Design of One-sided Modified *S* Control Charts for Monitoring a Finite Horizon Process

Mei Tuan Teng[1], Sin Yin Teh[2], Khai Wah Khaw[3*], XinYing Chew[4], Wai Chung Yeong[5]

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[1,2,3]School of Management, Universiti Sains Malaysia, 11800 Penang, Malaysia
[4]School of Computer Sciences, Universiti Sains Malaysia, 11800 Penang, Malaysia
[5]School of Mathematical Sciences, Sunway University, 47500 Petaling Jaya, Malaysia
[1] teng.meituan@gmail.com, [2]tehsyin@usm.my, [3]khaiwah@usm.my, [4]xinying@usm.my, [5]waichungy@sunway.edu.my
*Corresponding Author

Abstract

Control charting techniques are widely used in the manufacturing industry. One of the common charts that are used to monitor process variability is the S control chart. Finite horizon process monitoring has received great attention in the last decade. In the current literature, no attempt has been made to monitor the process variability in a finite horizon process. To fill this gap in research, this paper proposes two one-sided modified S charts for monitoring the standard deviation in a finite horizon process. The performance of the proposed charts is evaluated in terms of the truncated average run length and truncated standard deviation of the run-length criteria. The numerical performances of the proposed charts are shown with the selection of numerous process shifts. The effect of the sample sizes, the number of inspections and the process shifts are studied.

Keywords: Control charting technique, finite horizon process, process variability, statistical quality control, truncated average run length

I. Introduction

Control charting techniques are common-used techniques in the process of signal detection to improve the quality of process monitoring [1 – 2]. Generally, two types of variables control charts are commonly implemented in-process monitoring, i.e. (1) process mean and (2) process standard deviation. Shewhart $\bar{X}$ was introduced for monitoring the process mean whereas *R* and *S* charts are used to monitor the process variability of the quality characteristic. The statistical control charts are widely used in the manufacturing and service industries to monitor both products and processes. For instance, Li et al. [3] implemented the multivariate weighted Poisson chart to monitor the two-dimensional telecommunications data. Chew et al. [4] considered the multivariate variable parameter coefficient of variation chart for monitoring the spring manufacturing process. The yoghurt cup filling process was monitored by Shongwe et al. [5] using the run rules chart. Chew et al. [6] applied the run rules chart for the steel sleeve measurement while Castagliola et al. [7] utilized the variable sample size chart for monitoring the die casting hot chamber process.

Numerous charts were proposed for monitoring the process variance in the last few decades.

Page [8] proposed a one-sided cumulative sum (CUSUM) *S* chart based on the subgroup range. An exponentially weighted moving average (EWMA) *S* chart was then developed by Crowder and Hamilton [9] based on the logarithmic transformation of sample variance. Shu and Jiang [10] discussed a new EWMA chart to monitor the process standard deviation, where the latter chart's performances are better than the former chart. Klein [11] modified the conventional *S* chart by approaching the equal and unequal tail chi-square distribution probabilities. The modified *S* chart from Khoo [12] can circumvent the drawbacks of the conventional *S* chart by adjusting the type I error. Rakitzis and Antzoulakos [13] improved the *S* chart by varying the sample size and sampling interval (VSSI). This chart was then surpassed by the new proposed $VSSI_t$ chart with three sample intervals [14]. Kuo and Lee [15] designed the *S* chart with one of the best adaptive strategy, i.e. variable parameter strategy while Adeoti and Olaomi [16] investigated a moving average *S* chart. Moreover, Abujiya et al. [17] and Costa and Neto [18] suggested a new combined Shewhart-CUSUM *S* and variable charting statistics *S* charts, respectively.

Most of the traditional control charting techniques are implemented for a mass production process, which means its lot sizes are large. This process can be called an infinite horizon process. When dealing with low-volume and high-variety production, a finite horizon process arises. The finite horizon process has received great attention in the last decade as many companies have become more flexible and specialized in their products and services, based on the frequent changes in demand. A well-known example of the finite horizon process is the Just-in-Time manufacturing setting, which emphasizes the minimization of surplus inventory, time of waiting and costs of overproduction. Numerous research works on a finite horizon process were extended to a wide variety of control charts. For instance, CUSUM and variable sampling interval charts for monitoring the process mean in a finite horizon process were discussed by Nenes and Tagaras [19] and Nenes et al. [20], respectively. The Shewhart *t* and the EWMA *t* charts for a finite horizon process were suggested by Celano et al. [21] to address the problems of the estimation error of the process standard deviation due to the availability of limited reliable historical data. According to Amdouni et al. [22], their proposed variable sampling interval coefficient of variation short-run chart has better performance than the standard coefficient of variation short-run chart of Castagliola et al. [23]. More recently, Chong et al. [24] and Chew et al. [25] recommended a variable sample size Hotelling's T^2 and run rules T^2 charts for monitoring the multivariate finite horizon process.

Tuprah and Ncube [26] indicated that the *S* chart has better performance than the *R* chart, for the detection of small to moderate shifts in the standard deviation, when the sample sizes increase. Thus, monitoring the process standard deviation in a finite horizon process is considered very important in Statistical Process Control. However, none of the studies is available on the *S* chart for a finite horizon process in the existing literature. This makes it difficult for quality engineers who wish to monitor the process standard deviation in a finite horizon process. This paper aims to fill the gap in research by proposing the modified *S* charts in a finite horizon process. The numerical performance of the proposed charts will be measured in terms of the truncated average run length (TARL) and truncated standard deviation of the run length (TSDRL) criteria, for monitoring both the upward and downward shifts. The remainder of the sections are organized as follows: Section 2 shows the properties of the classical *S* chart. Section 3 discusses the design of the two one-sided modified *S* charts for monitoring a finite horizon process. The derivations of the formulae and algorithms to compute the TARL and TSDRL values are illustrated in Section 4. Statistical performances of the proposed charts are enumerated in Section 5. Lastly, the research findings and suggestions for future research are shown in the last section.

II. Methods

I. Properties of Classical *S* Chart

Montgomery [27] introduced the process variability that can be monitored with the sample standard deviation of the classical *S* chart, which is based on the $\pm 3\sigma$ limits underlying normal distribution.

Let $\{X_{i1}, X_{i2}, \ldots, X_{in}\}$, for i = 1, 2, …, I, be a sample of n independent random variables, having a normal $N\left(\mu,\sigma_0^{\ 2}\right)$ distribution, where μ is the process mean and $\sigma_0^{\ 2}$ is the nominal process variance. When the standard deviation, σ of a process, is known, the upper control limit (UCL), centerline (CL) and lower control limit (LCL) can be computed as [27]:

$$\mathrm{UCL} = c_4\sigma + 3\sigma\sqrt{1-c_4^{\ 2}} \tag{1}$$

$$\mathrm{CL} = c_4\sigma \tag{2}$$

and

$$\mathrm{LCL} = c_4\sigma - 3\sigma\sqrt{1-c_4^{\ 2}}, \tag{3}$$

where c_4 is a constant that depends on the sample size n, which can be obtained from Montgomery [27].

If σ is unknown, then it can be estimated through analyzing past data. Assume that there are m preliminary samples, each of size n and let S_i be the standard deviation of the i^{th} sample. The average sample standard deviation is

$$\overline{S} = \frac{1}{m}\sum_{i=1}^{m} S_i, i = 1, 2, \ldots, m \tag{4}$$

The statistics $\dfrac{\overline{S}}{c_4}$ is an unbiased estimator of σ. Thus, the UCL, CL and LCL of the *S* chart can be obtained as

$$\mathrm{UCL} = \overline{S} + 3\frac{\overline{S}}{c_4}\sqrt{1-c_4^{\ 2}} \tag{5}$$

$$\mathrm{CL} = \overline{S} \tag{6}$$

$$\mathrm{LCL} = \overline{S} - 3\frac{\overline{S}}{c_4}\sqrt{1-c_4^{\ 2}} \tag{7}$$

Subsequently, the sample standard deviation of the *S* chart can be denoted as

$$S_i = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}\left(X_{ij} - \overline{X}_i\right)^2} \tag{8}$$

where $\overline{X}_i$ is the mean of sample i. An out-of-control signal is indicated when S_i is being plotted outside the UCL or LCL.

II. The One-Sided Modified *S* Charts in a Finite Horizon Process

An industrial finite horizon process is scheduled to generate a small lot of N parts with finite length H, where I refers to the number of scheduled inspections within H. Subsequently, the sampling frequency between two consecutive inspections is denoted as $h = H/(I+1)$ hours due to absence of inspection at the end of the process. With the subgroup $\{X_{i1}, X_{i2}, \ldots, X_{in}\}$ defined in Section 2, the UCL and LCL of the one-sided modified *S* control charts with in-control ARL (ARL_0) equal to $\dfrac{1}{\theta}$ are given as [11]

$$\mathrm{UCL} = \sqrt{\frac{\sigma^2\left(\chi^2_{n-1,\theta}\right)}{n-1}} \tag{9}$$

and

$$\mathrm{LCL}=\sqrt{\frac{\sigma^2\left(\chi^2_{n-1,1-\theta}\right)}{n-1}}, \tag{10}$$

where $\chi^2_{n-1,\theta}$ and $\chi^2_{n-1,1-\theta}$ denote the $100\theta^{\text{th}}$ percentile of the chi-square distribution with n - 1 degrees of freedom and σ^2 is the process variance. Note that $0<\theta<1$ is selected to satisfy the desired in-control TARL (TARL_0) value. Additionally, if the computed LCL value is negative, then the value is rounded up to zero since $S_i > 0$.

III. Performance Measures of the Modified *S* Charts in a Finite Horizon Process

The statistical performance of the control chart is measured by average run length (ARL) and standard deviation of the run length (SDRL) criteria in an infinite horizon process monitoring. For monitoring a finite horizon process, the ARL criterion is replaced by TARL and TSDRL criteria. This is because the chart's performance measure must be a function of the finite number *I* of scheduled inspections. TARL can be denoted as the average number of plotted samples in the chart up till a signal is given or up till the process is completed, whichever occurs first. According to Nenes and Tagaras [19], the TARL value equals *I* + 1 if the production run is completed without detecting any signal in the *I* inspections. The TARL and TSDRL values of the modified *S* chart are given as

$$\mathrm{TARL}=(1-\beta)\sum_{l=1}^{I} l\beta^{l-1}+(I+1)\beta^{I}=\frac{1-\beta^{I+1}}{1-\beta} \tag{11}$$

and

$$\mathrm{TSDRL}=\frac{\sqrt{\beta\left(1-\beta^{2I+1}\right)-(1-\beta)\beta^{I+1}(1+2I)}}{1-\beta}, \tag{12}$$

respectively, where β represents the probability of Type-II error of the chart and its value can be obtained as $\beta=1-F_{\chi^2}(\mathrm{LCL})$ for the downward case and $\beta=F_{\chi^2}(\mathrm{UCL})$ for the upward case. Here, $F_{\chi^2}(\cdot)$ is the cumulative distribution function (cdf) of a non-central chi-square random variable.

III. Results

Tables 1 - 4 display the TARL_1 and TSDRL_1 values for the upward and downward modified *S* charts in a finite horizon process, for monitoring the upward and downward shifts, when sample size $n \in$ {5, 7, 10, 15}, $I \in$ {10, 20, 30, 40, 50}, $\delta \in$ {1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0} (for the upward case) and $\delta \in$ {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9} (for the downward case). Note that when $\delta = 1.0$ indicates the process is in-control, where the $\mathrm{TARL}_0 = I$. The results show that when the θ value, which is used to compute the UCL and LCL, is decreasing when the *I* value increases. For example, in Table 1, θ = 0.0193, 0.0050, 0.0022, 0.0013 and 0.0008 when *I* = 10, 20, 30, 40 and 50. In addition, another notable trend observed is that the larger shift δ provides smaller TARL_1 and TSDRL_1 values regardless of the *I* and *n* values. An example is shown for monitoring the upward shifts, in Tables 1 and 2, for $n \in$ {5, 7, 10, 15} and *I* = 10, where the $\mathrm{TARL}_1 \in$ {(8.81, 8.56, 8.23, 7.77), (3.64, 2.85, 2.19, 1.65), (1.76, 1.42, 1.19, 1.06)} and $\mathrm{TSDRL}_1 \in$ {(3.34, 3.45, 3.56, 3.68), (2.79, 2.20, 1.60, 1.03), (1.15, 0.77, 0.48, 0.25)}, when $\delta \in$ {1.1, 1.5, 2.0}. Another example is shown for monitoring the downward shifts, in Tables 3 and 4, for $I \in$ {10, 20, 30, 40, 50} and *n* = 10, the $\mathrm{TARL}_1 \in$ {(1.53, 2.89, 4.50, 6.30, 8.76), (5.01, 12.02, 20.21, 28.47, 37.79), (9.06, 18.85, 28.79, 38.64, 48.70)} and $\mathrm{TSDRL}_1 \in$ {(0.91, 2.25, 3.95, 5.74, 8.15), (3.46, 7.35, 10.91, 14.23, 17.11), (3.20, 5.03, 6.37, 7.67, 8,54)}, when $\delta \in$ {0.5, 0.7, 0.9}. When *n* and δ values are fixed, the TARL_1 and TSDRL_1 values increase consistently by increasing the *I* value. For example, in Tables 1 and 2, for $n = 10$, $\delta = 1.3$, the $TARL_1 \in \{4.10, 7.57, 11.14, 14.40, 18.16\}$ and $TSDRL_1 \in \{3.05, 6.02, 9.03, 11.88, 14.98\}$, when $I \in \{10, 20, 30, 40, 50\}$.When I and δ values are fixed while n value increases, the $TARL_1$ value decreases. For example, in Table 3, for $I = 30$ and $\delta = 0.8$, $TARL_1 \in \{28.68, 27.73, 26.03, 22.70\}$ when $n \in \{5, 7, 10, 15\}$. Figures 1 and 2 present the graphical view of $TARL_1$ values for the upward and downward modified *S* charts.

IV. Conclusion

In the existing literature, no attempt has been made to monitor the process variability in a finite horizon process. This paper proposes the one-sided upward and downward modified *S* charts for monitoring a finite horizon process. The performance of the proposed charts is evaluated in terms of the $TARL_1$ and $TSDRL_1$ criteria. The formulas of control limits for the modified *S* charts, $TARL_1$ and $TSDRL_1$ are discussed. Different parameter combinations, in terms of the sample size, the number of inspections and process shifts are applied to the proposed charts, for both the upward and downward cases. The results showed that the sample size and the number of inspections affected the $TARL_1$ and $TSDRL_1$ values, for monitoring the different upward and downward process shifts in a finite horizon process. Additionally, the two one-sided modified *S* charts can provide unbiased performance, in terms of $TARL_1$ and $TSDRL_1$ criteria. In future research, the study of proposed charts can be extended in the case of estimated parameters.

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Declaration of Conflict Interests

The Author(s) declare(s) that there is no conflict of interest.

Table 1. $TARL_1$ values of the upward modified *S* chart when $I \in \{10, 20, 30, 40, 50\}$, $n \in \{5, 7, 10, 15\}$ and $\delta \in \{1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0\}$

			δ										
θ	I	n	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.0193	10	5	10.00	8.81	7.31	5.82	4.58	3.64	2.98	2.51	2.18	1.94	1.76
		7	10.00	8.56	6.71	5.00	3.71	2.85	2.30	1.94	1.70	1.54	1.42
		10	10.00	8.23	5.98	4.10	2.89	2.19	1.78	1.53	1.37	1.26	1.19
		15	10.00	7.77	5.03	3.12	2.14	1.65	1.39	1.24	1.15	1.09	1.06
0.0050	20	5	19.98	18.06	15.00	11.49	8.41	6.15	4.64	3.66	3.01	2.56	2.24
		7	19.98	17.64	13.77	9.64	6.48	4.50	3.35	2.65	2.20	1.90	1.69
		10	19.98	17.08	12.19	7.57	4.70	3.20	2.39	1.93	1.64	1.46	1.33
		15	19.98	16.22	10.03	5.37	3.18	2.19	1.70	1.43	1.27	1.17	1.11
0.0022	30	5	30.00	27.58	23.10	17.44	12.25	8.50	6.11	4.63	3.68	3.05	2.61
		7	30.00	27.04	21.27	14.48	9.18	6.00	4.24	3.22	2.59	2.18	1.90
		10	30.00	26.28	18.83	11.14	6.41	4.08	2.91	2.25	1.86	1.61	1.44
		15	30.00	25.12	15.39	7.59	4.11	2.66	1.96	1.59	1.37	1.24	1.16
0.0013	40	5	39.95	37.03	31.08	23.13	15.74	10.51	7.30	5.39	4.20	3.42	2.89
		7	39.95	36.35	28.63	19.03	11.55	7.24	4.95	3.67	2.89	2.39	2.06
		10	39.95	35.41	25.32	14.40	7.85	4.79	3.31	2.50	2.02	1.72	1.52
		15	39.95	33.93	20.55	9.54	4.88	3.02	2.16	1.70	1.44	1.29	1.19
0.0008	50	5	49.99	46.79	39.66	29.47	19.68	12.75	8.61	6.21	4.75	3.81	3.17
		7	49.99	46.03	36.67	24.20	14.25	8.62	5.72	4.14	3.20	2.61	2.21
		10	49.99	44.97	32.54	18.16	9.49	5.58	3.73	2.75	2.19	1.83	1.60
		15	49.99	43.27	26.46	11.80	5.73	3.41	2.36	1.82	1.52	1.34	1.22

Table 2. TSDRL1 values of the upward modified *S* chart when $I \in \{10, 20, 30, 40, 50\}$, $n \in \{5, 7, 10, 15\}$ and $\delta \in \{1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0\}$

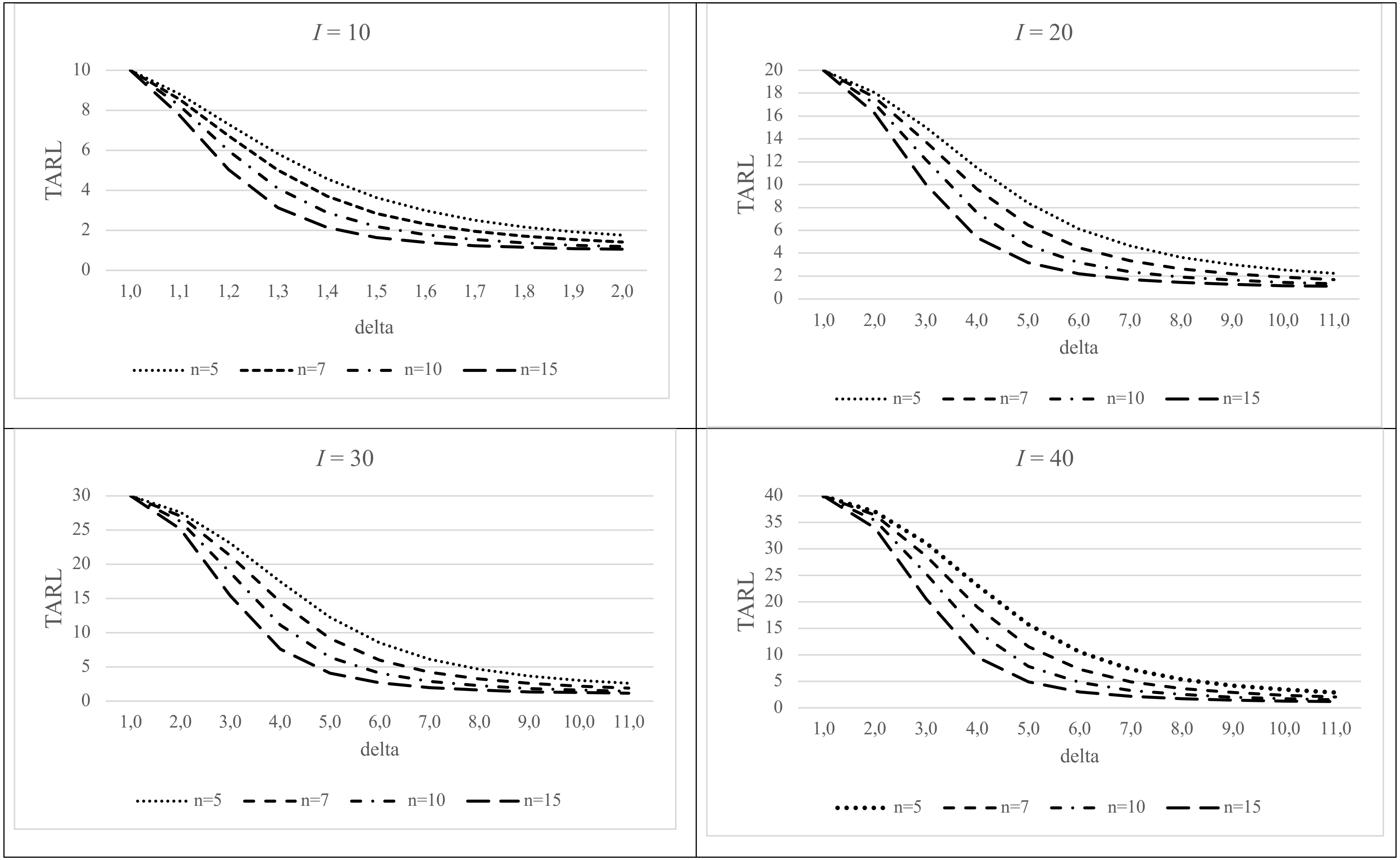
			δ										
θ	I	n	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.0193	10	5	2.48	3.34	3.75	3.68	3.29	2.79	2.30	1.90	1.59	1.34	1.15
		7	2.48	3.45	3.77	3.46	2.83	2.20	1.71	1.35	1.09	0.91	0.77
		10	2.48	3.56	3.71	3.05	2.23	1.60	1.18	0.90	0.71	0.58	0.48
		15	2.48	3.68	3.47	2.42	1.55	1.03	0.73	0.54	0.41	0.32	0.25
0.0050	20	5	3.60	5.70	7.12	7.29	6.43	5.16	3.99	3.09	2.45	2.00	1.67
		7	3.60	6.00	7.33	6.88	5.38	3.87	2.79	2.08	1.62	1.31	1.08
		10	3.60	6.33	7.36	6.02	4.04	2.64	1.83	1.34	1.03	0.82	0.67
		15	3.60	6.73	7.00	4.59	2.62	1.62	1.09	0.79	0.59	0.45	0.36
0.0022	30	5	4.41	7.69	10.31	10.92	9.56	7.41	5.47	4.08	3.14	2.50	2.05
		7	4.41	8.17	10.77	10.36	7.88	5.38	3.70	2.67	2.03	1.61	1.31
		10	4.41	8.74	10.98	9.03	5.74	3.54	2.35	1.68	1.26	0.99	0.80
		15	4.41	9.45	10.60	6.72	3.57	2.10	1.37	0.96	0.71	0.54	0.43
0.0013	40	5	5.23	9.66	13.49	14.51	12.54	9.39	6.69	4.85	3.67	2.88	2.34
		7	5.23	10.32	14.19	13.75	10.15	6.64	4.42	3.13	2.34	1.83	1.47
		10	5.23	11.11	14.58	11.88	7.20	4.26	2.76	1.93	1.44	1.11	0.89
		15	5.23	12.12	14.13	8.64	4.35	2.47	1.58	1.09	0.80	0.61	0.47
0.0008	50	5	5.74	11.23	16.42	18.15	15.71	11.54	8.01	5.68	4.22	3.27	2.63
		7	5.74	12.06	17.44	17.31	12.63	8.02	5.20	3.61	2.66	2.05	1.64
		10	5.74	13.07	18.12	14.98	8.82	5.05	3.19	2.20	1.61	1.23	0.98
		15	5.74	14.41	17.81	10.80	5.21	2.87	1.80	1.23	0.89	0.67	0.52

Table 3. TARL1 values of the upward modified *S* chart when $I \in \{10, 20, 30, 40, 50\}$, $n \in \{5, 7, 10, 15\}$ and $\delta \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$

			δ									
θ	I	n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0193	10	5	1.00	1.03	1.47	2.62	4.46	6.37	7.87	8.89	9.56	10.00
		7	1.00	1.00	1.06	1.47	2.57	4.49	6.62	8.28	9.35	10.00
		10	1.00	1.00	1.00	1.08	1.53	2.76	5.01	7.40	9.06	10.00
		15	1.00	1.00	1.00	1.00	1.10	1.64	3.23	6.07	8.61	10.00
0.0050	20	5	1.00	1.37	3.13	6.94	11.60	15.16	17.39	18.71	19.50	19.98
		7	1.00	1.01	1.38	2.83	6.27	11.23	15.34	17.86	19.24	19.98
		10	1.00	1.00	1.02	1.40	2.80	6.44	12.02	16.44	18.85	19.98
		15	1.00	1.00	1.00	1.03	1.42	2.95	7.38	13.94	18.20	19.98
0.0022	30	5	1.01	1.98	5.71	13.03	20.16	24.70	27.26	28.68	29.51	30.00
		7	1.00	1.05	1.89	4.77	11.39	19.35	24.79	27.73	29.23	30.00
		10	1.00	1.00	1.08	1.85	4.50	11.33	20.21	26.03	28.79	30.00
		15	1.00	1.00	1.00	1.09	1.82	4.59	12.66	22.70	28.04	30.00
0.0013	40	5	1.04	2.69	8.70	19.57	28.78	34.13	36.99	38.55	39.43	39.95
		7	1.00	1.12	2.44	6.92	16.92	27.57	34.15	37.48	39.12	39.95
		10	1.00	1.00	1.16	2.31	6.30	16.51	28.47	35.51	38.64	39.95
		15	1.00	1.00	1.00	1.15	2.21	6.28	18.14	31.44	37.78	39.95
0.0008	50	5	1.10	3.71	12.99	27.78	38.48	44.17	47.08	48.62	49.49	49.99
		7	1.00	1.23	3.20	9.96	24.04	36.95	44.11	47.55	49.18	49.99
		10	1.00	1.00	1.26	2.91	8.76	23.14	37.79	45.48	48.70	49.99
		15	1.00	1.00	1.00	1.24	2.71	8.52	24.96	41.00	47.81	49.99

Table 4. TSDRL1 values of the upward modified *S* chart when $I \in \{10, 20, 30, 40, 50\}$, $n \in \{5, 7, 10, 15\}$ and $\delta \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$

			δ									
θ	I	n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0193	10	5	0.00	0.19	0.84	2.01	3.24	3.76	3.66	3.30	2.87	2.48
		7	0.00	0.01	0.24	0.84	1.96	3.25	3.77	3.55	3.02	2.48
		10	0.00	0.00	0.03	0.29	0.91	2.13	3.46	3.74	3.20	2.48
		15	0.00	0.00	0.00	0.05	0.34	1.03	2.50	3.72	3.42	2.48
0.0050	20	5	0.02	0.71	2.58	5.67	7.31	7.08	6.16	5.17	4.31	3.60
		7	0.00	0.10	0.73	2.27	5.24	7.26	7.04	5.85	4.62	3.60
		10	0.00	0.00	0.16	0.75	2.25	5.35	7.35	6.64	5.03	3.60
		15	0.00	0.00	0.01	0.18	0.77	2.39	5.92	7.31	5.60	3.60
0.0022	30	5	0.09	1.39	5.11	9.88	10.92	9.66	7.98	6.51	5.33	4.41
		7	0.00	0.24	1.30	4.22	9.16	10.97	9.62	7.56	5.77	4.41
		10	0.00	0.01	0.30	1.25	3.95	9.13	10.91	8.91	6.37	4.41
		15	0.00	0.00	0.02	0.31	1.22	4.05	9.73	10.43	7.24	4.41
0.0013	40	5	0.19	2.13	7.94	13.90	14.16	12.00	9.70	7.80	6.34	5.23
		7	0.00	0.37	1.88	6.34	13.04	14.38	11.99	9.17	6.90	5.23
		10	0.00	0.02	0.42	1.73	5.74	12.87	14.23	11.03	7.67	5.23
		15	0.00	0.00	0.04	0.42	1.63	5.72	13.48	13.36	8.81	5.23
0.0008	50	5	0.32	3.18	11.73	18.00	16.88	13.74	10.88	8.66	7.00	5.74
		7	0.00	0.53	2.66	9.25	17.27	17.37	13.79	10.28	7.63	5.74
		10	0.00	0.04	0.57	2.36	8.15	17.01	17.11	12.60	8.54	5.74
		15	0.00	0.00	0.06	0.55	2.15	7.92	17.50	15.79	9.92	5.74


I = 10
TARL
10
8
6
4
2
0
1,0 1,1 1,2 1,3 1,4 1,5 1,6 1,7 1,8 1,9 2,0
delta
n=5 n=7 n=10 n=15
I = 20
TARL
20
18
16
14
12
10
8
6
4
2
0
1,0 2,0 3,0 4,0 5,0 6,0 7,0 8,0 9,0 10,0 11,0
delta
n=5 n=7 n=10 n=15
I = 30
TARL
30
25
20
15
10
5
0
1,0 2,0 3,0 4,0 5,0 6,0 7,0 8,0 9,0 10,0 11,0
delta
n=5 n=7 n=10 n=15
I = 40
TARL
40
35
30
25
20
15
10
5
0
1,0 2,0 3,0 4,0 5,0 6,0 7,0 8,0 9,0 10,0 11,0
delta
n=5 n=7 n=10 n=15

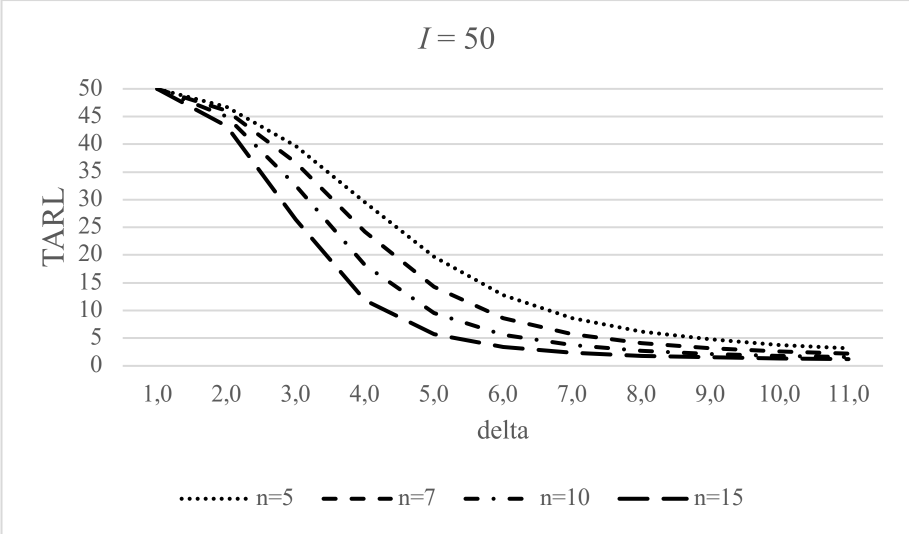
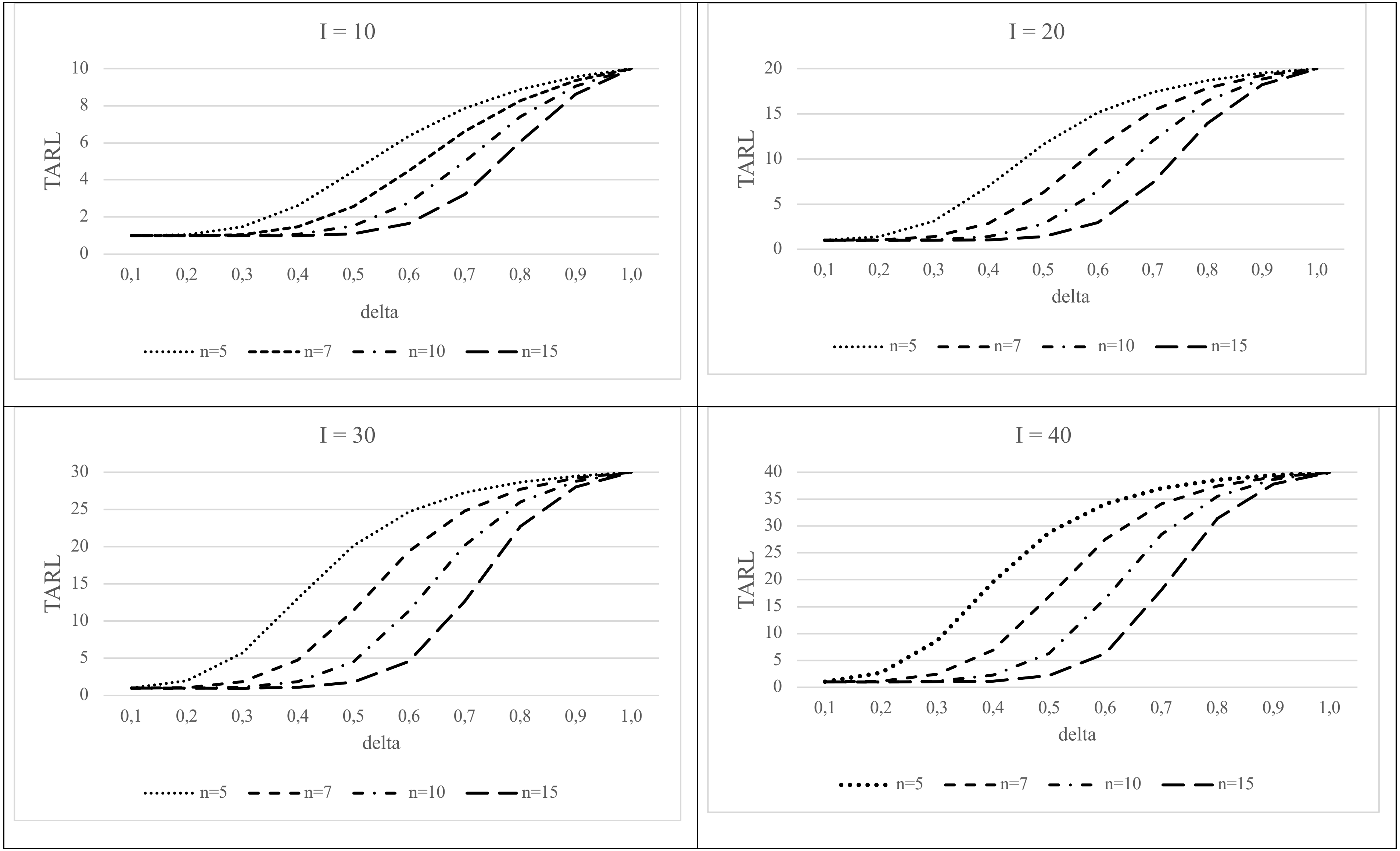


Figure 1. Comparison of TARL1 values for the upward modified *S* chart, when $I \in \{10, 20, 30, 40, 50\}$, $n \in \{5, 7, 10, 15\}$ and $\delta \in \{1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0\}$


I = 10
TARL
0
2
4
6
8
10
0,1 0,2 0,3 0,4 0,5 0,6 0,7 0,8 0,9 1,0
delta
n=5 n=7 n=10 n=15
I = 20
TARL
0
5
10
15
20
0,1 0,2 0,3 0,4 0,5 0,6 0,7 0,8 0,9 1,0
delta
n=5 n=7 n=10 n=15
I = 30
TARL
0
5
10
15
20
25
30
0,1 0,2 0,3 0,4 0,5 0,6 0,7 0,8 0,9 1,0
delta
n=5 n=7 n=10 n=15
I = 40
TARL
0
5
10
15
20
25
30
35
40
0,1 0,2 0,3 0,4 0,5 0,6 0,7 0,8 0,9 1,0
delta
n=5 n=7 n=10 n=15

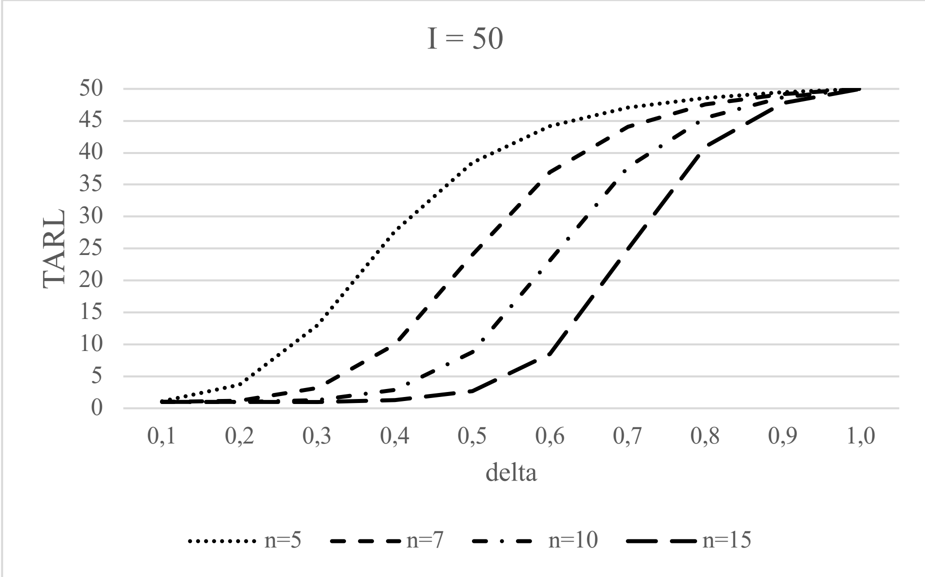


Figure 2. Comparison of $TARL_1$ values for the downward modified *S* chart, when $I \in \{10, 20, 30, 40, 50\}$, $n \in \{5, 7, 10, 15\}$ and $\delta \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$

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