On Estimating Standby Redundancy System in a MSS Model with GLFRD Based on Progressive Type II Censoring Data

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Abstract

Redundancy is an approach to improve the reliability system. There are three main models of redundancy. In a system with standby redundancy, there are number of components only one of which works at a time and the other remain as standbys. When an impact of stress exceeds the strength of the active component, for the first time, it fails and another from standbys, if there is any, is activated and faces the impact of stresses, not necessarily identical as faced by the preceding component and the system fails when all the components have failed. In This paper, we consider the problem of estimation the reliability of a multicomponent stress- strength system called N-M- cold - standby redundancy. This system includes N- subsystem consisting of M- independent distributed strength components only one of which works under the impact of stress. The system fails when all the components have failed. Assuming the stress and strength random variables have the generalized linear failure rate distribution with common scale parameters and different shape parameter. The reliability estimated based on progressive type II data. Simulation study is used to compare the performance of the estimators. Finally, real data set is used the proposed model in practice.

Keywords: N-M Standby Redundancy System; Progressive Type II Censoring; Generalized Linear Failure Rate Distribution (GLFRD); Multicomponent stress- strength (MSS); Simulation study; Bootstrap; Bayes estimator; Maximum Likelihood Method.

I. Introduction

Reliability is defined as the probability of not failing in an environment for a mission time. Reliability is a statistical probability and are no absolutes or guarantees. For stress and strength models, both the strength X of the system and stress Y are random variables. The stress-strength model describes the life of a component which has a random strength X and is subjected to random stress Y. The component fails at the instant that the stress applied to it exceeds the strength and the component will function satisfactory whenever $Y < X$. Thus $R = P[Y < X]$ is a measure of component reliability. The idea of stress-strength model was presented by Birnbaum [1], for more reference see Kotz et al [2]. Recently it studied by Basirat et al [2], Asgharzadeh et al [4] and Hassan [5-6]. The stress- strength model is applied in many fields such as quality control, engineering, medicine, biostatistics and economics. Also, the reliability is considered in a multicomponent stress-strength model, which is introduced by Bhattacharyya and Johnson [7] and recently see Hassan and Alohali [8] Sriwastav and Kakati [9] are considered the stress-strength reliability of standby redundancy, that is, there are number of components only one of which works at a time and the other remains as standby.

When an impact of stress exceeds the strength of the active component, for the first time, it fails and another component from standby. The system is fails when all the components has failed. The standby redundancy systems have many applications such as military satellite, standby redundancy system can improve the lifetime of satellite. For stress-strength of standby redundancy system see Khan and Jan [10] ke et al [11], Gogoi et al [12 and Liu et al [13]. In This paper we consider the estimation of N-M- cold- standby redundancy system in a multicomponent stress-strength model based on generalized failure rate distribution (GLFRD). Sarhan and Kundu [14] introduced this distribution has the following pdf and cdf are given by

$$f(x; a, b, \alpha) = \alpha\,(a + b\,x)\,\mathrm{Exp}\left[-\left(ax + \frac{b}{2}x^2\right)\right]\,(1 - \mathrm{Exp}\left[-\left(ax + \frac{b}{2}x^2\right)\right])^{\alpha-1}$$

and

$$F(x; ; a, b, \alpha\,) = \,(1 - \mathrm{Exp}\left[-\left(ax + \frac{b}{2}x^2\right)\right])^{\alpha}$$

Where $x > 0$, a, b>0 are the scale parameters and $\alpha > 0$ is the shape parameter.

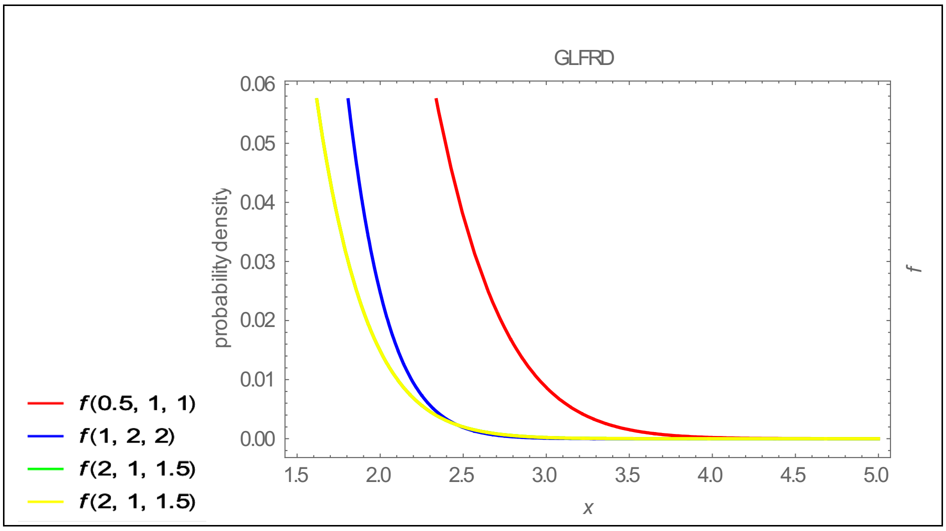

Figure (1): *Different shapes of pdf of GLFRD*

This distribution used as a lifetime model because it has increasing, decreasing or bathtub shaped hazard rate function. Figure (1) show the pdf of GLFRD which may have no mode at all.

II. METHODOLOGY

Reliability is an important concept at the planning, designing, manufacturing, and operating stages operating stages of systems ranging from simple to complex. For multicomponent system to make it more reliable use redundant parts. Redundancy plays an important role in enhancing system reliability. One of the commonly used forms of the redundancy is standby redundancy system. We consider the standby redundancy system which consists of certain number of same subsystems with series structure. Suppose $X_{i1} \dots \dots X_{iM}$, $i = 1 \dots \dots N$ are M independent strength random variables follow have $\mathrm{GLFRD}(a, b, \alpha)$ in i^{th} subsystem. Let $Z_i = \min\,(X_{i1} \dots \dots X_{iM})$, $i = 1 \dots \dots N$, then $Z_1 \dots \dots Z_N$ is the set of N independent strength random variables. Let $Y_1 \dots \dots Y_N$ be independent stress random variables follow have $\mathrm{GLFRD}(a, b, \beta)$. Then the reliability of the system is given by:

$$R = R(1) + \cdots + R(N). \tag{1}$$

The marginal reliability R(i) of i^{th} subsystem *is*

$$R(i) = P[Z_1 < Y_1, \dots\dots, Z_{i-1} < Y_{i-1}, Z_i > Y_i], \qquad i = 1 \dots\dots N.$$

$$= \int_0^\infty F_i(y_1)h_1(y_1)dy_1 \dots\dots \int_0^\infty F_{i-1}(y_{i-1})h_{i-1}(y_{i-1})dy_{i-1} \int_0^\infty (1 - F_i(y_i))h_i(y_i)dy_i. \qquad (2)$$

Since $Z_i's \approx GLFRD(a, b, \alpha)$, *then*

$$f_i(z; a, b, \alpha_i) = \alpha_i\ (a + b\,z)\ Exp\left[-\left(az + \frac{b}{2}\ z^2\right)\right]\ (1 - Exp\left[-\left(az + \frac{b}{2}\ z^2\right)\right])^{\alpha_i - 1}$$

and

$$F_i(z; ; a, b, \alpha_i\ ) = \ (1 - Exp\left[-\left(az + \frac{b}{2}\ z^2\right)\right])^{\alpha_i}\ , i = 1 \dots\dots N.$$

Since $Y_i's \approx GLFRD(a, b, \beta)$, *then*

$$h_i(y; a, b, \beta_i) = \beta_i\ (a + b\,y)\ Exp\left[-\left(ay + \frac{b}{2}\ y^2\right)\right]\ (1 - Exp\left[-\left(ay + \frac{b}{2}\ y^2\right)\right])^{\beta_i - 1}$$

and

$$H_i(y; a, b, \beta_i) = \ (1 - Exp\left[-\left(ay + \frac{b}{2}\ y^2\right)\right])^{\beta_i}\ , i = 1 \dots\dots N.$$

Hence,

$$R(i) = \frac{\beta_i}{\beta_i + \alpha_i} \prod_{j=1}^{i-1} \frac{\alpha_j}{\alpha_j + \beta_j} \qquad (3)$$

And the reliability system is

$$R = \frac{\beta_1}{\beta_1 + \alpha_1} + \sum_{i=2}^{N} \frac{\beta_i}{\beta_i + \alpha_i} \prod_{j=1}^{i-1} \frac{\alpha_j}{\alpha_j + \beta_j} \qquad (4)$$

I. Point Estimators of Standby Redundancy System in a MSS Model with GLFRD Based on Progressive Type II Censoring Sample:

we will derive the different point estimator for R based on progressive type II censored sample. First, we will introduce a brief description for this date type. It is very useful in lifetime studies. It saves cost and time, because it allows to cancel surviving units during the experiment time. In this progressive censoring scheme, let we want to study n units but m failures are completely observed. At the first failure time $Z_{1:m:n}$, S_1 of surviving units are randomly selected and removed from remaining $n - 1$ units, in the second failure time $Z_{2:m:n}$ observed, S_2 of the surviving units are randomly selected and removed from $n - 2 - S_1$, finally at the m^{th} failure time, when m^{th} failure $Z_{m:m:n}$ observed all the S_m surviving units are removed. This approach getting the censored sample of size m is called progressive type II censored sample with censoring scheme $S_1, \dots, S_m$ for more details see Balakrishnan and Aggarwala [15], Balakrishnan [16], and Krishna and Kumar [17].

1. Maximum Likelihood Estimator of R

To get the maximum likelihood estimator of R, Let the progressive type II censored sample $Z_{1:n_i:N_i.} \dots Z_{n_i:n_i:N_i}$ of Z_i random variables with censoring scheme $\{n_i, N_i, r_{i1} \dots r_{in_i}\}$ and similar $Y_{1:m_i:M_i.} \dots Y_{m_i:m_i:M_i}$ of Y_i random variables with censoring scheme $\{m_i, M_i, \grave{r}_{i1} \dots \grave{r}_{im_i}\}$. To simplify our symbols, we replace $Z_{1:n_i:N_i.} \dots Z_{n_i:n_i:N_i}$ by $Z_{1:n_i.} \dots Z_{n_i:n_i}$ and replace $Y_{1:m_i:M_i.} \dots Y_{m_i:m_i:M_i}$ by $Y_{1:m_i.} \dots Y_{m_i:m_i}$.

Let Z_{ij} and Y_{il}, $i = 1 \dots N$, $j = 1 \dots n_i$ and $l = 1 \dots m_i$ are independent random variables having $\mathrm{GLFRD}(a, b, \alpha_i)$ and $\mathrm{GLFRD}(a, b, \beta_i)$ respectively.

The Likelihood function of Z_{ij} and Y_{il} is

$$L(\alpha_i, \beta_i) = \prod_{i=1}^{N} f\left(z_{i1} \dots z_{in_i}\right) h\left(y_{i1} \dots y_{im_i}\right)$$

Where,

$$f\left(z_{i1} \dots z_{in_i}\right) = C_i\, \alpha_i^{n_i}\ Exp\left(-T_i(a,b) + S_i(a,b) + (\alpha_i - 1)\, Q_i(a,b) + \textstyle\sum_{j=1}^{n_i} r_{ij} Ln\left(1 - A_{ij}^{\alpha_i}(a,b)\right)\right),$$

$$h\left(y_{i1} \dots y_{im_i}\right) = \grave{C}_i\, \beta_i^{m_i}\ Exp\left(-\grave{T}_i(a,b) + \acute{S}_i(a,b) + (\beta_i - 1)\, \grave{Q}_i(a,b) + \sum_{l=1}^{m_i} \grave{r}_{il} Ln\left(1 - \grave{A}_{il}^{\beta}(a,b)\right)\right)$$

where,

$$C_i = N_i\,(N_i - 1 - r_{i1}) \dots (N_i - n_i + 1 - \textstyle\sum_{j=1}^{n_i - 1} r_{ij}),$$

$$\grave{C}_i = M_i\,(M_i - 1 - \grave{r}_{i1}) \dots (M_i - m_i + 1 - \textstyle\sum_{l=1}^{m_i - 1} \grave{r}_{il}),$$

$$T_i(a,b) = \textstyle\sum_{j=1}^{n_i} (a\, z_{ij} + b\, z_{ij}^2),\ \ \grave{T}_i(a,b) = \textstyle\sum_{l=1}^{m_i} (a\, y_{il} + b\, y_{il}^2)$$

$$S_i(a,b) = \textstyle\sum_{j=1}^{n_i} Ln(a + b\, z_{ij}),\ \grave{S}_i(a,b) = \textstyle\sum_{l=1}^{m_i} Ln(a + b\, y_{il})$$

$$Q_i(a,b) = \textstyle\sum_{j=1}^{n_i} Ln(A_{ij}(a,b)),\ \grave{Q}_i(a,b) = \textstyle\sum_{l=1}^{m_i} Ln(\grave{A}_{il}(a,b))$$

$$A_{ij}(a,b) = 1 - exp\left(-\left(a\, z_{ij} + \tfrac{b}{2}\, z_{ij}^2\right)\right),\ and\ \grave{A}_{il}(a,b) = 1 - exp\left(-\left(a\, y_{il} + \tfrac{b}{2}\, y_{il}^2\right)\right).$$

Then, the log-likelihood function is given by

$$\begin{aligned} Ln\left(L(\alpha_i, \beta_i)\right) = {} & Ln(k) + \sum_{i=1}^{N} n_i \ln(\alpha_i) - \sum_{i=1}^{N} T_i(a,b) + \sum_{i=1}^{N} S_i(a,b) \\ & + \textstyle\sum_{i=1}^{N} (\alpha_i - 1)\, Q_i(a,b) + \sum_{i=1}^{N} \sum_{j=1}^{n_i} r_{ij} Ln(1 - A_{ij}^{\alpha_i}(a,b)) + \\ & + \sum_{i=1}^{N} m_i \ln(\beta_i) - \sum_{i=1}^{N} \grave{T}_i(a,b) + \sum_{i=1}^{N} \grave{S}_i(a,b) + \\ & \textstyle\sum_{i=1}^{N} (\beta_i - 1)\ \grave{Q}_i(a,b) + \sum_{i=1}^{N} \sum_{l=1}^{m_i} \grave{r}_{il} Ln(1 - \grave{A}_{il}^{\beta_i}(a,b)) \end{aligned}$$

If shape parameters a and b are known, then the maximum likelihood estimators of α_i and β_i are the solution of the following nonlinear equations

$$\frac{n_i}{\alpha_i} + Q_i(a,b) - \sum_{j=1}^{n_i} r_{ij} \left(\frac{A_{ij}^{\alpha_i}(a,b) Ln\left(A_{ij}(a,b)\right)}{1 - A_{ij}^{\alpha_i}(a,b)}\right) = 0 \tag{5}$$

$$\frac{m_i}{\beta_i} + \grave{Q}_i(a,b) - \sum_{l=1}^{m_i} \grave{r}_{il} \left(\frac{\grave{A}_{il}^{\beta_i}(a,b) Ln\left(\grave{A}_{il}(a,b)\right)}{1 - \grave{A}_{il}^{\beta_i}(a,b)}\right) = 0 \tag{6}$$

using numerical nonlinear maximization techniques are $\hat{\alpha}_i$ and $\hat{\beta}_i$ then the maximum likelihood estimator of R is getting using the invariance property as

$$R_{MLE} = \frac{\hat{\beta}_1}{\hat{\beta}_1 + \hat{\alpha}_1} + \sum_{i=2}^{N} \frac{\hat{\beta}_i}{\hat{\beta}_i + \hat{\alpha}_i} \prod_{j=1}^{i-1} \frac{\hat{\alpha}_j}{\hat{\alpha}_j + \hat{\beta}_j} \tag{7}$$

2. Bayes estimator of R

To get the Bayes estimator of R, suppose that the prior distributions for

$\alpha_i \approx$ Gamma (t_{i1}, s_{i1}) and $\beta_i \approx$ Gamma (t_{i2}, s_{i2}), then the Bayes estimator of R under squared error loss function is defined by

$$R_{Bayes} = E[R(\alpha_i, \beta_i|\text{data})]$$

$$= \int_0^\infty \int_0^\infty R(\alpha_i, \beta_i)\pi(\alpha_i, \beta_i|\text{data})\ d\alpha_i\ d\,\beta_i$$

Where,

$$\pi(\alpha_i, \beta_i|\text{data}) = W\,L(\alpha_i, \beta_i|\text{data})\ \pi(\alpha_i)\ \pi(\beta_i)\ ;$$

$$W^{-1} = \int_0^\infty \int_0^\infty L(\alpha_i, \beta_i|\text{data})\ \pi(\alpha_i)\ \pi(\beta_i)\ d\alpha_i\ d\,\beta_i;$$

$$\pi(\alpha_i) = \frac{s_{i1}^{t_{i1}}}{\Gamma(t_{i1})}\ \alpha_i^{t_{i1}-1} e^{-\,s_{i1}\,\alpha_i},\ \ \pi(\beta_i) = \frac{s_{i2}^{t_{i2}}}{\Gamma(t_{i2})}\ \alpha_i^{t_{i2}-1} e^{-\,s_{i2}\,\beta_i}.$$

But this integral is difficult to calculate, we will use the Lindley approximation which introduced by Lindley [18] more details see Ahmed et al [19]. Then the Bayes estimator of R using the Lindley approximation is defined as

$$R_{Bayes} = R + \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{p}\left(R_{ij} + 2\,R_i\,\rho_j\right)\sigma_{ij} + \frac{1}{2}\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}\sum_{l=1}^{p} L_{ijk}R_i\ \sigma_{ij}\ \sigma_{kl} \tag{8}$$

Where all calculations are computed at by $\hat{\alpha}_i$ and $\hat{\beta}_i$. Let $\Theta = (\theta_1, \theta_2) = (\alpha_i, \beta_i)$, $\rho(\Theta)$ is the log of joint prior of Θ, then

$$R_i = \frac{\partial R}{\partial \theta_i},\ \rho_i = \frac{\partial \rho}{\partial \theta_i}\ ,\ i = 1,2.$$

$$R_{ij} = \frac{\partial^2 R}{\partial \theta_i\,\partial \theta_j}\,, i,j = 1,2.$$

$$L_{ijk} = \frac{\partial^3 \mathrm{Log}(L)}{\partial \theta_i\ \partial \theta_j\ \partial \theta_k}\,, i,j,k = 1,2$$

and σ_{ij} are the elements of the Fisher information matrix of Θ.

II. Interval Estimation of Standby Redundancy System in a MSS Model with GLFRD Based on Progressive Type II Censoring Sample:

we consider the interval estimation of R. Construct the asymptotic confidence interval (ACI) and bootstrap confidence interval (BCI) of R.

1. Bootstrap confidence interval of *R* (Boot-P Method).

Eforn [20] suggested confidence interval based on nonparametric bootstrap method called Boot-P. as follows:

1. Generate the progressive type II censored data $Z_{1:n_i}.\ ...\ Z_{n_i:n_i}$ for given parameters a, b, α_i of GLFRD.
2. Generate the progressive type II censored data $Y_{1:m_i}.\ ...\ Y_{m_i:m_i}$ for given parameters a, b, β_i of GLFRD.

3. Compute the maximum likelihood estimators of α_i, β_i and R according to equation (5).
4. Based on pre-specified progressive censoring schemes $\{r_{i1} \dots r_{in_i}\}$ and $\{\grave{r}_{i1} \dots \grave{r}_{im_i}\}$ to generate type II progressive censoring samples $Z^*_{1:n_i} \dots Z^*_{n_i:n_i}$ and $Y^*_{1:m_i} \dots Y^*_{m_i:m_i}$ from $\mathrm{GLFRD}(a, b, \alpha_i^*)$ and $\mathrm{GLFRD}(a, b, \beta_i^*)$ respectively.
5. Find the maximum likelihood estimators of α_i^*, β_i^* and R^*.
6. Repeat step 3 and 4 B-times.
7. Sort R_i^* , $i = 1 \dots B$ in ascending order $R^*_{(1)} \dots R^*_{(B)}$.
8. Compute the approximate $(1-\alpha)\%$ Boot-P confidence interval of R as $(\widehat{R}_{Boot-p(\alpha/2)}, \widehat{R}_{Boot-p(1-\alpha/2)})$ where $\alpha = 0.05$ and $\widehat{R}_{Boot-p}$ is the cumulative distribution of $R^*_{(i)}$ $i = 1 \dots B$.

2. Asymptotic confidence interval (ACI) of R

To obtain the asymptotic confidence interval of R, we must get the asymptotic distribution of R because the exact distribution of R does not exist. First, derive the asymptotic distribution of α_i, β_i, $i = 1 \dots N$. Then obtain the asymptotic distribution of R. To derive the asymptotic distribution, compute the Fisher information matrix of $(\alpha_1 \dots \alpha_N, \beta_1 \dots \beta_N)$ as

$$I(\alpha_1 \dots \alpha_N, \beta_1 \dots \beta_N) = \begin{bmatrix} I_{1,1} & \cdots & I_{1,2N} \\ \vdots & \ddots & \vdots \\ I_{2N,1} & \cdots & I_{2N,2N} \end{bmatrix}$$

$$= -\begin{bmatrix} E(\frac{\partial^2 \mathrm{Log(L)}}{\partial \alpha_1^2}) & \cdots & E(\frac{\partial^2 \mathrm{Log(L)}}{\partial \alpha_1\, \partial \beta_N}) \\ \vdots & \ddots & \vdots \\ E(\frac{\partial^2 \mathrm{Log(L)}}{\partial \alpha_N\, \partial \beta_1}) & \cdots & E(\frac{\partial^2 \mathrm{Log(L)}}{\partial \beta_N^2}) \end{bmatrix}$$

Where,

$$E\left(\frac{\partial^2 \mathrm{Log(L)}}{\partial \alpha_i^2}\right) = -\frac{n_i}{\alpha_i^2} + P_i, \quad P_i = NE\left(\sum_{j=1}^{n_i} r_{ij}\left(\frac{A_{ij}^{\alpha_i}(a,b)\mathrm{Ln}\left(A_{ij}(a,b)\right)^2}{(1-A_{ij}^{\alpha_i}(a,b))^2}\right)\right), \ i = 1 \dots N.$$

$$E\left(\frac{\partial^2 \mathrm{Log(L)}}{\partial \beta_i^2}\right) = -\frac{m_i}{\beta_i^2} + P'_i; \ P'_i = NE(\alpha \sum_{l=1}^{m_i} \grave{r}_{il}\left(\frac{\grave{A}_{il}^{\beta_i}(a,b)\mathrm{Ln}\left(\grave{A}_{il}(a,b)\right)^2}{(1-\grave{A}_{il}^{\beta_i}(a,b))^2}\right)), \ i = 1 \dots N.$$

$$E\left(\frac{\partial^2 \mathrm{Log(L)}}{\partial \alpha_i\, \partial \beta_j}\right) = E\left(\frac{\partial^2 \mathrm{Log(L)}}{\partial \alpha_i\, \partial \alpha_j}\right) = E\left(\frac{\partial^2 \mathrm{Log(L)}}{\partial \beta_i\, \partial \beta_j}\right) = 0, i \neq j, i, j = 1 \dots N.$$

Theorem: $n_i \to \infty$ and $m_i \to \infty$, $i = 1 \dots N$, then $(\widehat{\Theta} - \Theta) \approx N(0, I^{-1})$. Where, $\widehat{\Theta} = \left(\widehat{\alpha}_1 \dots \widehat{\alpha}_N, \widehat{\beta}_1 \dots \widehat{\beta}_N\right)$ and $\Theta = (\alpha_1 \dots \alpha_N, \beta_1 \dots \beta_N)$.

Proof: See, Ferguson[21].

The asymptotic distribution of R_{MLE} according to Delta method see Rao [22] and Wasserman [23] is $(R_{MLE} - R) \sim N\ (0, H^T\, I^{-1} H)$,

where

$$H^T = \left(\frac{\partial\, R}{\partial\, \alpha_1} \cdots \frac{\partial\, R}{\partial\, \alpha_N}, \frac{\partial\, R}{\partial\, \beta_1} \cdots \frac{\partial\, R}{\partial\, \beta_N}\right)$$

and,

$$\frac{\partial R}{\partial \alpha_i} = \frac{-\beta_1}{(\alpha_1+\beta_1)^2} + \sum_{i=2}^{N}((\frac{-\beta_i}{(\alpha_i+\beta_i)^2}\prod_{j=1}^{i-1}\frac{\alpha_j}{(\alpha_j+\beta_j)}) + (\frac{\beta_i}{(\alpha_i+\beta_i)}\prod_{j=1}^{i-1}\frac{\beta_j}{(\alpha_j+\beta_j)^2})$$

$$\frac{\partial R}{\partial \beta_i} = \frac{\alpha_1}{(\alpha_1+\beta_1)^2} + \sum_{i=2}^{N}((\frac{\alpha_i}{(\alpha_i+\beta_i)^2}\prod_{j=1}^{i-1}\frac{\alpha_j}{(\alpha_j+\beta_j)}) + (\frac{\beta_i}{(\alpha_i+\beta_i)}\prod_{j=1}^{i-1}\frac{-\alpha_j}{(\alpha_j+\beta_j)^2})$$

Hence, an asymptotic 100(1-α)% confidence interval of R

$$(R_{MLE} - Z_{\alpha/2}\sqrt{Var(R_{MLE})}, R_{MLE} + Z_{\alpha/2}\sqrt{Var(R_{MLE})})$$

Where $Z_{\alpha/2}$ the upper $\frac{\alpha}{2}^{th}$ quantile of standard normal distribution and $Var(R_{MLE})$ is the variance at the maximum likelihood estimator.

III. Results & Discussion

Monte-Carlo simulation study is present for 1-2- cold standby redundancy system to compare the performance of different point estimators using biases, mean square error and relative efficiency. Also, the comparison of different confidence intervals is made using the average length (ACL) and converge probability (CP). We use different parameters and different censoring schemes. For Bayes estimator three prior as follows

Prior 1	$s_i = 0$	$t_i = 0$	$i = 1, 2$	(non- informative gamma prior)
Prior 2	$s_i = 1$	$t_i = 2$	$i = 1, 2$	(informative gamma prior)
Prior 3	$s_i = 2$	$t_i = 3$	$i = 1, 2$	(informative gamma prior)

We use three censoring schemes as

r_1	$(0, 0, ..., M-m)$
r_2	$(M-m, 0, ..., 0)$
r_3	All take the same number

For the population parameters, we assume $\alpha_1 = \alpha_2 = \alpha$, $\beta_1 = \beta_2 = \beta$. Also, the censoring schemes of Z_1, Z_2, Y_1, Y_2 are the same i.e. $n_1 = n_2 = m_1 = m_2 = m = 10$, $N_1 = N_2 = M_1 = M_2 = M = 30$. Biases, MSE's and relative efficiency of two-point estimators are computed. For interval estimation, average converge length (A.C.L) and converge probability (Cp) are computed. To perform the simulation study, we use the following algorithm:

1. Use different value $(a, b, \alpha_1, \alpha_2, \beta_1, \beta_2)$= (0.5,1,1,1,1,1), (1,2,1,2,2,3) and (2,1,3,2,1.5,2.5).
2. Use the following three censoring schemes (CS)

$$r_1 = (0,0,0,020,0,0,0,0,0,)$$

$$r_2 = (20,0,0,0,0,0,0,0,0,0)$$

$$r_3 = (2,2,2,2,2,2,2,2,2,2)$$

3. For a, b use the numerical method to solve equation 5 and 6 to get the maximum likelihood estimators for $(\alpha_1, \alpha_2, \beta_1, \beta_2)$.
4. Use equation 7 to calculate the maximum likelihood estimator of R.

5. Use equation 8 to calculate the Bayes estimator of R as

$$R_{Bayes} = R_{MLE} + \frac{1}{2}[(R_{11} + 2\,R_1\,\rho_1)\sigma_{11} + (R_{22} + 2\,R_2\,\rho_2)\sigma_{22} + (R_{33} + 2\,R_3\,\rho_3)\sigma_{33} + (R_{44} + 2\,R_4\,\rho_4)\sigma_{44}] + \frac{1}{2}[L_{111}\,R_1\sigma_{11}\sigma_{11} + L_{222}\,R_2\sigma_{22}\sigma_{22} + L_{333}\,R_3\sigma_{33}\sigma_{33} + L_{444}\,R_4\sigma_{44}\sigma_{44}]$$

Where,

$R_1 = \frac{\partial R}{\partial \alpha_1} = \frac{-\beta_1}{(\alpha_1+\beta_1)^2}$, $R_2 = \frac{-\beta_1}{(\alpha_1+\beta_1)^2} - \frac{\beta_2\,\alpha_1}{(\alpha_1+\beta_1)(\alpha_2+\beta_2)^2} + \frac{\beta_1\,\beta_2}{(\alpha_2+\beta_2)(\alpha_1+\beta_1)^2}$, $R_3 = \frac{\alpha_1}{(\alpha_1+\beta_1)^2}$,

$R_4 = \frac{\alpha_1}{(\alpha_1+\beta_1)^2} + \frac{\alpha_2\,\alpha_1}{(\alpha_1+\beta_1)(\alpha_2+\beta_2)^2} - \frac{\alpha_1\,\beta_2}{(\alpha_2+\beta_2)(\alpha_1+\beta_1)^2}$, $R_{11} = \frac{\partial^2 R}{\partial \alpha_1^2} = \frac{2\,\beta_1}{(\alpha_1+\beta_1)^3}$,

$R_{22} = \frac{\partial^2 R}{\partial \alpha_2^2} = \frac{-\beta_2(-2\,\alpha_1^2 - 2\,\alpha_1\,\beta_1 + \beta_1(\alpha_2+\beta_2))}{(\alpha_1+\beta_1)^2(\alpha_2+\beta_2)^3}$, $R_{33} = \frac{\partial^2 R}{\partial \beta_1^2} = \frac{-2\,\alpha_1}{(\alpha_1+\beta_1)^3}$, $R_{44} = \frac{\partial^2 R}{\partial \beta_2^2} = \frac{-\alpha_1\alpha_2\,(2\,\alpha_1+\alpha_2+2\,\beta_1+\beta_2)}{(\alpha_1+\beta_1)^2(\alpha_2+\beta_2)^3}$,

$\rho_1 = \frac{\partial \rho}{\partial \alpha_1} = \frac{t_{11}-1}{\alpha_1} - s_{11}$, $\rho_2 = \frac{\partial \rho}{\partial \alpha_2} = \frac{t_{21}-1}{\alpha_2} - s_{21}$, $\rho_3 = \frac{\partial \rho}{\partial \beta_1} = \frac{t_{12}-1}{\beta_1} - s_{12}$, $\rho_4 = \frac{\partial \rho}{\partial \beta_2} = \frac{t_{22}-1}{\beta_2} - s_{22}$,

$$L_{111} = \frac{\partial^3 \mathrm{Log}\,L}{\partial \alpha_1^3} = \frac{2\,n_1}{\alpha_1^3} + \sum_{j=1}^{n_1} \frac{A_{ij}^{\alpha_1}(a,b)\left(1+A_{ij}^{\alpha_1}(a,b)\right)\mathrm{Log}[A_{ij}(a,b)]^3}{(A_{ij}^{\alpha_1}(a,b)-1)^3},$$

$$L_{222} = \frac{\partial^3 \mathrm{Log}\,L}{\partial \alpha_2^3} = \frac{2\,n_2}{\alpha_2^3} + \sum_{j=1}^{n_2} \frac{A_{ij}^{\alpha_2}(a,b)\left(1+A_{ij}^{\alpha_2}(a,b)\right)\mathrm{Log}[A_{ij}(a,b)]^3}{(A_{ij}^{\alpha_2}(a,b)-1)^3},$$

$$L_{333} = \frac{\partial^3 \mathrm{Log}\,L}{\partial \beta_1^3} = \frac{2\,m_1}{\beta_1^3} + \sum_{l=1}^{m_1} \frac{\grave{A}_{il}^{\beta_1}(a,b)\left(1+\grave{A}_{il}^{\beta_1}(a,b)\right)\mathrm{Log}[\grave{A}_{il}(a,b)]^3}{(\grave{A}_{il}^{\beta_1}(a,b)-1)^3},$$

$$L_{444} = \frac{\partial^3 \mathrm{Log}\,L}{\partial \beta_2^3} = \frac{2\,m_2}{\beta_2^3} + \sum_{l=1}^{m_2} \frac{\grave{A}_{il}^{\beta_2}(a,b)\left(1+\grave{A}_{il}^{\beta_2}(a,b)\right)\mathrm{Log}[\grave{A}_{il}(a,b)]^3}{(\grave{A}_{il}^{\beta_2}(a,b)-1)^3},$$

6. Calculate bias, mean square error and relative efficiency as $B = E[\hat{R} - R]$, $MSE = E[\hat{R} - R]^2$ and $RE = \frac{MSE(R_{Bayes})}{MSE(R_{MLE})}$. If $RE > 1$, then the maximum likelihood estimator of R is more efficient than the Bayes estimator and if $RE < 1$, then the Bayes estimator is more efficient than the maximum likelihood estimator.
7. Calculate the BCI and ACI use section 4.1 and section 4.2 respectively. Also calculate ACL and CP to compare between two interval estimations.

Note that the results of simulation study based on 1000 replications. The results of point estimators show in Table (1), we observe the biases of Bayes estimator for three priors are less than the biases of maximum likelihood estimator but for the biases of three priors, we get the biases of the third prior is smallest. Table (2) shows mean square errors and relative efficiency, The Bayes estimator may be the best for some cases and the maximum likelihood estimator may be the best for another cases. Table (3) shows interval estimation, we get ACL for asymptotic confidence interval is less than its counterpart for bootstrap confidence interval and for CP, we get it is for ACI are closer to nominal value for BCI counterparts. So, we can decide the confidence interval is more efficient than the bootstrap confidence interval.

IV. Application

In this section, we show the implementation of point and interval estimation procedure proposed in this paper. We use a real data sets from Lawless [24]. The data sets represent failure time in minutes. For two types of electrical insulation in an experiment in which insulation is subjected to a continuously increasing voltage stress. Twelve electrical insulations of each type are tested and recorded. The failure time of the first type (z) are 21.8-70.7-24.4-138.6-151.9-75.3-12.3-95.5-98.1-43.2-28.6-16.9 and the failure times of the second type (Y) are 219.3-79.4-86.0-150.2-21.7-18.5-121.9-40.5-147.1-35.1-42.3-48.7. First, we must check wither the GLFRD fit to the data (Z, Y) or not. For this check, we use the Kolmogorov–Smirnov test (K-S).

The results show in Table (4) as. So, at significant level 0.05 cannot reject that the hypothesis that the data are coming from GLFRD. We check graphically the adequacy of the GLFRD to the real data. The probability plot in Figure 2 and Figure 3 shows an excellent goodness of fit of GLFRD. Also, Now, consider 1-2-cold-standby redundancy system consisting of the first electrical insulation and system consisting of a single second type electrical insulation. The maximum likelihood estimator, Bayes estimator and asymptotic confidence interval are computed for the probability R of 1-2-cold-stand by redundancy system consisting of the first type electrical insulation with longer life for different censored schemes and the results show in Table (5).

V. Conclusions

In this paper, we consider the multicomponent system of reliability called N-M-cold- standby redundancy system. Where the distributions for stress and strength variables are GLFRD with different shape parameter, Sarhan et al [25] studied the statistical properties of this distribution and find many physical interpretations. The reliability system is estimated by maximum likelihood method, Bayes estimator and the ACI and BIC are computed.

All estimators calculated under progressive type-II censoring data. Simulation study is performed. It is showed that the bias of Bayes estimator is less than the bias of maximum likelihood estimator. The MSE and RE showed that the MLE may be more efficient for some cases and Bayes estimator is more efficient for another cases. In the context of interval estimation, the comparison between ACI and BCI is made. Finally, we discuss the real data set represents failure time in minutes to illustrate the implementation of point and interval estimation procedures which proposed in this paper.

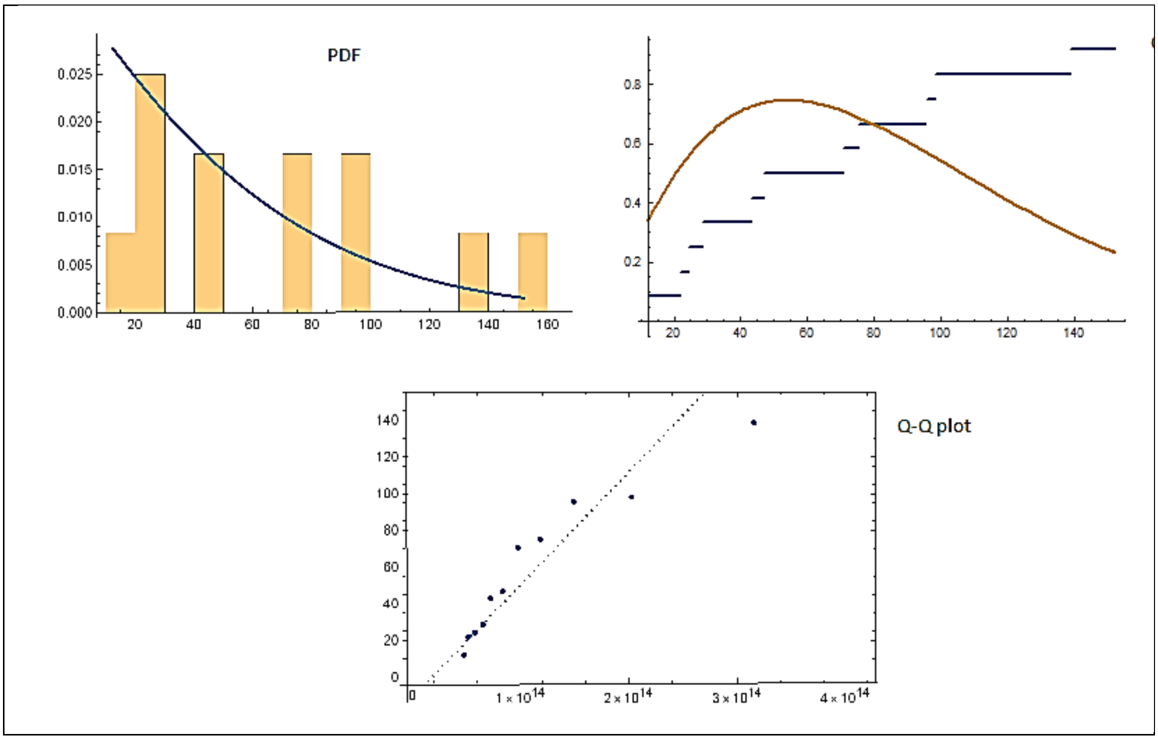

Figure (2): *Graphical goodness-of-fit on the failure time of the first type.*

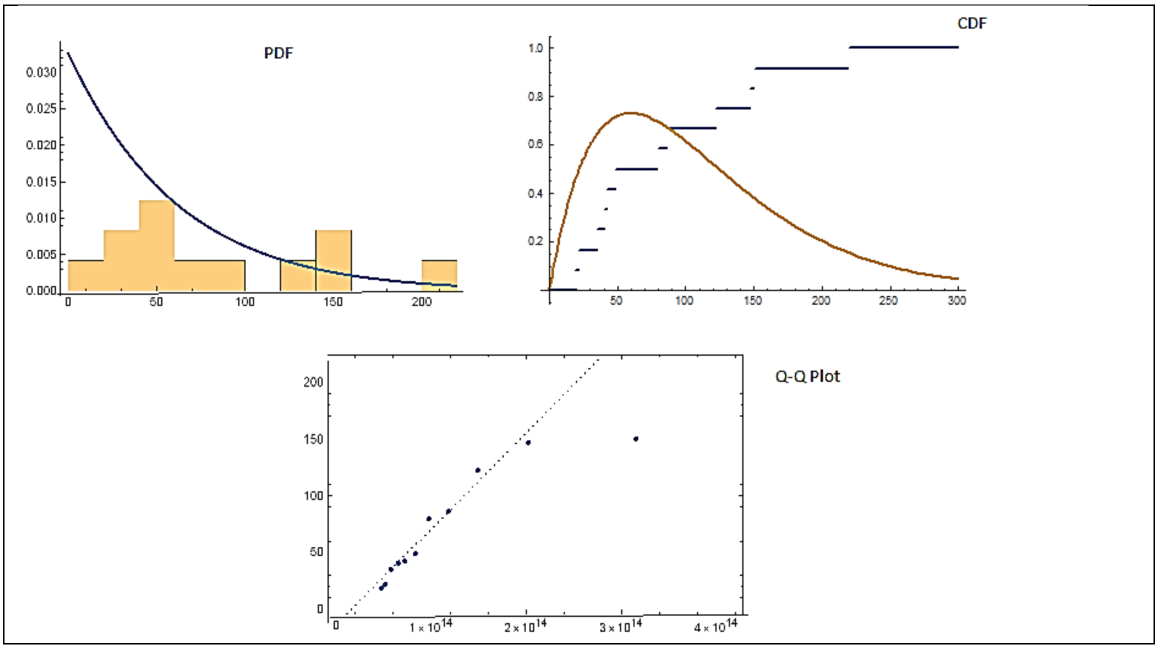

Figure (3): *Graphical goodness-of-fit on the failure time of the Second type.*

Table 1: *The Maximum likelihood and Bayes estimators of R and its biases*

CS	$(\hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_1, \hat{\beta}_2)$	R_{true}	MLE R_{MLE}	MLE Bias	Prior (1) R_{Bayes}	Prior (1) Bias	Prior (2) R_{Bayes}	Prior (2) Bias	Prior (3) R_{Bayes}	Prior (3) Bias
	$a = 0.5$, $b = 1$, $\alpha_1 = 1, \alpha_2 = 1$, $\beta_1 = 1$, $\beta_2 = 1$									
(r_1, r_1)	(0.4,0.4,0.5,0.5)	0.75	0.8024	0.0262	0.7933	0.0216	0.7643	0.0071	0.7029	-0.0235
(r_1, r_2)	(0.44,0.44,0.36,0.36)		0.6975	-0.0262	0.7494	-0.0002	0.7205	-0.0147	0.6590	-0.0454
(r_1, r_3)	(0.38,0.38,0.41,0.41)		0.7686	0.0093	0.7085	-0.0207	0.6870	-0.0314	0.6486	-0.0506
(r_2, r_2)	(0.5,0.5,0.55,0.55)		0.7732	0.0116	0.7853	0.0167	0.7803	0.0151	0.7685	0.0092
(r_2, r_3)	(0.37,0.37,0.47,0.47)		0.8059	0.0279	0.7248	-0.0125	0.6759	-0.0370	0.5836	-0.0831
(r_3, r_3)	(0.41,0.41,0.42,0.42)		0.7559	0.0029	0.7354	-0.0072	0.7303	-0.0098	0.7206	-0.0145
	$a = 1$, $b = 2$, $\alpha_1 = 1$, $\alpha_2 = 2, \beta_1 = 2$, $\beta_2 = 3$									
(r_1, r_1)	(0.35,0.33,0.37,0.54)	0.817	0.8445	0.0137	0.6104	-0.1032	0.5452	-0.1358	0.4328	-0.1920
(r_1, r_2)	(0.36,0.39,0.39,0.46)		0.8136	-0.0016	0.7374	-0.0397	0.7065	-0.0552	0.6509	-0.0830
(r_1, r_3)	(0.359,0.33,0.35,0.44)		0.8212	0.0021	0.6607	-0.0781	0.6378	-0.0895	0.6025	-0.1072
(r_2, r_2)	(0.35,0.38,0.35,0.48)		0.8503	0.0166	0.6322	-0.0923	0.5784	-0.1192	0.4884	-0.1642
(r_2, r_3)	(0.384,0.326,0.31,0.47)		0.7983	-0.0093	0.9302	0.0566	0.9420	0.0625	0.9563	0.0696
(r_3, r_3)	(0.385,0.371,0.383,0.492)		0.8200	0.0015	0.7695	-0.0237	0.7455	-0.0357	0.7007	-0.0581
	$a = 2$, $b = 1$, $\alpha_1 = 3$, $\alpha_2 = 2$, $\beta_1 = 1.5$, $\beta_2 = 2.5$									
(r_1, r_1)	(0.36,0.387,0.376,0.319)	0.809	0.6467	-0.0811	0.8562	0.0236	0.8972	0.0441	0.9581	0.0746
(r_1, r_2)	(0.309,0.338,0.339,0.355)		0.7085	-0.0502	02894	-0.2598	0.2239	-0.2925	0.1245	-0.3422
(r_1, r_3)	(0.356,0.394,0.363,0.375)		0.8094	0.0002	0.7853	-0.0118	0.7845	-0.0122	0.7830	-0.0129
(r_2, r_2)	(0.332,0.385,0.335,0.363)		0.8981	0.0445	0.9229	0.0569	0.9527	0.0718	0.9993	0.0951
(r_2, r_3)	(0.330,0.393,0.391,0.332)		0.8357	0.0133	0.7894	-0.0097	0.7812	-0.0138	0.7643	-0.0223
(r_3, r_3)	(0.320,0.356,0.362,0.336)		0.7033	0.0528	0.5245	-0.1422	0.4965	-0.1562	0.4572	-0.1788

Table 2: *MSE for The Maximum likelihood and Bayes estimators of R and RE*

CS	MSE				RE		
	MLE	Bayes					
		Prior (1)	Prior (2)	Prior (3)	RE1	RE2	RE3
$a = 0.5$, $b = 1$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\beta_1 = 1$, $\beta_2 = 1$							
(r_1, r_1)	0.0006	0.0004	0.0001	0.0005	0.6666	0.1666	0.8333
(r_1, r_2)	0.0006	0.0005	0.0002	0.0020	0.8333	0.3333	3.333
(r_1, r_3)	0.0001	0.0004	0.0009	0.0025	4	9	25
(r_2, r_2)	0.0001	0.0002	0.0002	0.0001	2	2	1
(r_2, r_3)	0.0007	0.0001	0.0013	0.0069	0.1428	1.85	9.8
(r_3, r_3)	0.0001	0.00005	0.00009	0.0002	5	9	2
$a = 1$, $b = 2$, $\alpha_1 = 1$, $\alpha_2 = 2$, $\beta_1 = 2$, $\beta_2 = 3$							
(r_1, r_1)	0.0001	0.0106	0.0184	0.0368	106	184	368
(r_1, r_2)	0.0002	0.0015	0.0030	0.0068	7.5	15	34
(r_1, r_3)	0.0004	0.0061	0.0080	0.0115	15.25	20	28.75
(r_2, r_2)	0.0002	0.0085	0.0142	0.0269	42.5	71	14.5
(r_2, r_3)	0.0001	0.0032	0.0039	0.0048	32	39	40
(r_3, r_3)	0.0003	0.0005	0.0012	0.0033	1.6	4	71
$a = 2$, $b = 1$, $\alpha_1 = 3$, $\alpha_2 = 2$, $\beta_1 = 1.5$, $\beta_2 = 2.5$							
(r_1, r_1)	0.0065	0.0005	0.0019	0.0055	0.07	0.29	0.84
(r_1, r_2)	0.0025	0.0674	0.0855	0.1171	26.9	34.2	46.84
(r_1, r_3)	0.0001	0.0001	0.0001	0.0001	1	1	1
(r_2, r_2)	0.0019	0.0032	0.0051	0.0090	1.6	2.6	4.7
(r_2, r_3)	0.0001	0.0001	0.0002	0.0004	1	2	4
(r_3, r_3)	0.0027	0.0202	0.0244	0.0319	7.4	8.8	11.8

Table 3: *Results of interval estimation*

CS	ACI		BCI	
	ACL	CP	ACL	CP
$a = 0.5$, $b = 1$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\beta_1 = 1$, $\beta_2 = 1$				
(r_1, r_1)	0.1170	0.9661	0.3344	0.9052
(r_1, r_2)	0.1344	0.9579	0.4498	0.8640
(r_1, r_3)	0.1178	0.9650	0.3581	0.8965
(r_2, r_2)	0.1449	0.9571	0.3840	0.8876
(r_2, r_3)	0.1104	0.9681	0.3246	0.9076
(r_3, r_3)	0.1244	0.9626	0.3820	0.8903
$a = 1$, $b = 2$, $\alpha_1 = 1$, $\alpha_2 = 2$, $\beta_1 = 2$, $\beta_2 = 3$				
(r_1, r_1)	0.0838	0.9765	0.3376	0.9047
(r_1, r_2)	0.0980	0.9719	0.3536	0.8991
(r_1, r_3)	0.0921	0.9737	0.3593	0.8977
(r_2, r_2)	0.0756	0.9789	0.3739	0.8899
(r_2, r_3)	0.1077	09687	0.4609	0.8637
(r_3, r_3)	0.1007	0.9712	0.3835	0.8898
$a = 2$, $b = 1$, $\alpha_1 = 3$, $\alpha_2 = 2$, $\beta_1 = 1.5$, $\beta_2 = 2.5$				
(r_1, r_1)	0.1290	0.9582	0.3500	0.8931
(r_1, r_2)	0.1216	0.9622	0.3185	0.9024
(r_1, r_3)	0.1103	0.9682	0.3687	0.8695
(r_2, r_2)	0.1051	0.9719	0.4323	0.8695
(r_2, r_3)	0.1007	0.9716	0.3261	0.9022
(r_3, r_3)	0.1186	0.9630	0.2747	0.9175

Table 4: ***(K-S) Test***

Data Set	Test Statistic	P-Value
Z	1	0.607
Y	5	0.082

Table 5: *Results of application*

CS	MLE	Bayes Estimator			ACI
		Prior (1)	Prior (2)	Prior (3)	
(r_1, r_1)	0.7420	0.8605	0.9394	0.9999	(0.7416,0.7709)
(r_1, r_2)	0.9666	0.1302	0.2895	0.3820	(0.8516,1)
(r_1, r_3)	0.9617	0.3648	0.1186	0.0523	(0.9579,0.9654)
(r_2, r_2)	0.9019	0.9922	0.7092	0.6240	(0.8935,0.9103)
(r_2, r_3)	0.9707	0.8009	0.8127	0.8245	(0.7511,1)
(r_3, r_3)	0.7510	0.7804	0.7366	0.7178	(0.7494,0.7539)

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