AN OPTIMUM RESOURCE SCORE ESTIMATION METHOD USING BIPARTITE GRAPH MODEL AND SINGLE NODE SYSTEMATIC SAMPLING

DEEPIKA RAJORIYA and D. SHUKLA

•

Dr. Harisingh Gour University, Sagar (M.P.) 470003
deepikarajoriya2112@gmail.com, diwakarshukla@rediffmail.com

Abstract

Consider a graphical population of vertices (nodes) and edges, where edges are connected with vertices to form a Bipartite graph. A complete Bipartite graph has vertices that can be partitioned into two subsets such that no edge has both endpoints in the same subset, and every possible edge connected to vertices in different subsets is a part of graph. In real life, there may hundreds of cities where at least one possible way exists reaching source to destinations. Several tourist places and small towns are the examples where the road transportation is available between origin and destination and these roads constitute Bipartite graph when they are like edges. The travel needs resource consumption who could be measured through resource-score. Walking at the hill station needs more energy consumption than at the plane area. This paper suggests an example to estimate the resource consumption by the values of score. Further, paper proposes a sample based methodology for calculating the average resource consumption between a pair of small town (city) and tourist place. Bipartite graph is used as a model tool. A single-node systematic sampling procedure is proposed under the Bipartite graph setup which is found useful for solution. The suggested estimation strategy is optimum at specific choice of parametric values. For quick selection, ready-reckoner tables are prepared who provide immediate optimum choice of constant. Results are numerically supported by the empirical study and proved by the calculation of confidence intervals.

Keywords: Graph; Bipartite Graph; Estimator; Bias; Mean Squared Error (MSE); Optimum Choice, Confidence intervals (C.I.).

1. Introduction

Traveling from one place to another, for pleasure and entertainment, is an integral part of human life. It is a diversified international phenomenon essential for recreation, interaction, exchange of thoughts and participation in events. Travel agencies offer world tour plans and holiday packages with maximum facilities at minimum cost. Private and public sector organizations, around the world, offer special leave to their employees for tour and travel event including financial support. For economists and scientists challenges are to get optimum estimates of traveling cost using appropriate models between any pair of tourist places. It is rather more difficult to pick up significant cost variables to incorporate in a cost model. Travel by road, by train or by flight need different types of resource consumption like time, fuel, energy, money etc. In a contribution of Moons[2], some equations for computing travel cost are suggested. More explicitly, travel cost methods are for estimation of economic use values associated with ecosystem or site who are frequently used for recreation. Such are important for valuation of economic benefits, useful in planning and decision making (see Ecosystem Valuation [3]) like :

(a) change in access cost of a recreational location,
(b) elimination of an existing tourist site from list,
(c) addition of new sites in tour plan lists.

The logical basis of a travel cost methods is cost and time, incurred to visit a site. This is analogous to estimating desire of people to pay for money to purchase goods based on quantities and prices (see Ecosystem Valuation[3]). Computing methodologies for travel expense, as suggested in [2] are 1 to 5 listed below:

1. $Travel\ Time = \sum_{i=1}^{8} \frac{distance(i)}{averagespeed(i)}$

2. $Fuel\ Cost = \frac{fuel\ cost(EURO/car\ per\ km)*distance(km)}{number\ of\ passengers}$

3. $Total\ car\ usage\ costs = \frac{total\ car\ usage\ cost(EURO/car\ pe\ rkm)*distance(km)}{number\ of\ passenger}$

4. $Total\ calculated\ costs = fuel\ cost + travel\ cost * 3$

5. $Total\ calulated\ cost\ = Total\ car\ usage\ cost + Trave\ time * 3.$

While travel by road, distinction may be among four wheelers, for example small car, gas car, petrol car and diesel car. Table 1.1 presents the calculated costs derived from [2].

Table 1.1 Car Type and Cost (see [2])

S. No.	Car Type	Small gasoline	Big gasoline	Small diesel	Big diesel
1.	Cylinder capacity(in cm^3)	< 1600	≥ 1600	< 2000	≥ 2000
2.	Fuel consumption(liter/km)	0.0716	0.0771	0.0534	0.0758
3.	Price of fuel(EURO/liter)	0.89	0.89	0.61	0.61
4.	Fuel cost(EURO/liter)	0.064	0.068	0.033	0.046
5.	Total car-usage cost(EURO/liter)	0.33	0.52	0.25	0.38

. Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

2. Mathematical Setup for Travel Cost

Let A, B, C, D are four locations at distances Φ_1 and Φ_2 scattered apart geographically. While reaching from A to B, the resource consumption is Φ_1' whereas from C to D it is Φ_2'. Figure 2.1 reveals graphical relationship to travel from origin A and C to destinations B and D with distances and resource consumptions.

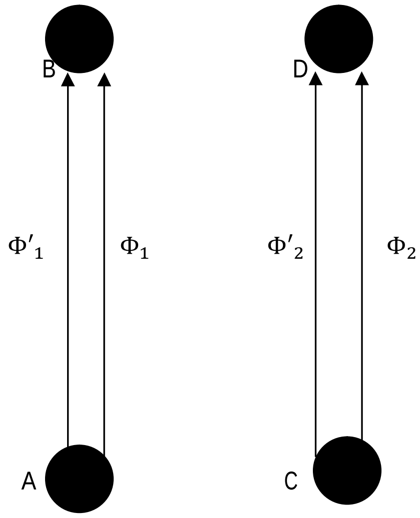

Fig 2.1 Resource consumption and distance for travel

Average resource consumption is $\bar{\Phi} = \frac{\Phi_1+\Phi_2}{2}$ which is an unknown quantity but useful for those who work as travel advisors. Government employees, in many countries, are entitled for availing the Leave Travel Concession (LTC) repeatedly in a block period of few years. They need to prepare and place an estimate of the likely expenditure to the government department for prior sanction of the advance money before the start of journey. This requires calculation of resource consumption in different segments of likely expenditure relating to travel plan. Table 2.1 presents an example of calculation of resource scores.

Table 2.1 Resource Score Traveling Plan from A to B and C to D

Segments	A to B Resource Score(0-25)	C to D Resource Score(0-25)
Fare(Bus/Train/Air)	x_{11}	x_{12}
Food	x_{21}	x_{22}
Boarding/Loading	x_{31}	x_{32}
Local Convenience	x_{41}	x_{42}
Energy Consumption	x_{51}	x_{52}
Time Consumption	x_{61}	x_{62}

Class-intervals for resource scores are as per perception like very low (0-5), low(5-10), medium(10-15), high(15-20), very high(20-25). Energy and time consumption in walking at hill station are higher than at cities located in plain area. Let x_{ij} be the resource score of i^{th} segment of j^{th} travel plan $(i = 1,2,3,4,5,6, j = 1,2)$ as per table 2.1. Average resource score of j^{th} plan is:

$$\bar{\Phi}'_j = \frac{\sum_{i=1}^{6} x_{ij}}{6}$$

In this study $\bar{\Phi}'_j$ is taken as a variable of main interest for different travel plans and packages and needs to be estimated regularly on sample basis over time domain. While travel plans are large in number, $\bar{\Phi}'_j$ are useful unknown parameters for travel agents to prepare the estimate of travel advance money for employees of private or public sectors where tour packages are part of privilege of the annual salary.

3. Graphical Model for $\bar{\Phi}_j$ Estimation

Let there are two groups of vertices, each having well connected edges. Every vertex of first group is connected to all vertices of other group once and only once. This constitutes a Bipartite graph and the mean-edge-length estimation using sampling techniques can be used with graph as a model tool for obtaining solution.

To extend further, assume that first group has even number of vertices, divided into two subgroups such that each contains equal number of vertices. First subgroup is of main interest while second contains well known correlated information, for example distances between cities. Figure 2.1 is a Bipartite graph structure useful as a model for average travel cost estimation problem considered in this paper.

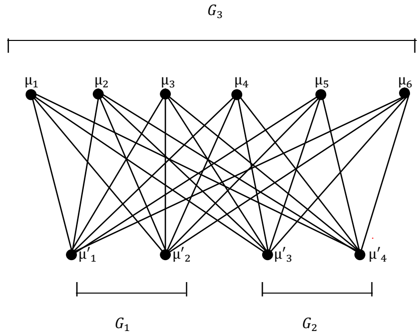

Fig 3.1 Bipartite Type Graph Structure

Remark 1. If the vertices of any graph G (say) can be split into two disjoint subsets V_1 and V_2 in such a way that each edge in the graph joins a vertex in V_1 to a vertex in V_2, and there are no edge in graph G that connect two vertices in V_1 or two vertices in V_2, then the graph is called Bipartite graph. For example, every tree structure is a bipartite graph (see [1]).

Remark 2. Suppose a graph G contains x vertices in the first set and y vertices in the second set . A complete Bipartite graph is a graph in which each vertex in the first set is joined with every single vertex in the second set (see fig 3.1 and [8] [14] [15][18] [19]).

3.1. Example of Bipartite Graph Application (as per fig 3.1)

Let there are six tourist places (destinations) and four origins. For each origin, the travel connectivity exists to all tourist places. Four origins form the first group and six tourist places constitute the second group. First group is further divided into two subgroups, each of two vertices (origins). Edge lengths of first subgroup reveal the total resources likely to consume to travel from origin to the tourist places. Second subgroup is a prototype of the first subgroup where edges represent travel distances who are well known through the travel booklets. Larger distance traveled leads to higher consumption of resources and corresponding requirement of more travel cost. The focus of this study is to know about what an average amount of resource consumption likely to occur while traveling between any pair of origins and tourist places.

Table 3.1 Node-Edge Matrix for Bipartite Graph (Fig 3.1)

	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	*row total*
μ'_1	1	1	1	1	1	1	6
μ'_2	1	1	1	1	1	1	6
μ'_3	1	1	1	1	1	1	6
μ'_4	1	1	1	1	1	1	6
							36

Table 3.1 displays the edge connectivity with vertices of groups where unit number denotes existence of an edge.

3.2. General Bipartite Graph Model

Let k origins (vertices) of even number denoted as $\{\mu'_1, \mu'_2, \mu'_3, ...\mu'_k\}$ divided into groups G_1 and G_2 each strictly of size $t = \frac{k}{2}$. The G_1 contains half of vertices (denoted as cities) while G_2 has remaining who are prototype of G_1(like taxi-stand of the same city in G_1).

$$G_1 : \{\mu'_1, \mu'_2, \mu'_3, ...\mu'_t\} \tag{1}$$

$$G_2 : \{\mu'_{t+1}, \mu'_{t+2}, \mu'_{t+3}, ...\mu'_k\} \tag{2}$$

Further, let there exist r tourist places (vertices) marked as third group G_3 .

$$G_3 : \{\mu_1, \mu_2, \mu_3, ...\mu_r\} \tag{3}$$

Each vertex of G_1 and G_2 is connected to every vertex of G_3 one and only once which constitutes a model like a Bipartite graph among groups G_1, G_2 and G_3. Similarity of identities exist in sequential pair $\{\mu'_1, \mu'_{t+1}\}$, $\{\mu'_2, \mu'_{t+2}\}$, $\{\mu'_3, \mu'_{t+3}\}$,...$\{\mu'_t, \mu'_k\}$ just as same origins or same objects or same organizations.
Assume ϵ_{ij} denotes edge with weights connecting to i^{th} vertex of G_1 to the j^{th} vertex of G_3 and ϵ'_{ij} the edge with weights of i^{th} vertex of G_2 to the j^{th} vertex of G_3 $(i = 1, 2, 3, ..., t, j = 1, 2, 3, ..., r)$. Further, k and r both are large integers and $r > k$ holds for an even k. Moreover, ϵ_{ij} represents weights as resource consumption score (unknown and to estimate) while ϵ'_{ij} are the known weights in advance like road distances. The problem undertaken in this paper is to estimate the average amount of resource consumption likely to occur between any pair of vertices of G_1 and G_3 with the help of edge weights prototype pair of vertices of G_2 and G_3 where information are priorly known.

Table 3.2 General Node-Edge Matrix

	Group (G_3)					
	Node	μ_1	μ_2	μ_3	--------- μ_r	Total
Group (G_1)	μ'_1	1	1	1	1	r
	μ'_2	1	1	1	1	r
	μ'_3	1	1	1	1	r
	⋮ μ'_t	1	1	1	1	r
Group (G_2)	μ'_{t+1}	1	1	1	1	r
	μ'_{t+2}	1	1	1	1	r
	μ'_{t+3}	1	1	1	1	r
	⋮ μ'_k	1	1	1	1	r

In table 3.2, if an edge exists between $(i, j)^{th}$ pair then $\epsilon_{ij} = \epsilon'_{ij} = 1$ else zero everywhere.

4. Single-Node-Systematic Random Sampling

A Single-Node Systematic Sampling scheme without replacement is proposed as under:

Step I For given r, even k, $t = \frac{k}{2}$, groups G_1, G_2, G_3, large k and r in population, construct population matrix as stated in table 3.2. Vertex-edge connectivity structure constitutes a model like a Bipartite graph.

Step II Assign labels to vertices in G_1 and G_2 like $1, 2, 3, ..., t, t + 1, t + 2, t + 3, ...k$.

Step III Draw one vertex randomly from label $1, 2, 3, ..., t$ (table 3.2). Assume it is l^{th} vertex $(l = 1, 2, 3, ..., t)$ from G_1. Using systematic sampling concept, the unit labeled $(l + t)$ is automatically selected in the sample from G_2. The resultant is single node (vertex) systematic random selection of units (μ'_l, μ'_{l+t}) from G_1 and G_2, each having r edges.

Step IV Note down weights, marked as edge-lengths, of μ'_l and μ'_{l+t} connecting to all vertices of G_3. Sampled weight values of r edges are as under:

$$\mu'_l : (s\epsilon_{lj},\ j = 1, 2, 3, ..., r)\ sample\ from\ G_1,\ G_3 \tag{4}$$

$$\mu'_{l+t} : (s\epsilon'_{l+t},\ j = 1, 2, 3, ..., r)\ sample\ from\ G_2,\ G_3 \tag{5}$$

Step V Apply an appropriate estimation procedure to obtain the estimate of unknown parameter (average resource consumption score).

5. Estimation

For a large Bipartite graph, the population parameters (means) are:

$$\bar{\Delta_1} = \frac{1}{rt}\sum_{i=1}^{t}\sum_{j=1}^{r}\epsilon_{ij} \tag{6}$$

$$\bar{\Delta_2} = \frac{1}{rt}\sum_{i=1}^{t}\sum_{j=1}^{r}\epsilon'_{ij} \tag{7}$$

Using step I to step V of Single-Node-Systematic Sampling, the two sample means are:

$$\bar{\delta_1} = \frac{1}{r}\sum_{j=1}^{r} s\epsilon_{ij} \tag{8}$$

$$\bar{\delta_2} = \frac{1}{r}\sum_{j=1}^{r} s\epsilon'_{ij} \tag{9}$$

where $s\epsilon_{ij}$ and $s\epsilon'_{ij}$ denote weights like edge-lengths appeared in sample. For very small numbers h_1, h_2, $|h_1| < 1$, $|h_2| < 1$, one can use large sample approximations discussed in [5],[6],[7].

$$\bar{\delta_1} = \Delta_1(1 + h_1)\ \ \bar{\delta_2} = \Delta_2(1 + h_2)\ \ with\ \ E(h_1) = E(h_2) = 0 \tag{10}$$

$$E(h_1^2) = \frac{t-1}{rt}(C^*\epsilon)^2,\ \ E(h_2^2) = \frac{t-1}{rt}(C^*\epsilon')^2,\ E(h_1h_2) = \frac{t-1}{rt}\rho(C^*\epsilon)(C^*\epsilon') \tag{11}$$

$$(C^*\epsilon) = \frac{(S\epsilon)}{\bar{\Delta_1}} \tag{12}$$

$$(C^*\epsilon') = \frac{(S\epsilon')}{\bar{\Delta_2}} \tag{13}$$

$$(C^*\epsilon\epsilon') = \frac{(S\epsilon\epsilon')}{(\bar{\Delta_1}.\bar{\Delta_2})} \tag{14}$$

$$(S\epsilon)^2 = \frac{1}{rt-1}\sum_{i=1}^{t}\sum_{j=1}^{r}(\epsilon_{ij} - \bar{\Delta_1})^2 \tag{15}$$

$$(S\epsilon')^2 = \frac{1}{rt-1}\sum_{i=1}^{t}\sum_{j=1}^{r}(\epsilon'_{ij} - \bar{\Delta_2})^2 \tag{16}$$

$$(S\epsilon\epsilon') = \frac{1}{rt-1}\sum_{i=1}^{t}\sum_{j=1}^{r}(\epsilon_{ij} - \bar{\Delta_1})(\epsilon'_{ij} - \bar{\Delta_2}) \tag{17}$$

$$\rho = \frac{S\epsilon\epsilon'}{[(S\epsilon)(S\epsilon')]} \tag{18}$$

Equation (5.13) is correlation coefficient between ϵ_{ij} and ϵ'_{ij} in Bipartite population.

5.1. Proposed method of estimation

Deriving motivation from [9],[10], [12], [14], [16], [17] and [18], estimation strategy E is proposed as under:

$$E = \bar{\delta_1}\left[f_1(\bar{\Delta_2}, \bar{\delta_2})\right]\left[f_2(\bar{\Delta_2}, \bar{\delta_2})\right]^{-1} \tag{19}$$

where

$$f_1(\bar{\Delta_2}, \bar{\delta_2}) = \left[(A+C)\bar{\Delta_2} + gB\bar{\delta_2}\right]$$

$$f_2(\bar{\Delta_2}, \bar{\delta_2}) = \left[(A+gB)\bar{\Delta_2} + C\bar{\delta_2}\right]$$

and $A = (P-1)(P-2);\ B = (P-1)(P-4)(P-5);\ C = (P-2)(P-3)(P-4);\ g = \frac{r}{rt} = \frac{1}{t};\ 0 < P < \infty$.

The proposed method of estimation is in accordance with Shukla et al. [9], [10],[12] with change in the term B which is now cubic in this paper but was considered quadratic in earlier contributions.

Theorem 1. Proposed method (5.14) could be expressed as
$E = \bar{\Delta_1}\left[(1+h_1) + \frac{(gB-C)}{(A+gB+C)}\left\{h_2 + h_1h_2 - \frac{Ch_2^2}{A+gB+C}\right\}\right]$
Proof. $E = \bar{\delta_1}\left[f_1(\bar{\Delta_2}, \bar{\delta_2})\right]\left[f_2(\bar{\Delta_2}, \bar{\delta_2})\right]^{-1}$

Using (5.5), $|h_1| < 1$, $|h_2| < 1$

$$f_1(\bar{\Delta_2}, \bar{\delta_2}) = [(A+C)\bar{\Delta_2} + gB\Delta_2(1+h_2)] \tag{20}$$

$$f_2(\bar{\Delta_2}, \bar{\delta_2}) = [(A+gB)\bar{\Delta_2} + C\Delta_2(1+h_2)] \tag{21}$$

The (5.15) expressed using (5.5) as

$$f_1(\bar{\Delta_2}, \bar{\delta_2}) = \bar{\Delta_2}\left[(A+gB+C) + gB(h_2)\right] = \bar{\Delta_2}\left[(A+gB+C)\right]\left[1 + \frac{gBh_2}{(A+gB+C)}\right] \tag{22}$$

Since $|h_2| < 1$, therefore $|\frac{gBh_2}{(A+gB+C)}| < 1$, due to $\forall\ g > 0,\ P > 0$
For (5.16), the expansion of $(1+x)^{-1}$ is used when $|x| < 1$ as under:

$$\left[f_2(\bar{\Delta_2}, \bar{\delta_2})\right]^{-1} = (\bar{\Delta_2})^{-1}\left[(A+gB+C) + Ch_2\right]^{-1}$$

$$= (\bar{\Delta_2})^{-1}\left[(A+gB+C)\right]^{-1}\left[1 + \frac{Ch_2}{(A+gB+C)}\right]^{-1}$$

$$Then, E = \bar{\Delta_1}(1+h_1)\left[1+\frac{gBh_2}{(A+gB+C)}\right]\left[1+\frac{Ch_2}{(A+gB+C)}\right]^{-1}$$

$$= \bar{\Delta_1}(1+h_1)\left[1+\frac{gBh_2}{(A+gB+C)}\right]\left[1-\frac{Ch_2}{(A+gB+C)}+\frac{C^2h_2^2}{(A+gB+C)^2}-\frac{C^3h_2^3}{(A+gB+C)^3}...\right]$$

$$= \bar{\Delta_1}(1+h_1)\left[1-\frac{Ch_2}{(A+gB+C)}+\frac{C^2h_2^2}{(A+gB+C)^2}-\frac{C^3h_2^3}{(A+gB+C)^3}...\right]$$
$$+\bar{\Delta_1}(1+h_1)\left[\left\{\frac{gBh_2}{(A+gB+C)}-\frac{gBCh_2^2}{(A+gB+C)^2}+\frac{gBC^2h_2^3}{(A+gB+C)^3}...\right\}\right]$$

The terms $[(h_1)^u(h_2)^v]$ could be ignored for $(u+v) > 2,\ u,v = 0,1,2,3,4,5...$ because of their low impact in ultimate estimate of parameter due to $|h_1| < 1$, $|h_2| < 1$, and $\left[\frac{1}{(A+gB+C)}\right] < 1$ (see [9] [10] [12])

Then above reduces to:

$$= \bar{\Delta_1}\left[(1+h_1)+\frac{(gB-C)h_2}{(A+gB+C)}-\frac{(gB-C)Ch_2^2}{(A+gB+C)^2}-\frac{(gB-C)h_1h_2}{(A+gB+C)}\right] \quad (23)$$

$$= \bar{\Delta_1}\left[(1+h_1)+\frac{(gB-C)}{(A+gB+C)}\left\{h_2+h_1h_2-\frac{Ch_2^2}{(A+gB+C)}\right\}\right] \quad (24)$$

■

Remark 3. Define Z = $[\frac{(gB-C)}{(A+gB+C)}]$.
Theorem 2. Using theorem 5.1, the bias of the proposed method is
$Bias(E) = B(E) = \bar{\Delta_1}(\frac{t-1}{rt})\left[\rho(C\epsilon)(C\epsilon') - \frac{C}{(A+gB+C)}(C\epsilon')^2\right]$
where ρ denotes correlation coefficients between ϵ_i and ϵ_i' at the Bipartite graph population level.
Proof. Let $E(.)$ denotes the expected value and $E(h_1) = 0$; $E(h_2) = 0$ (as in theorem 5.1 and using equation (5.5));

$Bias(E) = B(E) = E[E - \bar{\Delta_1}]$ (see [5],[6],[7])
$= E\left[\bar{\Delta_1}\left\{(1+h_1)+Z\left(h_2+h_1h_2-\frac{Ch_2^2}{(A+gB+C)}\right)\right\}-\bar{\Delta_1}\right]$ theorem 5.1, remark 5.1
$= E\left[\bar{\Delta_1}\left\{h_1+Z\left(h_2+h_1h_2-\frac{Ch_2^2}{(A+gB+C)}\right)\right\}\right]$
$= \bar{\Delta_1}\left[E(h_1)+Z\left\{E(h_2)+E(h_1h_2)-\frac{CE(h_2^2)}{(A+gB+C)}\right\}\right]$ [using (5.5), (5.6)]
$= \bar{\Delta_1}\left[Z\left\{E(h_1h_2)-\frac{CE(h_2^2)}{(A+gB+C)}\right\}\right]$
$Bias(E) = \bar{\Delta_1}Z(\frac{t-1}{rt})\left[\rho(C\epsilon)(C\epsilon') - \frac{C}{(A+gB+C)}(C\epsilon')^2\right]$ [using (5.5), (5.6)] ■
Theorem 3. The proposed methodology is almost unbiased for a given pair of (g, R) when following equation is satisfied at suitable choice of P
$R(A+gB)+C(R-1) = 0,\ where\ R = \frac{\rho(C\epsilon)}{(C\epsilon')},\ g = \frac{1}{t}$
Proof. $Bias(E) = 0 \implies \left[\rho(C\epsilon)(C\epsilon') - \frac{C(C\epsilon')^2}{(A+gB+C)}\right] = 0$

For given (g, R) , re-arranging above, one can get equation

$R(A+gB)+C(R-1) = 0$ ■

Theorem 4. For a given pair of (g, R), on maximum of three values of P, the proposed strategy E is almost unbiased.
Proof. Condition of unbiasedness is $R(A+gB)+C(R-1)=0$

This could be expressed in the power of P as:

$$(R-1)P^3+[R+gR+g]\,P^2+[23R+299R-26]\,P-[22R+20gR-24]=0 \tag{25}$$

Above equation is of degree three in P and has maximum of three roots at which E is almost unbiased. ■
Corollary 1. Few of maximum three roots of (5.20) may imaginary or overlapping depending upon given (g, R).
Corollary 2. The best choice of P, among maximum of three, is that bearing lowest mean squared error.
Theorem 5. The mean squared error of E under (5.5) to (5.13) and using (5.20) is

$MSE[E]=(\bar{\Delta_1})^2(\frac{t-1}{rt})\left[(C^*\epsilon)^2+Z^2(C^*\epsilon')^2+2Z\rho(C^*\epsilon)(C^*\epsilon')\right]$
Proof. $MSE[E]=E[E-\bar{\Delta_1}]^2$ (see [], [], [])
$=E\left[\bar{\Delta_1}\left\{(1+h_1)+\frac{(gB-C)h_2}{(A+gB+C)}+...\right\}-\bar{\Delta_1}\right]^2$ using theorem 5.1 and ignoring terms $[(h_1)^u(h_2)^v]$, $u,v=0,1,2,3,4$ of higher order $(u+v)>2$ as adopted in theorem 5.1

$=(\bar{\Delta_1})^2E\left[h_1+Zh_2\right]^2$
$=(\bar{\Delta})^2\left[E(h_1^2)+(Z)^2E(h_2^2)+(2Z)E(h_1h_2)\right]$

$MSE[E]=(\bar{\Delta})^2(\frac{t-1}{rt})\left[(C^*\epsilon)^2+Z^2(C^*\epsilon')^2+2Z\rho(C^*\bar{\epsilon})(C^*\bar{\epsilon'})\right]$ using (5.6). ■
Theorem 6. For a given pair of (g, R), the minimum mean squared error occurs at suitable choice of P when $RA+(R+1)gB+(R-1)C=0$ is satisfied.
Proof. Differentiating $MSE[E]$, in theorem 5.5, with respect to the term Z and equating to zero, one can get,

$$2Z=-2\left[\frac{\rho(C^*\epsilon)}{(C^*\epsilon')}\right]=-2R$$

$$\implies (gB-C)+R(A+gB+C)=0$$

$$\implies RA+(R+1)gB+(R-1)C=0 \tag{26}$$

■
Theorem 7. For given (g, R), the maximum three values of P exist at which MSE[E] is minimum.
Proof. The condition for minimum MSE is $RA+(R+1)gB+(R-1)C=0$
which could be expressed in the power of P as:

$$P^3[R(g+1)+(g-1)]-P^2[2R(5g+4)+(10g-9)]+P[R(29g+23)+(29g-26)]-2[R(10g+11)+2(5g-6)]=0$$

Above equation is of degree three in P, therefore, has at most the three roots. ■
Corollary 3. One can get three values of P, using theorem 5.7, having the minimum MSE. The best P is that having the lowest bias.

6. Best Selection Procedure of P for Almost Optimum MSE Strategy

Step I Find P for given pair of (g, R) producing almost unbiased strategy.

Step II Plot the range of P over the X-axis obtained by step I.

Step III Find P for given pair of (g, R) producing minimum MSE.

Step IV Plot the range of P over X-axis under step III.

Step V The overlapping range is the best choice of P for efficient estimation.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

7. Empirical Study

Consider six tourist places $(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6)$, two remote small towns (cities) (μ'_1, μ'_2). The μ'_3 is prototype of μ'_1 and μ'_4 is prototype of μ'_2 like taxi stands of μ'_1 and μ'_2 cities respectively. The weight of edges ϵ'_{ij} are assumed known like distances while weight of edges ϵ_{ij} are resource consumption scores likely during traveling by road and unknown.

Table 7.1 Population

Tourist Places		μ_1	μ_2	μ_3	μ_4	μ_5	μ_6
	μ'_1	$\epsilon_{11} = 09$	$\epsilon_{12} = 13$	$\epsilon_{13} = 21$	$\epsilon_{14} = 15$	$\epsilon_{15} = 14$	$\epsilon_{16} = 17$
Cities and	μ'_2	$\epsilon_{21} = 15$	$\epsilon_{22} = 07$	$\epsilon_{23} = 16$	$\epsilon_{24} = 19$	$\epsilon_{25} = 11$	$\epsilon_{26} = 14$
Prototype	μ'_3	$\epsilon'_{31} = 04$	$\epsilon'_{32} = 06$	$\epsilon'_{33} = 11$	$\epsilon'_{34} = 08$	$\epsilon'_{35} = 07$	$\epsilon'_{36} = 09$
	μ'_4	$\epsilon'_{41} = 07$	$\epsilon'_{42} = 03$	$\epsilon'_{43} = 08$	$\epsilon'_{44} = 09$	$\epsilon'_{45} = 05$	$\epsilon'_{46} = 06$

Table 7.3 First Sample (Random selection of μ'_1)

Tourist Places		μ_1	μ_2	μ_3	μ_4	μ_5	μ_6
Cities and	μ'_1	$\epsilon_{11} = 09$	$\epsilon_{12} = 13$	$\epsilon_{13} = 21$	$\epsilon_{14} = 15$	$\epsilon_{15} = 14$	$\epsilon_{16} = 17$
Prototype	μ'_3	$\epsilon'_{31} = 04$	$\epsilon'_{32} = 06$	$\epsilon'_{33} = 11$	$\epsilon'_{34} = 08$	$\epsilon'_{35} = 07$	$\epsilon'_{36} = 09$

Table 7.4 Second Sample (Random selection of μ'_2)

Tourist Places		μ_1	μ_2	μ_3	μ_4	μ_5	μ_6
Cities and	μ'_2	$\epsilon_{21} = 15$	$\epsilon_{22} = 07$	$\epsilon_{23} = 16$	$\epsilon_{24} = 19$	$\epsilon_{25} = 11$	$\epsilon_{26} = 14$
Prototype	μ'_4	$\epsilon'_{41} = 07$	$\epsilon'_{42} = 03$	$\epsilon'_{43} = 08$	$\epsilon'_{44} = 09$	$\epsilon'_{45} = 05$	$\epsilon'_{46} = 06$

Table 7.5 Bipartite Graph Population Parameters [table 7.1]

S.No.	Parameter	Value	Description/Equation no.
1.	rt	12	Population size
2.	t	2	Sample size
3.	$\bar{\Delta}_1$	14.25	Using (5.1)
4.	$\bar{\Delta}_2$	6.91	Using (5.2)
5.	$S\epsilon$	3.95	Using (5.10)
6.	$S\epsilon'$	2.27	Using(5.11)
7.	$C^*\epsilon$	0.27	Using (5.7)
8.	$C^*\epsilon'$	0.32	Using (5.8)
9.	ρ	0.98	(5.13)
10.	R	0.8285	Using Theorem (5.6)
11.	r	6	Large Integer

Table 7.6 Almost Unbiased Choice of P for given (g, R)

S.No.	R	g	Choice of P	Bias	MSE
1.	0.8285	0.5	$P_1 = 0.61$	-0.0001	0.1308
2.	0.8285	0.5	$P_2 = --$	–	–
3.	0.8285	0.5	$P_3 = --$	–	–

Table 7.7 Choice of P for Minimum MSE for given (g, R)

S.No.	R	g	Choice of q	MSE	Bias
1.	0.8285	0.5	$P_{1(opt)} = 0.8830$	0.0462	0.0100
2.	0.8285	0.5	$P_{2(opt)} = --$	–	–
3.	0.8285	0.5	$P_{3(opt)} = --$	–	–

Remark 4. As per data of table 7.1, and calculation of table 7.6 and table 7.7 when R=0.8285, g=0.5, the most suitable range of P is P= 0.61 to P=0.8830 to produce the best estimate of average resource consumption score using proposed E. This range of consistent P provides almost unbiased optimal estimate of average resource score.

Table 7.8 Comparison with Particular Cases of Proposed E (Using PRE)

S.No.	Choice of P	Bias(E) (Theorem 5.2)	MSE(E) (Theorem 5.5)	Comparison (PRE)
1.	At P=1	0.0220	0.1001	53.1468%
2.	At P=2	0.1066	6.1768	99.24407%
3.	At P=3	0.0533	3.2824	98.5711%
4.	At P=4	0.0000	1.3049	96.4058%
5.	At P=5	-0.0212	0.4958	90.5405%

Where, Percentage Relative Efficiency (PRE) at $P_{opt} = 0.8830$

$$= \frac{[MSE(E)_{P=1,2,3,4,5}] - [MSE(E)_{P_{(opt)}}]}{MSE(E)_{P=1,2,3,4,5}} X100$$

Remark 5. As per table 7.8, for given pair of (g,R)=(0.5, 0.8285), the best choice of P is $P = 0.8830$ which provides lowest bias and minimum MSE for the proposed strategy E in light of given data set of table 7.1.

Remark 6. The proposed strategy E is constantly better over particular cases at P= 1, 2, 3, 4, 5 as evident from table 7.8 using PRE.

Table 7.9 Calculation for First Sample Parameter and Confidence Interval (C.I.)

$\bar{\delta_1}$	$\bar{\delta_2}$	$(s_\epsilon)^2$	$(s'_\epsilon)^2$	(c_ϵ)	(c'_ϵ)	$(s_{\epsilon\epsilon'})$	ρ'	C.I.
14.8333	7.5	16.1660	5.9000	0.2711	0.3238	3.1144	0.3189	at P =0.61 (unbiasedness) [12.47, 17.18]
	–	–	–	–	–	–	–	at P =0.883 (Minimum MSE) [12.11, 17.47]

Table 7.10 Calculation for Second Sample Parameter and Confidence Interval (C.I.)

$\bar{\delta_1}$	$\bar{\delta_2}$	$(s_\epsilon)^2$	$(s'_\epsilon)^2$	(c_ϵ)	(c'_ϵ)	$(s_{\epsilon\epsilon'})$	ρ'	C.I.
13.6666	6.3333	17.4666	4.6666	0.3058	0.3410	2.9888	0.3310	at P =0.61 (unbiasedness) [11.28, 16.04]
	–	–	–	–	–	–	–	at P= 0.883 (minimum MSE) [11.03, 16.30]

Remark 7. $\bar{\delta_1}$ and $\bar{\delta_2}$ are in (5.3) and (5.4), and others are defined as
$(c_\epsilon) = \frac{(s_\epsilon)}{\bar{\delta_1}}$
$(c_{\epsilon'}) = \frac{(s_{\epsilon'})}{\bar{\delta_2}}$
$(c\epsilon\epsilon') = \frac{(s_{\epsilon\epsilon'})}{(\bar{\delta_1}.\bar{\delta_2})}$
$(s_\epsilon)^2 = \frac{1}{r-1}\sum_{j=1}^{r}(s_{\epsilon_{ij}} - \bar{\delta_1})^2$
$(s\epsilon')^2 = \frac{1}{r-1}\sum_{j=1}^{r}(s_{\epsilon'_{ij}} - \bar{\delta_2})^2$
$(s_{\epsilon\epsilon'}) = \frac{1}{r-1}\sum_{i=1}^{t}\sum_{j=1}^{r}(s_{\epsilon_{ij}} - \bar{\delta_1})(s_{\epsilon'_{ij}} - \bar{\delta_2})$
$\rho' = \frac{s_{\epsilon\epsilon'}}{[(s_\epsilon)(s_{\epsilon'})]}$

Remark 8. Let P[A] denotes the probability of event A then a 95% confidence interval in defined as:

$$P\left[(E)_P - 1.96\sqrt{MSE(E)_P},\ (E)_P + 1.96\sqrt{MSE(E)_P}\right] = 0.95$$

This interval indicates that there is 95% chance, the true value of population mean lies in the range $P\left[(E)_P - 1.96\sqrt{MSE(E)_P},\ (E)_P + 1.96\sqrt{MSE(E)_P}\right] = 0.95$. If the value of $P = P_{opt}$ then this interval will be the optimal confidence interval with respect to minimum MSE (or with respect to unbiasedness as the case may be)

Table 7.11 Ready Reckoner for P Value for Unbiasedness for given (g, R)

S.No.	R	g	Choice of P	Bias	MSE	S.No.	R	g	Choice of P	Bias	MSE
1.	0.2	0.3	$P_1 = 4.5641$	-0.0182	0.7089	46.	1.2	0.3	$P_1 = 1.2549$	0.0016	0.7877
2.	0.2	0.3	$P_2 = 4$	0.0000	1.3049	47.	1.2	0.3	$P_2 = 4$	0.0000	1.3049
3.	0.2	0.3	$P_3 = --$	–	–	48.	1.2	0.3	$P_3 = --$	–	–
4.	0.2	0.6	$P_1 = 0.45410$	-0.0202	0.6601	49.	1.2	0.6	$P_1 = 1.335$	0.0682	0.7085
5.	0.2	0.6	$P_2 = 4$	0.0000	1.3049	50	1.2	0.6	$P_2 = 4$	0.0000	1.3049
6.	0.2	0.6	$P_3 = --$	–	–	51.	1.2	0.6	$P_3 = --$	–	–
7.	0.2	0.9	$P_1 = 4.5264$	-0.0224	0.6019	52.	1.2	0.9	$P_1 = 1.0900$	0.0675	0.6823
8.	0.2	0.9	$P_2 = 4$	0.0000	0.1304	53.	1.2	0.9	$P_2 = 4$	0.0000	0.1304
9.	0.2	0.9	$P_3 = --$	–	–	54.	1.2	0.9	$P_3 = --$	–	–
10.	0.4	0.3	$P_1 = 5.2910$	-0.0209	0.4150	55.	1.4	0.3	$P_1 = 1.3560$	0.1407	2.2086
11.	0.4	0.3	$P_2 = 4$	0.0000	1.3049	56.	1.4	0.3	$P_2 = 4$	0.0000	1.3049
12.	0.4	0.3	$P_3 = --$	–	–	57.	1.4	0.3	$P_3 = --$	–	–
13.	0.4	0.6	$P_1 = 5.3389$	-0.0195	0.4573	58.	1.4	0.6	$P_1 = 1.2048$	0.1366	1.9884
14.	0.4	0.6	$P_2 = 4$	0.0000	1.3049	59.	1.4	0.6	$P_2 = 4$	0.0000	1.3049
15.	0.4	0.6	$P_3 = --$	–	–	60.	1.4	0.6	$P_3 = --$	–	–
16.	0.4	0.9	$P_1 = 5.4050$	-0.0177	0.5174	61.	1.4	0.9	$P_1 = 1.1430$	0.1345	1.9025
17.	0.4	0.9	$P_2 = 4$	0.0000	1.3049	62.	1.4	0.9	$P_2 = 4$	0.0000	1.3049
18.	0.4	0.9	$P_3 = --$	–	–	63.	1.4	0.9	$P_3 = --$	–	–
19.	0.6	0.3	$P_1 = 7.0810$	-0.0144	0.2561	64.	1.6	0.3	$P_1 = 1.4121$	0.2341	4.3451
20.	0.6	0.3	$P_2 = 4$	-0.0000	1.3049	65.	1.6	0.3	$P_2 = 4$	-0.0000	1.3049
21.	0.6	0.3	$P_3 = --$	–	–	66.	1.6	0.3	$P_3 = --$	–	–
22.	0.6	0.6	$P_1 = 12.0001$	-0.0085	0.5738	67.	1.6	0.6	$P_1 = 1.2490$	0.2259	3.9084
23.	0.6	0.6	$P_2 = 4$	0.0000	1.309	68.	1.6	0.6	$P_2 = 4$	0.0000	1.309
24.	0.6	0.6	$P_3 = --$	–	–	69.	1.6	0.6	$P_3 = --$	–	–
25.	0.6	0.9	$P_1 = 0.0001$	-0.0044	0.8903	70.	1.6	0.9	$P_1 = 1.1781$	0.2232	3.7648
26.	0.6	0.9	$P_2 = 4$	0.0000	1.3049	71.	1.6	0.9	$P_2 = 4$	0.0000	1.3049
27.	0.6	0.9	$P_3 = --$	–	–	72.	1.6	0.9	$P_3 = --$	–	–
28.	0.8	0.3	$P_1 = 0.0001$	0.0027	0.1015	73.	1.8	0.3	$P_1 = 1.4480$	0.3497	7.1827
29.	0.8	0.3	$P_2 = 4$	0.0000	1.3049	74.	1.8	0.3	$P_2 = 4$	0.0000	1.3049
30.	0.8	0.3	$P_3 = --$	–	–	75.	1.8	0.3	$P_3 = --$	–	–
31.	0.8	0.6	$P_1 = 0.6354$	-0.0020	0.1752	76.	1.8	0.6	$P_1 = 1.2800$	0.3391	6.5374
32.	0.8	0.6	$P_2 = 4$	0.0000	1.3049	77.	1.8	0.6	$P_2 = 4$	0.0000	1.3049
33.	0.8	0.6	$P_3 = --$	–	–	78.	1.8	0.6	$P_3 = --$	–	–
34.	0.8	0.9	$P_1 = 0.8201$	-0.0015	0.1456	79.	1.8	0.9	$P_1 = 1.2030$	0.3331	6.2609
35.	0.8	0.9	$P_2 = 4$	0.0000	1.3049	80.	1.8	0.9	$P_2 = 4$	0.0000	1.3049
36.	0.8	0.9	$P_3 = --$	–	–	81.	1.8	0.9	$P_3 = --$	–	–
37.	1.0	0.3	$P_1 = 0.9008$	0.0158	0.0592	82.	2.0	0.3	$P_1 = 1.4738$	0.4928	10.8517
38.	1.0	0.3	$P_2 = 4$	0.0000	1.3049	83.	2.0	0.3	$P_2 = 4$	0.0000	1.3049
39.	1.0	0.3	$P_3 = --$	–	–	84.	2.0	0.3	$P_3 = --$	–	–
40.	1.0	0.6	$P_1 = 0.9800$	0.0186	0.0766	85.	2.0	0.6	$P_1 = 1.3020$	0.4704	9.7348
41.	1.0	0.6	$P_2 = 4$	0.0000	1.3049	86.	2.0	0.6	$P_2 = 4$	0.0000	1.3049
42.	1.0	0.6	$P_3 = --$	–	–	87.	2.0	0.6	$P_3 = --$	–	–
43.	1.0	0.9	$P_1 = 0.9500$	0.0109	0.0470	88.	2.0	0.9	$P_1 = 1.2220$	0.4678	9.4728
44.	1.0	0.9	$P_2 = 4$	0.0000	1.3049	89.	2.0	0.9	$P_2 = 4$	0.0000	1.3049
45.	1.0	0.9	$P_3 = --$	–	–	90.	2.0	0.9	$P_3 = --$	–	–

Table 7.12 Ready Reckoner for P Value for Minimum MSE for given (g, R)

S.No.	R	g	Choice of P	Bias	MSE	S.No.	R	g	Choice of P	Bias	MSE
1.	0.2	0.3	$P_1 = 4.5001$	-0.0171	0.7561	46.	1.2	0.3	$P_1 = 1.1452$	0.0396	0.2993
2.	0.2	0.3	$P_2 = --$	–	–	47.	1.2	0.3	$P_2 = --$	–	–
3.	0.2	0.3	$P_3 = --$	–	–	48.	1.2	0.3	$P_3 = --$	–	–
4.	0.2	0.6	$P_1 = 4.5005$	-0.0196	0.6861	49.	1.2	0.6	$P_1 = 1.0743$	0.0408	0.2994
5.	0.2	0.6	$P_2 = --$	–	–	50.	1.2	0.6	$P_2 = --$	–	–
6.	0.2	0.6	$P_3 = --$	–	–	51.	1.2	0.6	$P_3 = --$	–	–
7.	0.2	0.9	$P_1 = 0.1964$	-0.0052	0.7840	52.	1.2	0.9	$P_1 = 1.0495$	0.0410	0.2969
8.	0.2	0.9	$P_2 = --$	–	–	53.	1.2	0.9	$P_2 = --$	–	–
9.	0.2	0.9	$P_3 = --$	–	–	54.	1.2	0.9	$P_3 = --$	–	–
10.	0.4	0.3	$P_1 = 5.5014$	-0.0203	0.3744	55.	1.4	0.3	$P_1 = 1.2340$	0.0621	0.6498
11.	0.4	0.3	$P_2 = --$	–	–	56.	1.4	0.3	$P_2 = --$	–	–
12.	0.4	0.3	$P_3 = --$	–	–	57.	1.4	0.3	$P_3 = --$	–	–
13.	0.4	0.6	$P_1 = 0.2300$	-0.0052	0.3826	48.	1.4	0.6	$P_1 = 1.1270$	0.0644	0.6452
14.	0.4	0.6	$P_2 = --$	–	–	59.	1.4	0.6	$P_2 = --$	–	–
15.	0.4	0.6	$P_3 = --$	–	–	60.	1.4	0.6	$P_3 = --$	–	–
16.	0.4	0.9	$P_1 = 0.6381$	-0.0057	0.3830	61.	1.4	0.9	$P_1 = 1.1201$	0.0991	1.2263
17.	0.4	0.9	$P_2 = --$	–	–	62.	1.4	0.9	$P_2 = --$	–	–
18.	0.4	0.9	$P_3 = --$	–	–	63.	1.4	0.9	$P_3 = --$	–	–
19.	0.6	0.3	$P_1 = 0.0001$	0.0023	0.1094	64.	1.6	0.3	$P_1 = 1.2928$	0.0888	1.1370
20.	0.6	0.3	$P_2 = --$	–	–	65.	1.6	0.3	$P_2 = --$	–	–
21.	0.6	0.3	$P_3 = --$	–	–	66.	1.6	0.3	$P_3 = --$	–	–
22.	0.6	0.6	$P_1 = 0.6900$	-0.0009	0.1416	67.	1.6	0.6	$P_1 = 1.1664$	0.0928	1.1367
23.	0.6	0.6	$P_2 = --$	–	–	68.	1.6	0.6	$P_2 = --$	–	–
24.	0.6	0.6	$P_3 = --$	–	–	69.	1.6	0.6	$P_3 = --$	–	–
25.	0.6	0.9	$P_1 = 0.8501$	-0.0001	0.1098	70.	1.6	0.9	$P_1 = 1.0990$	0.0756	0.8146
26.	0.6	0.9	$P_2 = --$	–	–	71.	1.6	0.9	$P_2 = --$	–	–
27.	0.6	0.9	$P_3 = --$	–	–	72.	1.6	0.9	$P_3 = --$	–	–
28.	0.8	0.3	$P_1 = 0.7102$	0.0093	0.0473	73.	1.8	0.3	$P_1 = 1.3360$	0.1203	1.7742
29.	0.8	0.3	$P_2 = --$	–	–	74.	1.8	0.3	$P_2 = --$	–	–
30.	0.8	0.3	$P_3 = --$	–	–	75.	1.8	0.3	$P_3 = --$	–	–
31.	0.8	0.6	$P_1 = 0.8863$	0.0081	0.0477	76.	1.8	0.6	$P_1 = 1.1970$	0.1259	1.7725
32.	0.8	0.6	$P_2 = --$	–	–	77.	1.8	0.6	$P_2 = --$	–	–
33.	0.8	0.6	$P_3 = --$	–	–	78.	1.8	0.6	$P_3 = --$	–	–
34.	0.8	0.9	$P_1 = 0.9301$	0.0079	0.0476	79.	1.8	0.9	$P_1 = 1.1395$	0.1282	1.7790
35.	0.8	0.9	$P_2 = --$	–	–	80.	1.8	0.9	$P_2 = --$	–	–
36.	0.8	0.9	$P_3 = --$	–	–	81.	1.8	0.9	$P_3 = --$	–	–
37.	1.0	0.3	$P_1 = 0.9700$	0.0198	0.0831	82.	2.0	0.3	$P_1 = 1.3690$	0.1567	2.5605
38.	1.0	0.3	$P_2 = --$	–	–	83.	2.0	0.3	$P_2 = --$	–	–
39.	1.0	0.3	$P_3 = --$	–	–	84.	2.0	0.3	$P_3 = --$	–	–
40.	1.0	0.6	$P_1 = 0.9801$	0.0187	0.0767	85.	2.0	0.6	$P_1 = 1.2215$	0.1638	2.5523
41.	1.0	0.6	$P_2 = --$	–	–	86.	2.0	0.6	$P_2 = --$	–	–
42.	1.0	0.6	$P_3 = --$	–	–	87.	2.0	0.6	$P_3 = --$	–	–
43.	1.0	0.9	$P_1 = 1.001$	0.0223	0.1022	88.	2.0	0.9	$P_1 = 1.1585$	0.1671	2.5642
44.	1.0	0.9	$P_2 = --$	–	–	89.	2.0	0.9	$P_2 = --$	–	–
45.	1.0	0.9	$P_3 = --$	–	–	90.	2.0	0.9	$P_3 = --$	–	–

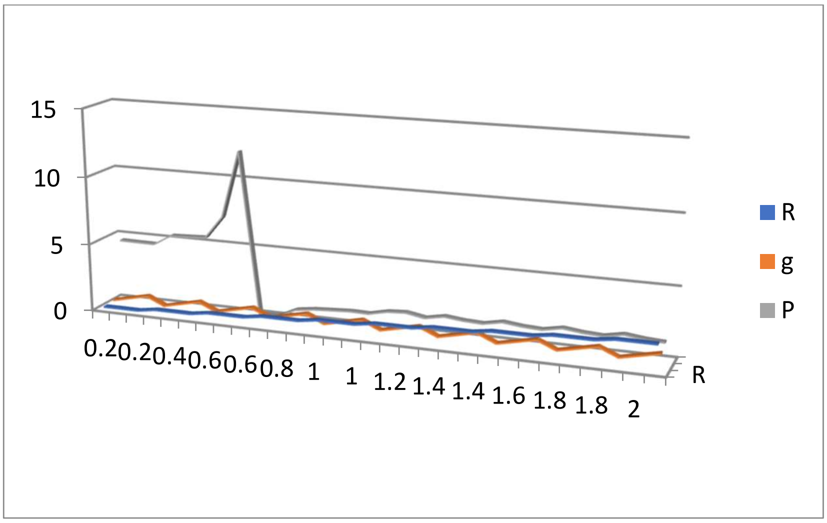

Fig 7.1 Three Dimensional Graph for Almost Unbiasedness

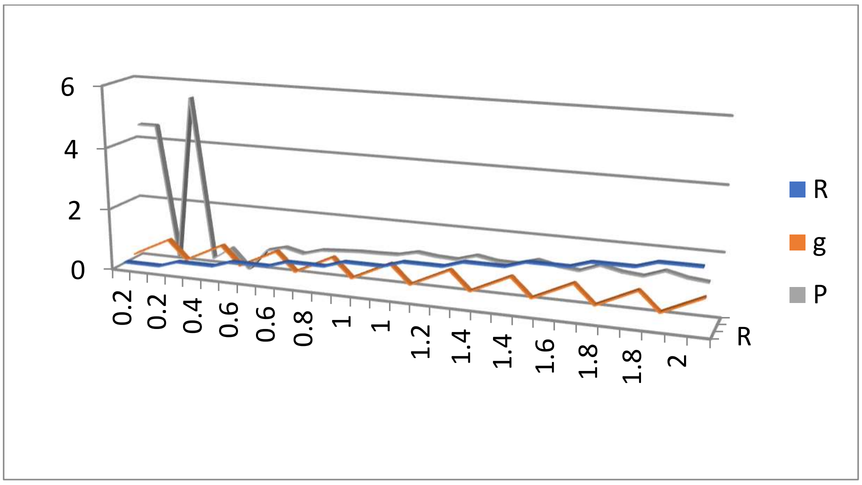

Fig 7.2 Three Dimensional Graph for Optimum MSE

7.1. About given (g, R)

To observe that $g = \frac{1}{t}$, where t is size of the group G_1 who is known and fixed before the draw of random sample using the single-node-systematic sampling procedure. The quantity R is R = $\frac{\rho(C^*\epsilon)}{(C^*\epsilon')}$ as described in theorem 5.6. The R is a ratio of $C^*\epsilon$ and $C^*\epsilon'$ who are coefficients of variations and remain almost stable over time occasions. Therefore, R could be guessed from the past data or by past reports or by the current sample. While minor fluctuations in R, the table 7.13 helps to find out most suitable values of P.

Table 7.13 Best Choice of P Under Fluctuations of R (independent of g)

Range of R	Best range of P for unbiasedness	Best range of P for opt. MSE
$0 < R \leq 0.6$	$0 < P < 5.4$	$0 < P < 5.5$
$0.6 < R \leq 1.0$	$0 < P < 0.95$	$0.71 < P < 1.001$
$1.0 < R \leq 1.6$	$1.09 < P < 1.7$	$1.0 < P < 1.29$
$1.6 < R \leq 2.0$	$1.22 < P < 1.45$	$1.15 < P < 1.36$

Note 7.2 The table 7.13 is independent of variations of g which is created using the Ready-reckoner tables 7.11 and 7.12. However, for any given (g, R), one can choose the best P using Ready-reckoner table 7.11 and 7.12.

Note 7.3 The most suitable value of P,for almost unbiased estimation, is ($P = 0.10$ to $P = 5.4$) for all $(g, 0 < g < 1)$ and for all R, $(0 < R \leq 2)$ (see table 7.13).

Note 7.4 The most suitable value of P for low MSE is ($P = 0.001$ to $P = 5.5$) for all g and $0 < R \leq 2$ (see table 7.13)

Note 7.5 The general recommended P is $P \in (0.1, 5.5)$ where one can get low bias and low MSE by the proposed strategy, whatever be the g, $(0 < g < 1)$ and whatever be R, $0 < R \leq 2$.

Note 7.6 This is beauty of the proposed strategy E because it is now independent to the known pair (g, R) to produce good estimate of average consumption of score in a Bipartite graph population when user has chosen $0.1 < P < 5.5$.

Note 7.7 The figure 7.2 presents three dimensional aspect of choice of P for given pair (g, R). The X- axis has R, Z- axis has g and Y- axis has calculated value of P.

8. Conclusion

On recapitulation, the content of the paper has a sample based methodology for evaluating the average resource consumption scores with the help of Bipartite graph. A single node systematic sampling procedure is suggested in the content which is a graph sampling based procedure, useful for parameter estimation in places where similar situation exists. This procedure opens up avenues for further researches in the area of sampling where population is synonymous to the graphical structure. The Bipartite graph is used for getting solution of the problem of estimation of the travel resource consumption parameter. An estimation strategy is proposed and its properties are derived. It is proved that they are bias and MSE controlled both, at the same time due to cubic equation. The proposed also converts to an almost unbiased optimum strategy at some appropriate choice of P.

The main difficulty occurs with the strategy is the selection of suitable value of constant incorporated in its structure. Two Ready-reckoner tables have been prepared who are useful for the quick selection of constant for give pair of (g, R) values. These tables provide to users, the population independency just as to utilize only (g, R), irrespective of the other population characteristics. Whatever may the distance and resource consumption, if (g, R) are given (or guessed or estimated or calculated from past data) ,the bias and MSE can be predicted through these Ready-reckoner tables who made easy to apply the suggested. It can be used for developing computer algorithms and softwares for enhancing the applications.

Two systematic samples are taken into account and their computed confidence intervals are $(12.47 - 17.18)$, $(12.11 - 17.47)$ for first sample , $(12.28 - 16.04)$, $(11.03 - 16.30)$ for second sample. All intervals are catching the true values of average resource consumption which is 14.25. The interval based prediction is sound enough indicating the efficiency of the suggested strategy. For $0 < R \leq 2$, a table has been developed, for the ease of users, towards rapid selection of best value of constant P whatever be the value of g and whatever be the population values in terms of distance and resource consumption scores . The general recommendation for best choice of P is the range $(0.1 - 5.5)$ irrespective of other population features and dependencies. Bipartite graph is used as a model tool for developing the single node systematic sampling procedure who can be extended further using other kinds of estimation procedures exiting in the concerned literature.

References

[1] Deo, N. (2001). *Graph theory with application to engineering and computer science*. Eastern Economy Edn, New Delhi, India .

[2] Moons, E., Loomis,J., Proost, S. Eggermont, K. and Hermy,M. (2001). *Travel cost and time measurement in travel cost models*. Katholieke Universiteit Leuven.

[3] Wikipedia, the Free Encyclopedia. *https://www.google.com/search?client=firefox-b-dq=Travel+Cost+Analysis*.

[4] Wikipedia, the Free Encyclopedia. *https://www.google.com/search?client=firefox-b-dq=ecosystem+valuation*.

[5] Shukla, D. (2002). *F-T estimator under two-phase sampling*. Metron, 60:97-106.

[6] Singh, S. (2003). *Advanced sampling theory with applications*. Kluwer Academic Publisher, Netherland, Springer, Dordrecht. htttps://doi.org/10.1007/978-94-007-0789-4.

[7] Cochran, W.G. (2005). *Sampling Techniques*. Jhon Willy and Sons.

[8] Zhang,B. , Chambers M. C. and Tabb, D.L. (2007). *Proteomic parsimony through Bipartite graph analysis improves accuracy and tranparency* Journal of Research Articles Proteome Research, 6.

[9] Shukla,D., Rajput, Y.S. and Thakur, N.S. (2009). *Estimation of spanning tree mean-edge using node sampling*. Model Assisted Statistics and Applications, 4:23-37.

[10] Shukla,D., Rajput, Y.S. and Thakur, N.S. (2010). *Edge estimation in population of planer graph using sampling*. Journal of Reliability and Statistical Studies, 3:13-29.

[11] Koutra,D. , Tong, H. and Lubensky, D. (2013). *Big-Align: fast Bipartite graph alignment*. 13th International Conference on Data Mining (IEEE), 389-398.

[12] Shukla,D., Rajput, Y.S. and Thakur, N.S.(2014). *Edge estimation in the population of a binary tree using node-sampling*. Communication in Statistics Theory and Methods, 43:2815"2829.

[13] F. Serratosa (2014). *Speeding up fast Bipartite graph matching through a new cost matrix*. International Journal of Pattern Recognition and Artificial Intellingence, 29, http://dx.doi.org/10.1016/j.patrec.2014.04.015.

[14] Shukla, D., Pathak, S. and Trivedi, S.K. (2016). *Graph sampling by Spanning tree under stratified setup*. Research and Review: Journal of Statistics, 5:11-31.

[15] Xiong, H., Zhau,Y., Hu,C., Wei, X. and Li,L. (2017). *A novel recommendation algorithm frame for tourist spots based on multi clustering Bipartite graph*. 2nd IEEE International Conference on Cloud Computing and Big Data Analysis, 276-282.

[16] Shukla,D., Trivedi,S.K., Pathak, S. and Rajoriya, D. (2018). *Mean edge estimation in population of Binary tree using stratified sampling procedure*. Research and Reviews:Journal of Statistics, 7:60-73.

[17] Shukla,D. Trivedi,S.K., Pathak, S. and Rajoriya,D. (2019). *Mean edge estimation in population of Planar graph using stratified sampling procedure*. Research and Reviews: Journal of Statistics, 8:11-31.

[18] Rajoriya, D. and Shukla, D. (2020). *Optimal estimation strategies of parameters in Hamiltonian circuit type population*. Journal of Critical Reviews, 7:3561-3573.

[19] Donga,J., Bhojak, P., Patel, K.and Shah, V. (2021). *An analysis of different computer science algorithms with the graph theory of mathematics*. Reliability: Theory and Methods, 16:376-383.