\infty) + h_6(\infty) + h_7(\infty) + h_8(\infty) + h_9(\infty) + h_{10}(\infty) + h_{11}(\infty) \tag{7}$$

All the derivatives of state probabilities are set equal to zero in the steady state, therefore equation (2) becomes:

$$M_1 H(t)^T = 0 \tag{8}$$

Which is

$$\begin{pmatrix} -3\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0 & 0 \\ \beta_0 & -\delta_0 & 0 & \alpha_0 & 0 & \alpha_0 & 0 & 0 & 0 & \alpha_0 & 0 & 0 \\ \beta_0 & 0 & -\delta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0 \\ 0 & \beta_0 & \beta_0 & -\delta_1 & 0 & 0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \\ h_{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing condition:

$$\sum_{n=0}^{11} h_n(\infty) = 1 \tag{9}$$

To compute the state probabilities $h_i(t)\ i = 0,1,2,...,11$, (9) is substituted in the last of (8) to give:

$$\begin{pmatrix} -3\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0 & 0 \\ \beta_0 & -\delta_0 & 0 & \alpha_0 & 0 & \alpha_0 & 0 & 0 & 0 & \alpha_0 & 0 & 0 \\ \beta_0 & 0 & -\delta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_0 \\ 0 & \beta_0 & \beta_0 & -\delta_1 & 0 & 0 & \alpha_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \\ h_{11} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

(10)

Solving (10) using MATLAB package to obtain $h_i(t)$, the expressions for the steady-state availability, busy period due to partial failure and complete failure given in (5) to (7) are respectively given by:

$$A_{V1}(\infty) = \frac{\alpha_0^3 + 2\alpha_0^2\beta_0 + \alpha_0\beta_0^2}{\alpha_0^3 + 3\alpha_0^2\beta_0 + 5\alpha_0\beta_0^2 + 3\beta_0^3} \tag{11}$$

$$B_{h1}(\infty) = \frac{2\alpha_0^2\beta_0 + \alpha_0\beta_0^2}{\alpha_0^3 + 3\alpha_0^2\beta_0 + 5\alpha_0\beta_0^2 + 3\beta_0^3} \tag{12}$$

$$B_{h2}(\infty) = \frac{3\beta_0^3 + \alpha_0^2\beta_0 + 4\alpha_0\beta_0^2}{\alpha_0^3 + 3\alpha_0^2\beta_0 + 5\alpha_0\beta_0^2 + 3\beta_0^3} \tag{13}$$

Profit Analysis of System Configuration I

The units are exposed to corrective maintenance due to partial and complete failure, while the repairman is busy performing maintenance action to the failed units. Let K_0, K_1 and K_2 be the revenue generated when the system is in working state and no income when in failed state, cost of each repair due to partial and complete failure respectively. The expected total profit of system configuration I per unit time incurred to the system in the steady-state is given by:

Profit =total revenue generated – cost incurred by the repair man due to partial failure – cost incurred due to complete failure.

$$PF_1 = K_0 A_{V1} - \left(K_1 B_{h1} + K_2 B_{h2}\right) \tag{14}$$

Formulation of System Configuration II

Mean time to failure (MTTF) Analysis of System Configuration II

Let $H(t) = \left[h_0(t), h_1(t), h_2(t), \ldots, h_{10}(t)\right]$ be the probability row vector at time t. The initial condition is given by: $h_j(0) = \begin{cases} 1, & j = 0 \\ 0, & j = 1,2,3,\ldots,10 \end{cases}$

The corresponding set of differential-difference equations for system configuration II are expressed as:

$$H'(t) = M_2 H(t) \tag{15}$$

Where:

$$M_2 = \begin{pmatrix} -2\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -(2\beta_0+\alpha_0) & 0 & 0 & \alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 \\ \beta_0 & 0 & -(2\beta_0+\alpha_0) & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & -(2\beta_0+\alpha_0) & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & \beta_0 & 0 & -(2\beta_0+2\alpha_0) & \alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & \beta_0 & -(2\beta_0+2\alpha_0) & 0 & 0 & 0 & \alpha_0 & \alpha_0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix}$$

Using similar procedure presented in subsection 3.1.1, the expression for the mean time to failure $(MTTF)$ of system configuration II is obtained through:

$$MTTF_2 = H(0)\left(-Q_2^{-1}\right)(1,1,1,1,1,1)^T \tag{16}$$

Thus,

$$MTTF_2 = \frac{4\alpha_0^5 + 29\alpha_0^4\beta_0 + 97\alpha_0^3\beta_0^2 + 119\alpha_0^2\beta_0^3 + 211\alpha_0\beta_0^4 + 100\beta_0^5}{\beta_0^2\left(4\alpha_0^4 + 25\alpha_0^3\beta_0 + 76\alpha_0^2\beta_0^2 + 112\alpha_0\beta_0^3 + 64\beta_0^4\right)} \tag{17}$$

Where $H(0) = [1,0,0,0,0,0]$ and

$$Q_2=\begin{pmatrix} -2\beta_0 & \beta_0 & \beta_0 & 0 & 0 & 0 \\ \alpha_0 & -\left(2\beta_0+\alpha_0\right) & 0 & 0 & \beta_0 & 0 \\ \alpha_0 & 0 & -\left(2\beta_0+\alpha_0\right) & \beta_0 & \beta_0 & 0 \\ 0 & 0 & \alpha_0 & -\left(2\beta_0+\alpha_0\right) & 0 & \beta_0 \\ 0 & \alpha_0 & \alpha_0 & 0 & -\left(2\beta_0+2\alpha_0\right) & \beta_0 \\ 0 & 0 & 0 & \alpha_0 & \alpha_0 & -\left(2\beta_0+2\alpha_0\right) \end{pmatrix}$$

Availability and Busy period Analysis of System Configuration II

To compute the availability of system configuration II, the differential-difference equations given in (14) are expressed in the form:

$$\begin{pmatrix} h_0' \\ h_1' \\ h_2' \\ h_3' \\ h_4' \\ h_5' \\ h_6' \\ h_7' \\ h_8' \\ h_9' \\ h_{10}' \end{pmatrix}=\begin{pmatrix} -2\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -\left(2\beta_0+\alpha_0\right) & 0 & 0 & \alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 \\ \beta_0 & 0 & -\left(2\beta_0+\alpha_0\right) & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & -\left(2\beta_0+\alpha_0\right) & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & \beta_0 & 0 & -\left(2\beta_0+2\alpha_0\right) & \alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & \beta_0 & -\left(2\beta_0+2\alpha_0\right) & 0 & 0 & 0 & \alpha_0 & \alpha_0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix}\begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \end{pmatrix}$$

The steady state availability (the proportion of the time the system is functioning or equivalently the sum of the probabilities of operational state), busy period due to partial failure and complete failure are given by:

$$A_{V2}\left(\infty\right)=h_0\left(\infty\right)+h_1\left(\infty\right)+h_2\left(\infty\right)+h_3\left(\infty\right)+h_4\left(\infty\right)+h_5\left(\infty\right) \tag{18}$$

$$B_{h3}=h_1\left(\infty\right)+h_2\left(\infty\right)+h_3\left(\infty\right)+h_4\left(\infty\right)+h_5\left(\infty\right) \tag{19}$$

$$B_{h4}\left(\infty\right)=h_6\left(\infty\right)+h_7\left(\infty\right)+h_8\left(\infty\right)+h_9\left(\infty\right)+h_{10}\left(\infty\right) \tag{20}$$

In the steady state, the derivatives of states probabilities become zero and therefore (15) becomes:

$$M_2H\left(t\right)^T=0 \tag{21}$$

In matrix form, we have:

$$\begin{pmatrix} -2\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -\left(2\beta_0+\alpha_0\right) & 0 & 0 & \alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 \\ \beta_0 & 0 & -\left(2\beta_0+\alpha_0\right) & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & -\left(2\beta_0+\alpha_0\right) & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & \beta_0 & 0 & -\left(2\beta_0+2\alpha_0\right) & \alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & \beta_0 & -\left(2\beta_0+2\alpha_0\right) & 0 & 0 & 0 & \alpha_0 & \alpha_0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix}\begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing condition

$$\sum_{n=0}^{10} h_n(\infty) = 1 \tag{22}$$

To obtain the state probabilities $h_i(t)\ i = 0,1,2,...,10$, we substitute (22) in (21) to get

$$\begin{pmatrix} -2\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -(2\beta_0+\alpha_0) & 0 & 0 & \alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 & 0 \\ \beta_0 & 0 & -(2\beta_0+\alpha_0) & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & -(2\beta_0+\alpha_0) & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & \beta_0 & 0 & -(2\beta_0+2\alpha_0) & \alpha_0 & 0 & 0 & \alpha_0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & \beta_0 & -(2\beta_0+2\alpha_0) & 0 & 0 & 0 & \alpha_0 & \alpha_0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \\ h_{10} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{23}$$

Solving (23) using MATLAB package to obtain $h_i(t)$, the explicit expressions for the steady-state availability, busy period due to partial failure and complete failure are given by:

$$A_{V2}(\infty) = \frac{\alpha_0^4 + 2\alpha_0^3\beta_0 + 2\alpha_0^2\beta_0^2 + \alpha_0\beta_0^3}{\alpha_0^4 + 2\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + 2\beta_0^4} \tag{24}$$

$$B_{h3}(\infty) = \frac{2\alpha_0^3\beta_0 + 2\alpha_0^2\beta_0^2 + \alpha_0\beta_0^3}{\alpha_0^4 + 2\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + 2\beta_0^4} \tag{25}$$

$$B_{h4}(\infty) = \frac{2\beta_0^4 + \alpha_0^2\beta_0^2 + \alpha_0\beta_0^3}{\alpha_0^4 + 2\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + 2\beta_0^4} \tag{26}$$

Profit Analysis of System Configuration II

Using similar procedure presented in subsection 3.1.3, the explicit expression for profit function of system configuration II is given by:

$$PF_2 = K_0 A_{V2} - \left(K_1 B_{h3} + K_2 B_{h4}\right) \tag{27}$$

Formulation of System Configuration III

Mean time to failure of Analysis System Configuration III

Let $H(t) = \left[h_0(t), h_1(t), ..., h_8(t)\right]$ be the probability row vector at time t with initial conditions

$$h_n(0) = \begin{cases} 1, & n = 0 \\ 0, & n = 1,2,3,...,8 \end{cases}$$

The corresponding set of differential-difference equations for system configuration III are expressed as:

$$H'(t) = M_3 H(t) \tag{28}$$

Where

$$M_3 = \begin{pmatrix} -3\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -(3\beta_0+\alpha_0) & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ 2\beta_0 & 0 & -(3\beta_0+\alpha_0) & 0 & 0 & \alpha_0 & \alpha_0 & 0 & 0 \\ 0 & \beta_0 & 0 & -(3\beta_0+\alpha_0) & 0 & 0 & 0 & \alpha_0 & \alpha_0 \\ 0 & 2\beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix}$$

Using similar procedure presented in subsection 3.1.1, the explicit expression for the mean time to failure $(MTTF)$ of System Configuration III is obtained through:

$$MTTF_3 = H(0)\left(-Q_3^{-1}\right)(1,1,1,1)^T \tag{29}$$

Thus,

$$MTTF_3 = \frac{\alpha_0^3 + 11\alpha_0^2\beta_0 + 41\alpha_0\beta_0^2 + 57\beta_0^3}{\beta_0^2\left(8\alpha_0^2 + 45\alpha_0\beta_0 + 81\beta_0^2\right)} \tag{30}$$

Where $H(0) = [1,0,0,0]$ and $Q_3 = \begin{pmatrix} -3\beta_0 & \beta_0 & 2\beta_0 & 0 \\ \alpha_0 & -(3\beta_0+\alpha_0) & 0 & \beta_0 \\ \alpha_0 & 0 & -(3\beta_0+\alpha_0) & 0 \\ 0 & \alpha_0 & 0 & -(3\beta_0+\alpha_0) \end{pmatrix}$

Availability and Busy period Analysis of System Configuration III

To compute the availability of system configuration III, the differential difference equations given in (28) are expressed in the form:

$$\begin{pmatrix} h_0' \\ h_1' \\ h_2' \\ h_3' \\ h_4' \\ h_5' \\ h_6' \\ h_7' \\ h_8' \end{pmatrix} = \begin{pmatrix} -3\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -y_0 & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ 2\beta_0 & 0 & -y_0 & 0 & 0 & \alpha_0 & \alpha_0 & 0 & 0 \\ 0 & \beta_0 & 0 & -y_0 & 0 & 0 & 0 & \alpha_0 & \alpha_0 \\ 0 & 2\beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{pmatrix}$$

The steady state availability (the proportion of the time the system is functioning), busy period due to partial failure and complete failure are given by:

$$A_{V3}(\infty) = h_0(\infty) + h_1(\infty) + h_2(\infty) + h_3(\infty) \tag{31}$$

$$B_{h5} = h_1(\infty) + h_2(\infty) + h_3(\infty) \tag{32}$$

$$B_{h6}(\infty) = h_4(\infty) + h_5(\infty) + h_6(\infty) + h_7(\infty) + h_8(\infty) \tag{33}$$

In the steady state, the derivatives of states probabilities become zero and therefore (28) becomes:

$$M_3 H(t)^T = 0 \tag{34}$$

In matrix form, we have:

$$\begin{pmatrix} -3\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -y_0 & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ 2\beta_0 & 0 & -y_0 & 0 & 0 & \alpha_0 & \alpha_0 & 0 & 0 \\ 0 & \beta_0 & 0 & -y_0 & 0 & 0 & 0 & \alpha_0 & \alpha_0 \\ 0 & 2\beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & 0 & 0 & -\alpha_0 \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing condition

$$\sum_{n=0}^{8} h_n(\infty) = 1 \tag{35}$$

To obtain the state probabilities $h_i(t)\ i = 0,1,2,...,8$, (35) is substituted in (34) to give:

$$\begin{pmatrix} -3\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -y_0 & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ 2\beta_0 & 0 & -y_0 & 0 & 0 & \alpha_0 & \alpha_0 & 0 & 0 \\ 0 & \beta_0 & 0 & -y_0 & 0 & 0 & 0 & \alpha_0 & \alpha_0 \\ 0 & 2\beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_0 & 0 & 0 & 0 & -\alpha_0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{36}$$

Solving (36) using MATLAB package to obtain $h_i(t)$,the explicit expressions for the steady-state availability, busy period due to partial failure and complete failure are given by:

$$A_{V3}(\infty) = \frac{\alpha_0^3 + 3\alpha_0^2\beta_0 + \alpha_0\beta_0^2}{\alpha_0^3 + 3\alpha_0^2\beta_0 + 9\alpha_0\beta_0^2 + 3\beta_0^3} \tag{37}$$

$$B_{h5}(\infty) = \frac{3\alpha_0^2\beta_0 + \alpha_0\beta_0^2}{\alpha_0^3 + 3\alpha_0^2\beta_0 + 9\alpha_0\beta_0^2 + 3\beta_0^3} \tag{38}$$

$$B_{h6}(\infty) = \frac{3\beta_0^3 + 8\alpha_0\beta_0^2}{\alpha_0^3 + 3\alpha_0^2\beta_0 + 9\alpha_0\beta_0^2 + 3\beta_0^3} \tag{39}$$

Profit analysis of System Configuration III

Using similar procedure presented in subsection 3.1.3, the explicit expression for profit function of configuration III is given by:

$$PF_3 = K_0 A_{V3} - \left(K_1 B_{h5} + K_2 B_{h6}\right) \tag{40}$$

Formulation of System Configuration IV

Mean time to failure of Analysis System Configuration IV

Define $H(t) = \left[h_0(t), h_1(t), h_2(t), ..., h_8(t)\right]$ to be the probability row vector at time t with initial conditions:

$$h_n(0)=\begin{cases}1, & n=0\\ 0, & n=1,2,3,\dots,8\end{cases}.$$

The corresponding set of differential-difference equations for system configuration IV is expressed as:

$$H'(t)=M_4H(t) \tag{41}$$

Where:

$$M_4=\begin{pmatrix}
-2\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_0 & -(2\beta_0+\alpha_0) & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0\\
\beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \beta_0 & 0 & -(2\beta_0+\alpha_0) & 0 & \alpha_0 & \alpha_0 & 0 & 0\\
0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \beta_0 & 0 & -(2\beta_0+\alpha_0) & 0 & \alpha_0 & \alpha_0\\
0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0\\
0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0
\end{pmatrix}$$

Using similar procedure presented in subsection 3.1.1, the explicit expression for the mean time to failure $(MTTF)$ of System Configuration IV is obtained through:

$$MTTF_4=H(0)\left(-Q_4^{-1}\right)(1,1,1,1)^T \tag{42}$$

Thus,

$$MTTF_4=\frac{\alpha_0^3+5\alpha_0^2\beta_0+12\alpha_0\beta_0^2+15\beta_0^3}{\alpha_0^3+5\alpha_0^2\beta_0+12\alpha_0\beta_0^2+16\beta_0^3} \tag{43}$$

Where $H(0)=[1,0,0,0]$ and $Q_4=\begin{pmatrix}-2\beta_0 & \beta_0 & 0 & 0\\ \alpha_0 & -(2\beta_0+\alpha_0) & \beta_0 & 0\\ 0 & \alpha_0 & -(2\beta_0+\alpha_0) & \beta_0\\ 0 & 0 & \alpha_0 & -(2\beta_0+\alpha_0)\end{pmatrix}$

Availability and Busy period Analysis of System Configuration IV

To compute the availability of system configuration IV, the differential difference equations given in (41) are expressed in the form:

$$\begin{pmatrix}h_0'\\h_1'\\h_2'\\h_3'\\h_4'\\h_5'\\h_6'\\h_7'\\h_8'\end{pmatrix}=\begin{pmatrix}
-2\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0\\
\beta_0 & -(2\beta_0+\alpha_0) & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0\\
\beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & \beta_0 & 0 & -(2\beta_0+\alpha_0) & 0 & \alpha_0 & \alpha_0 & 0 & 0\\
0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & \beta_0 & 0 & -(2\beta_0+\alpha_0) & 0 & \alpha_0 & \alpha_0\\
0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0\\
0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0
\end{pmatrix}\begin{pmatrix}h_0\\h_1\\h_2\\h_3\\h_4\\h_5\\h_6\\h_7\\h_8\end{pmatrix}$$

The steady state availability (the proportion of the time the system is functioning or equivalently the sum of the probabilities of operational states), busy period due to partial failure and complete failure are given by:

$$A_{V4}\left(\infty\right)=h_0\left(\infty\right)+h_1\left(\infty\right)+h_3\left(\infty\right)+h_5\left(\infty\right) \tag{44}$$

$$B_{h7}=h_1\left(\infty\right)+h_3\left(\infty\right)+h_5\left(\infty\right) \tag{45}$$

$$B_{h8}\left(\infty\right)=h_2\left(\infty\right)+h_4\left(\infty\right)+h_6\left(\infty\right)+h_7\left(\infty\right)+h_8\left(\infty\right) \tag{46}$$

In the steady state, the derivatives of states probabilities become zero and therefore (41) becomes:

$$M_4 H\left(t\right)^T=0 \tag{47}$$

These can be expressed in matrix form as:

$$\begin{pmatrix} -2\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -\left(2\beta_0+\alpha_0\right) & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & -\left(2\beta_0+\alpha_0\right) & 0 & \alpha_0 & \alpha_0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & -\left(2\beta_0+\alpha_0\right) & 0 & \alpha_0 & \alpha_0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Using the following normalizing condition

$$\sum_{n=0}^{8} h_n\left(\infty\right)=1 \tag{48}$$

To obtained the state probabilities $h_i(t)$ $i=0,1,2,\ldots,8$ using (48) in the last row of (47) we get:

$$\begin{pmatrix} -2\beta_0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_0 & -\left(2\beta_0+\alpha_0\right) & 0 & \alpha_0 & \alpha_0 & 0 & 0 & 0 & 0 \\ \beta_0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_0 & 0 & -\left(2\beta_0+\alpha_0\right) & 0 & \alpha_0 & \alpha_0 & 0 & 0 \\ 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_0 & 0 & -\left(2\beta_0+\alpha_0\right) & 0 & \alpha_0 & \alpha_0 \\ 0 & 0 & 0 & \beta_0 & 0 & 0 & -\alpha_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_0 & 0 & -\alpha_0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -\alpha_0 \end{pmatrix} \begin{pmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{49}$$

Explicit expressions for the steady-state availability, busy period due to partial failure and complete failure are given by:

$$A_{V4}=\frac{\alpha_0^4+\alpha_0^3\beta_0+\alpha_0^2\beta_0^2+\alpha_0\beta_0^3}{\alpha_0^4+2\alpha_0^3\beta_0+2\alpha_0^2\beta_0^2+2\alpha_0\beta_0^3+2\beta_0^4} \tag{50}$$

$$B_{h7}=\frac{\alpha_0^3\beta_0+2\alpha_0^2\beta_0^2}{\alpha_0^4+2\alpha_0^3\beta_0+2\alpha_0^2\beta_0^2+2\alpha_0\beta_0^3+2\beta_0^4} \tag{51}$$

$$B_{h8}=\frac{\alpha_0^3\beta_0+\alpha_0^2\beta_0^2+\alpha_0\beta_0^3+2\beta_0^4}{\alpha_0^4+2\alpha_0^3\beta_0+2\alpha_0^2\beta_0^2+2\alpha_0\beta_0^3+2\beta_0^4} \tag{52}$$

Profit analysis of System Configuration IV

Using similar procedure presented in subsection 3.1.3, the explicit expression for profit function of system configuration IV is given by:

$$PF_4 = K_0 A_{V4} - \left(K_1 B_{h7} + K_2 B_{h8}\right) \tag{53}$$

IV. Discussion

I. Comparison between the System Configurations

Analytical Comparisons

Here, the configurations are compared analytically in terms of their availability and mean time to failure to determine the optimal configuration by taking the difference between the configurations $\alpha_0, \beta_0 > 0$ using MAPLE software package.

$$A_{V2} - A_{V1} = \frac{\alpha_0\beta_0\left(\alpha_0^4 + 4\alpha_0^3\beta_0 + 5\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + \beta_0^4\right)}{\left(\alpha_0^3 + 3\alpha_0^2\beta_0 + 5\alpha_0\beta_0^2 + 3\beta_0^3\right)\left(\alpha_0^4 + 2\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + 2\beta_0^4\right)} > 0 \tag{54}$$

$$\Rightarrow A_{V2} > A_{V1} \qquad \forall\ \alpha_0, \beta_0 > 0$$

$$A_{V2} - A_{V3} = \frac{\alpha_0\beta_0^2\left(7\alpha_0^4 + 14\alpha_0^3\beta_0 + 13\alpha_0^2\beta_0^2 + 6\alpha_0\beta_0^3 + \beta_0^4\right)}{\left(\alpha_0^4 + 2\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + 2\beta_0^4\right)\left(\alpha_0^3 + 3\alpha_0^2\beta_0 + 9\alpha_0\beta_0^2 + 3\beta_0^3\right)} > 0 \tag{55}$$

$$\Rightarrow A_{V2} > A_{V3} \qquad \forall\ \alpha_0, \beta_0 > 0$$

$$A_{V2} - A_{V4} = \frac{\alpha_0^2\beta_0\left(\alpha_0^5 + 2\alpha_0^4\beta_0 + 2\alpha_0^3\beta_0^2 + 2\alpha_0^2\beta_0^3 + 2\alpha_0\beta_0^4 + \beta_0^5\right)}{\left(\alpha_0^4 + 2\alpha_0^3\beta_0 + 3\alpha_0^2\beta_0^2 + 3\alpha_0\beta_0^3 + 2\beta_0^4\right)\left(\alpha_0^4 + 2\alpha_0^3\beta_0 + 2\alpha_0^2\beta_0^2 + 2\alpha_0\beta_0^3 + 2\beta_0^4\right)} > 0 \tag{56}$$

$$\Rightarrow A_{V2} > A_{V4} \qquad \forall\ \alpha_0, \beta_0 > 0$$

$$A_{V3} - A_{V4} = \frac{\alpha_0\beta_0\left(\alpha_0^5 - 4\alpha_0^4\beta_0 - 6\alpha_0^3\beta_0^2 - 5\alpha_0^2\beta_0^3 - 4\alpha_0\beta_0^4 - \beta_0^5\right)}{\left(\alpha_0^3 + 3\alpha_0^2\beta_0 + 9\alpha_0\beta_0^2 + 3\beta_0^3\right)\left(\alpha_0^4 + 2\alpha_0^3\beta_0 + 2\alpha_0^2\beta_0^2 + 2\alpha_0\beta_0^3 + 2\beta_0^4\right)} > 0 \tag{57}$$

$\Rightarrow A_{V3}(\infty) > A_{V4}(\infty)$ if and only if $\alpha_0^5 > \left(4\alpha_0^4\beta_0 + 6\alpha_0^3\beta_0^2 + 5\alpha_0^2\beta_0^3 + 4\alpha_0\beta_0^4 + \beta_0^5\right)$ for some $\alpha_0 > \beta_0$

$$A_{V3} - A_{V1} = \frac{\alpha_0\beta_0^2\left(\alpha_0^2 - 2\alpha_0\beta_0 - \beta_0^2\right)}{\left(\alpha_0^3 + 3\alpha_0^2\beta_0 + 9\alpha_0\beta_0^2 + 3\beta_0^3\right)\left(\alpha_0^3 + 3\alpha_0^2\beta_0 + 5\alpha_0\beta_0^2 + 3\beta_0^3\right)} > 0 \tag{58}$$

$\Rightarrow A_{V3}(\infty) > A_{V1}(\infty)$ if and only if $\alpha_0^2 > \left(2\alpha_0\beta_0 + \beta_0^2\right)$ for some $\alpha_0 > \beta_0$

$$A_{V4} - A_{V1} = \frac{\alpha_0\beta_0^2\left(2\alpha_0^3 + 2\alpha_0^2\beta_0 + \alpha_0\beta_0^2 + \beta_0^3\right)}{\left(\alpha_0^4 + 2\alpha_0^3\beta_0 + 2\alpha_0^2\beta_0^2 + 2\alpha_0\beta_0^3 + 2\beta_0^4\right)\left(\alpha_0^2 + 2\alpha_0\beta_0 + 3\beta_0^2\right)} > 0 \tag{59}$$

$\Rightarrow A_{V4} > A_{V1} \qquad \forall\ \alpha_0, \beta_0 > 0$

Using availability models of all the system configurations, it is clear from (54) – (59) that

$$A_{V2} > A_{V3} > A_{V4} > A_{V1}$$

Thus, the optimal system configuration is configuration II

$$MTTF_2 - MTTF_1 = \frac{8\alpha_0^7 + 110\alpha_0^6\beta_0 + 643\alpha_0^5\beta_0^2 + 2125\alpha_0^4\beta_0^3 + 4421\alpha_0^3\beta_0^4 + 5870\alpha_0^2\beta_0^5 + 4589\alpha_0\beta_0^6 + 1617\beta_0^7}{\beta_0^2\left(2\alpha_0^2 + 15\alpha_0\beta_0 + 27\beta_0^2\right)\left(4\alpha_0^4 + 25\alpha_0^3\beta_0 + 76\alpha_0^2\beta_0^2 + 112\alpha_0\beta_0^3 + 64\beta_0^4\right)} > 0 \tag{60}$$

$\Rightarrow MTTF_2 > MTTF_1 \qquad \forall\ \alpha_0, \beta_0 > 0$

$$MTTF_2 - MTTF_3 = \frac{28\alpha_0^7 + 343\alpha_0^6\beta_0 + 1890\alpha_0^5\beta_0^2 + 6041\alpha_0^4\beta_0^3 + 12303\alpha_0^3\beta_0^4 + 16138\alpha_0^2\beta_0^5 + 12585\alpha_0\beta_0^6 + 4452\beta_0^7}{\beta_0^2\left(4\alpha_0^4 + 25\alpha_0^3\beta_0 + 76\alpha_0^2\beta_0^2 + 112\alpha_0\beta_0^3 + 64\beta_0^4\right)\left(8\alpha_0^2 + 45\alpha_0\beta_0 + 81\beta_0^2\right)} > 0 \quad (61)$$

$$\Rightarrow MTTF_2 > MTTF_3 \qquad \forall\ \alpha_0, \beta_0 > 0$$

$$MTTF_2 - MTTF_4 = \frac{4\alpha_0^8 + 45\alpha_0^7\beta_0 + 245\alpha_0^6\beta_0^2 + 839\alpha_0^5\beta_0^3 + 1942\alpha_0^4\beta_0^4 + 3088\alpha_0^3\beta_0^5 + 3284\alpha_0^2\beta_0^6 + 2128\alpha_0\beta_0^7 + 640\beta_0^8}{\beta_0^2\left(\alpha_0^3 + 5\alpha_0^2\beta_0 + 12\alpha_0\beta_0^2 + 16\beta_0^3\right)\left(4\alpha_0^4 + 25\alpha_0^3\beta_0 + 76\alpha_0^2\beta_0^2 + 112\alpha_0\beta_0^3 + 64\beta_0^4\right)} > 0 \quad (62)$$

$$\Rightarrow MTTF_2 > MTTF_4 \qquad \forall\ \alpha_0, \beta_0 > 0$$

$$MTTF_3 - MTTF_1 = \frac{2\alpha_0^5 + 21\alpha_0^4\beta_0 + 96\alpha_0^3\beta_0^2 + 233\alpha_0^2\beta_0^2 + 306\alpha_0\beta_0^4 + 162\beta_0^5}{\beta_0^2\left(2\alpha_0^2 + 15\alpha_0\beta_0 + 27\beta_0^2\right)\left(8\alpha_0^2 + 45\alpha_0\beta_0 + 81\beta_0^2\right)} > 0 \quad (63)$$

$$\Rightarrow MTTF_3 > MTTF_1 \qquad \forall\ \alpha_0, \beta_0 > 0$$

$$MTTF_3 - MTTF_4 = \frac{\alpha_0^6 + 8\alpha_0^5\beta_0 + 23\alpha_0^4\beta_0^2 + 8\alpha_0^3\beta_0^3 - 112\alpha_0^2\beta_0^4 - 307\alpha_0\beta_0^5 - 305\beta_0^6}{\beta_0^2\left(\alpha_0^3 + 5\alpha_0^2\beta_0 + 12\alpha_0\beta_0^2 + 16\beta_0^3\right)\left(8\alpha_0^2 + 45\alpha_0\beta_0 + 81\beta_0^2\right)} > 0 \quad (64)$$

$\Rightarrow MTTF_3 > MTTF_4$ if and only if

$\left(\alpha_0^6 + 8\alpha_0^5\beta_0 + 23\alpha_0^4\beta_0^2 + 8\alpha_0^3\beta_0^3\right) > \left(112\alpha_0^2\beta_0^4 + 307\alpha_0\beta_0^5 + 305\beta_0^6\right)$ for some $\alpha_0, \beta_0 > 0$

$$MTTF_4 - MTTF_1 = \frac{4\alpha_0^4 + 30\alpha_0^3\beta_0 + 96\alpha_0^2\beta_0^2 + 8\alpha_0^3\beta_0^3 + 169\alpha_0\beta_0^3 + 133\beta_0^4}{\left(\alpha_0^3 + 5\alpha_0^2\beta_0 + 12\alpha_0\beta_0^2 + 16\beta_0^3\right)\left(2\alpha_0^2 + 15\alpha_0\beta_0 + 27\beta_0^2\right)} > 0 \quad (65)$$

$$\Rightarrow MTTF_4 > MTTF_1 \qquad \forall\ \alpha_0, \beta_0 > 0$$

Similarly, using mean time to failure $(MTTF)$ models of all the system configurations, it is clear from (60) – (65) that

$$MTTF_2 > MTTF_3 > MTTF_4 > MTTF_1$$

Thus, the optimal system configuration is configuration II

II. Numerical examples

The purpose of this section is to rank the system configurations in terms of their availability and mean time to failure using MATLAB software package. The results are summarized in tables below

Table 1: *Ranking between the systems configurations in terms of their availability and mean time to failure.*

Case	Parameter Range	Results	Constant Value
1	$0 < \beta_0 < 0.3$	$A_2(\infty) > A_4(\infty) > A_1(\infty) > A_3(\infty)$	$\alpha_0 = 0.6$ $C_0 = 100000$ $C_1 = 500$ $C_2 = 1000$
	$0.3 < \beta_0 < 0.6$	$A_2(\infty) > A_4(\infty) > A_1(\infty) > A_3(\infty)$	
	$0.6 < \beta_0 < 0.9$	$A_2(\infty) > A_4(\infty) > A_1(\infty) > A_3(\infty)$	
	$0.9 < \beta_0 < 1.2$	$A_2(\infty) > A_4(\infty) > A_1(\infty) > A_3(\infty)$	
2	$0 < \beta_0 < 0.3$	$MTTF_2 > MTTF_3 > MTTF_4 > MTTF_1$	
	$0.3 < \beta_0 < 0.6$	$MTTF_2 > MTTF_4 > MTTF_3 > MTTF_1$	
	$0.6 < \beta_0 < 0.9$	$MTTF_2 > MTTF_4 > MTTF_3 > MTTF_1$	
	$0.9 < \beta_0 < 1.2$	$MTTF_2 > MTTF_4 > MTTF_3 > MTTF_1$	
3	$0 < \alpha_0 < 0.3$	$A_2(\infty) > A_3(\infty) > A_4(\infty) > A_1(\infty)$	
	$0.3 < \alpha_0 < 0.6$	$A_2(\infty) > A_3(\infty) > A_4(\infty) > A_1(\infty)$	

	$0.6 < \alpha_0 < 0.9$	$A_2(\infty) > A_3(\infty) > A_4(\infty) > A_1(\infty)$	$\beta_0 = 0.01$
	$0.9 < \alpha_0 < 1.2$	$A_2(\infty) > A_3(\infty) > A_4(\infty) > A_1(\infty)$	$C_0 = 100000$
4	$0 < \alpha_0 < 0.3$	$MTTF_2 > MTTF_3 > MTTF_4 = MTTF_1$	$C_1 = 500$
	$0.3 < \alpha_0 < 0.6$	$MTTF_2 > MTTF_3 > MTTF_4 = MTTF_1$	$C_2 = 1000$
	$0.6 < \alpha_0 < 0.9$	$MTTF_2 > MTTF_3 > MTTF_4 = MTTF_1$	
	$0.9 < \alpha_0 < 1.2$	$MTTF_2 > MTTF_3 > MTTF_4 = MTTF_1$	

Profit Comparison

In this section, numerically comparison with respect to the profit functions for all configurations are discussed. For consistency, we fix the following set of parameters values throughout the simulations: $\alpha_0 = 0.6$, $\beta_0 = 0.01$, $K_0 = 100,000$, $K_1 = 500$ and $K_2 = 1,000$

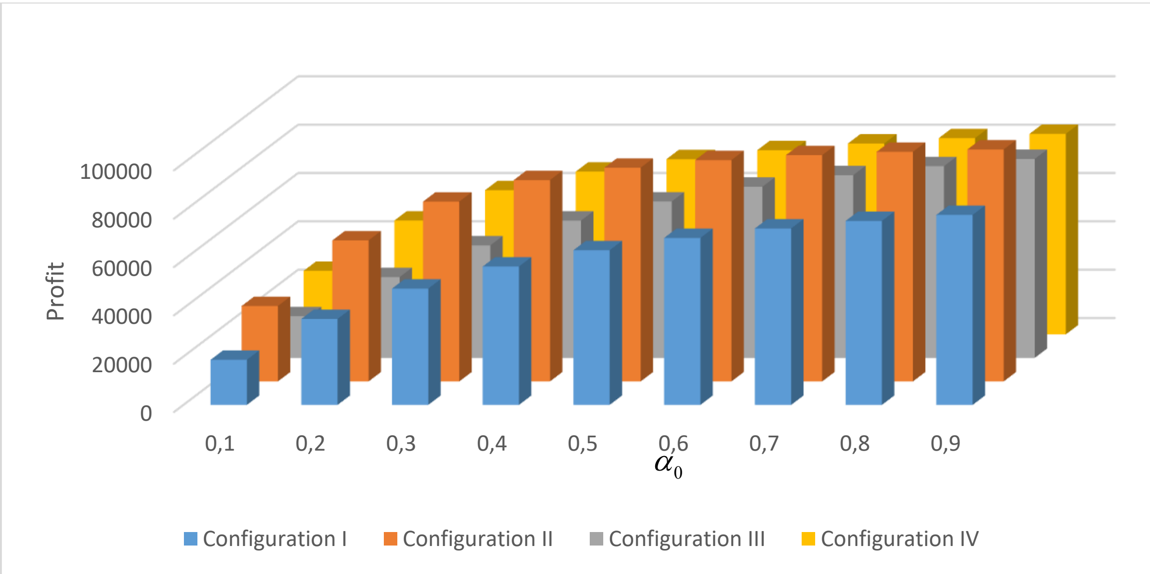

Figure 5: *Profit Comparison for all configuration using* α_0

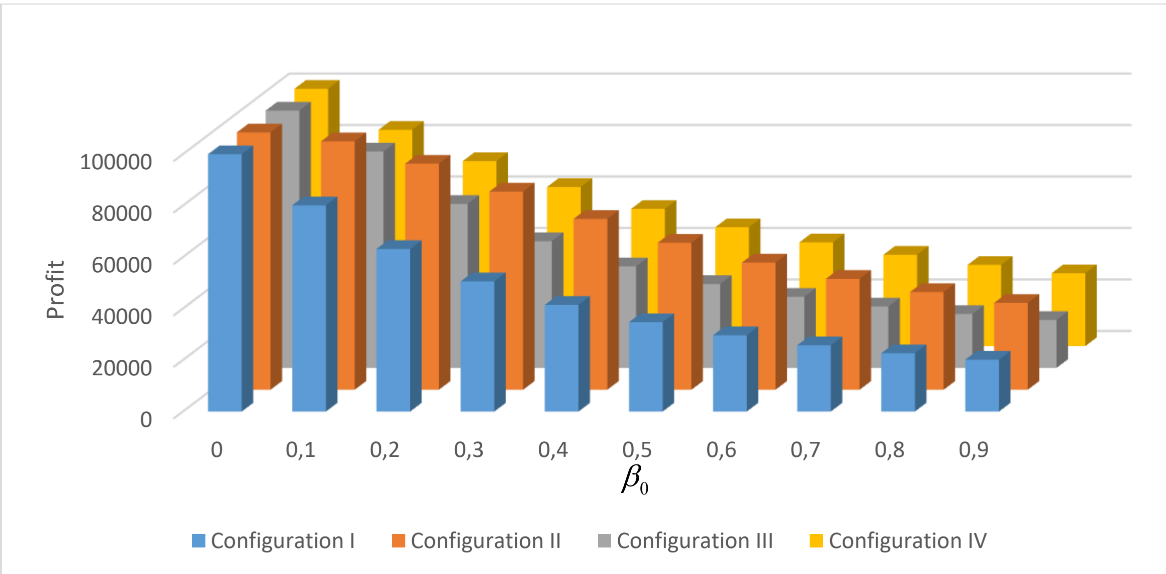

Figure 6: *Profit Comparison for all configuration using* β_0

Figures 5 and 6 depict the trends of profit for all the system configurations against the repair and failure rates α_0 and β_0 respectively. In both figures, it is seen that as repair rate α_0 increases, the profit increases, while with increase in failure rate β_0, the profit decreases. This means that preventive and major maintenance is significant in maximizing the system profit. It is also evident from these Figures that Configuration II has the highest profit as compared to the other three configurations.

Cost Benefit Comparison

In this section, the system configurations are compared based on their cost benefit, where the benefit is either availability or mean time to failure. Numerical values of Wang el al. (2006) parameter values are used to compare the configurations.

$C_1 = 48,000,000$, $C_2 = 39,000,000$, $C_3 = 42,000,000$ and $C_4 = 39,000,000$

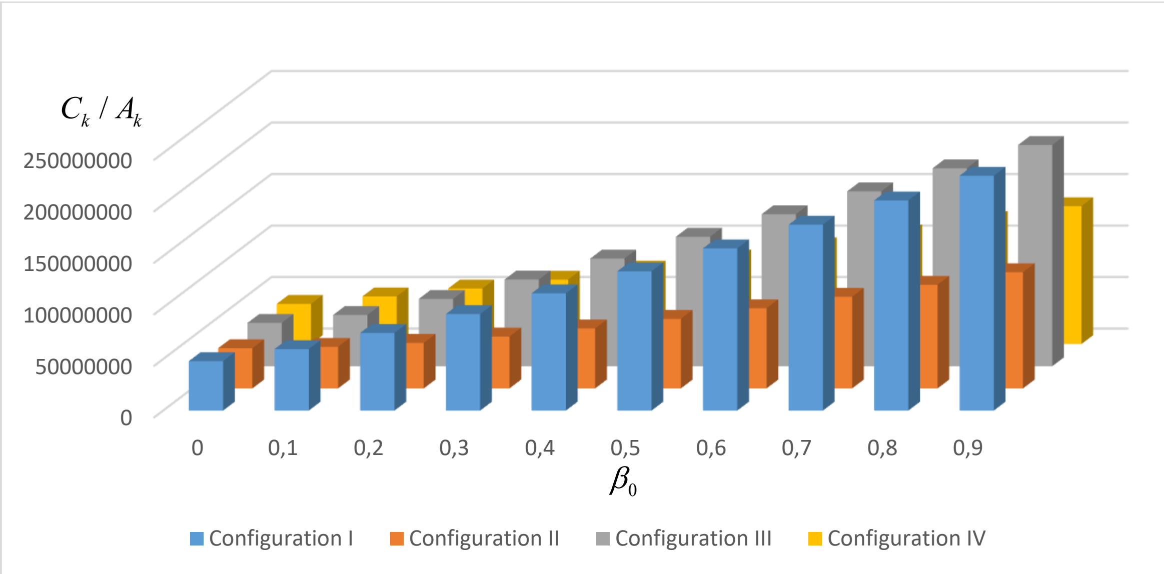

Figure 7: C_k / A_k *Comparison for all configuration using* β_0

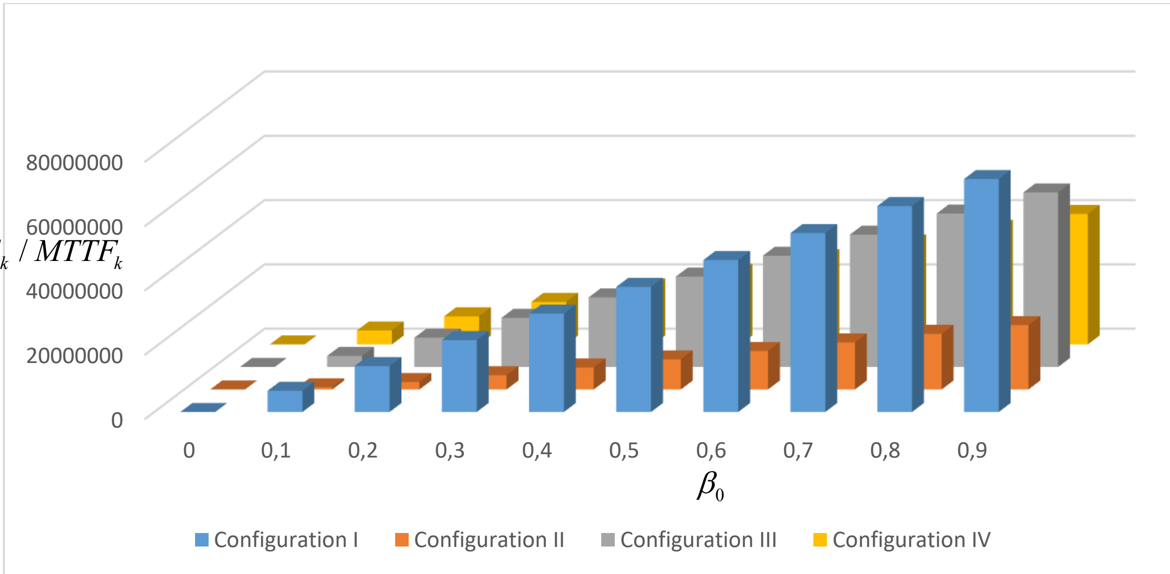

Figure 8: $C_k / MTTF_k$ *Comparison for all configuration using* β_0

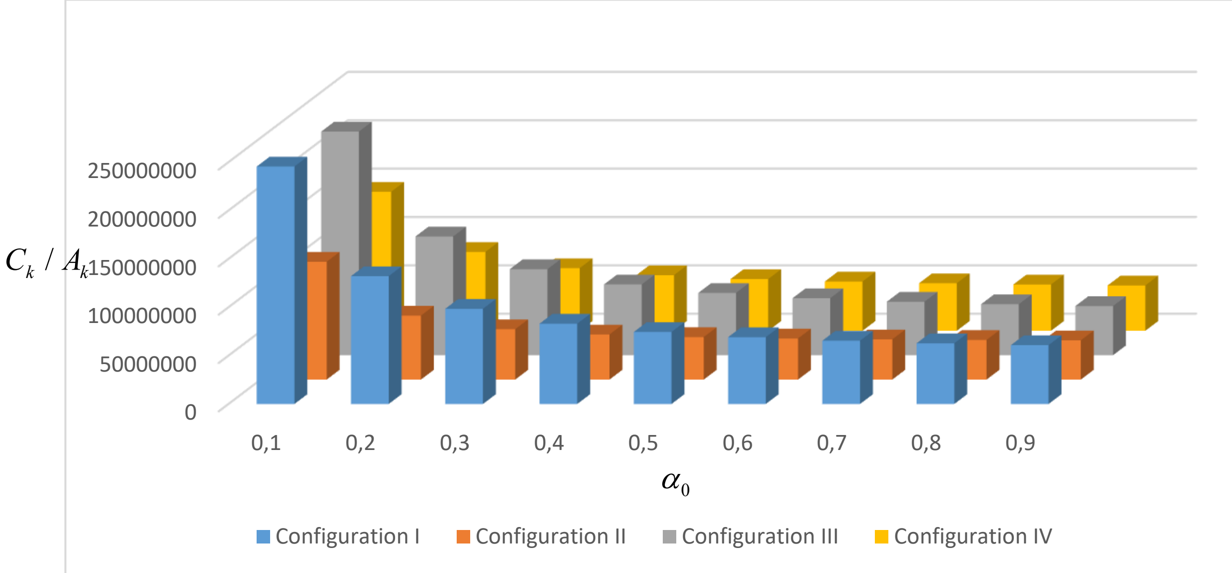

Figure 9: C_k / A_k ***Comparison for all configuration using*** α_0

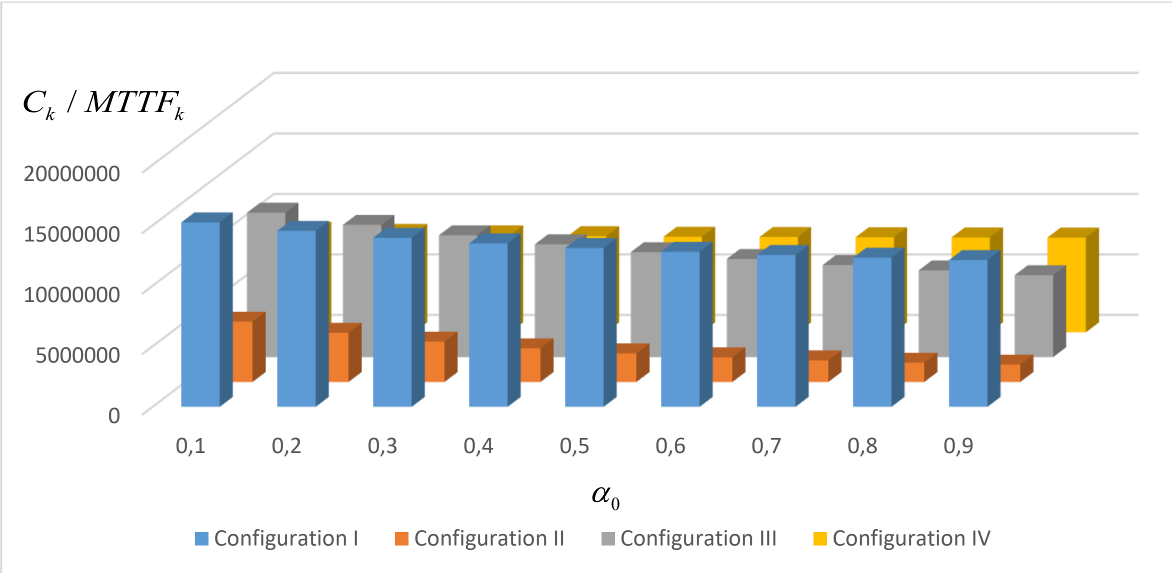

Figure 10: $C_k / MTTF_k$ *Comparison for all configuration using* α_0

Figures 7-10 present the trends of cost benefit C_k / A_k and $C_k / MTTF_k$ for all configurations against the repair and failure rates α_0 and β_0 respectively. It is observed from Figures 7 and 8 that C_k / A_k and $C_k / MTTF_k$ increases as β_0 increases for any system configuration. It is also observed from these figures that the optimal system configuration is configuration II. On the other hand, Figures 9 and 10 display the effects of C_k / A_k and $C_k / MTTF_k$ for all the system configurations against the repair rate α_0. These figures revealed that C_k / A_k and $C_k / MTTF_k$ decreases as α_0 increases for any system configuration. Also, from these figures, it is clear that the optimal configuration is configuration II.

V. CONCLUSION

In this paper, we have constructed four different standby serial systems each consisting of five units. The expressions for the system characteristics such as system availability, busy period of repairman due to partial and complete failure as well as profit functions for all the configurations have been obtained and validated by performing numerical experiments. Analysis of the effect of various system parameters on profit function and availability was performed. These are the main contributions of this study. On the basis of the numerical results obtained in Figures 5 – 10 and Tables 1-4 for a particular case, it is evident that the optimal system configuration is configuration II. This is supported from analytical comparison presented in terms of the availability and mean time to failure models obtained in which configuration II is the optimal configuration for all $\alpha_0, \beta_0 > 0$ contrary to some studies where the optimality among the system configuration is not uniform as it depends on some system parameters.

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