Parameter estimation for progressive censored data under accelerated life test with *k* levels of constant stress

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Abstract

Accelerated life testing (ALT) is a time-saving technique that has been used in a variety of sectors to get failure time data for test units in a relatively short time it takes to test them under regular operating circumstances. One of the primary goals of ALT is to estimate failure time functions and reliability under typical use. In this article, an ALT with k increasing stress levels that is stopped by a type II progressive censoring (TIIPC) scheme is considered. At each stress level, it is assumed that the failure times of test units follow a generalized Pareto (GnP) distribution. The link between the life characteristic and stress level is considered to be log-linear. The maximum likelihood estimation (MLE) method is used to obtain inferences about unknown parameters of the model. Furthermore, the asymptotic confidence intervals (ACIs) are obtained by utilizing the inverse of the fisher information matrix. Finally, a simulation exercise is presented to show how well the developed inferential approaches performed. The performance of MLEs is assessed in terms of relative mean square error (RMSE) and relative absolute bias (RAB), whereas the performance of ACIs is assessed in terms of their length and coverage probability (CP).

Keywords: Simulation, type-II progressive censoring, multiple constant stress accelerated life test

I. Introduction

Traditional reliability tests are designed to examine failure time data acquired under normal operating circumstances. However, due to restricted testing time and highly reliable products like as electronics systems, insulating materials, engines, and so on, such life data is not easy to obtain. As a result, the use of traditional reliability tests is inappropriate, time consuming and expensive. Therefore, ALT is widely used in the manufacturing and production industries due to its capacity to provide timely and adequate failure data for product reliability and design assessment. Furthermore, the growing competitiveness of innovation, as well as the desire to shorten product development time, have underlined the use and significance of ALT techniques. It is especially difficult to detect faults in highly reliable items or systems under typical operating conditions in a short period of time. Thus, in the manufacturing business, ALT has become an essential element of the product design and development process. In ALT, samples of test items are exposed to more intense levels of stress, such as temperature, voltage, humidity etc. to cause early failures, and the resultant failure times and the used censoring schemes are recorded. The data is then utilized to create an ALT model for extrapolating the product's reliability under normal operation conditions.

The stress loading in ALT may be applied in a various different way, however the most often utilized stress loadings are constant, step, and progressive stress loadings [1] (abbreviated as

CSALT, SSALT & PSALT). In CSALT, units are tested at two or more constant levels of high stress until they all fail or the test is stopped owing to some censoring scheme or other factors. Many researchers have investigated CSALT models, including [2-9]. In SSALT, the testing units are initially subjected to a starting high level of stress and the failures are noted and then the test items are removed at prespecified time to test at the next level of stress, and so on. Many scholars have looked at the SSALT models, including [10-19]. Progressive stress loading, often known as ramp stress loading, is a method of exposing test units to gradually increasing stress over time. PSALT designs were initially proposed by [20], who took into account both exponential and Weibull life distributions. Since then, several writers have explored PSALT for different distributions with different types of data, including [21-25].

Most items are subjected to constant stress when they are in use in general. The CSALT is straightforward in most tests, making it easier to maintain a consistent stress level and the CSALT mimics actual use of the product. For some materials and products, CSALT models are better designed and empirically validated. Furthermore, data analysis for estimating reliability is well established and automated. However, CSALT is the most commonly utilized ALT method because of its above-mentioned benefits, but the majority of ALT research has concentrated on statistical inference with only two or three levels of constant stress.

So far, there are only a few studies which have considered multi-stress CSALT. [26] discussed the reliability analysis of type-I censored Weibull failure data obtained from a system of multiple components connected in series under CSALT assuming a log-linear relation between scale parameter and the stress variable. [27] used TIIPC data to develop MLEs and Bayes estimates (BEs) of the parameters of the extended exponential distribution under CSALT. [28] examined several estimating methods for the parameters of the exponentiated distributions family, with the exponentiated inverted Weibull regarded as a particular example under CSALT. [29] estimated the parameters of a lower-truncated family of distributions using the MLE approach for simple CSALT under TIIPCS. Assuming mean life as a linear function of the stress, [30] considered hybrid type-I censored data from a CSALT and obtained the MLEs and approximation MLEs of the parameters of a generalized log-location-scale distribution. [31] investigated a multiple stress CSALT and utilized MLE and BE approaches to construct point and interval estimates of Weibull distribution parameters based on TIIP adaptive hybrid censored data. Assuming that failure under arithmetically increasing stress levels of CSALT, [32] employed MLE methods for estimating the Burr-X life distribution parameters. [33] studied CSALT and estimated the doubly truncated Burr-XII parameters using MLE and BE methods. [34] address the problem of statistical inference using MLE and BE approaches for TIIPC data under multiple stress CSALT, assuming that failure times follow the modified Kies exponential distribution and removal follows a binomial distribution.

In this paper, an ALT with k constant stress levels which is stopped by a TIIPC scheme is considered. The following is how the paper is structured. Section 2 provides fundamental terminology, failure distribution, and basic multi-stress CSALT assumptions. In Section 3, the MLE technique is employed to derive estimates of the parameters using TIIPC data. In Section 4, a simulation study with different test setting is conducted to compare the performance of the proposed model. Section 5 concludes the paper with some remarks.

II. Assumptions and procedure for k-stress level ALT

A k levels of CSALT is considered. Let Q_0 be the normal stress level and $Q_i, i = 1,2,\dots k$ are the k levels of applied higher constant stress levels. The following assumptions are used in this paper:

1. Suppose $n_i, i = 1, 2, \dots, k$ are samples containing independent and identical items put on test at the same time at stress levels $Q_i, i = 1,2,\dots k$ in such way that $\mathcal{N} = \sum_{i=1}^{k} n_i$, where $\mathcal{N}$ total number of items assigned on all stress levels to test.

2. The product's life has a GnP distribution under normal stress Q_0 and accelerated stress $Q_i, i = 1,2,\dots k$. The density function, cumulative distribution function, the reliability function and the hazard rate function of GnP distribution are as follows:

$$f(t_i, \phi_i, \psi_i) = \phi_i \psi_i (1 + \psi_i t_i)^{-(\phi_i+1)}, \;\; t_i, \phi_i, \psi_i > 0 \tag{1}$$

$$\mathcal{F}(t_i, \phi_i, \psi_i) = 1 - (1 + \psi_i t_i)^{-\phi_i}, \qquad t_i, \phi_i, \psi_i > 0 \tag{2}$$

$$R(t_i, \phi_i, \psi_i) = (1 + \psi_i t_i)^{-\phi_i}, \qquad t_i, \phi_i, \psi_i > 0 \tag{3}$$

$$h(t_i, \phi_i, \psi_i) = \frac{\phi_i \psi_i}{(1 + \psi_i t_i)}, \qquad t_i, \phi_i, \psi_i > 0 \tag{4}$$

where ϕ_i is the shape parameter and the scale parameter is ψ_i at stress level $Q_i, i = 1,2,\dots k$.

3. At each increased stress, the product's failure mechanism stays unchanged. Because ϕ_i specifies the failure mechanism, it follows that

$$\phi_0 = \phi_1 = \phi_2 = \dots = \phi_k = \phi \tag{5}$$

4. The parameter ψ_i has a log-linear relationship with the stress variable Q_i and may be described as follows:

$$log\, \psi_i = \alpha + \beta \mathcal{Z}_i, \;\; i = 0, 1, 2, \dots, k \tag{6}$$

where α and $\beta (> 0)$ are the unknown parameters of the relationship and their values usually depend on true nature of the test items. And $\mathcal{Z}_i = \mathcal{Z}(Q_i)$ is an increasing function of stress Q_i. Eq. (6) depends on the type of stress used for testing, e. g., if stress is temperature, then, the Arrhenius model is used and can written as $log\, \psi_i = \alpha + \beta / Q_i$, where Q_i is temperature stress. If stress is voltage, then the inverse power model is appropriate to be used and can written as $log\, \psi_i = \alpha + \beta(log(Q_i))$, where Q_i is voltage stress. For weather conditions, exponential model is used and can written as $log\, \psi_i = \alpha + \beta Q_i$. Above defined three well-known models may be converted into the linear form as in eq. (6) by transforming the stress with $\mathcal{Z}(Q_i) = 1/Q_i, log(Q_i)$ and Q_i respectively.

Let $t_{ij}, i = 1,2,\dots k;\; j = 1,2,\dots, n_i$ are observed ordered failures with progressive censoring scheme $w_{ij} = (w_{i1,} w_{i2}, \dots, w_{im_i})$, $i = 1,2,\dots k;\; j = 1,2,\dots, m_i$ at each Q_i stress level. Based on k stress ALT with TIIPC scheme w_{ij}, n_i units are put under accelerated testing condition Q_i and the experiment will be run until m_i failures at each stress level and the number of failures is prefixed. Now, the TIIPC can be implemented as follows: at each $Q_i, i = 1,2,\dots k$, at first failure time t_{i1}, w_{i1} items are omitted from the remaining $(n_i - 1)$ survivals randomly. Similarly at time t_{i2}, w_{i2} is the number of removed items from $(n_i - 2 - w_{i1})$ remaining survivals and so on until the desired number of failures $m_i,\; i = 0, 1, 2, \dots, k$ at each stress level obtained and then the test is terminated by removing all the remaining survivals $w_{ij} = n_i - m_i - \sum_{i=1}^{m_i - 1} w_{ij}$ from the test.

III. Parameter estimation

In statistics, MLEs and BE techniques are two of the most significant and commonly used approaches. The MLEs are asymptotically normal and consistent. The BE technique necessitates the selection of previous knowledge of unknown parameters, although this is generally a challenging task in reality. Furthermore, BE method frequently necessitates the use of complicated integral procedures. As a result, in a CSALT, this paper uses an MLE method utilizing TIIPC data.

Let the obtained observed failure TIIPC samples at i^{th} stress level in the considered ALT $t_{i1} \le t_{i2} \le \dots \le t_{im_i}$, $i = 0, 1, 2, \dots, k$, then the likelihood for the observed data under TIIPC scheme can be obtained in the following form

$$L(y,\xi,\delta) = \prod_{i=1}^{k}\left\{C_i \prod_{j=1}^{m_i} f_{T_{ij}}(t_{ij})\left(1 - F_{T_{ij}}(t_{ij})\right)^{w_{ij}}\right\} \tag{7}$$

where, $C_i = n_i(n_i - 1 - w_{i1})(n_i - 2 - w_{i1} - w_{i2})\ldots\sum_{i=1}^{m_i-1} w_{ij}$. Now, the log likelihood $\ell = L(y,\xi,\sigma,\delta)$ corresponding to Eq. (7) after substituting the values of $f_{T_{ij}}(t_{ij})$ & $F_{T_{ij}}(t_{ij})$ and taking log on both sides is obtained as follows:

$$\ell = \sum_{i=1}^{k}\sum_{j=1}^{m_i}\{log(\phi) + log(\psi_i) - (w_{ij}\phi + \phi + 1)log(1 + \psi_i t_{ij})\} \tag{8}$$

Now, from equation (6), we can drive

$$\psi_i = \psi_0 e^{(\beta Z_i - \beta Z_0)} = \psi_0 \vartheta^{\Omega_i}, \qquad i = 0,1,2,\ldots,k \tag{9}$$

Where, $\psi_0 = \alpha + \beta Z_0$ represents the GnP distribution's scale parameter at stress Q_0 and $\psi_1/\psi_0 = \vartheta = e^{\beta(Z_1 - Z_0)}$ denotes the acceleration factor from Q_1 to Q_0, and

$$\Omega_i = (Z_i - Z_0)/(Z_1 - Z_0), \qquad i = 0,1,2,\ldots,k \tag{10}$$

Because the transformation from (α,β,ϕ) to (ψ_0,ϑ,ϕ) is one-to-one, we can immediately calculate the product's life at Q_0 using the new transformed parameters. As a result, the likelihood function (8) may be rewritten as follows:

$$\ell = \sum_{i=1}^{k}\sum_{j=1}^{m_i}\{log(\phi) + log(\psi_0) + \Omega_i log(\vartheta) - (1 + \phi + w_{ij}\phi)log(1 + \psi_0\vartheta^{\Omega_i} t_{ij})\} \tag{11}$$

By solving the following likelihood equations, the MLEs of the parameters can now be calculated:

$$\frac{\partial \ell}{\partial \psi_0} = \frac{1}{\psi_0} - \frac{\vartheta^{\Omega_i}(1 + (1 + w_{ij})\phi)t_{ij}}{1 + \psi_0\vartheta^{\Omega_i} t_{ij}} = 0 \tag{12}$$

$$\frac{\partial \ell}{\partial \vartheta} = \frac{\Omega_i}{\vartheta} - \frac{\psi_0\vartheta^{\Omega_i - 1}\Omega_i(1 + (1 + w_{ij})\phi)t_{ij}}{1 + \psi_0\vartheta^{\Omega_i} t_{ij}} = 0 \tag{13}$$

$$\frac{\partial \ell}{\partial \phi} = \frac{1}{\phi} - (1 + w_{ij})log(1 + \psi_0\vartheta^{\Omega_i} t_{ij}) = 0 \tag{14}$$

We now have a system of three nonlinear equations with three unknowns, making it difficult to find closed-form solutions manually. Hence, numerical solution to equations is obtained using Newton Raphson iterative approach, the R programming language is used to get the solutions.

By using asymptotic characteristics of the MLEs, the ACIs of the parameters may now be estimated using TIIPC by mathematically inverting Fisher's information matrix. As a result, we can compute the estimates of 95% two-sided ACIs for ψ_0, ϑ and ϕ as follows:

$$\widehat{\psi_0} \pm 1.96\sqrt{var(\widehat{\psi_0})}\,;\ \ \hat{\vartheta} \pm 1.96\sqrt{var(\hat{\vartheta})};\ \ \hat{\phi} \pm 1.96\sqrt{var(\hat{\phi})} \tag{15}$$

Where, $var(\widehat{\psi_0})$, $var(\hat{\vartheta})$ and $var(\hat{\phi})$ are main diagonal entries of inverted Fisher matrix and the elements of the matrix are given by following equations:

$$\frac{\partial^2\ell}{\partial\phi^2} = -\frac{1}{\phi^2} \tag{16}$$

$$\frac{\partial^2\ell}{\partial\vartheta^2} = -\frac{\Omega_i(1+\psi_0\vartheta^{\Omega_i}t_{ij}(1-\phi(1+w_{ij})(1+\psi_0\vartheta^{\Omega_i}t_{ij})+\Omega_i(1+\phi+w_{ij}\phi)))}{\vartheta^2(1+\psi_0\vartheta^{\Omega_i}t_{ij})^2} \tag{17}$$

$$\frac{\partial^2\ell}{\partial\psi_0^2} = -\frac{1}{\psi_0^2} + \frac{\vartheta^{2\Omega_i}(1+(1+w_{ij})\phi)t_{ij}^2}{(1+\psi_0\vartheta^{\Omega_i}t_{ij})^2} \tag{18}$$

$$\frac{\partial^2\ell}{\partial\phi\partial\vartheta} = \frac{\partial^2\ell}{\partial\vartheta\partial\phi} = -\frac{\psi_0\Omega_i\vartheta^{\Omega_i-1}(1+w_{ij})t_{ij}}{1+\psi_0\vartheta^{\Omega_i}t_{ij}} \tag{19}$$

$$\frac{\partial^2\ell}{\partial\phi\partial\psi_0} = \frac{\partial^2\ell}{\partial\psi_0\partial\phi} = -\frac{\vartheta^{\Omega_i}(1+w_{ij})t_{ij}}{1+\psi_0\vartheta^{\Omega_i}t_{ij}} \tag{20}$$

$$\frac{\partial^2\ell}{\partial\psi_0\partial\vartheta} = \frac{\partial^2\ell}{\partial\vartheta\partial\psi_0} = -\frac{\Omega_i\vartheta^{\Omega_i-1}(1+\phi+w_{ij}\phi)t_{ij}}{(1+\psi_0\vartheta^{\Omega_i}t_{ij})^2} \tag{21}$$

IV. Simulation Study

In this section, the performance of the considered methodology for estimating the parameters of the GnP distribution based on the CSALT with k stresses for TIIPC data utilizing the log liner association between stress and life characteristic is examined through a Monte-Carlo simulation using the R-package. Two set of initial values ($\psi_0 = 1.2, \vartheta = 0.75, \phi = 0.5$), ($\psi_0 = 1.5, \vartheta = 0.5, \phi = 0.8$) of parameter with various sample combinations (n_i, m_i)= (20, 10), (20, 15), (30, 15), (30, 20), (30, 25), (40, 20), (40, 25), (40, 30), (50, 25), (50, 30), (50, 35), (60, 35), (60, 40), (60, 45) are selected for simulation. Apart from the normal stress level $Q_0 = 110$, three levels of constant stress are assumed as; $Q_1 = 150$, $Q_2 = 220$ and $Q_3 = 250$ under TIIPC scheme. Additionally, two distinct removal schemes $i.\, w_{i1}, w_{i2}, \ldots, w_{i(m-1)} = (n_i - m_i)$ & $w_{im_i} = 0$; $ii.\, w_{i1}, w_{i2}, \ldots, w_{i(m-1)} = 1$ & $w_{im_i} = n_i - m_s + 1$ are used to generate TIIPC samples with various combinations of (n_i, m_i) under three different constant-stress levels. For each test scheme, average RABs and RMSEs for point estimates, as well as lower and upper ACI limits (LACIL, UACIL) and ACIs lengths (ACIsL) of 95% ACIs with corresponding CPs are computed. Following steps are used to perform the simulation study:

Step 1: Set the initial values of the parameters ψ_0, ϑ and ϕ.

Step 2: Set the values of stress levels Q_i, $i = 0, 1, 2, \ldots, k$.

Step 3: Set the values of $(n_i, m_i), i = 0, 1, 2, \ldots, k$ at each stress levels Q_i.

Step 4: Now using the defined values in step 1-3, generate $i = 1, 2, \ldots, k$ random samples of size m_i of TIIPC data from Uniform (0, 1) distribution according to the steps outlined by [35].

Step 5: Using inverse CDF method, for each sample size m_i obtained in step 4, generate TIIPC data from GnP distribution using $(exp(-ln(1-u)/\phi) - 1)/\psi$.

Step 6: For each stress levels along with removal scheme, repeat the steps 1-5 for 10000 times.

Step 7: Compute the average of MLEs of ψ_0, ϑ and ϕ with their respective RABs and RMSEs.

Step 8: Compute LACIL, UACIL, ACIsL of 95% ACIs with corresponding CPs of ψ_0, ϑ and ϕ.

Table 1-6 displays the numerical findings of RMSEs and RABs of MLEs and LACIL, UACIL, and ACIsL, as well as the corresponding CPs of ACIs. Figures 1-3 represent the behavior of the RABs and RMSEs with respect to the sample size.

Table 1: *MLEs with RMSEs, RABs of* ψ_0 & *LACIL, UACIL, ACIsL, CPs of ACIs with* $(\psi_0 = 1.2, \vartheta = 0.75, \phi = 0.5)$

(n_i, m_i)	CS	MLE	RMSE	RAB	LACIL	UACIL	ACIsL	CP
20, 10	1	1.379815	0.960779	0.439453	-0.503310	3.262942	3.766253	0.950000
20, 15	1	1.439942	0.797518	0.433950	-0.123190	3.003076	3.126269	0.940000
30, 15	1	1.370340	0.705081	0.408689	-0.011620	2.752298	2.763916	0.960000
30, 20	1	1.486785	0.666933	0.357396	0.179597	2.793973	2.614376	0.950000
30, 25	1	1.312451	0.664671	0.392446	0.009696	2.615206	2.605510	0.950000
40, 20	1	1.291834	0.644747	0.383245	0.028131	2.555537	2.527407	0.950000
40, 25	1	1.370246	0.639035	0.389515	0.117737	2.622754	2.505017	0.940000
40, 30	1	1.365194	0.611388	0.358700	0.166874	2.563514	2.396640	0.980000
50, 25	1	1.371536	0.588579	0.350251	0.217920	2.525151	2.307231	0.970000
50, 30	1	1.053091	0.533587	0.382436	0.007259	2.098922	2.091663	0.940000
50, 35	1	1.244513	0.521590	0.330190	0.222196	2.266830	2.044634	0.950000
60, 35	1	1.275332	0.504480	0.344008	0.286551	2.264113	1.977562	0.940000
60, 40	1	1.294818	0.451734	0.283193	0.409419	2.180216	1.770797	0.950000
60, 45	1	1.244503	0.451180	0.266636	0.360191	2.128815	1.768624	0.950000
20, 10	2	1.359424	0.946580	0.432959	-0.495874	3.214721	3.710595	0.980000
20, 15	2	1.418662	0.785732	0.427537	-0.121372	2.958696	3.080068	0.940000
30, 15	2	1.350089	0.694661	0.402649	-0.011446	2.711624	2.723070	0.940000
30, 20	2	1.464813	0.657076	0.352115	0.176943	2.752682	2.575739	0.950000
30, 25	2	1.293055	0.654848	0.386646	0.009553	2.576557	2.567005	0.950000
40, 20	2	1.272743	0.635218	0.377582	0.027715	2.517771	2.490056	0.950000
40, 25	2	1.349996	0.629591	0.383759	0.115997	2.583995	2.467997	0.950000
40, 30	2	1.345019	0.602353	0.353399	0.164408	2.525630	2.361222	0.940000
50, 25	2	1.351267	0.579881	0.345075	0.214700	2.487834	2.273134	0.970000
50, 30	2	1.037528	0.525702	0.376785	0.007152	2.067903	2.060751	0.940000
50, 35	2	1.226121	0.513882	0.325310	0.218912	2.233330	2.014417	0.950000
60, 35	2	1.256485	0.497025	0.338925	0.282316	2.230653	1.948337	0.970000
60, 40	2	1.275682	0.445058	0.279008	0.403369	2.147996	1.744628	0.950000
60, 45	2	1.226111	0.444512	0.262695	0.354868	2.097355	1.742486	0.970000

Table 2: *MLEs with RMSEs, RABs of* ψ_0 & *LACIL, UACIL, ACIsL, CPs of ACIs with* $(\psi_0 = 1.5, \vartheta = 0.5, \phi = 0.8)$

(n_i, m_i)	CS	MLE	RMSE	RAB	LACIL	UACIL	ACIsL	CP
20, 10	1	1.594526	0.948091	0.458727	-0.263732	3.452785	3.716517	0.930000
20, 15	1	1.556388	0.844726	0.385720	-0.099275	3.212051	3.311326	0.950000
30, 15	1	1.582766	0.812384	0.367365	-0.009506	3.175038	3.184544	0.930000
30, 20	1	1.546824	0.807062	0.381692	-0.035018	3.128666	3.163684	0.970000
30, 25	1	1.189059	0.807045	0.483961	-0.392749	2.770867	3.163616	0.960000
40, 20	1	1.557939	0.806578	0.406983	-0.022955	3.138833	3.161788	0.940000
40, 25	1	1.589465	0.763197	0.372843	0.093600	3.085331	2.991731	0.920000
40, 30	1	1.520138	0.754312	0.365067	0.041687	2.998588	2.956901	0.950000
50, 25	1	1.560208	0.736059	0.399957	0.117533	3.002883	2.885350	0.970000
50, 30	1	1.147427	0.687020	0.415694	-0.199131	2.493985	2.693117	0.970000
50, 35	1	1.581000	0.661877	0.319240	0.283721	2.878278	2.594557	0.950000
60, 35	1	1.466991	0.645759	0.352575	0.201303	2.732678	2.531375	0.940000
60, 40	1	1.582213	0.589180	0.280259	0.427421	2.737005	2.309584	0.950000
60, 45	1	1.350998	0.574095	0.326686	0.225772	2.476224	2.250452	0.970000
20, 10	2	1.496527	0.812236	0.370885	-0.095460	3.088511	3.183967	0.940000
20, 15	2	1.521890	0.781138	0.353235	-0.009140	3.052921	3.062062	0.940000
30, 15	2	1.487331	0.776021	0.367011	-0.033670	3.008333	3.042004	0.950000
30, 20	2	1.143326	0.776005	0.465347	-0.377640	2.664295	3.041939	0.930000
30, 25	2	1.498018	0.775556	0.391330	-0.022070	3.018108	3.040180	0.940000
40, 20	2	1.528332	0.733843	0.358503	0.091020	2.966664	2.876664	0.950000
40, 25	2	1.461671	0.725300	0.351026	0.040084	2.883258	2.843174	0.950000
40, 30	2	1.500200	0.707749	0.384574	0.113013	2.887387	2.774375	0.960000
50, 25	2	1.103295	0.660596	0.399705	-0.191470	2.398063	2.589535	0.960000
50, 30	2	1.520192	0.636420	0.306962	0.272809	2.767575	2.494766	0.960000
50, 35	2	1.410568	0.620922	0.339014	0.193561	2.627575	2.434014	0.950000
60, 35	2	1.521359	0.566519	0.269480	0.410982	2.631736	2.220754	0.950000
60, 40	2	1.299036	0.552014	0.314121	0.217088	2.380985	2.163896	0.960000
60, 45	2	1.429823	0.520777	0.260784	0.409101	2.450546	2.041445	0.970000

Table 3: *MLEs with RMSEs, RABs of* ϑ *& LACIL, UACIL, ACIsL, CPs of ACIs with* $(\psi_0 = 1.2, \vartheta = 0.75, \phi = 0.5)$

(n_i, m_i)	CS	MLE	RMSE	RAB	LACIL	UACIL	ACIsL	CP
20, 10	1	0.956331	0.146834	0.116788	0.668536	1.244127	0.575591	0.960000
20, 15	1	0.970048	0.144884	0.116523	0.686076	1.254020	0.567944	0.950000
30, 15	1	0.964952	0.142178	0.112322	0.686283	1.243622	0.557339	0.970000
30, 20	1	0.938985	0.138562	0.116477	0.667403	1.210567	0.543164	0.950000
30, 25	1	0.964784	0.127289	0.102683	0.715298	1.214270	0.498972	0.940000
40, 20	1	0.937890	0.120433	0.104640	0.701840	1.173940	0.472099	0.950000
40, 25	1	0.948488	0.119736	0.097341	0.713805	1.183171	0.469366	0.950000
40, 30	1	0.951535	0.117819	0.098871	0.720610	1.182461	0.461852	0.960000
50, 25	1	0.936677	0.115799	0.101152	0.709711	1.163643	0.453932	0.960000
50, 30	1	0.879029	0.112284	0.099044	0.658952	1.099106	0.440154	0.950000
50, 35	1	0.884003	0.110710	0.103224	0.667011	1.100995	0.433983	0.960000
60, 35	1	0.951515	0.102543	0.079229	0.750531	1.152500	0.401969	0.950000
60, 40	1	0.962086	0.094635	0.077911	0.776601	1.147571	0.370970	0.940000
60, 45	1	0.948595	0.092008	0.079129	0.768259	1.128932	0.360673	0.960000
20, 10	2	1.004047	0.169615	0.129299	0.671601	1.336493	0.664892	0.910000
20, 15	2	0.993732	0.160594	0.128606	0.678968	1.308495	0.629528	0.940000
30, 15	2	0.919236	0.159483	0.138557	0.606649	1.231823	0.625174	0.970000
30, 20	2	0.975282	0.152032	0.125374	0.677299	1.273265	0.595966	0.960000
30, 25	2	0.987848	0.149513	0.121256	0.694803	1.280893	0.586090	0.960000
40, 20	2	0.945983	0.138707	0.117505	0.674117	1.217849	0.543732	0.960000
40, 25	2	0.967796	0.136775	0.109025	0.699717	1.235876	0.536159	0.970000
40, 30	2	0.908435	0.136498	0.121973	0.640898	1.175972	0.535073	0.940000
50, 25	2	0.854772	0.134755	0.124194	0.590652	1.118892	0.528240	0.930000
50, 30	2	0.963932	0.132995	0.111667	0.703263	1.224602	0.521339	0.940000
50, 35	2	0.970456	0.129270	0.109791	0.717087	1.223824	0.506737	0.960000
60, 35	2	0.950931	0.121757	0.106660	0.712287	1.189574	0.477287	0.960000
60, 40	2	0.977545	0.116375	0.096155	0.749450	1.205640	0.456190	0.940000
60, 45	2	1.003948	0.101925	0.080405	0.804175	1.203721	0.399546	0.970000

Table 4: *MLEs with RMSEs, RABs of* ϑ *& LACIL, UACIL, ACIsL, CPs of ACIs with* $(\psi_0 = 1.5, \vartheta = 0.5, \phi = 0.8)$

(n_i, m_i)	CS	MLE	RMSE	RAB	LACIL	UACIL	ACIsL	CP
20, 10	1	0.729710	0.156055	0.171920	0.423842	1.035579	0.611737	0.970000
20, 15	1	0.635912	0.147363	0.168628	0.347080	0.924744	0.577663	0.970000
30, 15	1	0.752896	0.146134	0.156156	0.466473	1.039318	0.572845	0.950000
30, 20	1	0.761058	0.141137	0.146475	0.484429	1.037687	0.553258	0.980000
30, 25	1	0.659879	0.139685	0.167380	0.386096	0.933663	0.547567	0.940000
40, 20	1	0.755815	0.138839	0.148709	0.483691	1.027939	0.544249	0.950000
40, 25	1	0.737196	0.134817	0.147778	0.472956	1.001437	0.528481	0.970000
40, 30	1	0.744293	0.128940	0.144283	0.491570	0.997016	0.505445	0.960000
50, 25	1	0.743150	0.122515	0.130935	0.503021	0.983279	0.480258	0.960000
50, 30	1	0.747007	0.120722	0.130088	0.510392	0.983622	0.473230	0.960000
50, 35	1	0.616800	0.118942	0.144958	0.383673	0.849927	0.466253	0.950000
60, 35	1	0.761865	0.113971	0.122099	0.538481	0.985249	0.446768	0.960000
60, 40	1	0.756613	0.105303	0.110844	0.550219	0.963008	0.412788	0.960000
60, 45	1	0.741342	0.098291	0.106389	0.548692	0.933991	0.385299	0.950000
20, 10	2	0.635545	0.121039	0.186381	0.398309	0.872781	0.474472	0.950000
20, 15	2	0.757236	0.120128	0.156847	0.521786	0.992686	0.470901	0.970000
30, 15	2	0.617029	0.110272	0.174321	0.400897	0.833161	0.432264	0.940000
30, 20	2	0.705092	0.106901	0.148531	0.495566	0.914619	0.419054	0.960000
30, 25	2	0.614613	0.099448	0.154738	0.419694	0.809531	0.389836	0.940000
40, 20	2	0.605555	0.099098	0.157103	0.411322	0.799788	0.388466	0.960000
40, 25	2	0.712096	0.098604	0.106531	0.518833	0.905359	0.386527	0.970000
40, 30	2	0.621327	0.097804	0.154010	0.429632	0.813023	0.383391	0.950000
50, 25	2	0.614977	0.083327	0.128538	0.451657	0.778298	0.326641	0.950000
50, 30	2	0.669440	0.074306	0.107218	0.523801	0.815080	0.291279	0.950000
50, 35	2	0.617426	0.067618	0.102392	0.484895	0.749956	0.265061	0.940000
60, 35	2	0.613535	0.064974	0.104036	0.486187	0.740883	0.254696	0.940000
60, 40	2	0.612077	0.064608	0.106285	0.485446	0.738708	0.253262	0.930000
60, 45	2	0.613641	0.063932	0.104994	0.488334	0.738948	0.250613	0.930000

Table 5: *MLEs with RMSEs, RABs of* ϕ *& LACIL, UACIL, ACIsL, CPs of ACIs with* $(\psi_0 = 1.2, \vartheta = 0.75, \phi = 0.5)$

(n_i, m_i)	CS	MLE	RMSE	RAB	LACIL	UACIL	ACIsL	CP
20, 10	1	0.356076	0.217232	0.331312	-0.069698	0.781851	0.851548	0.980000
20, 15	1	0.423038	0.170116	0.307633	0.089610	0.756466	0.666856	0.940000
30, 15	1	0.442701	0.103119	0.181128	0.240587	0.644815	0.404228	0.960000
30, 20	1	0.385605	0.096882	0.187231	0.195716	0.575493	0.379777	0.950000
30, 25	1	0.408111	0.095668	0.188336	0.220601	0.595620	0.375019	0.950000
40, 20	1	0.322006	0.092630	0.225161	0.140452	0.503560	0.363108	0.980000
40, 25	1	0.365803	0.080635	0.180866	0.207758	0.523847	0.316089	0.950000
40, 30	1	0.335572	0.078809	0.191962	0.181107	0.490037	0.308930	0.990000
50, 25	1	0.448864	0.077859	0.139534	0.296260	0.601468	0.305208	0.960000
50, 30	1	0.420924	0.064415	0.122822	0.294671	0.547176	0.252505	0.950000
50, 35	1	0.469790	0.062875	0.101525	0.346555	0.593024	0.246469	0.940000
60, 35	1	0.411541	0.053268	0.103181	0.307136	0.515946	0.208809	0.940000
60, 40	1	0.326189	0.046967	0.110134	0.234134	0.418244	0.184110	0.950000
60, 45	1	0.333336	0.035991	0.087729	0.262794	0.403879	0.141085	0.980000
20, 10	2	0.387373	0.124545	0.226787	0.143265	0.631482	0.488217	0.970000
20, 15	2	0.360833	0.105094	0.239976	0.154850	0.566817	0.411967	0.970000
30, 15	2	0.465610	0.104133	0.167382	0.261510	0.669710	0.408200	0.920000
30, 20	2	0.404602	0.101346	0.199987	0.205964	0.603241	0.397277	0.950000
30, 25	2	0.402596	0.094031	0.194124	0.218295	0.586897	0.368602	0.970000
40, 20	2	0.340660	0.093892	0.218835	0.156631	0.524689	0.368058	0.960000
40, 25	2	0.447136	0.078380	0.134325	0.293511	0.600761	0.307250	0.930000
40, 30	2	0.403542	0.077751	0.163517	0.251151	0.555934	0.304783	0.980000
50, 25	2	0.450346	0.073391	0.134952	0.306500	0.594192	0.287693	0.960000
50, 30	2	0.390654	0.061132	0.122158	0.270836	0.510473	0.239636	0.950000
50, 35	2	0.416645	0.059222	0.115242	0.300570	0.532720	0.232150	0.960000
60, 35	2	0.388899	0.050428	0.104176	0.290060	0.487738	0.197678	0.940000
60, 40	2	0.341503	0.037570	0.081691	0.267867	0.415140	0.147273	0.950000
60, 45	2	0.450346	0.030832	0.095481	0.195635	0.316495	0.120860	0.950000

Table 6: *MLEs with RMSEs, RABs of* ϕ *& LACIL, UACIL, ACIsL, CPs of ACIs with* $(\psi_0 = 1.5, \vartheta = 0.5, \phi = 0.8)$

(n_i, m_i)	CS	MLE	RMSE	RAB	LACIL	UACIL	ACIsL	CP
20, 10	1	0.534378	0.248217	0.339334	0.047871	1.020884	0.973012	0.940000
20, 15	1	0.628356	0.226171	0.291756	0.185060	1.071652	0.886592	0.970000
30, 15	1	0.716714	0.218887	0.253214	0.287696	1.145733	0.858036	0.960000
30, 20	1	0.568052	0.204952	0.291313	0.166346	0.969758	0.803412	0.950000
30, 25	1	0.788660	0.188148	0.175430	0.419890	1.157431	0.737541	0.940000
40, 20	1	0.567105	0.180587	0.266770	0.213155	0.921055	0.707900	0.970000
40, 25	1	0.763205	0.171400	0.166184	0.427261	1.099149	0.671888	0.950000
40, 30	1	0.677318	0.165821	0.198586	0.352309	1.002327	0.650018	0.950000
50, 25	1	0.552460	0.158731	0.226367	0.241347	0.863573	0.622227	0.940000
50, 30	1	0.541943	0.131083	0.189980	0.285021	0.798864	0.513844	0.970000
50, 35	1	0.622779	0.130376	0.170692	0.367242	0.878316	0.511074	0.960000
60, 35	1	0.603203	0.126617	0.150484	0.355034	0.851372	0.496338	0.970000
60, 40	1	0.628403	0.101000	0.126772	0.430444	0.826363	0.395920	0.950000
60, 45	1	0.578427	0.095940	0.128139	0.390385	0.766470	0.376086	0.960000
20, 10	2	0.562503	0.261281	0.357193	0.050391	1.074614	1.024223	0.940000
20, 15	2	0.661427	0.238075	0.307111	0.194800	1.128055	0.933255	0.950000
30, 15	2	0.754436	0.230407	0.266541	0.302838	1.206034	0.903196	0.940000
30, 20	2	0.597950	0.215739	0.306645	0.175101	1.020798	0.845697	0.960000
30, 25	2	0.830169	0.198051	0.184663	0.441990	1.218348	0.776359	0.950000
40, 20	2	0.596952	0.190091	0.280810	0.224373	0.969531	0.745158	0.940000
40, 25	2	0.803374	0.180421	0.174930	0.449749	1.156999	0.707250	0.970000
40, 30	2	0.712967	0.174548	0.209037	0.370852	1.055081	0.684230	0.970000
50, 25	2	0.581537	0.167086	0.238281	0.254049	0.909024	0.654975	0.950000
50, 30	2	0.519055	0.156485	0.257407	0.212344	0.825767	0.613423	0.940000
50, 35	2	0.570466	0.137982	0.199979	0.300022	0.840910	0.540888	0.940000
60, 35	2	0.655557	0.137238	0.179676	0.386571	0.924543	0.537972	0.960000
60, 40	2	0.634950	0.133281	0.158404	0.373720	0.896181	0.522461	0.940000
60, 45	2	0.661477	0.106316	0.133444	0.453098	0.869856	0.416758	0.940000

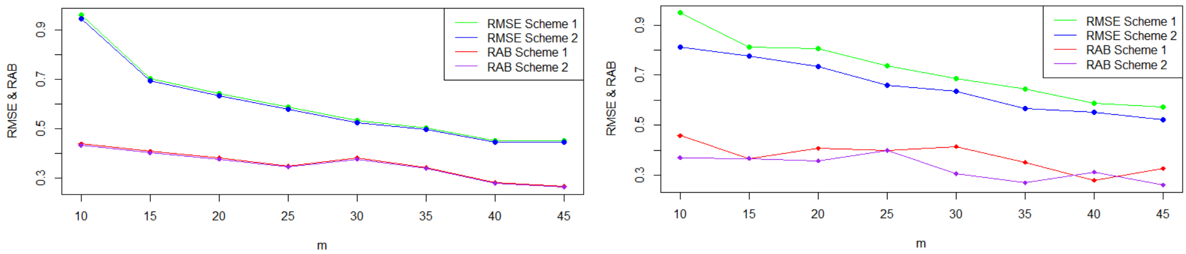

Figure 1: *RMSEs and RABs of the estimates of* ψ_0 *with* $(\psi_0 = 1.2, \vartheta = 0.75, \phi = 0.5)$ & $(\psi_0 = 1.5, \vartheta = 0.5, \phi = 0.8)$

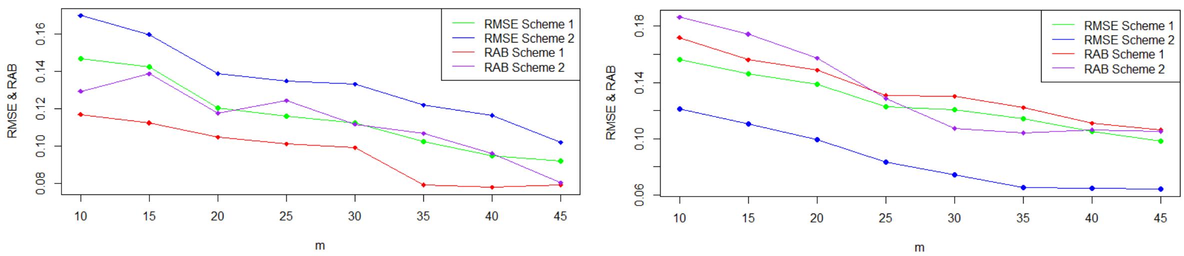

Figure 2: *RMSEs and RABs of the estimates of* ϑ *with* $(\psi_0 = 1.2, \vartheta = 0.75, \phi = 0.5)$ & $(\psi_0 = 1.5, \vartheta = 0.5, \phi = 0.8)$

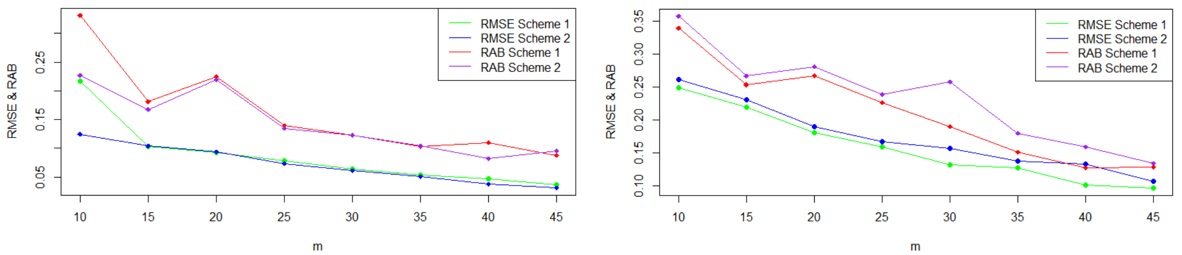

Figure 3: *RMSEs and RABs of the estimates of* ϕ *with* $(\psi_0 = 1.2, \vartheta = 0.75, \phi = 0.5)$ & $(\psi_0 = 1.5, \vartheta = 0.5, \phi = 0.8)$

It is evident from the results in tables 1-6 and Figures 1-3 that the results are consistent and that the estimates are quite closer to the real values of the parameters. The following observations can be made in general:

1. In all situations, the RMSEs and RABs decrease as the values of n_i and m_i increase, which is to be expected because greater samples produce more accurate results.
2. The lengths and CPs of 95% of ACIs are relatively precise in all situations, as shown in table 1-6. However, the ACIs are narrower for parameter setting 1 and removal scheme 2.
3. With increasing values of n_i and m_i, it can also be seen that the lengths of 95% percent ACIs are getting smaller and the CPs are getting larger.

As an outcome of the above observations, it is reasonable to infer that the proposed model and estimation method in this paper performed well, and that all statistical assumptions for fitting the model and estimation are suitable.

V. Conclusions

In this paper, the CSALT model has been considered with k levels of constant stress. The observed TIIPC failure data was assumed to come from a GnP distribution. The distribution's shape parameter has been assumed to be independent of the stress, whereas the scale parameter was assumed to have a log linear relationship with the stress variable. Model parameters are estimated using the MLE approach, and their performance is evaluated using the corresponding RABs and MSEs. The performance of MLEs has been found to be satisfactory, as the estimated values approaching real values as the sample size increases. The ACIs have also been constructed based on the asymptotic properties of the MLEs. The performance of ACIs was evaluated in terms of their corresponding CPs and ACIIL. An alternative lifetime distribution can be considered in the future research, and the corresponding inferences under various censoring methods can be developed using the BE technique.

References

[1] Nelson, W. (1990). Accelerated Testing: Statistical Models, Test Plans and Data Analysis. John Wiley & Sons, New York.

[2] Yang, G. B. (1994). Optimum constant-stress accelerated life-test plans. *IEEE transactions on reliability*, *43*(4), 575-581.

[3] Watkins, A. J., & John, A. M. (2008). On constant stress accelerated life tests terminated by Type II censoring at one of the stress levels. *Journal of statistical Planning and Inference*, *138*(3), 768-786.

[4] Zarrin, S., Kamal, M., & Saxena, S. (2012). Estimation in constant stress partially accelerated life tests for Rayleigh distribution using type-I censoring. *Reliability: Theory & Applications, 7(4)*, 41-52.

[5] Kamal, M. (2013). Application of geometric process in accelerated life testing analysis with type-I censored Weibull failure data. *Reliability: Theory & Applications, 8(3)*, 87-96.

[6] Ma, H., & Meeker, W. Q. (2010). Strategy for planning accelerated life tests with small sample sizes. *IEEE Transactions on Reliability*, *59*(4), 610-619.

[7] Kamal, M., Zarrin, S., & Islam, A. (2014). Design of accelerated life testing using geometric process for type-II censored Pareto failure data., *International Journal of Mathematical Modelling & Computations, 4(2)*, 125-134.

[8] Gao, L., Chen, W., Qian, P., Pan, J., & He, Q. (2016). Optimal time-censored constant-stress ALT plan based on chord of nonlinear stress-life relationship. *IEEE Transactions on Reliability*, *65*(3), 1496-1508.

[9] Han, D., & Bai, T. (2019). On the maximum likelihood estimation for progressively censored lifetimes from constant-stress and step-stress accelerated tests. *Electronic Journal of Applied Statistical Analysis*, *12*(2), 392-404.

[10] Miller, R., & Nelson, W. (1983). Optimum simple step-stress plans for accelerated life testing. *IEEE Transactions on Reliability*, *32*(1), 59-65.

[11] Bai, D. S., Kim, M. S., & Lee, S. H. (1989). Optimum simple step-stress accelerated life tests with censoring. *IEEE transactions on reliability*, *38*(5), 528-532.

[12] Saxena, S., Zarrin, S., Kamal, M., & & Islam, A. U. (2012). Optimum Step Stress Accelerated Life Testing for Rayleigh Distribution. *International journal of statistics and applications, 2(6)*, 120-125.

[13] Kamal, M., Zarrin, S. & Islam, A. (2013). Step stress accelerated life testing plan for two parameter Pareto distribution. *Reliability: Theory & Applications, 8(1)*, 30-40.

[14] Han, D., & Bai, T. (2020). Design optimization of a simple step-stress accelerated life test–Contrast between continuous and interval inspections with non-uniform step durations. *Reliability Engineering & System Safety*, *199*, 106875.

[15] Bai, X., Shi, Y., & Ng, H. K. T. (2020). Statistical inference of Type-I progressively censored step-stress accelerated life test with dependent competing risks. *Communications in Statistics-Theory and Methods*, 1-27.

[16] Kamal, M., Rahman, A., Ansari, S. I., & Zarrin, S. (2020). Statistical Analysis and Optimum Step Stress Accelerated Life Test Design for Nadarajah Haghighi Distribution. *Reliability: Theory & Applications, 15(4)*, 1-9.

[17] Hakamipour, N. (2021). Comparison between constant-stress and step-stress accelerated life tests under a cost constraint for progressive type I censoring. *Sequential Analysis*, *40*(1), 17-31.

[18] Nassar, M., Okasha, H., & Albassam, M. (2021). E-Bayesian estimation and associated properties of simple step–stress model for exponential distribution based on type-II censoring. *Quality and Reliability Engineering International*, *37*(3), 997-1016.

[19] Klemenc, J., & Nagode, M. (2021). Design of step-stress accelerated life tests for estimating the fatigue reliability of structural components based on a finite-element approach. *Fatigue & Fracture of Engineering Materials & Structures*, *44*(6), 1562-1582.

[20] Yin, X. K., & Sheng, B. Z. (1987). Some aspects of accelerated life testing by progressive stress. *IEEE transactions on reliability*, *36*(1), 150-155.

[21] Srivastava, P. W., & Mittal, N. (2012). Optimum multi-objective ramp-stress accelerated life test with stress upper bound for Burr type-XII distribution. *IEEE Transactions on Reliability*, *61*(4), 1030-1038.

[22] Abdel-Hamid, A. H., & AL-Hussaini, E. K. (2014). Bayesian prediction for type-II progressive-censored data from the Rayleigh distribution under progressive-stress model. *Journal of Statistical Computation and Simulation*, *84*(6), 1297-1312.

[23] Chen, Y., Sun, W., & Xu, D. (2017). Multi-stress equivalent optimum design for ramp-stress accelerated life test plans based on D-efficiency. *IEEE Access*, *5*, 25854-25862.

[24] El-Din, M. M., Amein, M. M., Abd El-Raheem, A. M., Hafez, E. H., & Riad, F. H. (2020). Bayesian inference on progressive-stress accelerated life testing for the exponentiated Weibull distribution under progressive type-II censoring. *J. Stat. Appl. Pro. Lett*, *7*, 109-126.

[25] Ma, Z., Liao, H., Ji, H., Wang, S., Yin, F., & Nie, S. (2021). Optimal design of hybrid accelerated test based on the Inverse Gaussian process model. *Reliability Engineering & System Safety*, *210*, 107509.

[26] Fan, T. H., & Hsu, T. M. (2014). Constant stress accelerated life test on a multiple-component series system under Weibull lifetime distributions. *Communications in Statistics-Theory and Methods*, *43*(10-12), 2370-2383.

[27] El-Din, M. M., Abu-Youssef, S. E., Ali, N. S., & Abd El-Raheem, A. M. (2016). Estimation in constant-stress accelerated life tests for extension of the exponential distribution under progressive censoring. *Metron*, *74*(2), 253-273.

[28] Abdel Ghaly, A. A., Aly, H. M., & Salah, R. N. (2016). Different estimation methods for constant stress accelerated life test under the family of the exponentiated distributions. *Quality and Reliability Engineering International*, *32*(3), 1095-1108.

[29] Wang, L. (2017). Inference of constant-stress accelerated life test for a truncated distribution under progressive censoring. *Applied Mathematical Modelling*, *44*, 743-757.

[30] Lin, C. T., Hsu, Y. Y., Lee, S. Y., & Balakrishnan, N. (2019). Inference on constant stress accelerated life tests for log-location-scale lifetime distributions with type-I hybrid censoring. *Journal of Statistical Computation and Simulation*, *89*(4), 720-749.

[31] Cui, W., Yan, Z., & Peng, X. (2019). Statistical analysis for constant-stress accelerated life test with Weibull distribution under adaptive Type-II hybrid censored data. *IEEE Access*, *7*, 165336-165344.

[32] Rahman, A., Sindhu, T. N., Lone, S. A., & Kamal, M. (2020). Statistical Inference for Burr Type X Distribution using Geometric Process in Accelerated Life Testing Design for Time censored data. *Pakistan Journal of Statistics and Operation Research, 16(3),* 577-586.

[33] Xin, H., Liu, Z., Lio, Y., & Tsai, T. R. (2020). Accelerated Life Test Method for the Doubly Truncated Burr Type XII Distribution. *Mathematics*, *8*(2), 162.

[34] Abd-El-Raheem, A. M., Almetwally, E. M., Mohamed, M., & Hafez, E. H. (2021). Accelerated life tests for modified Kies exponential lifetime distribution: binomial removal, transformers turn insulation application and numerical results. *AIMS Mathematics*, *6*(5), 5222-5255.

[35] Balakrishnan, N., & Sandhu, R. A. (1995). A simple simulation algorithm for generating progressive Type-II censored samples. *The American Statistician, 49(2),* 229-230.