Reliability Test Plan For The Marshall-Olkin Extended Inverted Kumaraswamy Distribution

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Abstract

This paper mainly interested in studying the wider range behavior of the Marshall-Olkin extended inverted Kumaraswamy distribution. The parameters of model are estimated by various estimation methods. A reliability sampling plan is proposed which can save the test time in practical situations. Some tables are also provided for the new sampling plans so that this method can be used conveniently by practitioners. The developed test plan is applied to ordered failure times of software release to provide its importance in industrial applications

Keywords: Reliability Test Plan, Kumaraswamy Distribution, Marshall-Olkin Family, Method of Maximum Likelihood, Method of Percentiles

I. Introduction

In statistical literature, there are numerous distributions but still remain many important problems where the real data does not follow any of the existing probability models. Because of this, significant strive has been taken in the development of generalizations of standard probability distributions along with relevant statistical methodologies. Kumaraswamy distribution introduced by Kumaraswamy (1980) is derived from beta distribution after fixing some parameters posses a closed-form cdf (cumulative density function) which is invertible. This distribution is applicable to many natural phenomena related to which outcomes have lower and upper bounds. The inverted Kumaraswamy model is the probability distribution of a random variable whose reciprocal has a Kumaraswamy distribution proposed by Abd AL-Fattah et al. (2017). Further, Iqbal et al. (2017) derived generalized form of inverted Kumaraswamy distribution by inserting another parameter to inverted Kumaraswamy distribution.

The method of addition of parameters has used to enhance the properties of existing family of distributions. This added new parameter improves the goodness-of-fit of the generated family. Parameters can be introduced by various methods, then we have new families such as exponentiated family of distributions (Gupta et al., 1998), transformedtransformer (T-X) family of distributions (Alzaatreh, 2011), Kumaraswamy family distributions (Cordeiro and Castro, 2011), geometric exponential-Poisson family family of distributions (Nadarajah et al., 2013), etc. Many researchers used the Marshall-Olkin method introduced by Marshall-Olkin (1997) to propose new distributions and established their distinct properties and characteristics.

This paper mainly focus on different methods of estimation and the reliability test plan for the Marshall-Olkin extended inverted Kumaraswamy distribution. The paper is organized as follows: Section 2 deals with the basic concepts of Marshall-Olkin Extended (MOE) inverted Kumaraswamy distribution (Tomy and Gillariose, 2017). Different methods of estimation discussed in Section 3. The reliability test plan is conducted in Section 4. This work is concluded in Section 5.

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II. Inverted Kumaraswamy Distribution

The cdf and probability density function (pdf) of MOE inverted Kumaraswamy (MOEIKum) distribution, respectively, are given by

$$G(x,\alpha,\beta,\gamma)=\frac{(1-(1+x)^{-\gamma})^{\beta}}{(\alpha+(1-\alpha)(1-(1+x)^{-\gamma})^{\beta})},\ x>0,\alpha,\beta,\gamma>0 \qquad (2.1)$$

and

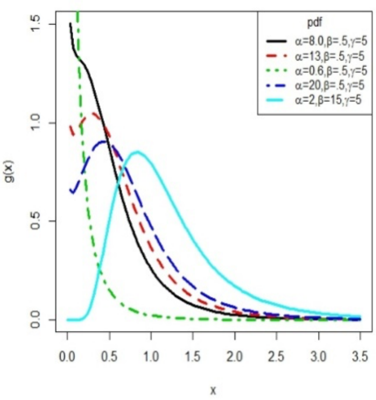
$$g(x,\alpha,\beta,\gamma)=\frac{\alpha\beta\gamma(1+x)^{-(\gamma+1)}(1-(1+x)^{-\gamma})^{\beta-1}}{[\alpha+(1-\alpha)(1-(1+x)^{-\gamma})^{\beta}]^{2}},\ x>0,\alpha,\beta,\gamma>0 \qquad (2.2)$$

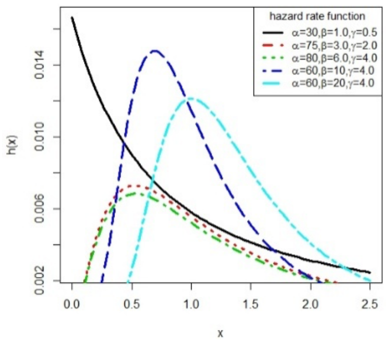
The hazard rate function of the MOEIKum distribution is given by the following equation

$$h(x,\alpha,\beta,\gamma)=\frac{\beta\gamma(1+x)^{-(\gamma+1)}(1-(1+x)^{-\gamma})^{\beta-1}}{[\alpha+(1-\alpha)(1-(1+x)^{-\gamma})^{\beta}][1-(1-(1+x)^{-\gamma})^{\beta}]},\ x>0,\alpha,\beta,\gamma>0$$

The different shapes of the pdf and hazard rate function of the MOEIKum distribution are displayed in Figure 1 for selected values of α, β and γ. From the figure we can see that hazard rate function accommodates increasing, decreasing, and unimodal shaped forms, that depend basically on the values of the shape parameters. This distribution can be expressed as a limiting case of some existing distributions and also from this distribution we can derive a number of sub-models for example, Lomax distribution, MOE Lomax distribution, log-logistic distribution etc. Consider the following theorem, which establish behavior of the MOEIKum distribution.

Figure 1: Graphs of pdf and hazard rate function of the MOEIKum distribution for different values of α, β and γ.





Theorem: 1*Let* $\{X_i,\ i \geq 1\}$ *be a sequence of i.i.d. random variables with common survival function* $\overline{F}(x)$. *Let* N *be a geometric random variable independently distributed of* $\{X_i,\ i \geq 1\}$ *such that* $P(N = n) = p(1-p)^{n-1}, n = 1,2,\ldots,0 < p < 1$. *Let* $U_N = min_{1 \leq i \leq n} X_i$. *Then* $\{U_N\}$ *is distributed as* $MOEIKum(p,\beta,\gamma)$ *iff* $\{X_i\}$ *follows* $IKum(\beta,\gamma)$.

Proof: The survival function of the random variable U_N is

$$\overline{W}(x) = P(U_N > x)$$
$$= \sum_{n=1}^{\infty} P(U_n > x)P(N = n)$$
$$= \sum_{n=1}^{\infty} [\overline{F}(x)]^n p(1-p)^{n-1}$$
$$= \frac{p\overline{F}(x)}{1-(1-p)\overline{F}(x)}$$
$$W(x) = \frac{(1-(1+x)^{-\gamma})^{\beta}}{(p+(1-p)(1-(1+x)^{-\gamma})^{\beta})}$$

which is cdf of a random variable with MOEIKum(p,β,γ) distribution.

Remark: 1 *Let* $\{X_i,\ i \geq 1\}$ *be a sequence of i.i.d. random variables with common survival function* $\overline{F}(x)$. *Let* N *be a geometric random variable independently distributed of* $\{X_i,\ i \geq 1\}$ *such that* $P(N = n) = p(1-p)^{n-1}, n = 1,2,\ldots,0 < p < 1$. *Let* $V_N = max_{1 \leq i \leq n} X_i$. *Then* $\{V_N\}$ *is distributed as* $MOEIKum(\frac{1}{p},\beta,\gamma)$ *iff* $\{X_i\}$ *follows* $IKum(\beta,\gamma)$ *distribution.*

III. Estimation

This section describes different estimation methods for estimating the parameters α, β, and γ of the MOEIKum distribution.

I. Method of Maximum Likelihood

Let $X_1, X_2, \ldots, X_n$ is a random sample of size n from MOEIKum(α,β,γ), then the log likelihood function is given by

$$logL(\alpha,\beta,\gamma) = nlog(\alpha\beta\gamma) - (\gamma+1)\sum_{i=1}^{n} log(x_i) + (\beta-1)\sum_{i=1}^{n} log(1-(1+x_i)^{-\gamma})$$
$$-2\sum_{i=1}^{n} log(1-\overline{\alpha}[1-[1-(1+x_i^{-\gamma})]^{\beta}])$$

The partial derivative of the log likelihood functions with respect to the parameters are

$$\frac{\partial \log L}{\partial \alpha} = \frac{n}{\alpha} - 2\sum_{i=1}^{n} \frac{1-[1-(1+x_i^{-\gamma})]^{\beta}}{1-\overline{\alpha}[1-[1-(1+x_i^{-\gamma})]^{\beta}]}$$

$$\frac{\partial \log L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} log(1-(x_i+1)^{-\gamma}) - 2\overline{\alpha}\sum_{i=1}^{n} \frac{[1-(1+x_i^{-\gamma})]^{\beta} log(1-[1+x_i^{-\gamma}])}{1-\overline{\alpha}[1-[1-(1+x_i^{-\gamma})]^{\beta}]}$$

$$\frac{\partial \log L}{\partial \gamma} = \frac{n}{\gamma} + \sum_{i=1}^{n} log(x_i) + (\beta-1)\sum_{i=1}^{n} \frac{[1-(1+x_i^{-\gamma})] log(1-(1+x_i^{-\gamma}))}{[1-(1+x_i^{-\gamma})]}$$
$$-2\sum_{i=1}^{n} \frac{[1-(1+x_i^{-\gamma})] log(1-(1+x_i^{-\gamma}))}{1-\overline{\alpha}[1-[1-(1+x_i^{-\gamma})]^{\beta}]}$$

The maximum likelihood estimates can be numerically obtained by solving the equations $\frac{\partial \log L}{\partial \alpha} = 0, \frac{\partial \log L}{\partial \beta} = 0, \frac{\partial \log L}{\partial \gamma} = 0$.

II. Methods of Ordinary and Weighted Least-Squares

A regression based method estimators of the unknown parameters suggested by Swain et al. (1988) to estimate the parameters of beta distributions. Let $X_1, X_2, \ldots, X_n$ is a random sample of size n from a distribution function with cdf G(x) and $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ denote the order statistics of the observed sample. It is well-known that $G(X_{(i)})$ behaves like the i^{th} order statistics of a sample of size n from U(0,1), therefore we have

$$E\big(G(X_{(j)})\big) = \frac{j}{n+1},\ Var\big(G(X_{(j)})\big) = \frac{j(n-j+1)}{(n+1)^2(n+2)},\ for i < j \tag{3.1}$$

Therefore, from equation (3.1) the least squares estimators of the unknown parameters α, β, and γ of MOEIKum(α, β, γ), can be obtained by minimizing

$$\sum_{i=1}^{n}\left(G(X_{(i)}) - \frac{j}{n+1}\right)^2 = \sum_{i=1}^{n}\left(\frac{(1-(1+X_{(i)})^{-\gamma})^{\beta}}{(\alpha+(1-\alpha)(1-(1+X_{(i)})^{-\gamma})^{\beta})} - \frac{j}{n+1}\right)^2$$

with respect to α, β, and γ.

By equation (3.1), the weighted least squares estimators of the unknown parameters of
MOEIKum(α, β, γ) can be obtained by minimizing

$$\sum_{i=1}^{n} \frac{1}{Var\big(G(X_{(i)})\big)}\left(G(X_{(i)}) - \frac{j}{n+1}\right)^2 = \sum_{i=1}^{n} \frac{(n+1)^2(n+2)}{j(n-j+1)}$$
$$\left(\frac{(1-(1+X_{(i)})^{-\gamma})^{\beta}}{(\alpha+(1-\alpha)(1-(1+X_{(i)})^{-\gamma})^{\beta})} - \frac{j}{n+1}\right)^2$$

with respect to α, β, and γ.

$$x = \left\{1 + \left[F(x;\alpha,\beta,\gamma)\alpha - \frac{\alpha}{1+\alpha}\right]^{\frac{1}{\beta}}\right\}^{-\frac{1}{\gamma}} - 1.$$

Let $X_{(i)}$ denoted as the i^{th} order statistic $X_{(1)} < X_{(2)} <\ldots< X_{(n)}$. If $p_i = \frac{i}{n+1}$ denotes some estimate of $F(x;\alpha,\beta,\gamma)$, then the estimates of α, β, and γ can be obtained by minimizing

$$\sum_{i=1}^{n}\left[X_{(i)} - \left\{1 + \left[F(x;\alpha,\beta,\gamma)\alpha - \frac{\alpha}{1+\alpha}\right]^{\frac{1}{\beta}}\right\}^{-\frac{1}{\gamma}} + 1\right]^2$$

with respect to α, β, and γ. These estimates of α, β, and γ were obtained by using R in method of maximum likelihood, methods of ordinary and weighted least-squares and Method of Percentiles.

III. Acceptance Sampling Plans

Reliability sampling plans are used for determining the acceptability of any product. In this section, we develop reliability test plan with the life time governed by an IMOEKu distribution with cdf

$$G(x,\alpha,\beta,\gamma) = \frac{(1-(1+x)^{-\gamma})^{\beta}}{(\alpha+(1-\alpha)(1-(1+x)^{-\gamma})^{\beta})}, x > 0, \alpha,\beta,\gamma > 0 \quad (4.1)$$

If a scale parameter $\theta > 0$ is introduced, the distribution function of IMOEKu is given by

$$G(x,\alpha,\beta,\gamma,\theta) = \frac{(1-(1+x/\theta)^{-\gamma})^{\beta}}{(\alpha+(1-\alpha)(1-(1+x/\theta)^{-\gamma})^{\beta})}, x > 0, \alpha,\beta,\gamma > 0 \quad (4.2)$$

A common practice in life testing is to terminate the life test by a pre-determined time *t* and note the number of failures (assuming that a failure is well-defined). One of the objectives of these experiments is to set a lower confidence limit on the average life. It is then desired to establish a specified

average life with a given probability of at least p^*. The decision to accept the specified average life occurs if and only if the number of observed failures at the end of the fixed time *t* does not exceed a given number c called the acceptance number. The test may get terminated before the time *t* is reached when the number of failures exceeds c in which case the decision is to reject the lot. For such a truncated life test and the associated decision rule, we are interested in obtaining the smallest sample sizes necessary to achieve the objective. Here, it is assume that α,β and γ are known while θ is unknown. So, average life time depends only on θ. A sampling plan consists of

- the number of units n on test,
- the acceptance number c,
- the maximum test duration t, and
- the ratio $\frac{t}{\theta_0}$ where θ_0 is the specified average life.

The consumer's risk, i.e., the probability of accepting a bad lot (the one for which the true average life is below the specified life θ_0) not to exceed $1-p^*$, so that p^* is a minimum confidence level with which a lot of true average life below θ_0 is rejected, by the sampling plan. For a fixed p^* our sampling plan is characterized by (n, c, $\frac{t}{\theta_0}$). Here, it is consider sufficiently large lots so that the binomial distribution can be applied. The problem is to determine for given values of p^*, $(0 < p^* < 1)$, θ_0 and c the smallest positive integer n such that

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \leq 1 - p^* \tag{4.3}$$

holds where $p = G(x, \alpha, \beta, \gamma, \theta_0)$ is given by (5.2) indicates the failure probabilities before time t which depends only on the ratio t/θ_0 it is sufficient to specify this ratio for designing the experiment. If the number of observed failures before t is less than or equal to c, from (5.3), we have:

$$G(t, \theta) \leq G(t, \theta_0) \Leftrightarrow \theta \geq \theta_0 \tag{4.4}$$

The minimum values of n satisfying the inequality (5.4) are obtained and displayed in Table1 for p^*=0.75, 0.90, 0.95 and t =1.0, 1.25, 1.5, 1.75, 2.0, 2.25, 2.5, 3.0, 3.5, 4.0 and $\alpha = \beta = \gamma = 2$. If $p = G(x, \alpha, \beta, \gamma, \theta_0)$ is small and n is large (as is true in some cases of our present work), the binomial probability may be approximated by Poisson probability with parameter $\lambda = np$ so that the left side of (4.3) can be written as

$$\sum_{i=0}^{c} \frac{e^{-\lambda} \lambda^x}{x} \leq 1 - p^* \tag{4.5}$$

where $p = G(x, \alpha, \beta, \gamma, \theta_0)$. The minimum values of n satisfying (4.5) are obtained for the same combination of p values as those used for (4.3). The results are given in Table 2. The operating characteristic function of the sampling plan (n, c, t/θ_0) gives the probability L(p) of accepting the lot with:

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{4.6}$$

where $p = G(x, \alpha, \beta, \gamma, \theta)$ is considered as a function of θ, i.e., the lot quality parameter. It can be seen that the operating characteristic is an increasing function of θ. For given p^*, t/θ_0 the choice of c and n is made on the basis of operating characteristics. Values of the operating characteristics as a function of θ/θ_0 for a few sampling plans are given in Table 3.

The producer's risk is the probability of rejecting lot when $\theta > \theta_0$. We can compute the producer's risk by first finding $p = F(t, \theta)$ and then using the binomial distribution function. For a given value of the producer's risk say 0.05, one may be interested in knowing what value of θ/θ_0 will ensure a producer's risk less than or equal to 0.05 if a sampling plan under discussion is adopted. It should be noted that the probability p may be obtained as function ofθ/θ_0, as

$$p = F\left(\frac{t}{\theta_0} \frac{\theta_0}{\theta}\right) \tag{4.7}$$

The value θ/θ_0 is the smallest positive number for which the following inequality hold:

$$\sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \geq .95 \tag{4.8}$$

For a given sampling plan (n, c, t/θ_0) and specified confidence level p^*. the minimum values of θ/θ_0 satisfying the inequality (4.8) are given in Table 4.

Example: Consider the following ordered failure times of the release of a software given in terms of hours from the starting of the execution of the software denoting the times at which the failure of the software is experienced (Wood, 1996). This data can be regarded as an ordered sample of size 10 with observations
(xi , i = 1, . . . , 10) = 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, 5218, 5823

Let the specified average life be 1000 hrs and the testing time be 1250 hrs, this leads to ratio of $t/\theta = 1.25$ with corresponding n and c as 10, 2 from Table 4.1 for $p^* = 0.9$. Therefore, the sampling plan for the above sample data is (n=10, c=2, $t/\theta_0 = 1.25$). Based on the observations, we have to decide whether to accept the product or reject it. We accept the product only, if the number of failures after 1250 hrs is less than or equal to 2. However the confidence level is assured by the sampling plan only if the given life times follow an MOEIKum distribution. In order to confirm that the given sample is generated by lifetimes following at least approximately the inverse Raleigh distribution, we have compared the sample quantiles and the corresponding population quantiles and found a satisfactory agreement. Thus, the adoption of the decision rule of the sampling plan seems to be justified. We see that in the sample of 10 failures there are 2 failures at 519 and 968 hrs before 1250 hrs. Therefore we accept the product.

Table 1: Minimum sample sizes necessary to assert the average life to exceed a given value t/θ_0 with probability p^* and the corresponding acceptance number c, $\alpha = \beta = \gamma = 2$ using Binomial probabilities.

					t/θ_0						
p^*	c	1	1.25	1.5	1.75	2	2.25	2.5	3	3.5	4
	0	3	3	2	2	2	2	2	1	1	1
	1	6	5	4	4	4	3	3	3	3	3
	2	9	8	7	6	5	5	5	4	4	4
	3	12	10	9	8	7	7	6	6	5	5
	4	15	12	11	10	9	8	8	7	7	6
.75	5	18	15	13	11	11	10	9	8	8	8
	6	21	17	15	13	13	11	11	10	9	9
	7	24	19	17	15	15	13	12	11	11	10
	8	27	22	19	17	16	14	14	12	12	11
	9	29	24	21	19	17	16	15	14	13	12
	10	32	26	23	20	19	17	16	16	14	14
	0	5	4	3	3	3	2	2	2	2	1
	1	9	7	6	5	5	4	4	4	3	2

	2	12	10	8	7	7	6	5	5	5	3
	3	16	13	11	9	9	7	7	7	6	5
	4	19	15	13	11	10	8	9	9	8	6
.90	5	22	18	15	13	12	11	11	10	9	7
	6	25	20	17	15	14	13	12	11	10	8
	7	28	23	19	17	16	15	14	12	12	9
	8	31	25	21	19	17	16	15	14	13	10
	9	34	27	24	21	19	18	17	15	14	11
	10	37	30	26	23	21	19	18	17	16	12
	0	7	5	4	4	3	3	3	2	2	1
	1	11	8	7	6	4	5	5	4	4	3
	2	15	11	10	8	8	7	6	6	5	4
	3	18	13	12	11	10	9	8	7	7	5
	4	21	17	14	13	11	11	10	9	8	6
.95	5	25	20	17	15	13	12	11	10	10	7
	6	28	22	19	17	15	14	13	12	11	8
	7	31	25	21	19	17	16	15	13	12	9
	8	34	27	24	21	19	17	16	15	14	10
	9	37	30	26	23	21	19	18	16	15	11
	10	40	32	28	24	22	21	19	18	16	12

Table 2: Minimum sample sizes necessary to assert the average life to exceed a given value t/θ_0 with probability p^* and the corresponding acceptance number c, α=β=γ=2 using Poisson probabilities. probabilities.

					t/θ_0					
p*	C	1	1.25	1.5	1.75	2	2.25	2.5	3	3.5	4
	0	4	3	3	4	4	3	3	2	2	2
	1	7	6	5	7	6	5	5	4	4	4
	2	11	9	8	10	8	7	7	6	6	5
	3	14	11	10	12	11	9	8	8	7	7
	4	17	14	12	14	13	11	10	10	9	8
.75	5	19	16	14	17	15	13	12	11	10	10
	6	22	19	16	19	17	15	14	13	11	11
	7	25	21	18	21	19	17	15	14	13	12
	8	28	23	21	24	21	18	17	16	14	14
	9	31	26	22	26	23	20	19	18	16	15
	10	34	28	24	28	24	22	20	19	18	16
	0	6	5	5	4	4	4	4	3	3	3
	1	10	9	8	7	6	6	6	5	5	5
	2	14	12	10	9	9	8	8	7	7	7
	3	18	15	13	12	11	10	10	9	9	8
	4	21	17	15	14	13	12	12	11	10	10
.90	5	24	20	18	16	15	14	14	12	12	11
	6	28	23	20	18	17	16	16	14	13	13
	7	31	25	22	20	19	17	17	16	15	14
	8	34	28	24	22	21	19	19	17	16	16
	9	37	30	27	24	22	21	21	19	18	17
	10	40	33	29	26	24	23	22	20	19	19

	0	8	7	6	5	5	5	5	4	4	4
	1	13	10	9	8	8	7	6	7	6	5
	2	17	14	12	11	10	10	8	9	8	7
	3	20	17	15	13	12	12	10	10	10	8
	4	24	20	17	16	15	13	11	12	12	10
.95	5	27	23	20	18	17	16	13	14	13	11
	6	31	26	22	20	19	18	14	16	15	13
	7	34	29	25	23	21	19	16	17	17	14
	8	38	31	27	24	23	21	17	19	18	15
	9	41	34	29	27	25	23	19	21	20	17
	10	44	34	32	29	26	25	20	22	21	18

Table 3: Operating characteristic values of the sampling plan (n, c, t/θ_0 for given p^* and $\alpha = \beta = \gamma = 2$ under MOEIKum probabilities.

						$\boldsymbol{\theta/\theta_0}$			
$\boldsymbol{p^*}$	N	c	t/θ_0	2	4	6	8	10	12
	9	2	1	0.7833	0.97365	0.9965	0.9991	0.9997	0.9998
	8	2	1.25	0.7053	0.9653	0.9934	0.9982	0.9994	0.9997
	7	2	1.5	0.6622	0.9538	0.9905	0.9973	0.999	0.9996
	6	2	1.75	0.6589	0.9496	0.9891	0.9968	0.9988	0.9995
.75	5	2	2	0.6974	0.9546	0.9899	0.997	0.9989	0.9995
	5	2	2.25	0.6207	0.9332	0.984	0.995	0.99813	0.9991
	5	2	2.5	0.547	0.907	0.9765	0.9992	0.997	0.9988
	4	2	3	0.6169	0.9212	0.979	0.9929	0.9971	0.9987
	4	2	3.5	0.518	0.879	0.9642	0.9871	0.9946	0.9975
	4	2	4	0.4315	0.8306	0.9448	0.979	0.9909	0.9956
	12	2	1	0.6208	0.9543	0.9916	0.9978	0.9992	0.9997
	10	2	1.25	0.5583	0.9357	0.987	0.9964	0.9987	0.9995
	8	2	1.5	0.5688	0.9329	0.9857	0.9959	0.9985	0.9994
	7	2	1.75	0.5458	0.9217	0.9822	0.9947	0.998	0.9992
.9	7	2	2	0.4389	0.8807	0.9701	0.9905	0.9964	0.9984
	6	2	2.25	0.4723	0.887	0.9709	0.9906	0.9964	0.9984
	5	2	2.5	0.547	0.9067	0.9762	0.9922	0.997	0.9986
	5	2	3	0.4158	0.8456	0.9546	0.981	0.9934	0.997
	5	2	3.5	0.3107	0.7739	0.9251	0.9715	0.9877	0.9941
	3	2	4	0.7214	0.994	0.9993	0.9939	0.9974	0.9988
	15	2	1	0.4664	0.91911	0.9841	0.9957	0.9985	0.9994
	11	2	1.25	0.4892	0.917	0.9828	0.9952	0.9983	0.9993
	10	2	1.5	0.4088	0.8809	0.9724	0.9918	0.997	0.9987
	8	2	1.75	0.4419	0.8886	0.9733	0.9918	0.997	0.9987
.95	8	2	2	0.3338	0.8341	0.9558	0.9857	0.9945	0.9976
	7	2	2.25	0.3465	0.8324	0.9587	0.9846	0.994	0.9973
	6	2	2.5	0.3919	0.8473	0.9574	0.9856	0.9943	0.99747
	6	2	3	0.2633	0.7533	0.9214	0.9709	0.9878	0.9943
	5	2	3.5	0.3107	0.7739	0.9251	0.9715	0.9877	0.9941
	4	2	4	0.4315	0.8303	0.9448	0.979	0.9909	0.9995

Table 4: Minimum ratio of true θ and required θ_0 for the acceptability of a lot with producer's risk of 0.05 for $\alpha = \beta = \gamma = 2$ under MOEIKum probabilities.

p^*	c	1	1.25	1.5	1.75	2	2.25	2.5	3	3.5	4
	0	9.83	12.29	11.45	13.36	15.26	17.18	19.08	15.45	18.025	20.025
	1	4.56	5.19	5.33	6.22	7.11	6.36	7.07	8.48	9.89	11.31
	2	3.56	4.18	4.71	4.52	4.6	5.17	5.74	5.88	6.86	7.84
	3	3.14	3.22	3.23	4.33	4.03	4.54	4.52	5.43	5.32	6.08
	4	2.69	2.87	3.32	3.53	3.72	3.88	4.31	4.64	5.42	5.46
.75	5	2.58	2.77	3.02	3.16	3.62	3.79	3.87	4.16	4.86	5.55
	6	2.47	2.6	2.86	3.02	3.03	3.41	3.79	4.23	4.42	5.05
	7	2.47	2.52	2.78	2.95	3.37	3.36	3.53	3.91	4.56	4.84
	8	2.29	2.45	2.65	2.82	3.09	3.29	3.53	3.79	4.42	4.59
	9	2.14	2.38	2.59	2.77	2.92	3.12	3.31	3.79	4.12	4.43
	10	2.14	2.32	2.52	2.66	2.93	3.02	3.2	3.79	3.97	4.54
	0	12.76	15.13	15.89	18.54	21.19	17.17	19.08	22.9	26.72	20.6
	1	5.81	6.36	6.84	7.27	8.31	8.01	8.89	10.68	9.9	8.07
	2	4.15	4.78	4.71	5.2	5.95	5.81	5.74	6.89	8.04	5.86
	3	3.56	3.93	4.24	4.33	4.95	4.54	5.04	6.05	6.33	6.08
	4	3.33	3.37	3.72	3.88	4.92	3.97	4.9	5.88	6.18	5.37
.9	5	2.97	3.22	3.32	3.53	3.81	3.97	4.41	4.94	5.31	4.91
	6	2.82	2.97	3.12	3.43	3.62	3.88	4.03	4.55	4.86	4.59
	7	2.58	2.77	3.02	3.16	3.53	3.79	4.03	4.16	4.86	4.32
	8	2.47	2.68	2.86	3.09	3.22	3.48	3.72	4.16	5.42	4.18
	9	2.38	2.6	2.86	3.02	3.16	3.48	3.65	3.96	4.42	4.05
	10	2.38	2.52	2.78	2.95	3.16	3.29	3.47	4.03	4.42	3.39
	0	15.5	16.9	17.31	20.2	19.66	22.12	24.58	22.9	26.72	20.6
	1	6.87	6.77	7.54	7.98	7.06	8.95	9.95	10.6	12.37	13.34
	2	3.85	3.92	4.46	4.94	5.16	6.69	6.45	7.74	8.04	7.43
	3	3.86	3.93	4.46	4.94	5.16	5.57	5.54	6.05	7.06	6.07
	4	3.37	3.71	3.87	4.33	4.32	4.86	5.04	5.57	6.04	5.37
.95	5	3.21	3.53	3.71	3.88	4.03	4.29	4.52	4.94	5.77	4.91
	6	2.92	3.16	3.38	3.62	3.86	4.07	4.26	4.833	5.23	4.59
	7	2.78	3.03	3.2	3.44	3.63	3.91	4.13	4.45	4.89	4.33
	8	2.63	2.82	3.12	3.31	3.52	3.69	3.87	4.45	4.89	4.18
	9	2.53	2.78	3.01	3.22	3.43	3.59	3.8	4.17	4.61	4.04
	10	2.46	2.68	2.89	3.03	3.22	3.52	3.63	4.17	4.42	3.93

IV. Conclusion

In this paper, a comprehensive description of properties of MOEIKum distribution are provided with the hope that it will attract wider applications in the area of research. Different methods for estimating unknown parameters of MOEIKum distribution are derived. Additionally, acceptance sampling plan is developed based on the truncated life test when the life distribution of the test items follows an MOEIKum distribution

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