Performance Analysis of the Water Treatment Reverse Osmosis Plant

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Abstract

In this research paper, profit analysis of a Water Treatment Reverse Osmosis (RO) Plant is carried out by using the Regenerative Point Graphical Technique (RPGT) under specific conditions for system parameters. The paper analyzes the behavior of a water treatment RO plant consisting of subunits namely Multimedia filter (MMF), Cartridge filter (CF), High-pressure pump (HPP), RO System (ROS). The system is in a working state when all subunits are in good condition. A repair facility is accessible for all subunits. Availability of the plant, Busy Period of the Server (BPS) and Expected number of inspection by the repairman (ENIR) is calculated by using the RPGT technique. Finally, numerical analysis is carried out for calculating the performance measures and their comparisons.

Keywords: Regenerative Point Graphical Technique, Profit Analysis, Availability, Water Treatment Reverse Osmosis (RO) Plant.

I. Introduction

Reliability performance measures have incredible importance in the modern system such as the bread-making system, power plants and engineering systems. For making the system more significant, it is necessary to keep reliability measures up in the framework. In the majority of the systems, significant levels are kept up by giving skilled repair facility and upkeep activities. In some cases, redundant standby units are introduced to obtain the highest significant level.

In today's scenario, 3% of water is fresh on earth out of which 2.5% is unapproachable as it is in the form of glaciers, polar ice caps, atmosphere and soil, so only 0.5% of the water is accessible as freshwater. With only 0.5% water available, it's crucial to have Water Treatment Plant (WTP) to treat the wastewater and provide us freshwater for our daily use. For continuous working of these resources, it is essential to have timely maintenance of these systems to reduce the failure rate and keep the machines up and running. For upgrading and maintaining the efficiency of WTP's, unproductive time due to servicing (breakdown, jam of membrane, low pressure etc.) have to be minimized and assure maximum availability. Generally, the fundamental problem in the WTP is the low maintenance and poor quality material of the components used at the time of manufacturing. The solution to these problems is the regular use of safety measures and maintenance techniques.

Thus Reliability, Availability and Maintainability (RAM) analysis of WTP's become a thoughtful issue for making the system more efficient and productive. Water treatment RO plant comprises of the following components which include Raw Water Forwarding Pump (RWFP), Flow Indicators (FI), Pressure Indicators (PI), Multi-Media Filter (MMF), Cartridge Filter (CF), Antiscalant Dosing pump with Tank (ASD), High-Pressure Pump (HPP), RO System (ROS), Product Water Storage Tank (PWST), Reject Water Storage Tank (RWST) and ancillary elements such as valves and gauges. The sub-system will fail if the primary and standby redundant units fail, thus producing total system failure. Cold standby excess units are switched in with the help of a perfect switch over the frameworks, which distinguishes the failure unit and switched in redundant standby unit.

Asi et al. (2021) studied a relative investigation of five productive dependability techniques to drive common rules for probabilistic evaluation of bridge pier. Li et al. (2020) discussed the time-dependent analysis with testing in practical engineering applications. Four models are developed to exhibit the effectiveness and exactness of the Improved Composite Limit state (ICLS) technique for the time-subordinate dependability analysis. Kumar et al. (2019) studied the behavior of the washing units in the paper industry by using the RPGT technique and noticing the framework's performance having all kinds of failures and test the workability of replacement of the breakdown structure. Kumar et al. (2018, 2017) have studied the behavior of a bread system and edible oil refinery plant. Zhai et al. (2015) developed an analytical technique based on a multi-valued verdict diagram to analyze the reliability of the system. Kumar et al. (2019) analyzed maintenance for a cold reserve framework that contains two identical subunits with server failure by using RPGT. Rajbala et al. (2019) studied the analysis and modeling: a case study EAEP industrial plant. Garg et al. (2009) analyzed the performance of a screw plant by using MATLAB Tool and cattle feed plant. Garg et al. (2010) articulated the crank availability of the component of the automobile industry taking the failure/repair rate of units as independent and solved the problem by using probability consideration and supplementary technique. Garg et al. (2010) discussed redundancy allocation in the pharmaceutical Plant. Wang et al. (2012) used some of the non-protective variables of distributions to demonstrate uncertainty, which was generally considered as stochastic factors for reliable models.

The main motive of this paper is to find the significant and critical parameters for the behavior and profit analysis of the water treatment RO plant by using the RPGT technique. For this purpose, State transition probabilities, availability, busy period of the server (BPS), maintenance specialist and profit analysis are evaluated. Finally, the numerical analysis is carried out for comparisons and comparing the results for making the system more efficient and productive.

II. Problem Description and Assumptions

I. System Description

The process diagram of the water treatment RO plant is shown in Figure 1.

- Multi-Media Filter (A):- It filters macro particles from the feed water. It consists of graded quartz and anthracite.
- Cartridge Filter (B):- This is a five-micron filter that filters micro particles from the feed water to enhance the membrane life by minimizing fouling on the membranes.
- High-Pressure Pump(C):- This pump creates the pressure above the osmotic pressure for reverse osmosis to take place.
- RO System (D):-It consists of RO Pressure vessels and RO Membranes.
    - RO Pressure Vessels (D1):- These are vessels that can take the load of the high-pressure created by the high-pressure pump and are also used to house

the RO membranes.

- RO Membranes (D2):- This is the heart of the system and the purification of the water is done by reverse osmosis process. The feed water is split into two streams; one is the stream of low TDS water called permeate and the other is the stream of high TDS water called Reject.

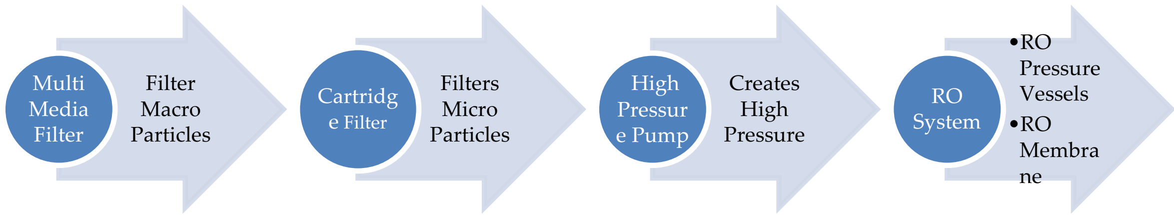

Figure 1: *Process Diagram of the Water Treatment RO Plant*

II. Notations

A, B, C, D : Working states
a, b, c, d : Failed states of A, B, C, D respectively
D_1, D_2 : Cold standby redundant D unit
s_i/r_i : Repair/Failure rates respectively; i = 1,2,3,4
$q_{i,j}(t)$: Probability distribution function from state S_i to S_j
$p_{i,j}$: Transition probability from state S_i to S_j
Ri(t) : Reliability of the system at time t, for the regenerative state S_i
μ_i : Mean sojourn time consumed in state S_i, before going in any other states
* : Laplace transform
T_0 : Mean Time to System Failure
A_0 : Availability of the System
B_0 : Mean Busy Period of the Server
V_0 : Expected Number of Inspections by the Repairman
P_0 : Profit Function
D_1 : Revenue per unit up-time of the system
D_2 : Cost per unit time in which system is under repair
D_3 : Cost due to inspection by the repairman

III. Assumptions

- The repair process begins soon after a unit fails.
- Failure and repair events are all statistically independent.
- The Repair unit is a new one.

IV. State Transition Diagram

S_1 : Initial Working state when all the four units are working; so system is working
S_5, S_9 : Reduced working states when units A, B, C are working; unit D is down and under repair; cold standby redundant units D_1, D_2 are working in place of unit D
S_2; S_6; S_{10} : Failed states when unit A fails and units B,C,D; D_1 ; D_2 are working
S_3; S_7; S_{11} : Failed states when unit B fails and units B, C, D; D_1 ; D_2 are working
S_4; S_8; S_{12} : Failed states when unit C fails and units B,C,D; D_1 ; D_2 are working
S_{13} : Failed state when unit D fails and units B, C, D are working

State S_1 is taken as the base state. By considering all the above annotations and assumptions, the

State Transition Diagram of the framework is shown in Figure 2.

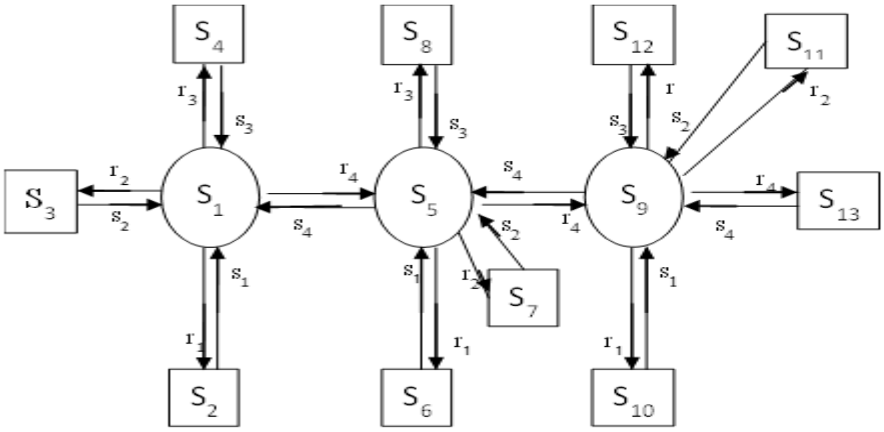

Figure 2: *Transition Diagram of the system*

S_1 = ABCD, S_2 = aBCD, S_3 = AbCD, S_4 = ABcD,
S_5 = ABCD$_1$, S_6 = aBCD$_1$, S_7 = AbCD$_1$, S_8 = ABcD$_1$,
S_9 = ABCD$_2$, S_{10} = aBCD$_2$, S_{11} = AbCD$_2$, S_{12} = ABcD$_2$,
S_{13} = ABCd

V. Transition Probabilities and Mean Sojourn Times (MST)

Table 1 and Table 2 represents the Transition probabilities and MST for the states i, j respectively.

Table 1: *Transition Probabilities*

$q_{i,j}(t)$	$p_{ij} = q^*_{i,j}(0)$
$q_{1,i}(t) = r_j e^{-(r_1+r_2+r_3+r_4)t}$; i =2,3,4,5 & j = 1,2,3,4	$p_{1,i} = r_j/(r_1+r_2+r_3+r_4)$ i =2,3,4,5 & j = 1,2,3,4
$q_{2,1} = s_1 e^{-s_1 t}$	$p_{2,1} = 1$
$q_{3,1} = s_2 e^{-s_2 t}$	$p_{3,1} = 1$
$q_{4,1} = s_3 e^{-s_3 t}$	$p_{4,1} = 1$
$q_{5,1}(t) = s_4 e^{-(r_1+r_4+r_3+r_2+s_4)t}$ $q_{5,i}(t) = r_j e^{-(r_1+r_4+r_3+r_2+s_4)t}$ i =6,7,8,9 & j = 1,2,3,4	$p_{5,1} = s_4/(r_1+r_3+r_2+r_4+s_4)$ $p_{5,i} = r_j/(r_1+r_3+r_2+r_4+s_4)$ i =6,7,8,9 & j = 1,2,3,4
$q_{6,5} = s_1 e^{-s_1 t}$	$p_{6,5} = 1$
$q_{7,5} = s_2 e^{-s_2 t}$	$p_{7,5} = 1$
$q_{8,5} = s_3 e^{-s_3 t}$	$p_{8,5} = 1$

$q_{9,5}(t) = s_4e^{-(r_1+r_3+r_2+r_4+s)t}$	$p_{9,5}= s_4/(r_1+r_4+r_3+r_2+s_4)$
$q_{9,i}(t) = r_je^{-(r_1+r_2+r+r_4+s_4)t}$	$p_{9,i}= r_j/(r_1+r_3+r_2+r_4+s_4)$
i =10,11,12,13 & j = 1,2,3,4	i =10,11,12,13 & j = 1,2,3,4
$q_{10,9}= s_1e^{-s_1t}$	$p_{10,9}= 1$
$q_{11,9}= s_2e^{-s_2t}$	$p_{11,9}= 1$
$q_{12,9}= s_3e^{-s_3t}$	$p_{12,9}= 1$
$q_{13,9}= s_4e^{-s_4t}$	$p_{13,9}= 1$

Table 2: *Mean Sojourn Time (MST)*

$R_i(t)$	**$\mu_i=R_i^*(0)$**
$R_1(t)= e^{-(r_1+r_3+r_2+r_4)t}$	$\mu_1 = 1/(r_1+r_3+r_2+r_4)$
$R_2(t)= e^{-s_1t}$	$\mu_2= 1/s_1$
$R_3(t)= e^{-s_2t}$	$\mu_3= 1/s_2$
$R_4(t)= e^{-s_3t}$	$\mu_4= 1/s_3$
$R_5(t)= e^{-(r_1+r_3+r_2+r_4+s_4)t}$	$\mu_5= 1/(r_1+r_3+r_2+r_4+s_4)$
$R_6(t)= e^{-s_1t}$	$\mu_6= 1/s_1$
$R_7(t)= e^{-s_2t}$	$\mu_7= 1/s_2$
$R_8(t)= e^{-s_3t}$	$\mu_8= 1/s_3$
$R_9(t)= e^{-(r_1+r_3+r_2+r_4+s_4)t}$	$\mu_9= 1/(r_1+r_3+r_2+r_4+s_4)$
$R_{10}(t)= e^{-s_1t}$	$\mu_{10}= 1/s_1$
$R_{11}(t)= e^{-s_2t}$	$\mu_{11}= 1/s_2$
$R_{12}(t)= e^{-s_3t}$	$\mu_{12}= 1/s_3$
$R_{13}(t)= e^{-s_4t}$	$\mu_{13}= 1/s_4$

III. Evaluation of Path Probabilities

Implementing the RPGT technique and considering 'S_1' as the starting state of the framework. Path Probabilities from state 'S_1' to various vertices are stated below:

$V_{1,1} = 1$ (1)

$V_{1,i} = (1,i) = p_{1,i;}$ where i = 2,3,4 (2)

$V_{1,5}= p_{1,5}/(1-p_{5,6}p_{6,5})(1-p_{5,7}p_{7,5})(1-p_{5,8}p_{8,5})\{(1-p_{5,9}p_{9,5})/(1-p_{9,10}p_{10,9})(1-p_{9,11}p_{11,9})(1-p_{9,12}p_{12,9})(1-p_{9,13}p_{13,9})\}$ (3)

$V_{1,i}= p_{1,5}p_{5,i}/(1-p_{5,6}p_{5,6})(1-p_{5,7}p_{7,5})(1-p_{5,8}p_{8,5})\{(1-p_{5,9}p_{9,5})/(1-p_{9,10}p_{10,9})(1-p_{9,11}p_{11,9})(1-p_{9,12}p_{12,9})(1-p_{9,13}p_{13,9})\}$;
i = 6, 7, 8 (4)

$V_{1,9}= p_{1,5}p_{5,9}/(1-p_{5,6}p_{6,5})(1-p_{5,7}p_{7,5})(1-p_{5,8}p_{8,5})(1-p_{9,10}p_{10,9})(1-p_{9,11}p_{11,9})(1-p_{9,12}p_{12,9})$
$(1-p_{9,13}p_{13,9})\{(1-p_{5,9}p_{9,5})/(1-p_{9,10}p_{10,9})(1-p_{9,11}p_{11,9})(1-p_{9,12}p_{12,9})(1-p_{9,13}p_{13,9})\}$ (5)

$V_{1,i} = p_{1,5}p_{5,9}p_{9,i}/(1-p_{5,6}p_{6,5})(1-p_{5,7}p_{7,5})(1-p_{5,8}p_{8,5})(1-p_{9,10}p_{10,9})(1-p_{9,11}p_{11,9})(1-p_{9,12}p_{12,9})$

$(1-p_{9,13}p_{13,9})\{(1-p_{5,9}p_{9,5})/(1-p_{9,10}p_{10,9})(1-p_{9,11}p_{11,9})(1-p_{9,12}p_{12,9})(1-p_{9,13}p_{13,9})\}$; where i = 10,11,12,13 (6)

Path Probabilities from state S9' to various vertices are stated below:

$V_{9,1}= p_{9,5}p_{5,1}/(1-p_{5,6}p_{6,5})(1-p_{5,7}p_{7,5})(1-p_{5,8}p_{8,5})(1-p_{1,2}p_{2,1})(1-p_{1,3}p_{3,1})(1-p_{1,4}p_{4,1})$
$\{(1-p_{5,1}p_{1,5})/(1-p_{1,2}p_{2,1})(1-p_{1,3}p_{3,1})(1-p_{1,4}p_{4,1})\}$ (7)

$V_{9,i} = p_{9,5}p_{5,1}p_{1,i}/(1-p_{5,6}p_{6,5})(1-p_{5,7}p_{7,5})(1-p_{5,8}p_{8,5})(1-p_{1,2}p_{2,1})(1-p_{1,3}p_{3,1})(1-p_{1,4}p_{4,1})$
$\{(1-p_{5,1}p_{1,5})/(1-p_{1,2}p_{2,1})(1-p_{1,3}p_{3,1})(1-p_{1,4}p_{4,1})\}$; where i = 2, 3, 4 (8)

$V_{9,5} = p_{9,5}/(1-p_{5,6}p_{6,5})(1-p_{5,7}p_{7,5})(1-p_{5,8}p_{8,5})\{(1-p_{5,1}p_{1,5})/(1-p_{1,2}p_{2,1})(1-p_{1,3}p_{3,1})(1-p_{1,4}p_{4,1})\}$ (9)

$V_{9,i} = p_{9,5}p_{5,i}/(1-p_{5,6}p_{6,5})(1-p_{5,7}p_{7,5})(1-p_{5,8}p_{8,5})\{(1-p_{5,1}p_{1,5})/(1-p_{1,2}p_{2,1})(1-p_{1,3}p_{3,1})(1-p_{1,4}p_{4,1})\}$; where i = 6,7,8(10)

$V_{9,9} = 1$ (11)

$V_{9,i} = p_{9,i}$; where i = 10,11, 12, 13 (12)

IV. Evaluation of System Parameters

The MTSF and other parameters are evaluated under steady-state conditions by using S_1 as the base state.

- Mean Time to System Failure (T_0): Regenerative working states to which the framework can transit (primary state 'S_1'), before arriving any failed state are 'i' = 1, 5, 9.
  $T_0 = (V_{1,1}\mu_1+V_{1,5}\mu_5+V_{1,9}\mu_9)/\{1-V(1,5,1)\}(1-p_{1,5}p_{5,1})$ (13)
- Availability of the System (A_0): Regenerative state at which framework is accessible are 'j' = 1, 5, 9, ; 'i' = 1 to 13.
  $A_0 = \left[\sum_j V_{\xi,j}, f_j, \mu_j\right]/\left[\sum_i V_{\xi,i}, f_j, \mu_i^1\right]$ (14)
  $A_0 = (V_{9,1}\mu_1+V_{9,5}\mu_5+V_{9,9}\mu_9)/D$ (15)
  Where $D = V_{1,i}\mu_i$, $\xi = 0$; $1 \le i \le 13$
- Busy Period of the Server (B_0): Regenerative positions where server is busy are j = 2 to 13; 'i' = 1 to 13. Considering $\xi = 0$
  $B_0 = \left[\sum_j V_{\xi,j}, n_j\right]/\left[\sum_i V_{\xi,i}, \mu_i^1\right]$ (16)
  $B_0 = (V_{1,j}\mu_j)/D$; $2 \le j \le 13$. (17)
- Expected Number of Inspections by the Repairman (V_0): Regenerative positions where the technician visit is j = 2 to 13; i = 0 to 13. Considering $\xi = 0$
  $V_0 = \left[\sum_j V_{\xi,j}\right]/\left[\sum_i V_{\xi,i}, \mu_i^1\right]$ (18)
  $V_0 = (V_{1,j})/D$; $2 \le j \le 13$. (19)

V. Results and Discussions

Particular Cases:- $s_i = s$ $(0 \le i \le 4)$, $r_i = r$ $(0 \le i \le 4)$

I. Mean Time to System Failure (MTSF) (T_0)

Table 3 shows the values of T_0 for varying repair/failure rates. Figure 3 displays the increasing decreasing trend of T_0 for varying repair/failure rates.

Table 3: *Mean Time to System Failure (MTSF)*

Repair rates/ Failure rates	s = .50	s = .60	s = .70
r = .10	2.86	2.80	2.79
r = .20	1.66	1.53	1.47
r = .30	0.52	0.47	0.42

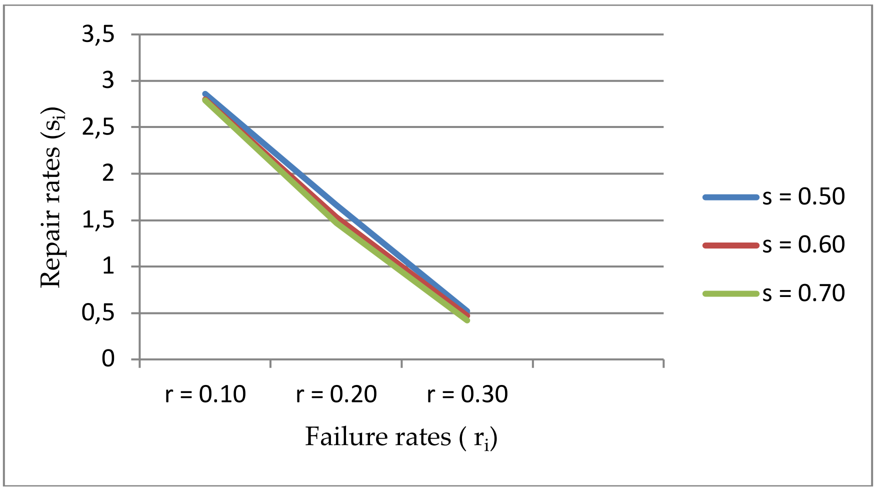

Figure 3: *Mean Time to System Failure (MTSF)*

II. Availability of the System (A_0)

Table 4 presents the values of A_0 for varying repair/failure rates. Figure 4 displays the increasing decreasing trend of A_0 for changing repair/failure rates.

Table 4: *Availability of the System (A_0)*

Repair rates/ Failure rates	s = .50	s = .60	s = .70
r = .10	.66	.70	.73
r = .20	.48	.51	.56
r = .30	.31	.40	.51

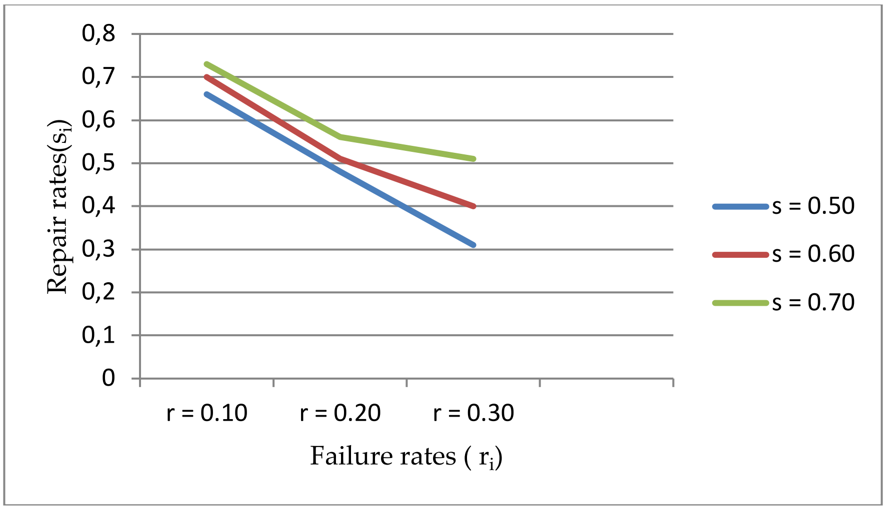

Figure 4: *Availability of the System (A_0)*

III. Busy Period of the Server (BPS) (B_0)

Table 5 shows the values of B_0 for varying repair/failure rates. Figure 5 displays the increasing decreasing trend of B_0 for varying repair/failure rates.

Table 5: *Busy Period of the Server (BPS)*

Repair rates/ Failure rates	s = .50	s = .60	s = .70
r = .10	.33	.28	.24
r = .20	.53	.47	.41
r = .30	.79	.63	.55

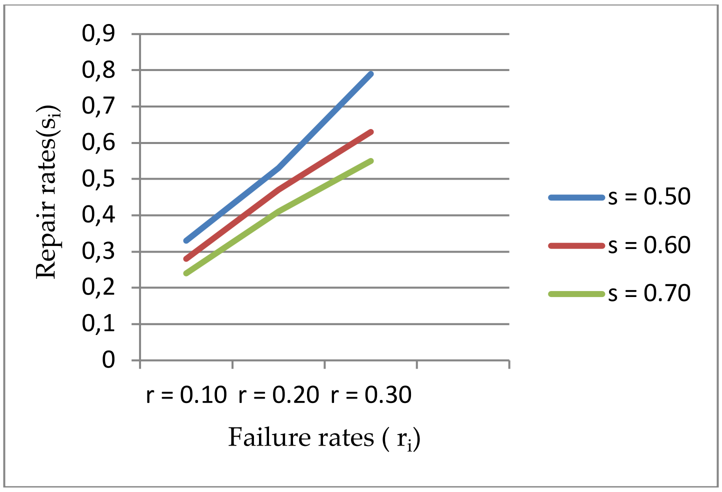

Figure 5: *Busy Period of the Server (BPS)*

IV. Expected Number of Inspection by the Repairman (ENIR) (V_0)

Table 6 shows the values of V_0 for varying repair/failure rates. Figure 6 displays the increasing decreasing trend of V_0 for changing repair/failure rates.

Table 6: *Expected Number of Inspection by Repairman*

Repair rates/ Failure rates	s = .50	s = .60	s = .70
r = .10	.12	.16	.19
r = .20	.14	.19	.23
r = .30	.21	.28	.33

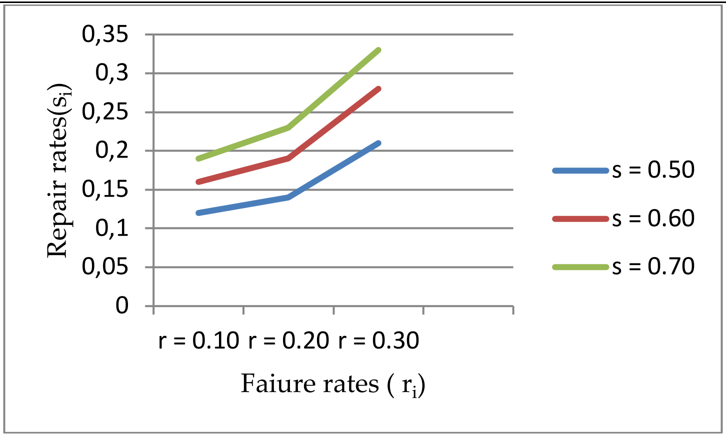

Figure 6: *Expected Number of Inspection by Repairman*

V. Profit Function

Profit analysis of the framework is calculated by applying the profit function given below

$P_0 = D_1A_0 – D_2B_0 - D_3V_0$ (20)

Assuming $D_1 = 2000, D_2 = 50, D_3 = 100$

Table 7 represents the values of profit function for varying repair/failure rates. Figure 7 shows the increasing decreasing trend of the profit function for varying repair/failure rates.

Table 7: *Profit Function*

Repair rates/ Failure rates	s = .50	s = .60	s = .70
r = .10	1291.5	1370.0	1429.0
r = .20	919.5	977.5	1076.5
r = .30	559.5	740.5	959.5

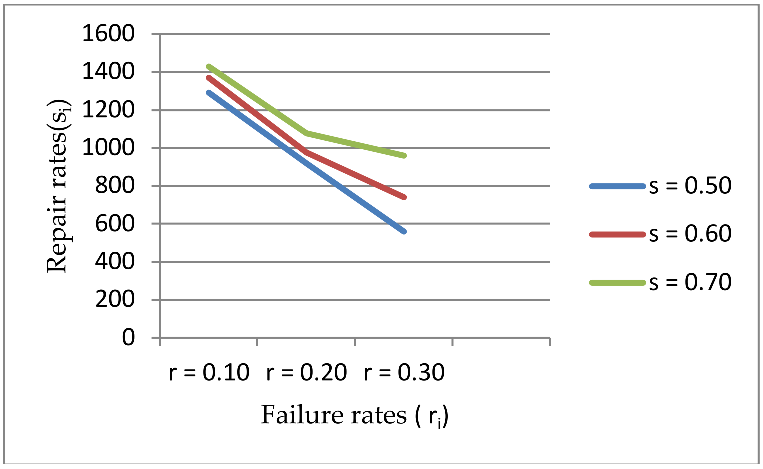

Figure 7: *Profit Function*

VI. Conclusion

Reliability, Availability and Maintainability (RAM) analysis of WTP's becomes an essential aspect for making the system more efficient and productive. The above calculations and graphs conclude that the availability of the system and the profit function reduces with the rise in failure rate and increases with the rise in repair rate. It is also observed that the expected no. of inspections by the repairman increases with the rise in failure rate while BSP and MTSF reduce with the rise in repair rates. Thus the effectiveness and the reliability of the plant can be improved by increasing the repair rate and decreasing the failure rate.

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