Availability And Performance Analysis Of Computer Network With Dual-Server Using Gumbel-Hougaard Family Copula Distribution

Ismail Tukur

•

Department of Art and Humanities, Kano State Polytechnic, Kano, Nigeria.
ismailatukur35@gmail.com

Kabiru H. Ibrahim

•

Department of Mathematics, Aminu Kano College of Islamic and Legal Studies Kano, Nigeria
kbrhamis032@gmial.com

Muhammad Salihu Isa

•

Department of Mathematics, Federal University Dutse, Jigawa, Nigeria
salihu.muhd.isa@gmail.com

Ibrahim Yusuf

•

Department of Mathematical Sciences, Bayero University, Kano, Nigeria
iyusuf.mth@buk.edu.ng

Abstract

The determination of this paper is to study reliability measures and routine analysis of computer network, which is a combination of four subsystems A, B, C and D and all the subsystem connected in series parallel pattern, the subsystem A is client, the subsystem B is load balancer and subsystem C is servers which is divided in to two subsystem (i.e. subsystem C1 and subsystem C2) and C1andC2 served as computer servers together with two unit each and working 1-out-of-2: G policy, and subsystem D is centralized server. The system has two types of failure, degraded and complete failure. The system can completely fail due to failure of one of the following subsystems A, B, C and D. The system is at partial failed state if at least one unit is working in either subsystem C1 or subsystem C2.The system is examined using supplementary variables techniques and Laplace transform. General distribution and copula family are employed to restore degraded and complete failed state respectively. Calculated results have been highlighted by the means of tables and graphs.

Keywords: Reliability, Availability, Sensitivity, (MTTF) mean time to system failure, Gumbel − Hougaard, family, cost Analysis and supplementary variable techniques.

I. Introduction

The study of reliability modeling was started During the World War II in 1939, later then, substantial efforts have been made in this direction to create the comprehensive theoretical background for reliability modeling. The discipline is mainly anxious with requirement and valuations of the probability of device performing its purpose sufficiently for the period intended under the encountered functioning condition. Reliability has introduced new dimensions in recent years because of the difficulty of larger systems and suggestions of their failure. Undependability in the modern age of technology causes inefficiency of a system, overgenerous maintenance and can also risk human life. In today's technological world, nearly everyone be subject to upon the continued function of a wide array of machinery and equipment for our safety, security, mobility and financial welfare. We receive our electronic agreements from illuminations, hospital monitoring control, next generation aircraft, nuclear power plants data exchange system and aerospace application, to function whenever we need them when they fail, the results can be a catastrophic, injurious or eventual loss of life. The theory of reliability is the scientific discipline that studies the general regularity that must be maintained under design, research, manufacture, acceptance, and use of units/ components to acquire the highest effectiveness of their use.

The most common vital mode of redundancy is k-out-of-n redundancy. This type of redundancy is more branded into two classifications, these are k-out-of-n: G and k-out-of-n: F. A k-out-of-n: G redundancy indicates that for successful operation of the system at least k units out of n units are essential to be good (i.e. essential to work properly). If less than k units are good then the system fails. A k-out-of-n: F redundancy indicates that if k units out of n units have failed then the system has failed. The system reliability has been widely studied and used by numerous authors like, Yusuf et al [1] studied performance analysis of multi computer system consisting of active parallel homogeneous, Abubakar and Singh [2] have studied assessment and performance of industrial system using Gumbel Hougaard copula approach, Singh et al. [3] have studied the reliability characteristic for Internet data center with a redundant server including a main mail sever, M. Ram et.al. [4] Have discussed the reliability of a system with different failure and common cause failure under the preemptive resume policy using Gumbel-Hougaard family copula distribution. Minjae Park [5] analyzed the multi-component system with imperfect repair during warranty using renewal process. Zhang [6] analyzed on computer network reliability analysis based on intelligent cloud computing method. Kudeep et al [7] studied tree topology network environment analysis under reliability approach, nonlinear. Muhammad et al [8] studied cost benefit analysis of tree different series parallel dynamo configurations. Dillon et al. [9] have discussed the common cause's failure analysis of k-out-of-n: G systems which consisting repairable units. Ibrahim Yusuf [10] evaluates the performance of a repairable system with the concept of minor deterioration under imperfect repair. Tseng- Chang Yen et al. [11] studied Reliability and sensitivity analysis of the controllable repair system with warm standbys and working breakdown. v. v. Singh et al [12] evaluates the performance and cost assessment of repairable complex system with two subsystems connected in series configuration. v. v. Singh and Jyoti Gulati. [13] Studied performance assessment of computer Centre at Yobe state university Nigerian under different policies using copula. Kabiru H. Ibrahim et al. [14] have studied the availability and cost analysis of complex tree topology of computer network with multi-server using Gumbel-Hougaard family approach. Geon Yoon, Dae Hyun et al [15] focused on ring topology-based redundancy Ethernet for industrial network. Pratap et al [16] have examined on the assessment of complex system with two subsystems and multi types failure and repair. Rawal et al. [17] have discussed the reliability of Internet Data Centre having one mail server and one redundant server especially for the use of Interne.

Ibrahim Yusuf and Hussaini [18] examine a three-unit redundant system with three types of failure and general repair. Negi and Singh [19] studied reliability characteristics of a non-repairable complex system connected in series. C. K. Goel.et al. [20] performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics. However, researchers have considered different models and scrutinized the performances and availability of a complex system to forecast better performance. Most of the authors studied the complex repairable systems that are treated as single repair between two contiguous transition states. In the present paper, several reliability measures of a complex repairable system consist of four subsystems together with k-out-of-n; G configuration using two types of repair have been studied. The designed structure of the system consist of four subsystems A, B, C, and D. where C consist of C1 and C2 served as computer servers together with two unit each and working 1-out-of-2: G policy respectively. The system both are working in a series and parallel arrangement. Gumbel Hougaard family copula distribution employed for calculation and illustration.

Lastly, {S1, S3, S5, S6} represents the states of operation in degraded/partial failure while {S2, S4, S7, S_8 and S9} are entirely failed states and S_0 is at flawless operational state. The degraded points have repaired by general repair and completely failed states have repaired under Gumbel Hougaard family copula. the supplementary variables and Laplace transformation use analyze the system. Measures in reliability among availability, reliability, and MTTF and cost analysis are all treated by the means of tables and graphs.

II. State Description, Assumption and Notations

Table1: State description

State	State Description
S_0	The state S_0 indicates a perfect state in which both the subsystems are in outstanding working condition.
S_1	The state S_1 reveals a degraded state with partial failure in subsystem C, due to the failure of first server in subsystem C_1.
S_2	In this state S_2 indicate a complete failure state, due to the failure of first and second servers in subsystem C_1. The system is under repair using copula.
S_3	The state S_3 account a degraded state with partial failure in subsystem C, due to the failure of third server in subsystem C_2.
S_4	In this state S_4 indicate a complete failure state, due to the failure of third and fourth servers in subsystem C_2. The system is under repair using copula.
S_5	The state S_5 account a degraded state with partial failure in subsystem C, due to the failure of third server in subsystem C_2.
S_6	The state S_6 represents a degraded state with partial failure in subsystem C, due to the failure of first server in subsystem C_1.
S_7	The state S_7 represents a complete failed state, due to the failure of subsystem D.The system is under repair using copula.
S_8	The state S_8 represents a complete failed state, due to the failure of subsystem A. The system is under repair using copula.
S_9	The state S_9 represents a complete failed state, due to the failure of subsystem B.The system is under repair using copula.

The state description indicates, that S_0 is a perfect state where both subsystems are in good working condition. The states S_1, S_3, S_4, and S_6 are the operational states. The states S_2, S_4, S_8, and S_9 of this modeling are a complete failed state in which the system is inoperative mode.

I. Assumption

The conventions of the model is discussion below:

- At initial state, all subsystems are in good working condition.
- One unit of each subsystem C1 and subsystem C2 is necessary for operational mode.
- The system will be at complete fail state if both subsystems and C1 and C2 fail.
- Failure of all if subsystems A, B, C and D will damage the entire system.
- Failed unit of the system can be repaired when it is in operative or failed state.
- All failure rates are constant and assumed to follow exponential distribution
- The repairs follow a general distribution.
- It is assumed that a repaired system works like a new system and no damage appears during repair.
- As soon as the failed unit gets repaired, it is ready to perform the task.

Table2: Notations

t:	Time variable on time scale.
S:	A variable for Laplace transform for all expressions.
$\lambda_{s_1} / \lambda_{s_2}$:	Failure rates of servers of subsystems C_1
$\lambda_{s_3} / \lambda_{s_4}$:	Failure rates of servers of subsystems C_2
$\lambda_B / \lambda_C / \lambda_{Cs}$:	Failure rates of subsystems B, A, and D
$\phi(x) : \mu(z)$	Repair rates for all subsystems i.e. A, B, C and D
$u_0(x), \mu(y), \mu(\alpha)$:	Repair rates for complete failed states.
$p_i(x,t)$:	The probability that the system is in Si state at instant's' for I =0 to 9.
$p_i(s)$	Laplace transformation of state transition probability P (t).
$E_P(t)$	Expected profit during the time interval [0, t).
K1, K2:	Revenue and service cost per unit time in the interval [0, t) respectively.
$s_\phi(x)$	Standard repair distribution function $s_\phi(x) = \phi(x)\,\ell^{-\int_0^\infty \phi(x)\partial(x)}$
$u_0(x) = C_\theta(u_1(x), u_2(x))$	The expression of joint probability (failed state Si to good state S0) according to Gumbel Hougaard family copula is given as $C_\theta(u_1(x), u_2(x)) = \exp[x^\theta + \{\log\phi(x)\}]^{\frac{1}{\theta}}$ where, u1 = ϕ(x), and u2 = ex, where θ as a parameter, 1≤ θ≤ ∞.

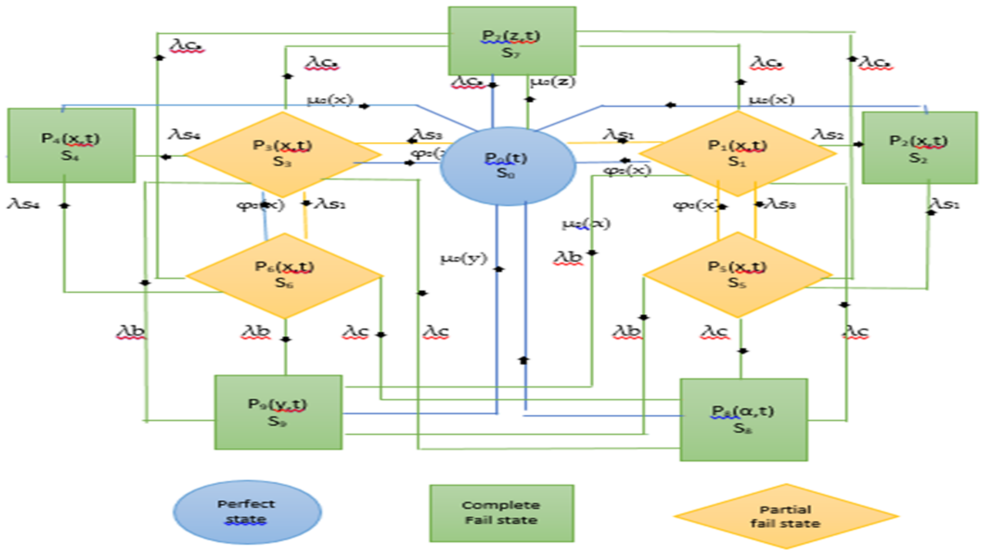

Figure 1: *Transition Diagram*

III. Formulation and Solution of Mathematical Model

By the related literature review, the following differential equation were generated for the mathematical classical.

$$\left(\frac{\partial}{\partial t}+\lambda_{s1}+\lambda_{s3}+\lambda_{Cs}\right)P_0(t)$$
$$=\int_0^\infty \Phi_0(x)P_1(x,t)dx+\int_0^\infty \Phi_0(x)P_3(x,t)dx+\int_0^\infty \mu_0 P_4(x,t)dx$$
$$+\int_0^\infty \mu_0 P_2(x,t)dx+\int_0^\infty \mu_0 P_7(z,t)dz+\int_0^\infty \mu_0 P_9(y,t)dy$$
$$+\int_0^\infty \mu_0 P_8(\alpha,t)d\alpha \quad (1)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_{s2}+\lambda_{s3}+\lambda_B+\lambda_c+\lambda_{cs}+\Phi(x)\right)P_1(x,t)=0 \quad (2)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)P_2(x,t)=0 \quad (3)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_{s4}+\lambda_{s1}+\lambda_B+\lambda_c+\lambda_{cs}+\Phi(x)\right)P_3(x,t)=0 \quad (4)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)P_4(x,t)=0 \quad (5)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_{s1}+\lambda_B+\lambda_c+\lambda_{cs}+\Phi(x)\right)P_5(x,t)=0 \quad (6)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_{s4}+\lambda_B+\lambda_c+\lambda_{cs}+\Phi(x)\right)P_6(x,t)=0 \quad (7)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\mu_0(z)\right)P_7(z,t)=0 \quad (8)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial \alpha}+\mu_0(\alpha)\right)P_8(\alpha,t)=0 \quad (9)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)P_9(y,t)=0 \quad (10)$$

Boundary conditions

$$P_1(0,t)=\lambda_{s1}P_0(t) \quad (11)$$
$$P_2(0,t)=\lambda_{21}\lambda_{s1}P_0(t) \quad (12)$$
$$P_3(0,t)=\lambda_{s3}P_0(t) \quad (13)$$
$$P_4(0,t)=\lambda_{s4}\lambda_{s3}P_0(t) \quad (14)$$
$$P_5(0,t)=\lambda_{s3}\lambda_{s1}P_0(t) \quad (15)$$
$$P_6(0,t)=\lambda_{s1}\lambda_{s3}P_0(t) \quad (16)$$
$$P_7(0,t)=(1+\lambda_{s1}+\lambda_{s3}+2\lambda_{s1}\lambda_{s3})\lambda_{cs}P_0(t) \quad (17)$$
$$P_8(0,t)=(\lambda_{s1}+\lambda_{s3}+2\lambda_{s1}\lambda_{s3})\lambda_c P_0(t) \quad (18)$$
$$P_9(0,t)=(\lambda_{s1}+\lambda_{s3}+2\lambda_{s1}\lambda_{s3})\lambda_B P_0(t) \quad (19)$$

I. Solution of the Model

Equation 20 to 38 below is obtained by taking Laplace transformation of equations 1 to 19 with the help of initial condition$P_0(0)=1$,

$$\begin{aligned}(s+\lambda_A+\lambda_B+2\lambda_C+2\lambda_D)\bar{P}_0(s) &= 1 + \int_0^\infty \Phi_0(x)P_1(x,s)dx+\int_0^\infty \Phi_0(x)P_3(x,s)dx+\int_0^\infty \mu_0(x)P_2(x,s)dx \\ &+ \int_0^\infty \mu_0(x)P_4(x,s)dx+\int_0^\infty \mu_0(z)P_7(z,s)dz+\int_0^\infty \mu_0(\alpha)P_8(\alpha,s)d\alpha \\ &+ \int_0^\infty \mu_0(y)P_9(y,s)dy\end{aligned} \quad (20)$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_{s2}+\lambda_{s3}+\lambda_c+\lambda_{cs}+\lambda_B+\Phi_0(x)\right)\bar{P}_1(x,s)=0 \quad (21)$$
$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\bar{P}_2(x,s)=0 \quad (22)$$
$$(s+\frac{\partial}{\partial x}+\lambda_{s4}+\lambda_{s1}+\lambda_c+\lambda_{cs}+\lambda_B+\Phi_0(x))\bar{P}_3(x,s)=0 \quad (23)$$
$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\bar{P}_4(x,s)=0 \quad (24)$$
$$\left(s+\frac{\partial}{\partial x}++\lambda_{s1}+\lambda_c+\lambda_{cs}+\lambda_B+\Phi_0(x)\right)\bar{P}_5(x,s)=0 \quad (25)$$
$$\left(s+\frac{\partial}{\partial x}++\lambda_{s4}+\lambda_c+\lambda_{cs}+\lambda_B+\Phi_0(x)\right)\bar{P}_6(x,s)=0 \quad (26)$$
$$\left(s+\frac{\partial}{\partial x}+\mu_0(z)\right)\bar{P}_7(z,s)=0 \quad * \quad (27)$$
$$\left(s+\frac{\partial}{\partial x}+\mu_0(\alpha)\right)\bar{P}_8(\alpha,s)=0 \quad (28)$$
$$\left(s+\frac{\partial}{\partial x}+\mu_0(y)\right)\bar{P}_9(y,s)=0 \quad (29)$$

The Laplace transformations of the boundary conditions are:

$$\bar{P}_1(0,s)=\lambda_{s1}\overline{P_0}(s) \quad (30)$$
$$\bar{P}_2(0,s)=\lambda_{s2}\lambda_{s1}\overline{P_0}(s) \quad (31)$$
$$\bar{P}_3(0,s)=\lambda_{s3}\overline{P_0}(s) \quad (32)$$
$$\bar{P}_4(0,s)=\lambda_{s4}\lambda_{s3}\overline{P_0}(s) \quad (33)$$
$$\bar{P}_5(0,s)=\lambda_{s3}\lambda_{s1}\overline{P_0}(s) \quad (34)$$
$$\bar{P}_6(0,s)=\lambda_{s1}\lambda_{s3}\bar{P}_0(s) \quad (35)$$
$$\bar{P}_7(0,s)=(\lambda_{s1}+\lambda_{s3}+2\lambda_{s1}\lambda_{s3})\,\lambda_{cs}\,\bar{P}_0(s) \quad (36)$$
$$\bar{P}_8(0,s)=(\lambda_{s1}+\lambda_{s3}+2\lambda_{s1}\lambda_{s3})\,\lambda_c\bar{P}_0(s) \quad (37)$$
$$\bar{P}_9(0,s)=(\lambda_{s1}+\lambda_{s3}+2\lambda_{s1}\lambda_{s3})\,\lambda_B\,\bar{P}_0(s) \quad (38)$$

Now solving equations (20) to (38) with the help of equations (11) to (19), yields,

$$\bar{P}_0(s)=\frac{1}{L(s)} \quad (39)$$
$$\bar{P}_1(s)=\frac{\lambda_{s1}}{L(s)}\left\{\frac{1-S_\phi(S+\lambda_{s2}+\lambda_{s3}+\lambda_B+\lambda_C+\lambda_{CS})}{S+\lambda_{s2}+\lambda_{s3}+\lambda_B+\lambda_C+\lambda_{CS}}\right\} \quad (40)$$

$$\bar{P}_2(s) = \frac{\lambda_{s1}\lambda_{s2}}{L(s)}\left\{\frac{1-S\mu_0(x)}{S}\right\} \tag{41}$$

$$\bar{P}_3(s) = \frac{\lambda_{s1}}{L(s)}\left\{\frac{1-S_\phi(S+\lambda_{s4}+\lambda_{s1}+\lambda_B+\lambda_C+\lambda_{CS})}{S+\lambda_{s4}+\lambda_{s1}+\lambda_B+\lambda_C+\lambda_{CS}}\right\} \tag{42}$$

$$\bar{P}_4(s) = \frac{\lambda_{s4}\lambda_{s3}}{L(s)}\left\{\frac{1-S\mu_0(x)}{S}\right\} \tag{43}$$

$$\bar{P}_5(s) = \frac{\lambda_{s1}\lambda_{s3}}{L(s)}\left\{\frac{1-S_\phi(S+\lambda_{s1}+\lambda_B+\lambda_C+\lambda_{CS})}{S+\lambda_{s1}+\lambda_B+\lambda_C+\lambda_{CS}}\right\} \tag{44}$$

$$\bar{P}_6(s) = \frac{\lambda_{s1}\lambda_{s3}}{L(s)}\left\{\frac{1-S_\phi(S+\lambda_{s4}+\lambda_B+\lambda_C+\lambda_{CS})}{S+\lambda_{s4}+\lambda_B+\lambda_C+\lambda_{CS}}\right\} \tag{45}$$

$$\bar{P}_7(s) = \frac{\lambda_{cs}}{L(s)}\,(1+\lambda_{s1}+\lambda_{s3}+2\lambda_{s1}\lambda_{s3})\left\{\frac{1-S\mu_0(x)}{S}\right\} \tag{46}$$

$$\bar{P}_8(s) = \frac{\lambda_C}{L(s)}\,(\lambda_{s1}+\lambda_{s3}+2\lambda_{s1}\lambda_{s3})\left\{\frac{1-S\mu_0(x)}{S}\right\} \tag{47}$$

$$\bar{P}_9(s) = \frac{\lambda_B}{L(s)}\,(\lambda_{s1}+\lambda_{s3}+2\lambda_{s1}\lambda_{s3})\left\{\frac{1-S\mu_0(x)}{S}\right\} \tag{48}$$

$$\bar{P}_{up}(s) = \bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_2(s)+\bar{P}_3(s)+\bar{P}_5(s)+\bar{P}_6(s) \tag{49}$$

$$\bar{P}_{down}(s) = 1-\bar{P}_{up}(s) \tag{50}$$

Where,

$$L(s)=\left\{\left(s+\lambda_{S1}+\lambda_{S3}+\lambda_{CS}+\right)-\left(\begin{pmatrix}\lambda_{S1}\left(s+\lambda_{s2}+\lambda_{s3}+\lambda_B+\lambda_C+\lambda_{CS}\right)+\lambda_{S3}\left(s+\lambda_{s4}+\lambda_{s1}+\lambda_B+\lambda_C+\lambda_{CS}\right)\\ +\lambda_{S2}\lambda_{S1}S_{\mu0}(s)+\lambda_{S3}\lambda_{S1}S_{\mu0}(s)+\left(1+\lambda_{S1}+\lambda_{S3}+2\lambda_{S2}\lambda_{S1}\right)\lambda_{cs}S_{\mu0}(s)+\\ \left(1+\lambda_{S1}+\lambda_{S3}+2\lambda_{S1}\lambda_{S3}\right)\lambda_c S_{\mu0}(s)+\left(1+\lambda_{S1}+\lambda_{S3}+2\lambda_{S1}\lambda_{S3}\right)\lambda_B S_{\mu0}(s)\end{pmatrix}+\right)\right\} \tag{51}$$

The $\bar{P}_{up}\,(s)$ and $\bar{P}_{dawn}(s)$ are the system Laplace transform of the state probabilities in operative and failed state. Then,

$$\overline{P_{up}}(s)=\sum_{i=0}^{12}\bar{P}_i(s) \quad and \quad \bar{P}_{dawn}(s)=1-\bar{P}_{up}(s)$$

$$P_{up}(t)=\begin{pmatrix}1+\lambda_{S1}\left(\dfrac{1-S_\phi\left(s+\lambda_{s2}+\lambda_{s3}+\lambda_B+\lambda_C+\lambda_{CS}\right)}{\left(s+\lambda_{s2}+\lambda_{s3}+\lambda_B+\lambda_C+\lambda_{CS}\right)}\right)+\lambda_{S3}\left(\dfrac{1-S_\phi\left(s+\lambda_{s4}+\lambda_{s1}+\lambda_B+\lambda_C+\lambda_{CS}\right)}{\left(s+\lambda_{s4}+\lambda_{s1}+\lambda_B+\lambda_C+\lambda_{CS}\right)}\right)\\ \lambda_{S3}\lambda_{S1}\left(\dfrac{1-S_\phi\left(s+\lambda_{s1}+\lambda_B+\lambda_C+\lambda_{CS}\right)}{\left(s+\lambda_{s1}+\lambda_B+\lambda_C+\lambda_{CS}\right)}\right)+\lambda_{S3}\lambda_{S1}\left(\dfrac{1-S_\phi\left(s+\lambda_{s4}+\lambda_B+\lambda_C+\lambda_{CS}\right)}{\left(s+\lambda_{s4}+\lambda_B+\lambda_C+\lambda_{CS}\right)}\right)\end{pmatrix} \tag{52}$$

III. Numerical Study of the Model

I. Availability Analysis

By Setting $S_{\mu_0}(s)=\bar{S}_{\exp[x^\theta+\{\log\varphi(x)\}^\theta]^{1/\theta}}(s)=\dfrac{\exp[x^\theta+\{\log\varphi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\varphi(x)\}^\theta]^{1/\theta}}$, $\bar{S}_{\varphi_S}(s)=\dfrac{\varphi_S}{s+\varphi_S}$,

The expression of availability is obtained by taking the inverse Laplace transform of equation 52 together with the values of failure rates, λ_{s1}=0.0001, λ_{s2}=0.0002, , λ_{s3}=0.0003, , λ_{s4}=0.0004,λ_B=0.0005, λ_c=0.0006, λ_{Cs} = 0.0007 at $\phi(x)=\theta=x=1$ and $\mu_0(x)=\mu_0(y)=2.781$

$$A(x)=\begin{cases}-5.99867865610^{-9}e^{-1.002200000t}+1.000000381e^{-0.0006999977873 64t}+3.16650339610^{-7}e^{-2.718300861t}\\ -6.76892596410^{-7}e^{-1.002699361t}-1.49939921910^{-8}e^{-1.001900000t}\end{cases} \tag{53}$$

The values of $P_{up}(t)$ through variation of time t= 0, 10, 20, 30, 40, 50, 60, 70, 80, 90. as shown in Table 1 and figure 2.

Table 3: *Variation of Availability with respect to time (t)*

Time(t)	Availability
0	1
10	0.993
20	0.986
30	0.979
40	0.972
50	0.965
60	0.958
70	0.952
80	0.945
90	0.938
100	0.932

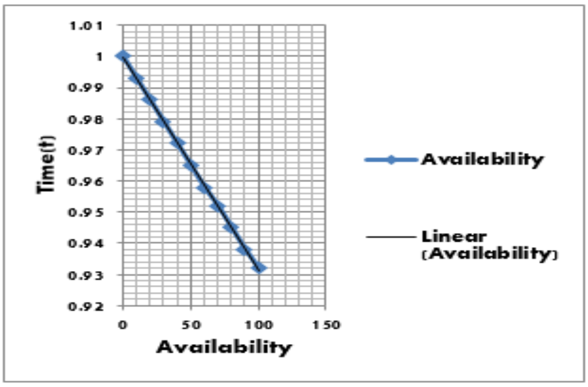

Figure 2: *Variation of Availability with respect to time (t)*

II. Reliability Analysis

Compelling all repair rates to zero with the same value of failure and repair rates in equation 53, i.e $\phi(x)$ and $\mu_0(x)$ and λ_{s1}=0.0001, λ_{s2}=0.0002, λ_{s3}=0.0003, λ_{s1}=0.0004 λ_B=0.0005, λ_C=0.0006, λ_s=0.0007, and then taking inverse Laplace transform, we obtained the expression of reliability.

$$R(t)=\begin{Bmatrix} -0.0000375000000e^{-0.00190000000t}-0.3333333333e^{-0.00230000000t}-0.00002727272727e^{-0.002200000000t} \\ +1.333398106e^{-0.001100000000t} \end{Bmatrix} \tag{54}$$

Aimed at different values of time t= 0, 10, 20, 30, 40, 50, 60, 70, 80, 90 units of time, we get different values of Reliability as shown in Table 2. and Figure. 3.

Table 4: *Variation of reliability with respect to time (t)*

Time(t)	Reliability
0	1
10	0.993
20	0.986
30	0.979
40	0.972
50	0.965
60	0.958
70	0.951
80	0.944
90	0.937
100	0.93

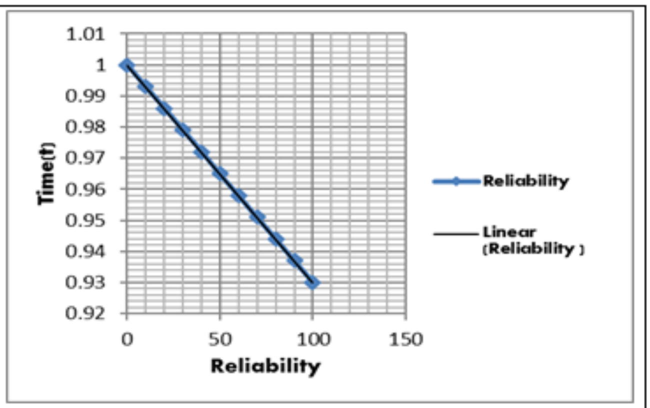

Figure 3: *Variation of reliability with respect to time (t)*

III. Mean Time To Failure (MTTF) Analysis

The expression for MTTF is found by pleasing all repairs zero in equation (53), and set the limit of s tends to zero:

$$M.T.T.F.=\lim_{s\to 0}\bar{P}_{up}(s)=\frac{1}{\{(\lambda_{s1}+\lambda_{s3}+\lambda_{Cs})\}}\left\{1+\frac{\lambda_{s1}}{\lambda_{s2}+\lambda_{s3}+\lambda_B+\lambda_c+\lambda_{cs}}+\frac{\lambda_{s3}}{\lambda_{s4}+\lambda_{s1}+\lambda_B+\lambda_c+\lambda_{cs}}+\frac{\lambda_{s1}\lambda_{s3}}{\lambda_{s1}+\lambda_B+\lambda_c+\lambda_{cs}}+\frac{\lambda_{s1}\lambda_{s3}}{\lambda_{s4}+\lambda_B+\lambda_c+\lambda_{cs}}\right\}$$

Location λ_{s1}=0.0001, λ_{s2}=0.0002, λ_{s3}=0.0003, λ_{s4}= 0.0004, λ_B=0.0005, λ_c=0.0006, λ_{Cs}=0.0007 and changing

λ_A, λ_B, λ_C and λ_D respectively as, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, in (53), the variation of M.T.T.F. is found with respect to failure rates as shown in
Table.3 and corresponding Figure.4.

Table 5: *Variation of MTTF with respect to failure rate*

Failure Rate	MTTF λ_{s1}	MTTF λ_{s2}	MTTF λ_{s3}	MTT F λ_{s4}	MTTF λ_b	MTTF λ_c	MTTF λ_{cs}
0.01	488.6	1035.2	496	971.6	939.9	940.2	99.47
0.02	462.5	1031.8	466.5	961.1	925.8	925.9	49.93
0.03	453.5	1030.5	456.2	957.2	920.5	920.6	33.31
0.04	448.9	1029.9	450.9	955.1	917.8	917.8	24.99
0.05	446.1	1029.4	447.8	953.9	916.1	916.1	19.91
0.06	444.3	1029.2	445.6	953	915	915	16.66
0.07	442.9	1028.1	444.1	952	914.1	914.2	14.28
0.08	441.9	1028.8	443	952	913.5	913.5	12.41
0.09	441.2	1028.7	442.1	951.6	913.1	913.1	11.11

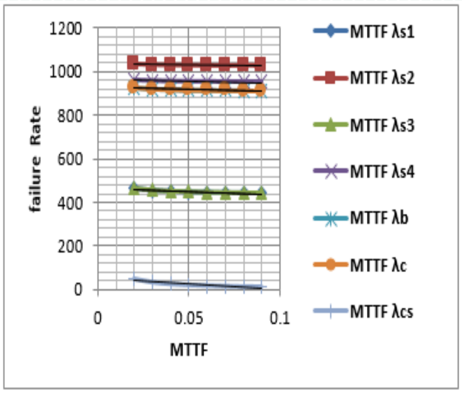

Figure 4: *Variation of MTTF with respect to failure rate*

IV. Sensitivity Analysis of MTTF

The calculation of sensitivity MTTF is studied through the partial differentiation of MTTF with respect to the failure rates of the system, by introducing the set of parametric variation of the failure rates λ_{s1} = 0.0001 λ_{s2}=0.0002, λ_{s3}=0.03, λ_{s1}=0.0004., λ_B=0.0005, λ_c=0.0006, λ_{cs}=0.0001, from the resulting expression, the MTTF sensitivity is calculated as shown in Table 4 and the corresponding value in Figure.5 However the failure rate with respect to λ_B=0.0005 and λ_{s3}=0.03 that is the load balancer and a server from subsystem two must be given a special consideration.

Table 6: *Sensitivity of MTTF as a function of failure rates*

Failure Rate	$\frac{\partial MTTF}{\partial \lambda_{s1}}$	$\frac{\partial MTTF}{\partial \lambda_{s2}}$	$\frac{\partial MTTF}{\partial \lambda_{s3}}$	$\frac{\partial MTTF}{\partial \lambda_{s4}}$	$\frac{\partial MTTF}{\partial \lambda_b}$	$\frac{\partial MTTF}{\partial \lambda_c}$	$\frac{\partial MTTF}{\partial \lambda_{cs}}$
0.01	-5058	-621	-5726	-1926	-2611	-2657	-9850
0.02	-1342	-186	-1528	-569	-765	-772	-2490
0.03	-607	-88	-694	-268	-360	-362	-1109
0.04	-345	-51	-395	-155	-208	-209	-624
0.05	-222	-33	-254	-101	-136	-136	-400
0.06	-155	-24	-177	-71.2	-95.2	-95.5	-278
0.07	-114	-17	-131	-52.8	-70.4	-70.7	-204
0.08	-87.3	-13	-100	-40.7	-54.4	-54.5	-156
0.09	-69.1	-11	-79.4	-32.3	-43.2	-43.3	-123

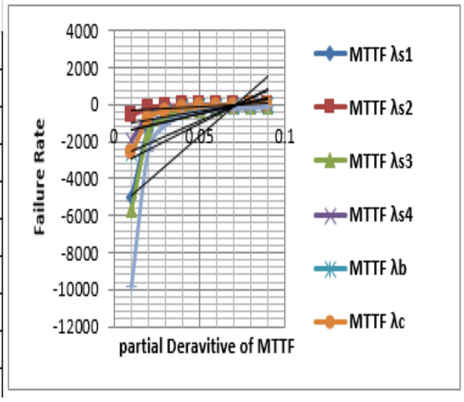

Figure 5: *Sensitivity of MTTF as a function of failure rates*

V. Cost Analysis

The predictable profit over the time interval [0, t), can be estimate by the following relation

$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$. If the service facility of the system is always available, where k1 is revenue generated and k2 service cost per unit time. For the same set of the parameter of failure and repair rates in (53), the expression of cost benefit analysis is attained.

$$E_p(t) = \begin{Bmatrix} 5.98551053310^{-9}\,\mathrm{e}^{-1.0022000000t} - 1429.023674\,\mathrm{e}^{-0.00069977787364t} + 6.75070337910^{-7}\,\mathrm{e}^{-1.002699361t} \\ -1.16488334410^{-7}\,\mathrm{e}^{-2.718300861t} + 1.49655576310^{-8}\,\mathrm{e}^{-1.001900000t} + 1429.023673 \end{Bmatrix} \quad (55)$$

By fixing the revenue K_1= 1 and taking the values K_2= 0.6, 0.5, 0.4, 0.3, 0.2 and 0.1 respectively together with the variation of t =0, 1, 2, 3, 4, 5, 6, 7, 8, 9, Units of time, we obtained the results for expected profit as shown in Table 5 and Figure.6

Table 7: *Expected Profit in [0,t) t=0,1 ,2, 3, 4, 5,6.7,8,9 & 10*

Time (t)	Ep(t): K_2=0.6	Ep(t): K_2=0.5	Ep(t): K_2=0.4	Ep(t): K_2=0.3	Ep(t): K_2=0.2	Ep(t): K_2=0.1
0	0	0	0	0	0	0
1	0.4	0.5	0.6	0.7	0.8	0.9
2	0.8	1	1.2	1.4	1.6	1.8
3	1.2	1.5	1.8	2.1	2.4	2.7
4	1.59	1.99	2.39	2.79	3.19	3.59
5	1.99	2.49	2.99	3.49	3.99	4.49
6	2.39	2.99	3.59	4.19	4.79	5.39
7	2.78	3.48	4.18	4.88	5.58	6.28
8	3.18	3.98	4.78	5.58	6.38	7.18
9	3.57	4.47	5.37	6.27	7.17	8.07
10	3.97	4.97	5.97	6.97	7.97	8.97

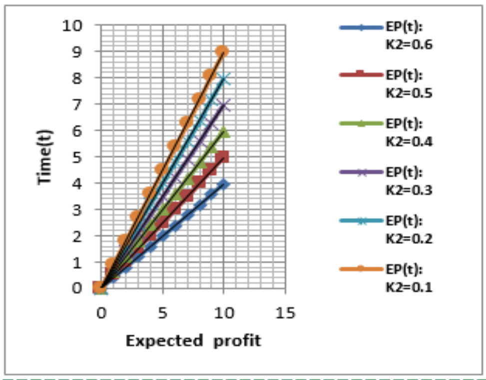

Figure 6: *Expected Profit in [0,t) t=0,1 ,2, 3, 4, 5, 6, 8,9 & 10*

IV. Discussion

The presentation of the system under the valuation of reliability measures for different values of failure and repair rates. Table.1 and figure 2 shows the evidence of availability of the complex with respect to time when the failure rates are fixed at different values mainly, λ_{s1}=0.0001, λ_{s2}=0.0002, , λ_{s3}=0.0003, , λ_{s4}=0.0004,λ_B=0.0005, λ_c=0.0006, λ_{Cs} = 0.0007. The availability of the system decreases slowly, as the probability of failure increases, the system availability will tend to zero at t very large. However, one can simply predict the future behavior of the complex system at any stage for any given set of parametric values.

Table.2 and figure.3 figure out the reliability of the system if there is no repair. The figure shown clearly that the reliability of the system is decreasing faster compare to availability, which evidently proved that when the repairs provided the performance of the system is quite better.

Table.3 and figure.4 assess the information of mean time to failure of the system (MTTF) with respect to variation of failure rates. The value change of MTTF is directly propotional to the system reliability. The computations MTTF for different values of failure rates, λ_{s1}, λ_{s2}, , λ_{s3}, λ_{s4},λ, λ_c, and λ_{Cs} from the figure the variation in MTTF corresponding to failure rates λ_{s1} is high compared to other failure which indicates that the system will not be affected with higher variations in values.

Table.4 and figure.5 demonstrations of sensitivity MTTF with respect to the values of parameters. which obtained from partial derivative of MTTF with respect to the corresponding failure rate, Moreover the variation of sensitivity MTTF corresponding to failure rates λ_B=0.0005 and λ_{s3}=0.03 that is the load balancer and a server from subsystem two, must be given a special consideration.

Table.5 and figure.6 afford the information on how the profit has been generated, by fixing revenue cost per unit time K_1= 1, and varies the service costs K_2 = 0.6, 05. 0.4, 0.3, 0.2 and 0.1, if we examine critically from Figure.6 we can reveals that the expected profit increases for low service cost. Which finally shows the Networking system of tree topology system is reliable.

Reference

[1] Yusuf Ibrahim, Sunusi Abdullahi, Ismail Abdulkareem Lado, muhaammmad Salihu Isa, K. Suleiman, Bala Shehu and Ali U.a (2020). Performance analysis of multi computer system consisting of active parallel homogeneous. *Annals of optimization Theory and Practice, volume1,no1.pp1-8. DOI:10.22121/aotp.2020.239383.1032*

[2] M.I. Abubakar and V. V. Singh (2019). Performance Assessment of an Industrial System (African Textile Manufacturers, LTD). Through Copula Approach. *Journal of Operations Research and Decisions. No. 4. Doi: 10.37190/ord190401*

[3] V. V. Singh, S. B. Singh, C. K. Goel, Stochastic analysis of internet data centre, International J. of Math. Sci& Engg. Appls. IJMESA, Vol. 3(4), (2009) 231-244.

[4] Lirong Cui, Haijun Li, (2007), Analytical method for reliability and MTTF assessment of coherent systems with dependent components, Reliability Engineering & System Safety 300-307.

[5] Minjae Park, Warranty cost analysis for multi- component systems with imperfect repair, *International Journal of Reliability and Applications*, **15**(1), (2014), 51-64.

[6] Zhang, F. (2019). Research on reliability Analysis of Computer Network Based on Intelligent Cloud Computing Method, *International Journal of Computers and Applications, 41:4, 283-288.*

[7] Kudeep Nagiya,Mangey Ram and Ayush Kumar Dua.(2017). A Tree Topology Network Environment Analysis Under reliability Approach, *Nonlinear Studies, 24(1), 199-202.*

[8] Muhammad Salihu Isa, U.A.Ali, Bashir Yusuf and Ibrahim Yusuf (2020) cost benefit analysis of tree different series parallel dynamo configurations. Life Cycle Reliability and Safety Engineering (2020) 9:413-423.*https://doi.org/10.1007/s41872-020-00141-0*

[9] B.S. Dhillon and O.C. Anude, Common cause failure analysis of a K-out-of-N: G system with repairable unit, *Microelectronics Reliability*, **34**(3), (1994), 429-442.

[10] Ibrahim Yusuf, Availability modeling and evaluation of a repairable system subject to minor Deterioration under imperfect repairs, *International journal Mathematics in Operational Research*, **7**(1), (2015), 45-51.

[11] Tseng- Chang Yen, Wu- Lin Chen and Jia-Yu Chen, Reliability and sensitivity analysis of the controllable repair system with warm standby and working breakdown, *Computers and Industrial Engineering*, **97**(3), (2016), 84-97 .

[12] Abdulkareem Lado, V. V. Singh, Kabiru, H, Ibrahim, Yusuf Ibrahim, Performance and Cost assessment of repairable complex system with two subsystems connected in series configuration, International journal of reliability and Applications. Vol.19, No1, pp.22-42.2018.

[13] V. V. Singh, and Jyoti Gulati (2015), performance assessment of computer centre at yobe state University Nigeria under Different repair policies using copula, International conference on energy, environment and chemical engineering (ICEECE2015) ISBN: 978-1-60595-257-4

[14] Kabiru, H. Ibrahim, Muhammad Salihu Isa, M. I. Ibrahim Yusuf, Ismail Tukur, Availability and cost Analysis of complex Tree Topology of computer network with Multi-server using Gumbel-hougaard family copula approach. R T AND A, NO1 (61) VOLUME 16, MARCH 2021.

[15] Geon Yoon, Dae Hyun Kwan Soon Chang Kwon, Yong Oon Park,Young Joon Lee (2006) Ring Topology-based Redundancy Ethernet for Industrial Network. *Siceicase International Joint Conference, pp.1404-1407, 18-21.*

[16] Pratap Kumar, Kabiru H. Ibrahim, M.I. Abubakar and V.V. Singh (2020) Probabilistic Assessment of Complex System with Two Subsystems in Series Arrangement With MultiTypes Failure and Two Types of Repair Using Copula. *Strategic System Assurance and Business Analytics, tttps://doi.org/10.1007/978-981-15-3647-2-2.*

[17] D. R. Cox, The analysis of non-Markov stochastic processes by the inclusion of supplementary variables, Proc. Comb. Phil. Soc. (Math. Phys. Sci.), Vol. 51, (1995) 433-44.

[18] Yusuf, I. and Hussaini, N. (2014) 'A comparative analysis of three unit redundant systems with three types of failures', *Arabian Journal for Science and Engineering*, Vol. 39, No. 4, pp.3337–3349.

[19] Negi, S. and Singh, S.B. (2015) 'Reliability analysis of non-repairable complex system with weighted subsystems connected in series', *Applied Mathematics and Computations*, Vol. 262, pp.79–89.

[20] Vijay Vir Singh, Hamisu Ismail Ayagi, C. K. Goel, Performance assessment of repairable system in series configuration under different types of failure and repair policies using copula linguistics *Int. J. Reliability and Safety, Vol. 12, No. 4, 2018*