Truncated Shukla Distribution: Properties And Applications

Kamlesh Kumar Shukla[1] and Rama Shanker[2]

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[1] Department of Community Medicine, Noida International Institute of Medical Science, NIU, Gautam Buddh Nagar, India, kkshukla22@gmail.com

[2]Department of Statistics, Assam University, Silchar, India
Shankerrama2009@gmail.com

Abstract

In this paper, Truncated Shukla distribution has been proposed. Some statistical properties including moments, coefficient of variation, skewness and index of dispersion have been derived. Survival and Hazard functions are derived and its behaviors are presented graphically. Maximum likelihood method of estimation has been used to estimate the parameter of proposed model. Simulation study of proposed distribution has also been discussed. It has been applied on three data sets and compares its superiority over two parameter Power Lindley, Gamma, Weibull, Shukla distributions and one parameter Truncated Akash, Truncated Lindley, Lindley and Exponential distributions

Keywords: Truncated distribution, Shukla distribution, Maximum likelihood Estimate

I. Introduction

In the recent decades, life time modeling has been becoming popular in distribution theory, where many statisticians are involved in introducing new models. Some of the life time models are very popular and applied in biological, engineering and agricultural areas, such as Lindley distribution suggested by Lindley [1], weighted Lindley distribution introduced by Ghitany et al. [2], Akash distribution of Shanker [3], Ishita distribution proposed by Shanker and Shukla [4], Pranav distribution of Shukla [5] etc, are some among others and extension of these lifetime distributions has also been capturing the attention of researchers in different areas of statistics, medical statistics, biomedical engineering, etc. Since each distribution is based on certain assumption and when these assumptions are not satisfied in the stochastic nature of the data, there is a need for another distribution. In search of a new distribution to fit the nature of some data sets where previously introduced lifetime distribution did not gives good fit, recently Shukla and Shanker [6] have introduced a lifetime distribution named Shukla distribution which is convex combination of exponential (θ) and gamma (α,θ) distributions and is defined by its probability density function (pdf) and cumulative distribution function (cdf) as

$$f(x;\theta,\alpha)=\frac{\theta^{\alpha+1}}{\theta^{\alpha+1}+\Gamma(\alpha+1)}(\theta+x^{\alpha})e^{-\theta x};x>0,\theta>0,\alpha\geq 0 \tag{1.1}$$

$$F\left(x;\theta,\alpha\right)=1-\frac{\theta^{\alpha}\left(\theta+x^{\alpha}\right)e^{-\theta x}+\alpha\Gamma\left(\alpha,\theta x\right)}{\theta^{\alpha+1}+\Gamma\left(\alpha+1\right)};x>0,\theta>0,\alpha\geq 0 \tag{1.2}$$

Shukla and Shanker [6] have discussed in details about its mathematical and statistical properties, estimation of parameters and application to model lifetime.

Truncated type of distribution are more effective in application to modeling life time data because its limits used as bound either upper or lower or both according to given data. Truncated normal distribution is well explained in Johnson et al [7]. It has wide application in economics and statistics. Many researchers have proposed truncated type of distribution and applied it in different areas of statistics, especially in censor data such as truncated Weibull distribution by Zange and Xie [8] , truncated Lomax distribution by Aryuyuen and Bodhisuwan [9], truncated Pareto distribution by Janinetti and Ferraro[10], truncated Lindley distribution by Singh et al [11], are some among others. Truncated version of distribution can be defined as

Definition 1. Let X be a random variable that is distributed according to some pdf $g(x;\theta)$ and cdf $G(x;\theta)$, where θ is a parameter vector of X.

Let X lies within the interval $[a,b]$ where $-\infty<a\leq x\leq b<\infty$ then the conditional on $a\leq x\leq b$ is distributed as truncated distribution and thus the pdf of truncated distribution as reported by Singh et al [11]is defined by

$$f(x;\theta)=g(x/a\leq x\leq b;\theta)=\frac{g(x;\theta)}{G(b;\theta)-G(a;\theta)} \tag{1.3}$$

where $f(x;\theta)=g(x;\theta)$ for all $a\leq x\leq b$ and $f(x;\theta)=0$ elsewhere.

Notice that $f(x;\theta)$ in fact is a pdf of X on interval $[a,b]$. We have

$$\int_a^b f(x;\theta)dx=\frac{1}{G(b;\theta)-G(a;\theta)}\int_a^b g(x;\theta)dx$$

$$=\frac{1}{G(b;\theta)-G(a;\theta)}G(b;\theta)-G(a;\theta)=1 \tag{1.4}$$

The cdf of truncated distribution is given by

$$F(x;\theta)=\int_a^x f(x;\theta)dx=\frac{G(x;\theta)-G(a;\theta)}{G(b;\theta)-G(a;\theta)} \tag{1.5}$$

The main objectives of this paper are (i) to propose new truncated distribution using Shukla distribution, which is called as Truncated Shukla distribution (TSD) (ii) to know statistical performance and its suitability, it has been compared with classical distributions of two- parameter as well as one parameter using three lifetime datasets. It has been divided in eight sections. Introduction about the paper is described in the first section. In the second section, Truncated Shukla distribution has been derived. Mathematical and statistical properties including its moment have been discussed in third section. Behavior of hazards rate has been presented mathematically as well as graphically in fourth section. Moments and its related expression have been discussed in fifth section. Simulation study of the presented distribution has been discussed to check estimation parameters using Bias and Mean square error in sixth section. Estimation of parameter of proposed distribution has been discussed in seven section where its applications and comparative study of other classical two parameter life time distributions as well as one parameter distributions have been illustrated using various fields of life time data. In the last, conclusions have been drawn according to studied of behavior and properties of Truncated Shukla distribution (TSD).

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II. Truncated Shukla Distribution

In this section, pdf and cdf of new truncated distribution is proposed and named Truncated Shukla distribution , using (1.3) & (1.4) of definition1 and from (1.1) & (1.2) , which is defined as :

Definition 2: Let X be random variable which is distributed as Truncated Shukla distribution (TSD) with scale parameter a, b and θ , and shape parameter α, will be denoted by TSD (a,b,θ,α). The pdf and cdf of X are respectively:

$$f(x;\theta,\alpha)=\frac{\theta^{\alpha+1}(x^{\alpha}+\theta)e^{-\theta x}}{\theta^{\alpha}\left((\theta+a^{\alpha})e^{-\theta a}-(\theta+b^{\alpha})e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)} \tag{1.6}$$

$$F(x;\theta,\alpha)=\frac{\theta^{\alpha}\left((\theta+a^{\alpha})e^{-\theta a}-(\theta+x^{\alpha})e^{-\theta x}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta x)\right)}{\theta^{\alpha}\left((\theta+a^{\alpha})e^{-\theta a}-(\theta+b^{\alpha})e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)} \tag{1.7}$$

Where $-\infty < a \le x \le b < \infty$, and $\theta > 0$

Performance of pdf of TSD for varying values of parameter has been illustrated in the figure 1.From the pdf plots of TSD, it is quite obvious that TSD is suitable for datasets of various nature.

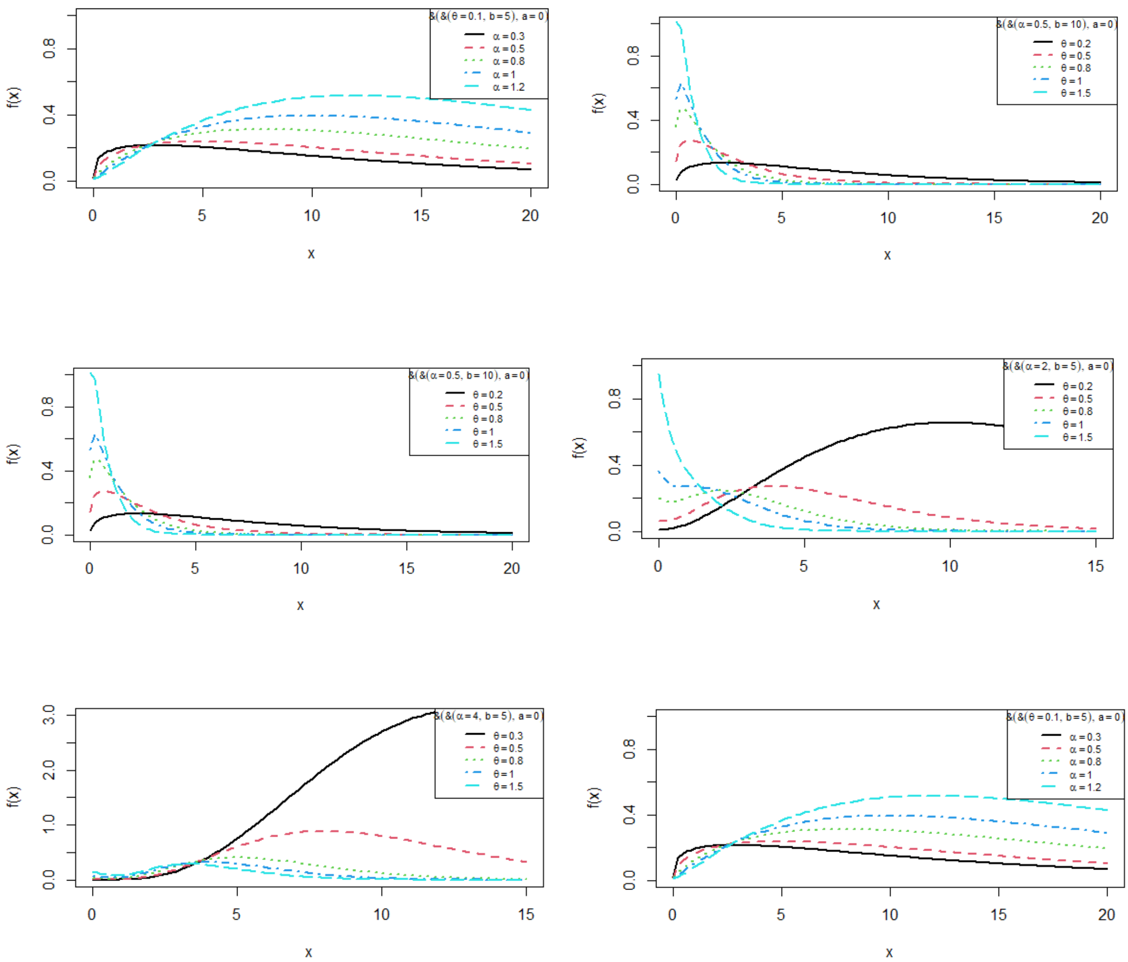

Figure 1: *pdf plots of TSD for varying values of parameters continued…*

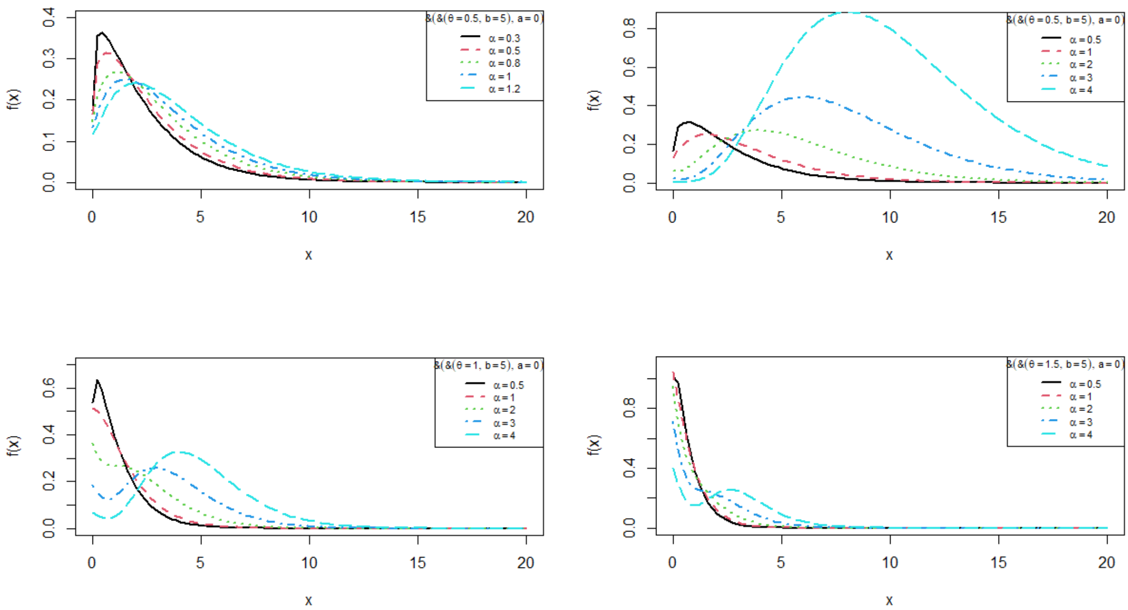

Figure 1: *pdf plots of TSD for varying values of parameters*

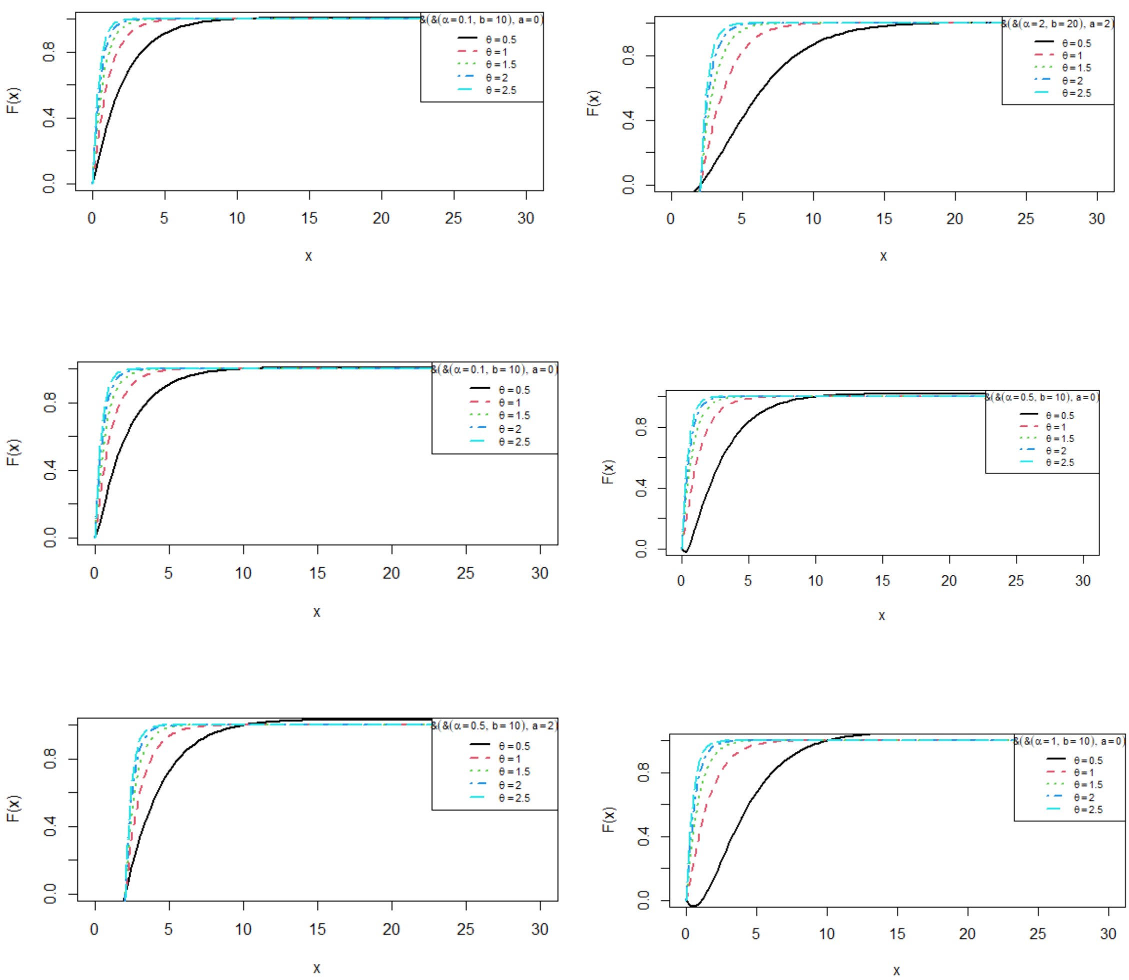

Figure 2: *cdf plots of TSD for varying values of parameters continued…*

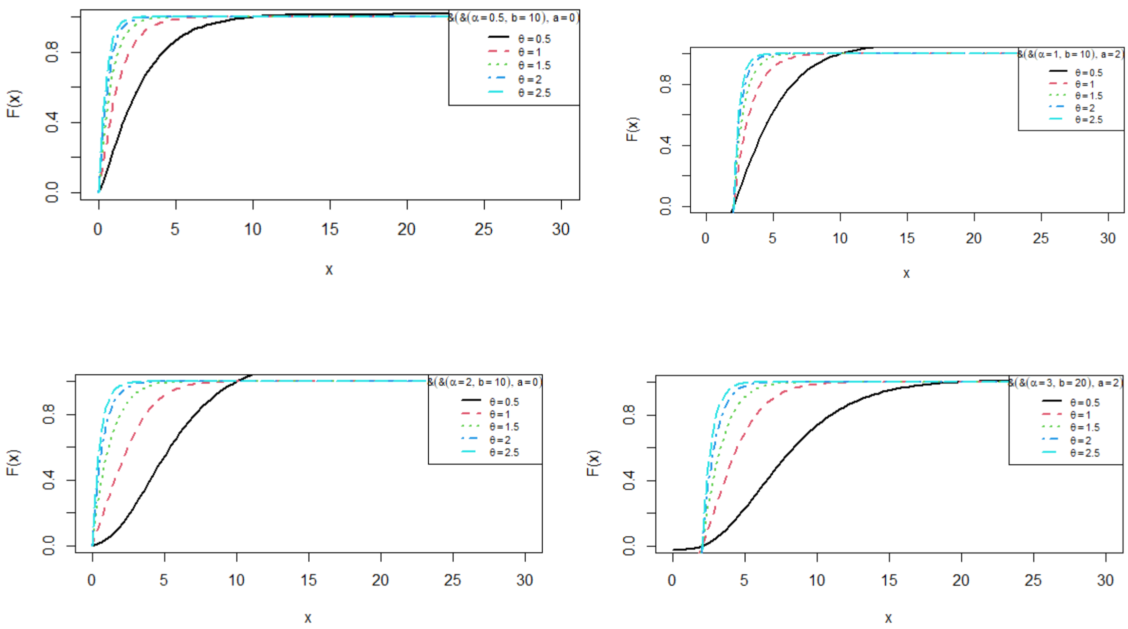

Figure 2: *cdf plots of TSD for varying values of parameters*

III. Survival and hazard function

The survival function $S(x)=S(x;\theta,\alpha)$ and hazard function $h(x)=h(x;\theta,\alpha)$ of TSD are respectively, obtained as

$$S(x;\theta,\alpha)=1-F(x;\theta,\alpha)=\frac{\theta^{\alpha}\left((\theta+x^{\alpha})e^{-\theta x}-(\theta+b^{\alpha})e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta x)-\Gamma(\alpha,\theta b)\right)}{\theta^{\alpha}\left((\theta+a^{\alpha})e^{-\theta a}-(\theta+b^{\alpha})e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)}$$

$$h(x)=\frac{f(x;\theta,\alpha)}{S(x;\theta,\alpha)}=\frac{\theta^{\alpha+1}(x^{\alpha}+\theta)e^{-\theta x}}{\theta^{\alpha}\left((\theta+x^{\alpha})e^{-\theta x}-(\theta+b^{\alpha})e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta x)-\Gamma(\alpha,\theta b)\right)}$$

It is quite obvious that the hazard rate function $h(x)$ is independent of parameter 'a'. Behavior of hazard function of TSD for varying values of parameter is presented in figure 3.

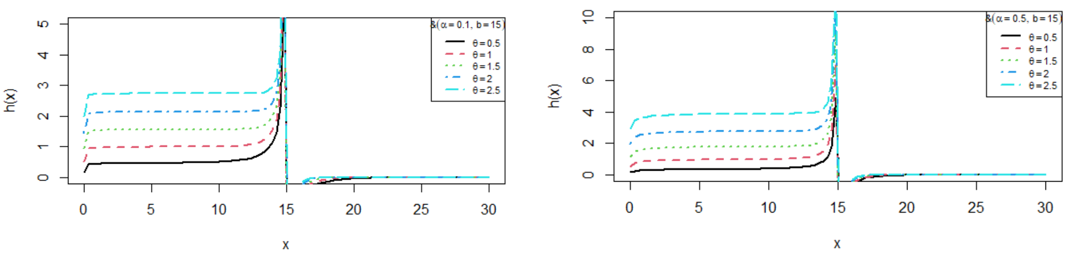

Figure 3: *h(x) plots of TSD for varying values of parameters continued…*

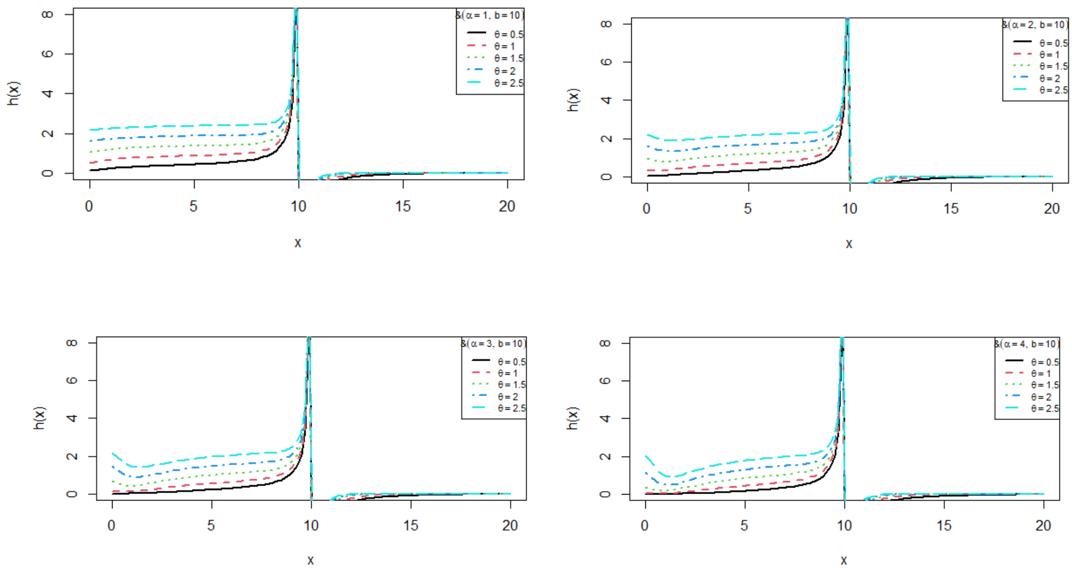

Figure 3: *h(x) plots of TSD for varying values of parameters*

IV. Moments and Mathematical Properties

The rth moment about origin of TSD is defined as

$$\mu_r' = \frac{\left[\theta^{\alpha+1}\Gamma(\alpha+1)+\Gamma\left(\alpha+r+1\right)\right]}{\theta^r\left\{\theta^\alpha\left((\theta+a^\alpha)e^{-\theta a}-(\theta+b^\alpha)e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)\right\}}\ ;r=1,2,3,...$$

The first four moments about origin of TSD can thus be given by

$$\mu_1' = \frac{\left[\theta^{\alpha+1}\Gamma(\alpha+1)+\Gamma\left(\alpha+2\right)\right]}{\theta\left\{\theta^\alpha\left((\theta+a^\alpha)e^{-\theta a}-(\theta+b^\alpha)e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)\right\}}$$

$$\mu_2' = \frac{\left[\theta^{\alpha+1}\Gamma(\alpha+1)+\Gamma\left(\alpha+3\right)\right]}{\theta^2\left\{\theta^\alpha\left((\theta+a^\alpha)e^{-\theta a}-(\theta+b^\alpha)e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)\right\}}$$

$$\mu_3' = \frac{\left[\theta^{\alpha+1}\Gamma(\alpha+1)+\Gamma\left(\alpha+4\right)\right]}{\theta^3\left\{\theta^\alpha\left((\theta+a^\alpha)e^{-\theta a}-(\theta+b^\alpha)e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)\right\}}$$

$$\mu_4' = \frac{\left[\theta^{\alpha+1}\Gamma(\alpha+1)+\Gamma\left(\alpha+5\right)\right]}{\theta^4\left\{\theta^\alpha\left((\theta+a^\alpha)e^{-\theta a}-(\theta+b^\alpha)e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)\right\}}.$$

The variance of TSD can be obtained as

$$\mu_2 = \left(\frac{\left[\theta^{\alpha+1}\Gamma(\alpha+1)+\Gamma\left(\alpha+3\right)\right]}{\theta^2\left\{\theta^\alpha\left((\theta+a^\alpha)e^{-\theta a}-(\theta+b^\alpha)e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)\right\}}\right)$$

$$-\left(\mu_1' = \frac{\left[\theta^{\alpha+1}\Gamma(\alpha+1)+\Gamma\left(\alpha+2\right)\right]}{\theta\left\{\theta^\alpha\left((\theta+a^\alpha)e^{-\theta a}-(\theta+b^\alpha)e^{-\theta b}\right)+\alpha\left(\Gamma(\alpha,\theta a)-\Gamma(\alpha,\theta b)\right)\right\}}\right)^2$$

Expressions of other central moments are not being given here because they have lengthy expressions. However, they can be easily obtained. Natures of the mean and the variance for varying values of parameters of TSD are presented in figures 4 and 5 respectively.

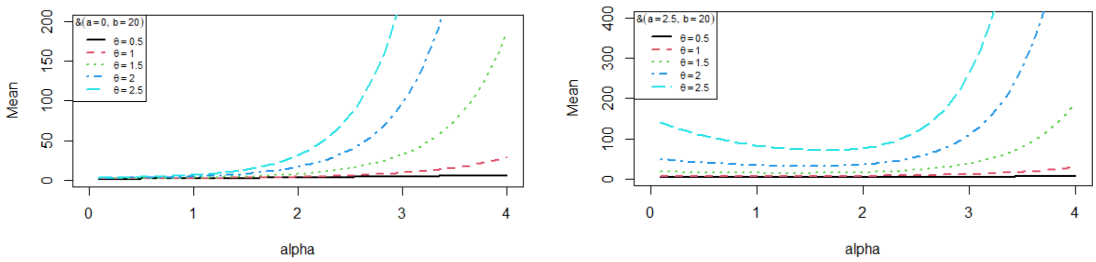

Figure 4: *Mean of TSD on varying value of parameters*

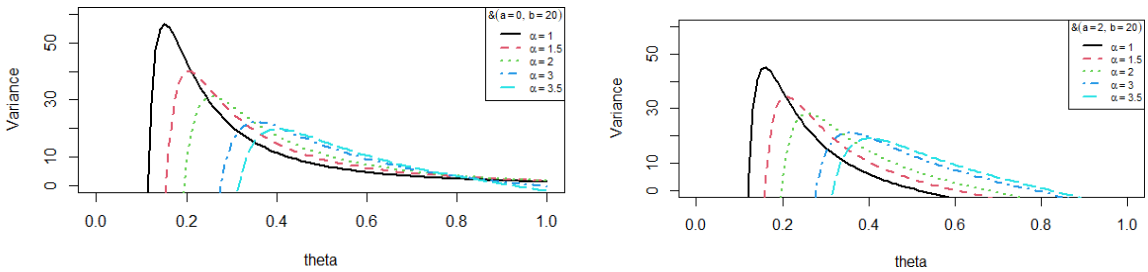

Figure 5: *Variance of TSD on varying value of parameters*

V. Maximum Likelihood Estimation

Let $(x_1, x_2, x_3, \dots, x_n)$ be a random sample of size n from (1.6). The likelihood function, L of TSD is given by

$$L = \left(\frac{\theta^{\alpha+1}}{\theta^{\alpha}\left((\theta+a^{\alpha})e^{-\theta a} - (\theta+b^{\alpha})e^{-\theta b}\right) + \alpha\left(\Gamma(\alpha,\theta a) - \Gamma(\alpha,\theta b)\right)} \right)^n \prod_{i=1}^{n}\left(\theta + x_i^{\alpha}\right) e^{-n\theta\bar{x}}$$

and its log likelihood function is thus obtained as

$$\ln L = n \ln\left(\frac{\theta^{\alpha+1}}{\theta^{\alpha}\left((\theta+a^{\alpha})e^{-\theta a} - (\theta+b^{\alpha})e^{-\theta b}\right) + \alpha\left(\Gamma(\alpha,\theta a) - \Gamma(\alpha,\theta b)\right)} \right) + \sum_{i=1}^{n} \ln\left(\theta + x_i^{\alpha}\right) - n\theta\bar{x}$$

Taking $\hat{a} = Min(x_1, x_2, x_3, \dots, x_n)$, $\hat{b} = Max(x_1, x_2, x_3, \dots, x_n)$, the maximum likelihood estimate $\hat{\theta}$ of parameter θ is the solution of the log-likelihood equation $\frac{\partial \ln L}{\partial \theta} = 0$. It is obvious that $\frac{\partial \ln L}{\partial \theta} = 0$ will not be in closed form and hence some numerical optimization technique can be used to solve the equation for θ. In this paper the nonlinear method available in R software has been used to find the MLE of the parameter θ.

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VI. Simulation Study

In this section, simulation of study of (1.6) has been carried out. Acceptance and Rejection method has been used to generate random number. Bias Error and Mean square Error have been calculated for varying values parameters θ and α whereas parameter a and b kept constant. It is obvious that as the sample size increases and values of parameters increases, the bias error and the mean square errors of both parameters decrease.

Table 1. *Simulation of TSD at a=0, b=20 and* $\theta = 0.1$ & $\alpha = 0.5$

Sample Size (n)	θ	α	Bias Error(θ)	MSE(θ)	Bias Error(α)	MSE(α)
40	0.1	0.5	0.02709	0.02935	0.071393	0.203882
	0.5	1.0	0.017091	0.011685	0.058893	0.138738
	1.0	2.0	0.004591	0.000843	0.033893	0.045951
	1.5	3.0	-0.007908	0.002501	0.008893	0.003163
60	0.1	0.5	0.005875	0.009437	0.027591	0.0456763
	0.5	1.0	0.017091	0.002071	0.019257	0.022251
	1.0	2.0	-0.002458	0.0003625	0.002591	0.0004028
	1.5	3.0	-0.010791	0.006987	-0.014075	0.0118871
80	0.1	0.5	0.007277	0.004236	0.015552	0.0193501
	0.5	1.0	0.0022772	0.000414	0.0093024	0.0069227
	1.0	2.0	-0.003972	0.0012626	-0.00319	0.0008179
	1.5	3.0	-0.010222	0.0083604	-0.015697	0.019713
100	0.1	0.5	0.0060863	0.0037043	0.013617	0.0185445
	0.5	1.0	0.0020863	0.0004352	0.0086178	0.0074267
	1.0	2.0	-0.002913	0.0008489	-0.001382	0.0001910
	1.5	3.0	-0.007913	0.0062625	-0.011382	0.0129553

Table 2. *Simulation of TSD at a=0, b=20 and* $\theta = 0.5$ & $\alpha = 1$

Sample Size (n)	θ	α	Bias Error(θ)	MSE(θ)	Bias Error(α)	MSE(α)
40	0.1	0.5	0.02900024	0.0336405	0.07806883	0.2437896
	0.5	1.0	0.01900024	0.0144403	0.06556883	0.1719708
	1.0	2.0	0.00650024	0.0016901	0.04056883	0.0658331
	1.5	3.0	-0.00599976	0.0014398	0.01556883	0.0096955
60	0.1	0.5	0.01367068	0.0112132	0.03260302	0.0637774
	0.5	1.0	0.00700401	0.0029433	0.02426968	0.0353410
	1.0	2.0	-0.0013293	0.0001060	0.00760302	0.0034683
	1.5	3.0	-0.0096626	0.0056020	-0.00906366	0.0049289
80	0.1	0.5	0.00681314	0.0037135	0.01293939	0.0133942
	0.5	1.0	0.00181314	0.0002629	0.00668939	0.0035798
	1.0	2.0	-0.00443685	0.0015748	-0.00581060	0.0027010
	1.5	3.0	-0.01068685	0.0091367	-0.01831060	0.0268222
100	0.1	0.5	0.00630281	0.0039725	0.01477387	0.0218267
	0.5	1.0	0.00230281	0.0005302	0.00977387	0.0095528
	1.0	2.0	-0.00269718	0.0007274	-0.00022612	0.0000511
	1.5	3.0	-0.00769718	0.0059246	-0.01022613	0.0104573

Table 3. *Simulation of TSD at a=0, b=20 and* $\theta = 1$ & $\alpha = 2$

Sample Size (n)	θ	α	Bias Error(θ)	MSE(θ)	Bias Error(α)	MSE(α)
40	0.1	0.5	0.02307633	0.0213006	0.04443256	0.0789700
	0.5	1.0	0.01307633	0.0068396	0.03193256	0.0407875
	1.0	2.0	0.000576330	0.0000132	0.00693256	0.0019224
	1.5	3.0	-0.0119236695	0.0056869	-0.01806743	0.0130572
60	0.1	0.5	0.01594010	0.0152452	0.03033473	0.0552117
	0.5	1.0	0.00927343	0.0051597	0.02200140	0.0290436
	1.0	2.0	0.00094010	0.0000530	0.00533473	0.0017075
	1.5	3.0	-0.00739322	0.0032795	-0.01133193	0.0077047
80	0.1	0.5	0.00993921	0.0079030	0.01602242	0.0205374
	0.5	1.0	0.00493921	0.0019516	0.00977242	0.0076400
	1.0	2.0	-0.00131078	0.0001374	-0.00272757	0.0005951
	1.5	3.0	-0.00756078	0.0045732	-0.01522757	0.0185503
100	0.1	0.5	0.00943759	0.0089068	0.01685769	0.0284181
	0.5	1.0	0.00543759	0.0029567	0.01185769	0.0140604
	1.0	2.0	0.00043759	0.0000191	0.00185769	0.0003451
	1.5	3.0	-0.00456240	0.0020815	-0.00814230	0.0066297

Table 4. *Simulation of TSD at a=0, b=20 and* $\theta = 1.5$ & $\alpha = 3$

Sample Size (n)	θ	α	Bias Error(θ)	MSE(θ)	Bias Error(α)	MSE(α)
40	0.1	0.5	0.04484480	0.08044228	0.09300719	0.34601349
	0.5	1.0	0.03484480	0.04856643	0.08050719	0.0407875
	1.0	2.0	0.022344809	0.01997162	0.05550719	0.25925630
	1.5	3.0	0.009844809	0.00387681	0.03050719	0.03722754
60	0.1	0.5	0.02788600	0.046657753	0.05440362	0.17758525
	0.5	1.0	0.02121933	0.027015617	0.04607029	0.12734829
	1.0	2.0	0.01288600	0.009962946	0.02940362	0.05187438
	1.5	3.0	0.00455267	0.001243609	0.01273696	0.00973380
80	0.1	0.5	0.01902251	0.028948472	0.03411478	0.09310547
	0.5	1.0	0.01402251	0.015730463	0.02786478	0.06211569
	1.0	2.0	0.00777251	0.004832953	0.01536478	0.01888612
	1.5	3.0	0.00152251	0.000185443	0.00286478	0.00065655
100	0.1	0.5	0.01418562	0.02012320	0.02460816	0.06055620
	0.5	1.0	0.01018562	0.002689072	0.01960816	0.03844803
	1.0	2.0	0.00518562	0.000093457	0.00960816	0.00923169
	1.5	3.0	0.00018562	0.0020815	-0.00039183	0.00015353

VII. Applications on life time data

In this section, TSD has been applied on three real lifetime datasets, where maximum likelihood method of estimation has been used for the estimation of its parameter. Booth parameters and θ and α are estimated using maximum likelihood estimation whereas another parameters a, and b are considered as lowest and highest values of data. i. e. *a=max(x)* and *b=max(x),* where x is data set. Goodness of fit has been decided using Akaike information criteria (AIC), Bayesian Information criteria (BIC) and Kolmogorov Simonov test (KS) values respectively, which are calculated for each distribution and also compared with p-value. As we know that best goodness of fit of the distribution can be decide on the basis minimum value of KS, AIC and BIC.

The datasets considered for testing goodness of fit of the TSD are as follows:

Data Set 1: The data is given by Birnbaum and Saunders [12] on the fatigue life of 6061 – T6 aluminum coupons cut parallel to the direction of rolling and oscillated at 18 cycles per second. The data set consists of 100 observations with maximum stress per cycle 31,000 psi. The data ($\times 10^{-3}$)are presented below (after subtracting 65**).**

5	25	31	32	34	35	38	39	39	40	42
43	43	43	44	44	47	47	48	49	49	49
51	54	55	55	55	56	56	56	58	59	59
59	59	59	63	63	64	64	65	65	65	66
66	66	66	66	67	67	67	68	69	69	69
69	71	71	72	73	73	73	74	74	76	76
77	77	77	77	77	77	79	79	80	81	83
83	84	86	86	87	90	91	92	92	92	92
93	94	97	98	98	99	101	103	105	109	136
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Data Set 2: This data set is the strength data of glass of the aircraft window reported by Fuller *et al* [13]:

18.83	20.8	21.657	23.03	23.23	24.05	24.321	25.5	25.52	25.8	26.69	26.77
26.78	27.05	27.67	29.9	31.11	33.2	33.73	33.76	33.89	34.76	35.75	35.91
36.98	37.08	37.09	39.58	44.045	45.29	45.381					

Data Set 3: The following data represent the tensile strength, measured in GPa, of 69 carbon fibers tested under tension at gauge lengths of 20mm (Bader and Priest[14]):

1.312	1.314	1.479	1.552	1.700	1.803	1.861	1.865	1.944	1.958	1.966
1.997	2.006	2.021	2.027	2.055	2.063	2.098	2.140	2.179	2.224	2.240
2.253	2.270	2.272	2.274	2.301	2.301	2.359	2.382	2.382	2.426	2.434
2.435	2.478	2.490	2.511	2.514	2.535	2.554	2.566	2.570	2.586	2.629
2.633	2.642	2.648	2.684	2.697	2.726	2.770	2.773	2.800	2.809	2.818
2.821	2.848	2.880	2.954	3.012	3.067	3.084	3.090	3.096	3.128	3.233
3.433	3.585	3.858								

The values of MLE's, Standard Errors, - 2ln L, AIC, K-S and p-values for the fitted distributions for the three datasets are presented in tables 5, 6 and 7, respectively. It crystal clear that the TSD gives much closer fit than the considered distributions.

Table 5: *MLE's, Standard Errors, - 2ln L, AIC, K-S and p-values of the fitted distributions for data set-1*

Distributions	**ML Estimates**	$-2\ln L$	**AIC**	**BIC**	**K-S**	**p-value**
TSD	$\hat{\theta}=0.10677$ $\hat{\alpha}=6.36414$	**914.72**	**918.72**	**923.93**	**0.095**	0.320
Power Lindley	$\hat{\theta}=0.00266$ $\hat{\alpha}=1.55583$	925.41	929.41	934.62	0.155	**0.015**
Gamma	$\hat{\theta}=0.11400$ $\hat{\alpha}=7.78985$	915.77	919.77	924.98	0.097	**0.281**
Weibull	$\hat{\theta}=0.00272$ $\hat{\theta}=1.39558$	989.35	993.35	998.56	0.294	**0.000**
Shukla	$\hat{\theta}=0.11253$ $\hat{\alpha}=6.6892$	915.76	919.76	924.97	0.099	**0.271**
TAD	$\hat{\theta}=0.03917$	939.13	941.13	942.05	0.153	**0.017**
TLD	$\hat{\theta}=0.02199$	958.88	960.88	962.31	0.186	**0.001**
Lindley	$\hat{\theta}=0.02886$	983.10	985.10	986.54	0.252	**0.000**
Exponential	$\hat{\theta}=0.01463$	**1044.87**	**1046.87**	**1048.30**	**0.336**	**0.000**

Table 6: *MLE's, Standard Errors, - 2ln L, AIC, K-S and p-values of the fitted distributions for data set-2*

Distributions	ML Estimates	$-2\ln L$	AIC	BIC	K-S	p-value
TSD	$\hat{\theta}=0.21682$ $\hat{\alpha}=5.93067$	**201.61**	**205.61**	**208.48**	**0.106**	0.834
Power Lindley	$\hat{\theta}=0.00243$ $\hat{\alpha}=1.9439$	220.14	224.14	226.13	0.198	**0.152**
Gamma	$\hat{\theta}=0.61482$ $\hat{\alpha}=18.9433$	208.22	212.22	216.05	0.134	**0.578**
Weibull	$\hat{\theta}=0.00203$ $\hat{\theta}=1.80566$	241.61	245.61	247.61	0.353	**0.000**
Shukla	$\hat{\theta}=0.6144$ $\hat{\theta}=17.9299$	208.23	212.23	216.05	0.134	**0.580**
TAD	$\hat{\theta}=0.08776$	201.96	203.96	205.58	0.112	**0.786**
TLD	$\hat{\theta}=0.05392$	202.18	204.18	205.61	0.117	**0.738**
Lindley	$\hat{\theta}=0.06299$	253.98	255.98	256.98	0.365	**0.000**
Exponential	$\hat{\theta}=0.032452$	**274.52**	**276.52**	**277.52**	**0.458**	**0.000**

Table 7: *MLE's, Standard Errors, - 2ln L, AIC, K-S and p-values of the fitted distributions for data set-3*

Distributions	**ML Estimates**	$-2\ln L$	**AIC**	**BIC**	**K-S**	**p-value**
TSD	$\hat{\theta}=8.0439$ $\hat{\alpha}=18.8613$	**99.71**	**103.71**	**106.58**	**0.061**	0.985
Power Lindley	$\hat{\theta}=0.05447$ $\hat{\alpha}=3.75881$	101.52	105.52	110.73	0.055	**0.988**
Gamma	$\hat{\theta}=9.2873$ $\hat{\alpha}=22.8030$	101.97	105.97	111.18	0.057	**0.977**
Weibull	$\hat{\theta}=0.00643$ $\hat{\theta}=5.16972$	103.47	107.47	112.68	0.065	**0.928**
Shukla	$\hat{\theta}=5.9922$ $\hat{\theta}=17.1611$	184.35	188.35	193.56	0.290	**0.000**
TAD	$\hat{\theta}=0.70314$	110.76	112.76	114.68	0.152	**0.079**
TLD	$\hat{\theta}=0.28986$	112.19	114.19	115.63	0.157	**0.065**
Lindley	$\hat{\theta}=0.65450$	238.38	240.38	241.37	0.401	**0.000**
Exponential	$\hat{\theta}=0.40794$	**261.73**	**263.73**	**264.73**	**0.448**	**0.000**

Fitted pots of considered distributions for the datasets 1, 2 and 3 are given in the figures 6, 7 and 8 respectively which also supports the claim that TSD is the best distribution among the considered datasets.

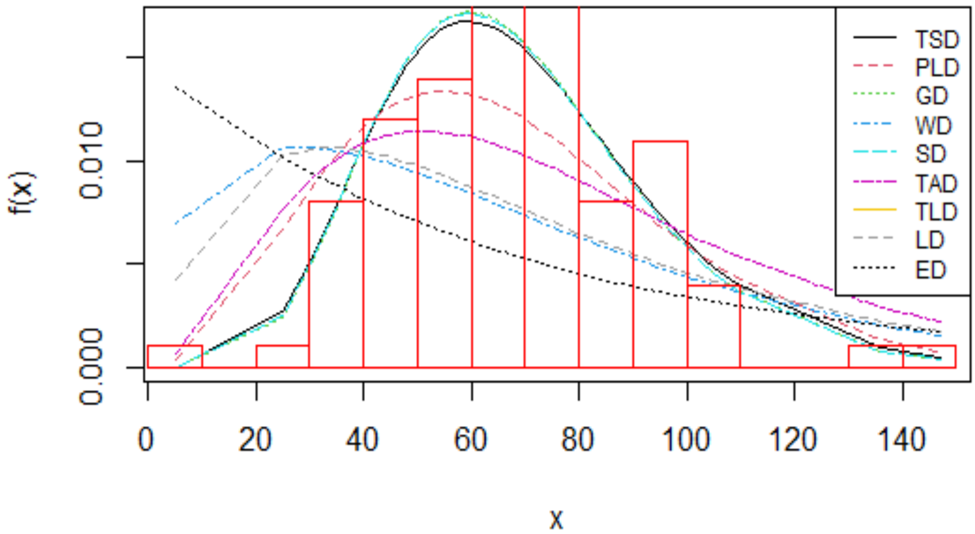

Figure.6. *Fitted plots of distributions for the dataset-1*

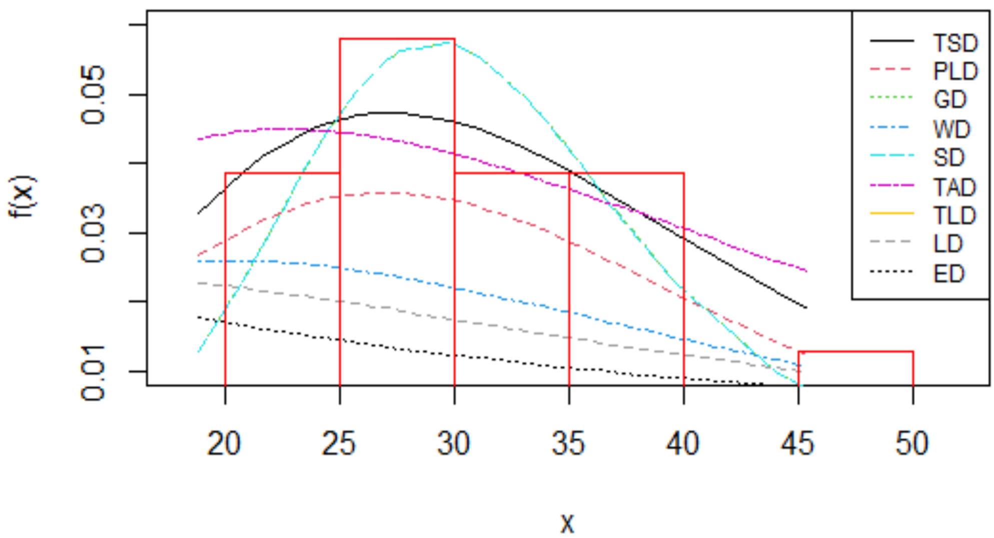

Figure.7. *Fitted plots of distributions for the dataset-2*

VIII. Conclusions

In this paper, Truncated Shukla distribution (TSD) has been proposed. Its mathematical and statistical properties have been discussed. Maximum likelihood method has been used for the estimation of its parameters. Goodness of fit of TSD has been discussed with three life time data sets and compared with Gamma, Weibull, Power Lindley distribution (PLD), truncated Lindley, Truncated distribution (TLD), truncated Akash distribution (TAD), Lindley and exponential distribution. It has been observed that TSD gives good fit over the considered distributions on all the data sets.

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