On Discrete Scheduled Replacement Model of a Series-Parallel System

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Abstract

This paper investigated the properties of discrete scheduled replacement model of a series-parallel system, with six units. The six units of the system formed three subsystems, which are subsystems A, B and C. Subsystem A is having three parallel units, subsystem B is having a single unit and subsystem C is having two parallel units. It is assumed that, the repairable system is subjected to two categories of failures (Category I and Category II). The mathematical expressions for both reliability function and failure rates, and an elementary renewal theorem were used based on some assumptions in constructing the discrete scheduled replacement model for a series- parallel system. A simple illustrative numerical example where made available, so as to study the properties of the replacement model constructed.

Keywords: category, discrete, replacement, scheduled, time

I. Introduction

Almost all systems deteriorate owing to age and usage, and experience stochastic failures during actual operation. Deterioration raises operating costs and produces less competitive goods. Moreover, consecutive failures are dangerous to the whole system, so timely preventive maintenance is beneficial for supporting normal and continuous system operation. But sometimes an operating systems cannot be replaced at the exact optimum times due to some reasons, such as : shortage of spare units, lack of money or workers, or inconvenience of time required to complete the replacement, but can be rather replaced in idle times, e.g., weekend, month-end, or year-end.

There is an extensive literature on preventive replacement models with their modified versions. Briš *et al.* [1] introduced a new approach for optimizing a complex system's maintenance strategy that respects a given reliability constraint. Chang [2] considered a device that faces two types of failures (repairable and non-repairable) based on a random mechanism. Coria *et al*. [3] proposed a method of analytical optimization for preventive maintenance policy with historical failure time data. Enogwe *et al.* [4] used the distribution of the probability of failure times and come up with a replacement model for items that fails un-notice. Fallahnezhad and Najafian [5] investigated the number of spare parts and installations for a unit and parallel systems, so as cut down the average cost per unit time. Jain and Gupta [6] studied optimal replacement policy for a repairable system with multiple vacation and imperfect coverage. Lim *et al.* [7] studied the characteristics of some age substitution policies. Liu *et al.* [8] developed mathematical models of uncertain reliability of some multi-component systems. Malki *et al.* [9] analyzed age replacement policies of a parallel system with stochastic dependency. Murthy and Hwang [10] discussed that, the failures can be reduced (in a probabilistic sense) through effective maintenance actions, and such maintenance actions can occur either at discrete time instants or continuously over time. Nakagawa [11] modified the continuous standard age replacement for a unit, and come up with a discrete replacement model for the unit. Nakagawa *et al.* [12] explored the advantages of some replacement policies. Safaei *et al.* [13] investigated the optimal preventive maintenance action for a system based on some conditions. Sudheesh *et al.* [14] studied age replacement policy in discrete approach. Tsoukalas and Agrafiotis [15] presented a new replacement policy warrant for a system with correlated failure and usage time. Waziri *et al.* [16] presented some discounted age replacement models with discounting factor for a serial system exposed to two forms of failures. Waziri and Yusuf [17] presented an age replacement model for a parallel-series system based on some proposed policies. Xie *et al.* [18] assessed the effects of safety barriers on the prevention of cascading failures. Yaun and Xu [19] studies a cold standby repairable system with two different components and one repairman who can take multiple vacations, where they assumed that, if there is a component which fails and the repairman is on vacation, the failed component will wait for repair until the repairman is available. Yusuf and Ali [20] considered two parallel units in which both units operate simultaneously, and the system is subjected to two types of failures. Yusuf *et al.* [21] modified the standard age replacement model by introducing random working time Y in the model, for which the system is replaced at a planned time T, at a random working time Y, or at the first non-repairable type 2 failure whichever occurs first. Zaharaddeen and Bashir [22] developed a planned time replacement model for a unit exposed to two different forms of failures. Zhao *et al.* [23] gathered some recently proposed policies on planned time replacement decision.

This paper is organized in five sections. The present section described the introductory part. Section 2 described the system and its notations. Furthermore, section 2 contained some assumptions based on which, the author will develop the proposed replacement model. Section 3 discussed the proposed replacement model. Section 4 presents some numerical results. Section 5 presents the discussion of the results obtained from both examples 1 and 2. Finally, section 6 presents the summary, conclusion and recommendations.

II. Methods

Reliability measures namely reliability function and failure rates are used to obtain the expressions of discrete scheduled replacement model involving minimal repair based on some assumptions. A numerical example was given for the purpose of investigating the characteristics of the model constructed.

III. Notations

- $r_i^*(t)$: Category I failure rate of unit A_i of subsystem A, for $i = 1, 2, 3$.
- $r_b(t)$: Category II failure rate of subsystem B.
- $r_i(t)$: Category II failure rate of unit C_i of subsystem C, for $i = 1, 2$.
- $R_i^*(t)$: reliability function of unit A_i of subsystem A, for $i = 1, 2, 3$.
- C_b : cost of minimal repair of subsystem B due to Type II failure.
- C_{1m} : cost of minimal repair of unit C_1 of subsystem C due to Category II failure.

- C_{2m} : cost of minimal repair of unit C_2 of subsystem C due to Category II failure.
- C_p : cost of scheduled replacement of the system at NT, for $N = 1, 2, 3 \dots$.
- C_r : cost of un-scheduled replacement of the system due to Category I failure.
- N^*: the system's optimum discrete scheduled replacement time.

IV. Description of the System

Consider a system comprising of three subsystems A, B and C in series. Subsystem A consist of three active parallel units, which are A_1, A_2 and A_3. Subsystem B consist of a single active unit. While, subsystem C consist of two active units, which are $C_1$and C_2. The three units A_1, A_2 and A_3 are all subjected to Category I failure, which is an un-repairable failure. Subsystem B is only subjected to Category II failure, which is repairable failure. Also, the two units $C_1$and C_2, are only subjected to Category II failure. The system fails due to Category I failure, if all the three units of subsystem A fails due to Category I failure, at such failure, the system is replaced completely. While the system fails due to Category II failure, if subsystem B or all the two units of subsystem C fails due to Category II failure, at such failure the system is minimally repaired. Figure 1 below is the diagram of the system.

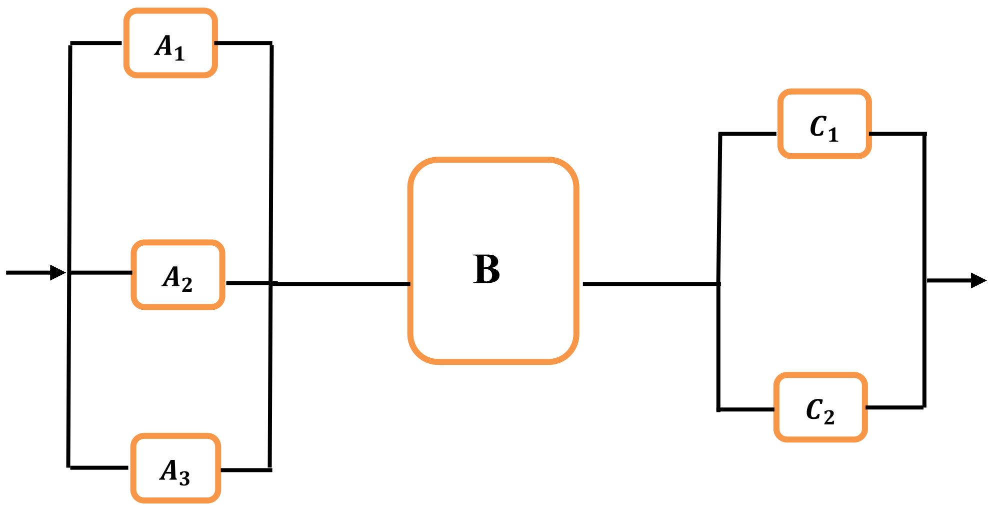

Figure 1: *Reliability block diagram of the system*

V. Discrete Scheduled Replacement Model

This section considers a fundamental discrete scheduled replacement model involving minimal repair.

Assumptions for this model:

1. If subsystem B failed due to Category II failure, then the failed subsystem undergoes minor repair, and allow the system operating from where it stopped.

2. If all the two units of subsystem C failed due to Category II failure, then the failed units will undergoes minor repair, and allow the system operating from where it stopped.
3. The system is replaced completely at scheduled time $NT(N = 1, 2, 3 \dots)$ for a fixed T or where all the three units of subsystem A fails due to Category I failure, whichever arrives first.
4. The cost of scheduled replacement of the system is less than the cost of un-scheduled replacement.
5. Both Category I failure rate and Category II failures rate arrives according to a non-homogeneous Poisson process
6. The cost of minor repair, planned replacement and un-planned replacement are all positive numbers.

Based on the assumptions, the reliability function of component A_i with respect to Category I failure is

$$R_i^*(T) = e^{-\int_0^T r_i^*(t)dt}, \quad \text{for } i = 1,\ 2,\ 3. \tag{1}$$

Based on the assumptions, the reliability function of the system with respect to Category I failure is

$$R_A^*(NT) = 1 - \left(1 - R_1^*(NT)\right)\left(1 - R_2^*(NT)\right)\left(1 - R_3^*(NT)\right), \tag{2}$$

where $N = 1, 2, 3 \dots$ and T is fixed.

The cost of unscheduled replacement of the system in one replacement cycle is

$$C_r\left(1 - R_A^*(NT)\right), \tag{3}$$

where $N = 1, 2, 3 \dots$ and T is fixed.

The cost of scheduled replacement of the system in one replacement cycle is

$$C_p R_A^*(NT), \tag{4}$$

where $N = 1, 2, 3 \dots$ and T is fixed.

The cost of minimal repair of subsystem B in one replacement cycle is

$$\int_0^{NT} C_b r_b(t) R_A^*(t) dt, \tag{5}$$

where $N = 1, 2, 3 \dots$ and T is fixed.

The cost of minimal repair of unit C_1 of subsystem C in one replacement cycle is

$$\int_0^{NT} C_{1m} r_1(t) R_A^*(t) dt, \tag{6}$$

where $N = 1, 2, 3 \dots$ and T is fixed.

The cost of minimal repair of unit C_2 of subsystem C in one replacement cycle is

$$\int_0^{NT} C_{2m} r_2(t) R_A^*(t)dt, \tag{7}$$

where $N = 1, 2, 3 \ldots$ and T is fixed.

Using equations (3) to (7), the replacement cost rate of the system in one replacement cycle is

$$C(N)\ \frac{C_r\left(1-R_A^*(NT)\right)+C_p R_A^*(NT)+\int_0^{NT} C_b r_b(t)R_A^*(t)dt+\int_0^{NT} C_{1m} r_1(t)R_A^*(t)dt+\int_0^{NT} C_{2m} r_2(t)R_A^*(t)dt}{\int_0^{NT} R_A^*(t)dt}. \tag{8}$$

Noting the following:

1. If the value of T is taking as one (that is , T = 1), then $C(N)$ will be a continuous standard age replacement model with minimal repair.
2. $C(N)$ is adopted as an objective function of an optimization problem, and the main goal is to obtain an optimal discrete scheduled replacement time N^* that minimizes $C(N)$.

VI. Numerical Example

In this section, we will give two numerical example, so as to illustrate the characteristics of the constructed discrete scheduled replacement model.

Let the rate of Category I failure of the three units of subsystem A follows Weibull distribution:

$$r_i^*(t) = \lambda_i^* \propto_i^* t^{\propto_i^*-1},\ t \geq 0,\ i = 1, 2, 3, \tag{9}$$

where $\propto_i^* > 1$.

Again, let the rate of Category II failure of subsystem B follows Weibull distribution:

$$r_b(t) = \lambda_b \propto_b t^{\propto_b-1}, t \geq 0\,, \tag{10}$$

where $\propto_b > 1$.

Also, let the rate of Category II failure of the two units of subsystem C follows Weibull distribution:

$$r_i(t) = \lambda_i \propto_i t^{\propto_i-1}, t \geq 0\,,\ i = 1, 2, \tag{11}$$

where $\propto_i > 1$.

Let the set of parameters and cost of repair/replacement be used throughout this particular example:

1. $\propto_b = 4, \propto_2 = 3, \propto_3 = 3$.
2. $\lambda_b = 0.03, \lambda_2 = 0.002, \lambda_3 = 0.03$.
3. $\propto_1^* = 2, \propto_2^* = 3, \propto_3^* = 4$.
4. $\lambda_1^* = 0.0014,\ \lambda_2^* = 0.003, \lambda_3^* = 0.004$.
5. $C_r = 50, C_p = 40$.
6. $C_b = 3, C_{1m} = 2\,, C_{2m} = 2$.

Consequently, by substituting the parameters in equations (9), we obtained the rate of Category I failure of three units of subsystem A as follows below:

$$r_1^*(t) = 0.0028t, \quad (12)$$

$$r_2^*(t) = 0.009t^2, \quad (13)$$

and

$$r_3^*(t) = 0.016t^3. \quad (14)$$

Also by substituting the parameters in equation (10), we obtained the rate of Category II failure of subsystem B as follows:

$$r_b(t) = 0.12t^3. \quad (15)$$

Also by substituting the parameters in equation (11), we obtained the rates of Category II failure of the two units of subsystem C as follows:

$$r_2(t) = 0.006t^2, \quad (16)$$

and

$$r_3(t) = 0.09t^2. \quad (17)$$

Table 1 below is obtained, by substituting the assumed cost of replacement/repair and failure rates of Category I, Category II and Category III failures obtained above (equations (12), (13), (14), (15), (16) and (17)) in equation (8), so as to evaluate the system's optimal discrete scheduled replacement time. When obtaining table 1, the value of $T = 1$, $T = 2$, $T = 3$, and $T = 4$ are considered so as to investigate the properties of the system's optimal discrete scheduled replacement time. Figure 2 is the graph of $C(N)$ against N, as $T = 1$. Figure 3 is the graph of $C(N)$ against N, as $T = 2$. Figure 4 is the graph of $C(N)$ against N, as $T = 3$. Figure 5 is the graph of $C(N)$ against N, as $T = 4$. Figure 6 is the graph comparing the values of $C(N)$ against N, as $T = 1, T = 2, T = 3$ and $T = 4$.

Table 1: *Values of C(N) for T = 1, T = 2, T = 3 and T = 4, versus N* (1, 2, 3 ...).

N	C(N) as T=1	C(N) as T=2	C(N) as T=3	C(N) as T=4
1	175.78	91.22	68.69	66.79
2	91.22	66.79	112.03	223.26
3	68.69	112.03	292.11	562.90
4	66.79	223.26	562.90	1089.46
5	80.61	369.52	944.26	1709.67
6	112.03	562.90	1395.91	2289.30
7	161.45	806.97	1863.78	2703.20
8	223.26	1089.46	2289.30	2874.25
9	292.11	1395.91	2620.56	2790.45
10	369.52	1709.67	2821.38	2497.61
11	459.33	2013.13	2876.24	2075.48
12	562.90	2289.30	2790.45	1609.72
13	679.30	2523.24	2586.36	1170.14
14	806.97	2703.20	2297.18	800.04
15	944.26	2821.38	1960.01	516.19

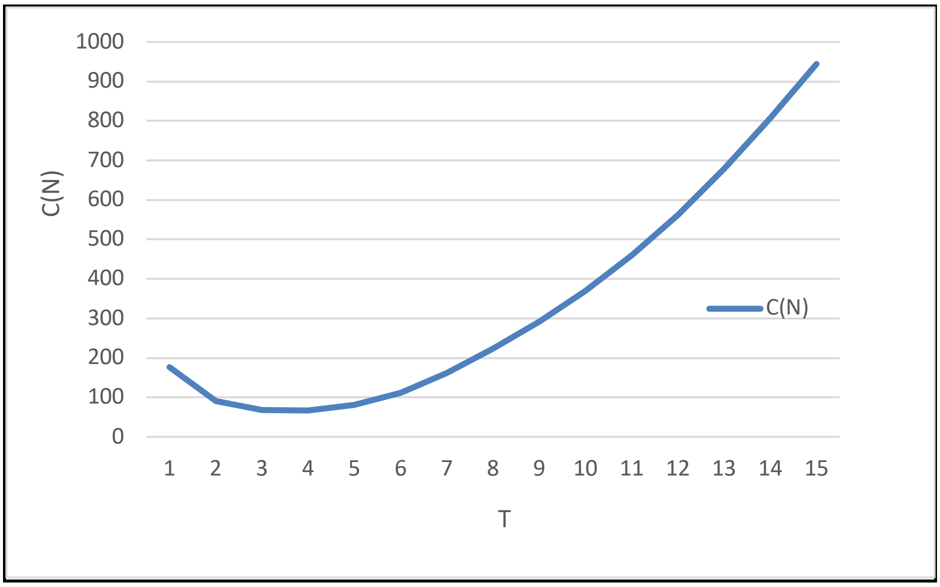

Figure 2: *C(N) against N, as T=1*

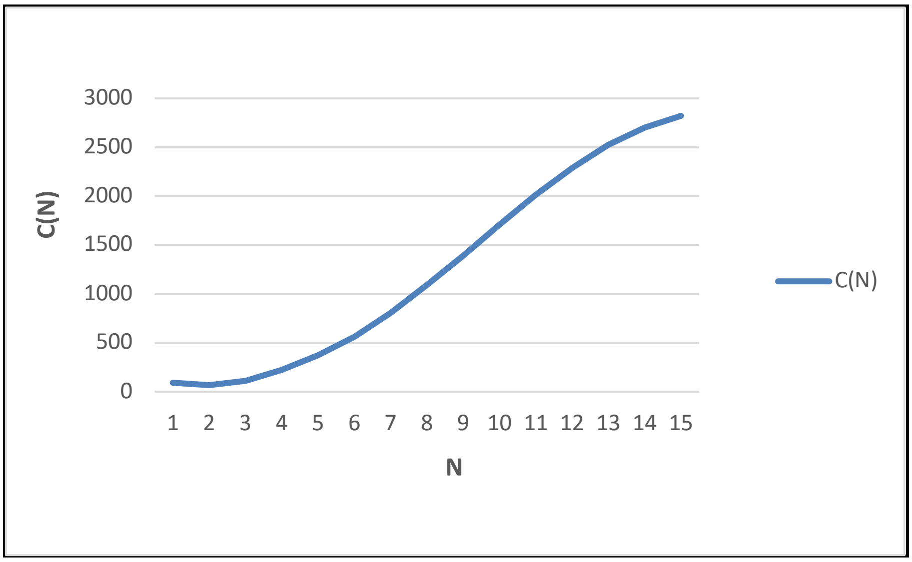

Figure 3: *C(N) against N, as T=2*

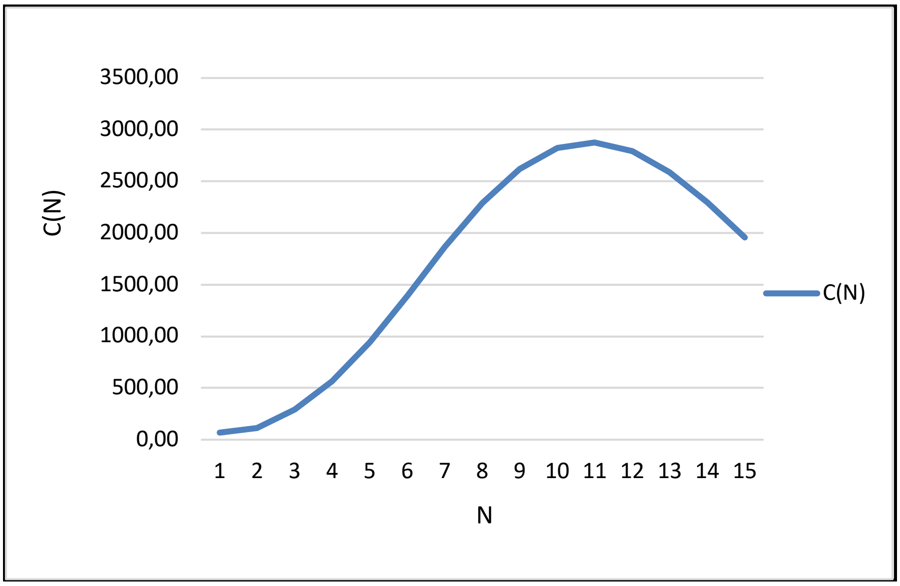

Figure 4: *C(N) against N, as T=3*

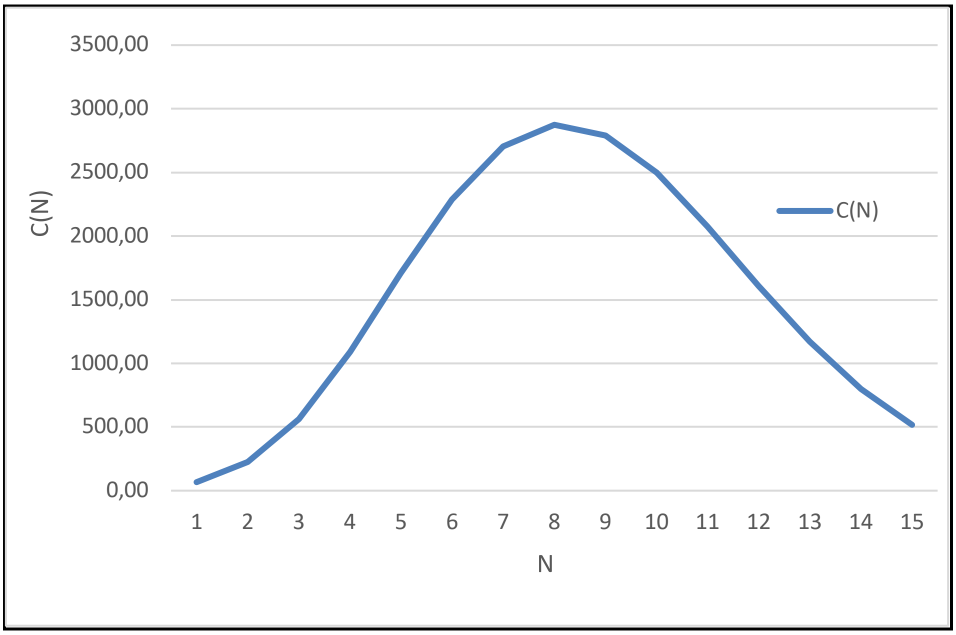

Figure 5: *C(N) against N, as T=4*

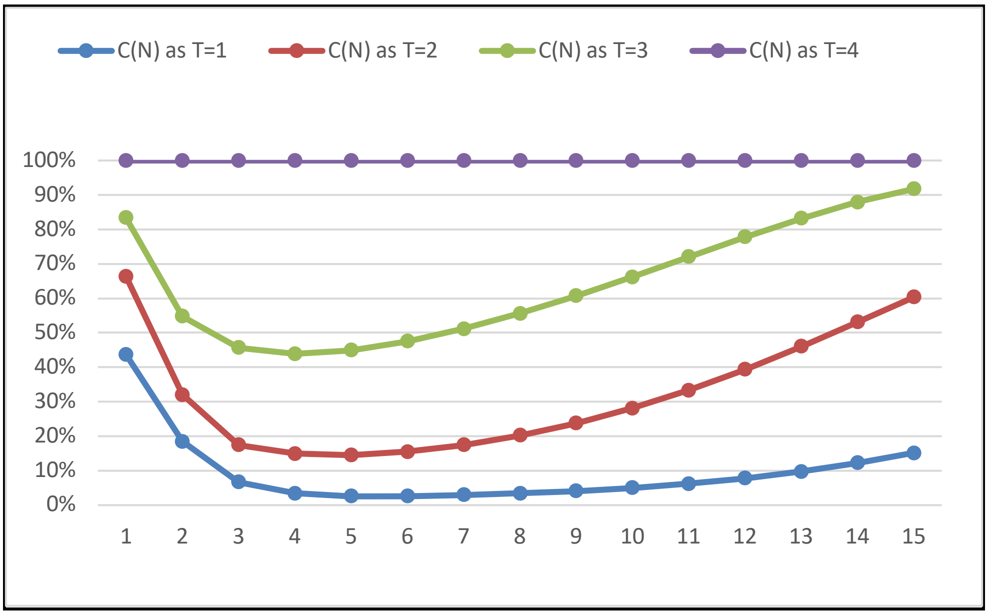

Figure 6: *Comparing C(N) as T=1, T=2, T=3 and T=4*

Some observations from the results obtained are as follows

1. Observe table 1, the optimum discrete scheduled replacement time is 4, when $T = 1$, that is, $N^* = 4$, with $C(N^* = 4) = 66.79$, when $T = 1$. See figure 2 below for the plot of $C(N)$ against N, as $T = 1$.
2. Observe table 1, the optimum discrete scheduled replacement time is 2, when $T = 2$, that is, $N^* = 2$, with $C(N^* = 3) = 66.79$, when $T = 2$. See figure 3 below for the plot of $C(N)$ against T, as $T = 2$.
3. Observe table 1, the optimum discrete scheduled replacement time is 1, when $T = 3$, that is, $N^* = 1$, with $C(N^* = 1) = 68.69$, when $T = 3$. See figure 4 below for the plot of $C(N)$ against T, as $T = 3$..
4. Observe table 1, the optimum discrete scheduled replacement time is 1, when $T = 4$, that is, $N^* = 1$, with $C(N^* = 1) = 66.79$, when $T = 4$. See figure 5 below for the plot of $C(N)$ against T, as $T = 4$.
5. Observe figure 6, $(C(N),\ T = 1) < (C(N)\,, T = 2) < (C(N), T = 3) < (C(N)\,, T = 4)$.
6. Observe figure 2 and figure 3, the graphs are in convex shape.
7. Observe figure 4, the graph is in s-shape.
8. Observe figure 5, the graph is in concave shape.

VII. Conclusion and Recommendations

This paper presented some properties of discrete scheduled replacement model involving minimal repairs. In trying to do that, it is assumed that a system is subjected to two categories of failures (Category I and Category II), such that, Category I is an un-repairable failure, while Category II is a repairable one. A numerical example was provided for simple illustrations. From the results obtained, it is discovered or verified that, the value of T have an effect on the discrete scheduled replacement model, because of the following reasons:

1. as the value of T decreases, the optimal discrete replacement time (N^*) increases, while as the value of T increases, the optimal discrete replacement time (N^*) decreases.
2. as the value of T increases, $C(N)$ increases, while as the value of T decreases, $C(N)$ increases decreases.

With such reasons above, it can be easily seen that, continuous scheduled replacement model (continuous age replacement model) is better than discrete scheduled replacement model (discrete age replacement model). This paper is important to engineers, maintenance managers and plant management in maintaining multi-component systems at idle times, such as weekend, month-end or year-end.

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