Analysis on Dual Supply Inventory Model having Negative Arrivals and Finite Life Time Inventory

M. L. Soujanya[a,*], P. Vijaya Laxmi[b]

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[a]Department of Mathematics,
Vignan's Institute of Information Technology (Autonomous),
Visakhapatnam, India.
drmlsoujanya@gmail.com
[b]Department of Applied Mathematics,
Andhra University, Visakhapatnam, India.
vijayalaxmiau@gmail.com

Abstract

In this paper the impact of dual supply chain on a perishable inventory model with negative arrivals is evaluated. The perishable and replenishment rates of dual suppliers are distributed exponentially. Arrival process follows Poisson distribution and the probability for an ordinary customer is p and for the negative customer is q. Limiting distribution of the assumed model is obtained. Numerical results are presented for cost function and various system performance parameters. The impact of dual suppliers on the optimal reorder points will be useful in developing strategies for handling various perishable inventory problems with replenishment rates.

Keywords: Matrix analytic method, Steady state distributions, Perishable inventory, Replenishment time, Negative arrivals, Dual supply inventory model, Cost function optimization.

I. Introduction

In an (s, S) inventory policy, an order of quantity $Q(= S - s)$ is placed if inventory drops to s, so that the maximum inventory level is S. This policy has been widely discussed for almost a century. However in inventory models with more than one supplier we can improve the quality of service, develop strong relationship with the customers, reduce loss of sales due to stock shortages, enhanced profits, etc. In dual supply (s, S) inventory policy, two orders of quantities Q_1 and Q_2 are placed whenever inventory level drops to r and s respectively. For literature on inventory models with dual supply chains one can refer [9] and [8].

A review of the literature on fixed time perishable inventory models was given by the life time of inventory items is indefinitely long in many classic inventory models, like vegetables, food items, medical products, etc., which become unusable after a certain span. That means there exists a real - life inventory system which consists of products having a finite lifetime. These types of products are called as perishable products and the corresponding inventory system can be considered as a perishable inventory system. [10] studied inventory models for perishable items with and without backlogging. A deterministic inventory model for perishable items with time dependent arrivals is developed by [5]. [1] discussed a perishable inventory model with style goals.

Finite waiting hall inventory model with negraive arrivals is introduced by [4]. For more literature on negative arrivals, one may refer [2],[3] and [6, 7].

In the present paper, an (s, S) inventory policy with dual supply chains for replenishment in which one having a shorter lead time is considered. Demands occur according to Poisson distribution. The arriving person may join the system with possibility p or remove one customer from the queue with probability q. The perishable and service rates are exponentially distributed. Limiting distributions are found. Several system performance parameters of the assumed model are presented. Also the analysis of the cost function is also carried out using direct search method.

II. Model description

In dual supply inventory model, when the inventory shrinks to a fixed level $r(> \frac{S}{2})$ an order of quantity $Q_1(= S - r)$ is placed from the first supplier and are replenished with an exponential rate η_1. If it drops to a prefixed level $s(< Q_1)$ an order of quantity $Q_2(= S - s > s + 1)$ is placed from the second supplier and is replenished with an exponential rate $\eta_2(\eta_2 > \eta_1)$. Arrival process follows Poisson distribution with rate λ. The arrived customer joins the queue with probability p and removes an existing customer from the end with probability $q(= 1 - p)$. Service time follows exponential distribution with rates μ. Let us assume that, the server stays idle at empty queue. Perishable rate follows exponential distribution with rate γ.

Let $N(t)$ be the queue length, $L(t)$ be the quantity of inventory and $\zeta(t)$ be the server state, which is defined as

$$\zeta(t) = \begin{cases} 1, & \text{server is idle at time } t; \\ 2, & \text{server is busy at time } t. \end{cases}$$

Therefore, the stochastic process $\{(\mathtt{N}(t), \zeta(t), L(t) \mid t \geq 0\}$ is a state space model with $E = \{(0,1,k) \mid 0 \leq k \leq S\} \bigcup \{(i,2,k) \mid i \geq 1, 1 \leq k \leq S\}\}$ and it is subdivided into level $\bar{i}$ defined as $\bar{0} = \{(0,1,0), (0,1,1), \ldots, (0,1,S)\}$ and $\bar{i} = \{(i,2,0), (i,2,1), \ldots, (i,2,S)\}$, for $i \geq 1$. Then the transition rate matrix P is

$$P = \begin{array}{c} \\ <0> \\ <1> \\ <2> \\ <3> \\ \vdots \end{array} \begin{array}{c} \begin{array}{ccccc} <0> & <1> & <2> & <3> & \ldots \end{array} \\ \begin{bmatrix} A_0 & C & 0 & 0 & \ldots \\ B & A & C & 0 & \ldots \\ 0 & B & A & C & \ldots \\ 0 & 0 & B & A & \ldots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \end{array}$$

where

$$[A_0] = \begin{cases} -(p\lambda + \eta_1 + \eta_2), & x = y, \quad y = 0; \\ -(p\lambda + \eta_1 + \eta_2 + y\gamma), & x = y, \quad y = 1, 2, \ldots, s; \\ -(p\lambda + \eta_1 + y\gamma), & x = y, \quad y = s+1, \ldots, r; \\ -(p\lambda + y\gamma), & x = y, \quad y = r+1, \ldots, S; \\ y\gamma, & x = y - 1, \quad y = 1, \ldots, S; \\ \eta_1, & x = y + Q_1, \; y = 0, \ldots, r; \\ \eta_2, & x = y + Q_2, \; y = 0, \ldots, r; \\ 0, & \text{otherwise.} \end{cases},$$

$$[C] = \begin{cases} p\lambda, & x = y, \; y = 0, \ldots, S; \\ 0, & \text{otherwise.} \end{cases}, \quad [B] = \begin{cases} \mu, & x = y - 1, y = 1, \ldots, S; \\ q\lambda, & x = y, \; y = 0, \ldots, S; \\ 0, & \text{otherwise.} \end{cases},$$

$$[A] = \begin{cases} -(p\lambda + q\lambda + \eta_1 + \eta_2), & x = y, \quad y = 0; \\ -(p\lambda + q\lambda + \eta_1 + \eta_2 + y\gamma + \mu), & x = y, \quad y = 1, 2, \ldots, s; \\ -(p\lambda + +q\lambda + \eta_1 + y\gamma + \mu), & x = y, \quad y = s+1, \ldots, r; \\ -(p\lambda + q\lambda + y\gamma + \mu), & x = y, \quad y = r+1, \ldots, S; \\ y\gamma, & x = y - 1, \quad y = 1, \ldots, S; \\ \eta_1, & x = y + Q_1, \; y = 0, \ldots, r; \\ \eta_2, & x = y + Q_2, \; y = 0, \ldots, r; \\ 0, & \text{otherwise.} \end{cases},$$

III. Analysis of the Model

Initially the stability condition of the defined model is determined and then the limiting probabilities are derived in this section.

I. Stability condition

For the stability condition, consider the matrix $G = A + B + C$ as

$$[G] = \begin{cases} -(\eta_1+\eta_2), & x = y, \quad y = 0; \\ -(\eta_1+\eta_2+\mu_b+y\gamma), & x = y, \quad y = 1; \\ -(\eta_1+\eta_2+y\gamma), & x = y, \quad y = 2,\dots,s; \\ -(\eta_1+y\gamma), & x = y, \quad y = s+1,\dots,r; \\ -y\gamma, & x = y, \quad y = r+1,\dots,S; \\ y\gamma, & x = y-1, \quad y = 2,\dots,S; \\ \eta_1, & x = y+Q_1, \ y = 1,\dots,r; \\ \eta_2, & x = y+Q_2, \ y = 1,\dots,r; \\ 0, & \text{otherwise.} \end{cases},$$

Let Π be the limiting distribution of G, i.e., $\Pi G = 0$ and $\Pi e = 1$ where $\Pi = (\Pi(2,0), \Pi(2,1), \dots, \Pi(2,S))$. From $\Pi G = 0$, we get

$$-(\eta_1+\eta_2)\Pi(2,0) + \gamma\Pi(2,1) = 0$$

$$-(\eta_1+\eta_2+y\gamma)\Pi(2,l) + y\gamma\Pi(2,l+1) = 0, 1 \le y \le s$$

$$-(\eta_1+y\gamma)\Pi(2,l) + y\gamma\Pi(2,l+1) + \eta_2\Pi(2,l+Q_2) = 0, s+1 \le y \le r$$

$$-(\eta_1+y\gamma)\Pi(2,l) + y\gamma\Pi(2,l+1) + \eta_2\Pi(2,l+Q_1) = 0, r+1 \le y \le S$$

On solving the above two equations and by using $\Pi e = 1$, one can get the limiting probability values.

II. Computation of Steady State Vectors

The limiting distribution for the defined model is

$$\pi_i^{(j,k)} = \lim_{t\to\infty} Pr\Big[\mathtt{N}(t) = i, \zeta(t) = j, L(t) = k \ |\mathtt{N}(0), \zeta(0), L(0)\Big]$$

where $\pi_i^{(j,k)}$ is the probability of i^{th} demand at j^{th} state of the server with k inventories. These probabilities are shortly represented as π_i. The limiting distribution is given by $\pi_i = \pi_0 R^i, \ i \ge 1$.

For finding π_0 and π_1, we have from $\pi P = 0$,

$$\pi_1 = -\pi_0 C(A+RB)^{-1} = \pi_0 w,$$

where

$$w = -C(A+RB)^{-1}.$$

Further, $\pi_0 A_0 + \pi_1 B = 0, i.e., \pi_0(A_0 + wB) = 0$.

First take π_0 as the limiting distribution of $A_0 + wB$. Then π_i, for $i \ge 1$ can be found using $\pi_1 = \pi_0 w, \ \pi_i = \pi_1 R^{i-1}, i \ge 2$. Therefore the limiting distribution of the system is obtained as follows.

$$(\pi_0 + \pi_1 + \pi_2 + \dots)e = \pi_0(1 + w(I-R)^{-1})e.$$

IV. System performance measures

(i) **Average inventory level:** The average inventory level (E_{IL}) is defined as

$$E_{IL} = \sum_{k=0}^{S} k\pi_0^{(1,k)} + \sum_{i=1}^{\infty}\sum_{k=0}^{S} \pi_i^{(2,k)}.$$

(ii) **Average service rate:** Let E_{SR} denote the average service rate and is given by

$$E_{SR} = \sum_{i=1}^{\infty} \mu\pi_i^{(2,k)}.$$

(iii) **Average reorder rate:** Average reorder rate (E_{OR}) is given by

$$E_{OR} = \begin{cases} \sum_{k=0}^{S} \gamma\left[\pi_0^{(1,s+1)} + \pi_0^{(1,r+1)}\right] + \\ \sum_{i=1}^{\infty}\sum_{k=0}^{S} (\mu+\gamma)\left[\pi_i^{(2,s+1)} + \pi_i^{(2,r+1)}\right]. \end{cases}$$

(iv) **Average negative arrivals:** The average negative arrivals (E_{NA}) is given by

$$E_{NA} = \sum_{i=1}^{\infty}\sum_{k=1}^{S} q\lambda\pi_i^{(2,k)}.$$

(v) **Average arrival rate:** The average arrival rate (E_{AR}) is given by

$$E_{AR} = \sum_{i=1}^{\infty}\sum_{k=0}^{S} p\lambda\pi_i^{(2,k)}.$$

(vi) **Average replenishment rate from 1^{st} supplier:** The average replenishment rate from 1^{st} supplier (E_{RR1}) is

$$E_{RR1} = \sum_{k=0}^{r} \eta_1\pi_0^{(1,k)} + \sum_{i=1}^{\infty}\sum_{k=0}^{r} \eta_1\pi_i^{(2,k)}.$$

(vii) **Average replenishment rate from 2^{nd} supplier:** The average replenishment rate from 2^{nd} supplier (E_{RR2}) is

$$E_{RR2} = \sum_{k=0}^{s} \eta_2\pi_0^{(1,k)} + \sum_{i=0}^{\infty}\sum_{k=0}^{s} \eta_2\pi_i^{(2,k)}.$$

(viii) **Average lifetime:** The average lifetime (E_{FR}) is defined as

$$E_{FR} = \sum_{k=1}^{S} k\gamma\pi_0^{(1,k)} + \sum_{i=1}^{\infty}\sum_{k=1}^{S} \pi_i^{(2,k)}.$$

I. Cost analysis

Let
C_H = Carrying cost,
C_S = Set up cost,
C_{S1} = cost for the 1^{st} supplier,
C_{S2} = cost for the 2^{nd} supplier,
C_F = Failure cost,

C_N = Loss incurred due to negative customer,

C_A = Fixed cost for arrivals,

C_{ST} = service cost.

Therefore, the total average cost is defined as

$$TC(s,r) \quad = \quad \begin{cases} C_H\ E_{IL} + C_S\ E_{OR} + C_{S1}\ E_{RR1} + C_{S2}\ E_{RR2} + \\ C_F E_{PR} + C_N\ E_{NA} + C_A\ E_{AR} + C_{ST}\ E_{ST}. \end{cases}$$

V. Numerical analysis

For this section, let us fix the parameters as $S = 14, p = 0.6, q = 0.4, \lambda = 2.3, \mu = 1.2, \eta_1 = 2.4, \eta_2 = 4.7, \gamma = 0.2$.

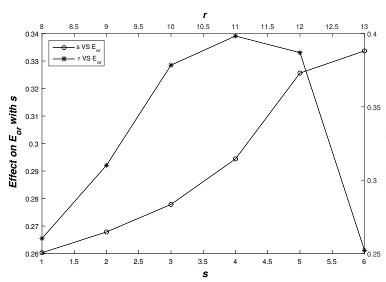

Figure 1: Impact of (s,r) on E_{OR}

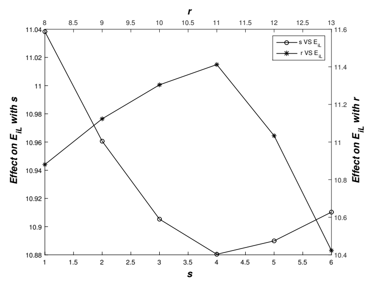

Figure 2: Impact of (s,r) on E_{IL}

Figure 1 and Figure 2 presents the influence of reordering points (s,r) on E_{OR} and E_{IL} respectively. As we know that, E_{OR} and E_{IL} increases with the increase of reorder points, which is evident from Figure 1 and Figure 2. However, one can observe that E_{OR} decreases from $r = 11$ as reordering is done very near to S. From Figure 2, one can also observe that E_{IL} is decreased till $s = 4$ since reordering is done very slowly.

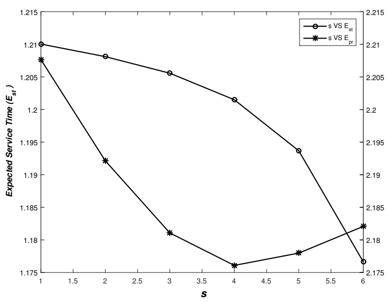

Figure 3: Impact of s on E_{PR} & E_{ST}

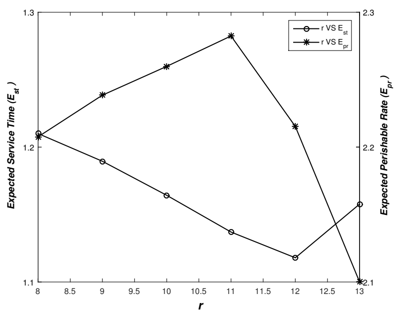

Figure 4: Impact of r on E_{PR} & E_{ST}

The influence of s and r on E_{PR} and E_{ST} are shown in Figure 3 and Figure 4, respectively. According to our assumption $s < S - r$, first order is replenished when the inventory reaches to r. Also $\eta_2 > \eta_1$, the replenishment time of the first supplier is greater than that of the second supplier, due to this E_{ST} decreases with increase in s and r, E_{PR} decreases with s and increases with r.

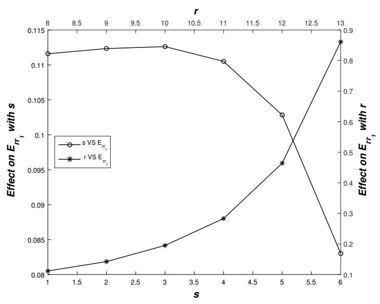

Figure 5: Impact of (s, r) on E_{RR1}

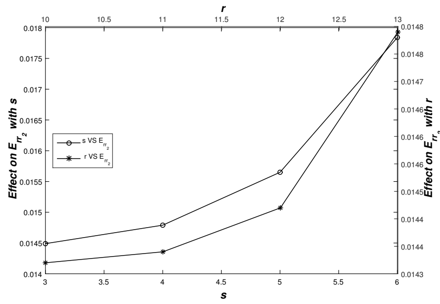

Figure 6: Impact of (s, r) on E_{RR2}

Figure 5 and Figure 6 presents the influence of s and r on E_{RR1} and E_{RR2} respectively. Since E_{RR1} is effected with r, it rises as r increases and falls as s increases as shown in Figure 5. Even though E_{RR2} is effected with s, the second replenishment order is placed only after the first replenishment order is done. Due to this E_{RR2} rises with the increase in both s and r which is clearly evident from Figure 6.

Table 1: *The values of* s^*, r^* *and* $TC(s^*, r^*)$

Case 1	(s^*, r^*)	(4,8)	(5,9)	(7,10)	(7,11)	(4,12)	(7,13)
	$TC(s^*, r^*)$	893.33	**891.62**	895.59	889.00	872.29	648.25
Case 2	(s^*, r^*)	(4,8)	(5,9)	(7,10)	(6,11)	(4,12)	(7,13)
	$TC(s^*, r^*)$	1437.35	**1436.10**	1462.06	1455.07	1423.96	1373.02
Case 3	(s^*, r^*)	(4,8)	(5,9)	(7,10)	(7,11)	(4,12)	(7,13)
	$TC(s^*, r^*)$	899.22	**897.79**	903.77	897.88	979.79	854.92
Case 4	(s^*, r^*)	(4,8)	(5,9)	(7,10)	(7,11)	(4,12)	(7,13)
	$TC(s^*, r^*)$	893.88	**892.27**	896.37	890.19	874.56	851.97
Case 5	(s^*, r^*)	(4,8)	(5,9)	(7,10)	(7,11)	(4,12)	(7,13)
	$TC(s^*, r^*)$	893.40	**891.69**	895.71	889.12	872.38	848.39

Table 1 gives s^* and r^* that minimize $TC(s, r)$, for different numerical examples which are defined as the following cases.

1: $C_H = 30,\ C_S = 100\ C_{S1} = 10,\ C_{S2} = 30,\ C_F = 13,\ C_N = 150,\ C_A = 250,\ C_{ST} = 20$

2: $C_H = 80,\ C_S = 100\ C_{S1} = 10,\ C_{S2} = 30,\ C_F = 13,\ C_N = 150,\ C_A = 250,\ C_{ST} = 20$

3: $C_H = 30,\ C_S = 120\ C_{S1} = 10,\ C_{S2} = 30,\ C_F = 13,\ C_N = 150,\ C_A = 250,\ C_{ST} = 20$

4: $C_H = 30,\ C_S = 100\ C_{S1} = 15,\ C_{S2} = 30,\ C_F = 13,\ C_N = 150,\ C_A = 250,\ C_{ST} = 20$

5: $C_H = 30,\ C_S = 100\ C_{S1} = 10,\ C_{S2} = 35,\ C_F = 13,\ C_N = 150,\ C_A = 250,\ C_{ST} = 20$

One can observe that the optimum reorder point in all the cases is $(s, r) = (5, 9)$ as 5the optimum total cost exists. We know that the average inventory level is more when compared to other performance measures discussed in Section 4. However the total cost function increases with increase in holding cost which is evident form Case 2.

VI. Conclusion

In this paper, some investigations are done on the impact of two suppliers on an inventory model having negative arrivals and finite lifetimes. The limiting distribution of the assumed model is derived. Various system performance parameters are discussed and analyzed the assumed cost function to obtain s^* and r^*.

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Declaration of Conflicting Interests

The Authors declare that there is no conflict of interest.

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