RELIABILITY OF A BIG CITY SEWER NETWORK

Baranov L. A.

•

Russian University of Transport (MIIT)
baranov.miit@gmail.com

Ermolin Y.A.

•

Russian University of Transport (MIIT)
ermolin.y@yandex.ru

Shubinsky I. B.

•

NIIAS
igor-shubinsky@yandex.ru

Abstract

The ramified sewerage system for receiving and transferring household and industrial sewage typical for a large city is considered. Consideration is restricted to the sub-system of sewage conveyance (sewer network). A sewer network is defined as a combination of underground pipes (sewers) passing sewage through the force of gravity. A review of the literature reveals that there is currently no universally acceptable definition or measure for the reliability of urban sewer network. The aim of this article is to propose the physically obvious reliability index, and to develop an engineering methodology for its calculating. The relative raw sewage volume that could be potentially discharged to the environment as a result of component failures in the sewer network is proposed as a measure of overall system reliability. A simple method for quick and proper calculation of this volume is presented. The basis for this method is a representation of the sewer network by a combination of Y-like fragments. Each such fragment is formally substituted by a fictitious equivalent sewer that has a failure rate leading to the same output for the same input. A sequential application of this approach reduces the problem of estimating the discharged sewage volume to an elementary sub-problem with a simple solution is. The proposed approach is based on the reliability theory. The notions "failure flow" and "repair flow" are used. These flows are taken stationary with known parameters. Numerical examples are used to demonstrate the proposed approach.

Keywords: Sewer network; Reliability; Sewage discharge; Y-like network fragment; Decomposition-equivalence method.

I. Introduction

The proper functioning of the urban sewage disposal system is a primary determinant of the city's ecological and sanitary-hygienic conditions. Confronting problems associated with the sewer network maintenance as a subsystem of an entire sewage disposal system, is a necessary step for improving operation efficiency in an urban waste water disposal system as a whole. In recent years, in response to increasing congestion in urban sewer networks and the adverse environmental impact of such congestion, substantial attention has been focused on working out the proposals to improve waste water disposal processes. A critical issue in the evaluation and effective implementation of these proposals is the development of the best, in some specified sense, sewage disposal strategies. In practice sometimes, very significant improvements in management efficiency could be accomplished simply by better maintenance of the waste water disposal system.

There is a great deal of research dedicated to the reliability problems of water supply systems reported in the literature [1-6]. From the latest publications we emphasize the work [2], which provides an in-depth review of the relevant research literature in the context of the mathematical methods for measuring water distribution system reliability. However, as note in other works, for example, [4]: "A review of the literature reveals that there is currently no universally acceptable definition or measure of the reliability of water distribution systems … For a large system … it is extremely difficult to analytically compute the mathematical reliability".

By contrast, the reliability problems of the sewage disposal systems are still uninvestigated [7-13]. Therefore, any effort to comprehend, set up and refine the issue of sewer network reliability takes on great significance. The final objective of these investigations is to develop sewer network design, reconstruction and maintenance methods with due regard for reliability.

II. Short description of the object and problem statement

An urban wastewater disposal system is a network of structurally and technologically interconnected structures intended for sewage collection and its conveyance to the purification facilities.

Usually the city sewage disposal system is designed and constructed according to the head-and-gravity concept. This means that the sewage passes through underground sewers having a specified fall by gravity, and pumping stations lift sewage in areas where gravity flow is impossible. (As a rule, the sewage pumping station is designed as a system providing a redundancy of the pumping equipment. Because of this, in the following, we assume that the pumping stations are absolutely reliable). By this means the sewer network, by nature, is a peculiar water distribution system. The reliability of such systems is often defined by heuristic guidelines, like having all pipe diameters greater than a minimum prescribed value. By using such guidelines it is implicitly assumed that reliability will be assured, but the level of reliability provided is not quantified or measured. Thus, the question: "Is the system reliable?" is usually well understood and easy to answer, while the question "What is its reliability level?" is not straightforward. As a result, only limited confidence can be placed on such rules, as reliability is not considered explicitly.

The underground pipes, as sewer network components, are subject to so many influences that it is difficult, if not impossible, to predict their combined effect in advance. These influences include the corrosive action of the soil and sewage, ground movements, the weather, etc. Most of these factors are random, and are characterized by significant variability. These circumstances adversely affect sewer network reliability. Currently, traditional wastewater disposal system design and maintenance methods usually fail to account for this situation.

A determination of the timeline and the sequence of a sewerage modernization plan is an important problem of the applied reliability theory. The strategy development for the object reconstruction falls into two stages. At the first stage, the object technical condition is established, and a need for renovation is determined. The second stage is job scheduling for the specific network elements requiring repair or replacements.

Depending on the purpose of the study and the specifics of an object, its technical state may be estimated, from the viewpoint of a reliability, using different quantitative measures: for example, by the average time between failures or by the probability of trouble-free functioning over a given period of time. We note that these traditional measures accepted in theory, as applied to sewer networks, provide not enough information because it is very hard to interpret them physically.

Here the specific reliability index is proposed. This index is intended for functioning

efficiency estimation of tree-like hierarchical structures; an urban sewer network is a typical representative of such structure. The damage due to raw sewage discharged to the environment resulting from sewer network failures is considered as a quantitative reliability measure [7].
At the moment there are no universally accepted procedures for assessment of the economic and ecological damage due to raw sewage discharge resulting from sewer network failures. Within any particular region, sewer basin or city district this damage and the methods of assessing it may differ significantly and may change with the time. What is considered acceptable for one area or time period may not be appropriate for another area or time. In any case however, it is evident that this damage is dependent on the volume of raw sewage discharged to the environment (in actual practice this sewage is usually pumped over into a suitable nearly manhole by a mobile emergency pumping plant).

For this reason, the volume of raw sewage potentially discharged from the sewer network to the environment over some time period (for example, one year) may be taken as a measure of the damage caused by the network unreliability, and, therefore, as an indirect measure of the sewer network reliability.
Thus, the problem reduces to finding of the raw sewage volume potentially discharged from the sewer network.

III. Reliability analysis of the sewer network fragment

Systems like a urban sewer network, are often described in terms of a graph, with links representing the pipes (sewers), and nodes representing connections between the pipes. The behavior of a sewer network is governed by the physical laws that describe the flow relationships in the pipes (laws of conservation), and the network layout.

Two features of a sewer network should be pointed out: 1) a sewage gravitates through each sewer in one direction only, and 2) the hydraulic elements used to link different sewer basins are lacking. This means particularly that the sewage entering into any network inlet, may be piped to a certain its outlet by a strictly specified sequence of sewers, i.e. along the only path. Thus, mathematically, the graph of an urban sewer network is a simply connected, oriented, and acyclic graph; in theory such graphs are also known as tree-like graphs.

We consider the three-component Y-like sewer network fragment shown in Fig. 1.

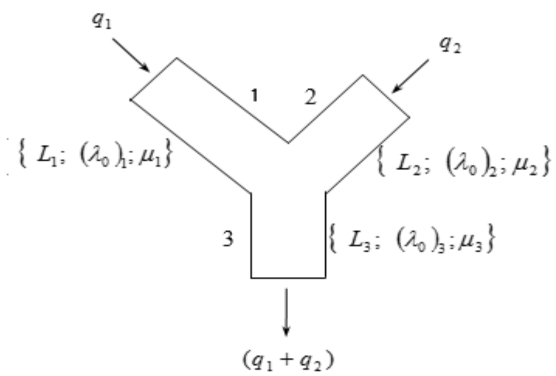

Figure. 1: *Y-like sewer network fragment.*

Each enumerated sewer of this fragment is characterized by its length $(L_1;L_2;L_3)$; in addition, we suppose that the unidirectional sewage flow rates in the inlets of sewer 1and 2 (q_1 and q_2, respectively) are known, constant and equal to the mean values calculated by averaging the historical data obtained over a long period of time. We assume that, from time to time, each sewer fails, is repaired and, thereafter, put back in service again. Thus, each sewer can be either up (operable) or down (failed). In terms of the reliability theory, this means that so-called failure and repair flows are both acting on each sewer.

For a mathematical description of these flows, what is meant by term "failure" must be ascertained.

The exact definition of failure is somewhat fluid and depends on the level of detail of the required analysis, and has a variety of meanings to different individuals. In actual practice, a disturbance of the normal operation of the sewer can be manifested as a reduction of its capacity caused by cracks in the pipe, sewer breaks under extreme mechanical load, increasing rates of infiltration, repeated overflows, etc. Here, we shall define "failure" as an event implying a need for immediate overhaul or replacement of the pipe. In other words, the failure of a sewer is defined as an event when the sewer capacity becomes equal to zero, and consequently, all sewage entering into the sewer discharges to the environment.

The repair is taken here to mean that a renewal process reaches completion and the sewer is returned to service.

Usually, such events are documented with accompanying parameters. This information is systematically renewed, statistically processed and stored in relevant data bases. In the following, we assume that these data (in particular, the mean time to failure and mean time to repair) are known and available for analysis.

Both of these flows are characterized by their rates. Physically, the failure rate is the mean number of failures in a unit of time. The repair rate is defined similarly. In line with a much used assumption, we suppose that the failure flow as well as the repair flow are exponentially distributed flows [4]. From this it follows that the specific failure rate (the failure rate per unit sewer length) for each sewer $((\lambda_0)_1, (\lambda_0)_2, (\lambda_0)_3$, respectively) is constant. Analogously, the repair rates for sewers 1, 2 and 3 (μ_1, μ_2, μ_3) are constant as well. We assume that all these values are given.

The problem is stated as follows: given the values of all quantities listed above, it is necessary to estimate the volume of raw sewage discharged from the sewer network to the environment over some time period (one year in this study).

In order to solve this problem, we must first bring out the possible states of the system taken as a whole. These states are enumerated and listed below; what is meant by each state is explained in parentheses, and, next, the associated probability p_i of the system residing in state i is introduced:

0: (sewers 1, 2 and 3 up) - p_0;

1: (sewer 1 down, sewers 2 and 3 up) - p_1;

2: (sewer 2 down, sewers 1 and 3 up) - p_2;

3: (sewer 3 down, sewers 1 and 2 up) - p_3;

4: (sewers 1 and 2 down, sewer 3 up) - p_4;

5: (sewers 1 and 3 down, sewer 2 up) - p_5;

6: (sewers 2 and 3 down, sewer 1 up) - p_6;

7: (sewers 1, 2 and 3 down) - p_7.

With time, under the influence of failure and repair flows, the system goes from one state to another accidentally. This process is conveniently described by the use of the state space graph [14] (see Fig. 2), in which the possible system states are represented by circles with their number inside. The arrows indicate the transitions between states. The associated failure or repair rate is placed by an arrow; in this case $\lambda_1 = (\lambda_0)_1 L_1, \lambda_2 = (\lambda_0)_2 L_2$, and $\lambda_3 = (\lambda_0)_3 L_3$.

Such a graph gives a descriptive idea of the changing system states. As an example, we consider state 4 (sewers 1 and 2 down, sewer 3 up). From this state the system departs to state 1 if the renewal process of sewer 2 reaches completion (the repair rate is μ_2), to state 2 when sewer 1 is returned to service (μ_1), and to state 7 when sewer 3 breaks down as well (λ_3). We note incidentally that the transition of the system, for example, from state 4 to state 0 is impossible due to the features of the exponentially distributed flow.

With the state space graph in hand it becomes possible to find all state probabilities $p_i(t)$ as functions of time. For this purpose so-called Kolmogorov's equations are formed [14].

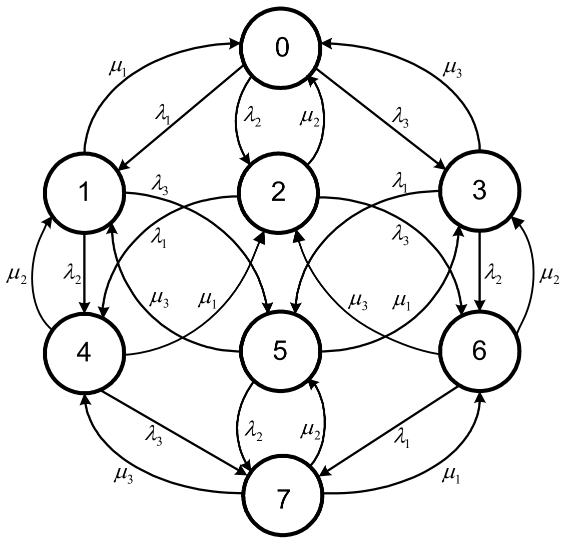

Figure. 2: *State space graph for the Y-like network fragment.*

For the graph shown in Fig. 2 these equations take the form:

$$\begin{cases} \dfrac{dp_0(t)}{dt} = \mu_1 p_1(t) + \mu_2 p_2(t) + \mu_3 p_3(t) - (\lambda_1 + \lambda_2 + \lambda_3) p_0(t), \\ \dfrac{dp_1(t)}{dt} = \lambda_1 p_0(t) + \mu_2 p_4(t) + \mu_3 p_5(t) - (\lambda_2 + \lambda_3 + \mu_1) p_1(t), \\ \dfrac{dp_2(t0}{dt} = \lambda_2 p_0(t) + \mu_1 p_4(t) + \mu_3 p_6(t) - (\lambda_1 + \lambda_3 + \mu_2) p_2(t), \\ \dfrac{dp_3(t)}{dt} = \lambda_3 p_0(t) + \mu_1 p_5(t) + \mu_2 p_6(t) - (\lambda_1 + \lambda_2 + \mu_3) p_3(t), \\ \dfrac{dp_4(t)}{dt} = \lambda_2 p_1(t) + \lambda_1 p_2(t) + \mu_3 p_7(t) - (\lambda_3 + \mu_1 + \mu_2) p_4(t), \\ \dfrac{dp_5(t)}{dt} = \lambda_3 p_1(t) + \lambda_1 p_3(t) + \mu_2 p_7(t) - (\lambda_2 + \mu_1 + \mu_3) p_5(t), \\ \dfrac{dp_6(t)}{dt} = \lambda_3 p_2(t) + \lambda_2 p_3(t) + \mu_1 p_7(t) - (\lambda_1 + \mu_2 + \mu_3) p_6(t), \\ \dfrac{dp_7(t)}{dt} = \lambda_3 p_4(t) + \lambda_2 p_5(t) + \lambda_1 p_6(t) - (\mu_1 + \mu_2 + \mu_3) p_7(t). \end{cases} \quad (1)$$

When a system state is changed, transition processes on the probabilities $p_i(t)$ take place. But, as shown this in [15] for real values λ_i and μ_i, these processes are very rapid. Usually, engineering practice uses so-called stationary probabilities. The probability of the system residing in state i assumes that stochastic transition process is stationary. By this is meant that all probabilities are independent of time (otherwise, they are also known as the stationary or limiting probabilities [14]. They may be obtained from Eqs. (1) taking all derivatives with respect to time equal to zero. In line with a common procedure [14], we form a set of linear algebraic equations for stationary probabilities p_i:

$$\begin{cases} p_0(\lambda_1+\lambda_2+\lambda_3)=\mu_1 p_1+\mu_2 p_2+\mu_3 p_3; \\ p_1(\lambda_2+\lambda_3+\mu_1)=\lambda_1 p_0+\mu_2 p_4+\mu_3 p_5; \\ p_2(\lambda_1+\lambda_3+\mu_2)=\lambda_2 p_0+\mu_1 p_4+\mu_3 p_6; \\ p_3(\lambda_1+\lambda_2+\mu_3)=\lambda_3 p_0+\mu_1 p_5+\mu_2 p_6; \\ p_4(\lambda_3+\mu_1+\mu_2)=\lambda_2 p_1+\lambda_1 p_2+\mu_3 p_7; \\ p_5(\lambda_2+\mu_1+\mu_3)=\lambda_3 p_1+\lambda_1 p_3+\mu_2 p_7; \\ p_6(\lambda_1+\mu_2+\mu_3)=\lambda_3 p_2+\lambda_2 p_3+\mu_1 p_7; \\ p_7(\mu_1+\mu_2+\mu_3)=\lambda_3 p_4+\lambda_2 p_5+\lambda_1 p_6. \end{cases} \tag{2}$$

Due to the fact that the set of Eqs. (2) fails to involve constant terms, there are infinitely many different solutions satisfying Eqs. (2). In order to be able to choose the unique solution in terms of p_i, it is necessary to substitute any one of the equations in (2) by the normalizing condition:

$$\sum_{i=0}^{7} p_i = 1 \qquad 3)$$

which reflects the fact that the considered system is in any one state at all time.

We can notice the following rules for forming each individual equation by inspecting the set (2) and the associated graph (Fig. 2). The left-hand side of the equation contains the product of the probability of residing in state i and of the summarized rate of all flows departing the system from the i th state. The right-side of the equation is the sum of products of the probability of the state from which it is possible to arrive to state i, and of the corresponding failure or repair flow rate. Thus, given the state space graph, forming a set of equations allows us to calculate the stationary probabilities.

Solving the set of Eqs. (2), we have the values of all stationary probabilities. Physically, the value of $p_i (0 \le p_i \le 1)$ obtained is the relative mean time of the system residing in state i. We point out that this method is known as the state-enumeration method [14].

We calculate the stationary probabilities for eight possible states for the graph shown in Fig. 2. We assume: L_1= 1 km, L_2= 1.5 km, L_3= 2 km. Let also $(\lambda_0)_1$= 0.42 1/(yr$\cdot$km), $(\lambda_0)_2$ = 0.37 1/(yr$\cdot$km), $(\lambda_0)_3$= 0.3 1/(yr$\cdot$km). Then $\lambda_1=(\lambda_0)_1 L_1$= 0.42 1/yr, $\lambda_2=(\lambda_0)_2 L_2$= 0.56 1/yr and $\lambda_3=(\lambda_0)_3 L_3$=0.6 1/yr.1/yr. For the sake of calculation simplicity it is assumed as well that $\mu_1=\mu_2=\mu_3=\mu$. We take μ = 0.02 1/h = 175.2 1/yr. By substituting these values in Eqs.(2) (replacing one of them by (3)) and solving the set (2) for p_i, we obtain: p_0= 0.9; p_1= 2.376 $\cdot$ 10

$^{-3}$; p_2= $3.168 \cdot 10^{-3}$; p_3= $3.394 \cdot 10^{-3}$; p_4=$7.594 \cdot 10^{-6}$; p_5= $8.136 \cdot 10^{-6}$; p_6= $1.085 \cdot 10^{-5}$; p_7= $2.596 \cdot 10^{-8}$.

With every system state one can associate a certain volume ΔQ_i of raw sewage discharged to the environment that can be represent by the following correspondence relations:

$$\begin{aligned} &0 \rightarrow \Delta Q_0 = 0; && 4 \rightarrow \Delta Q_4 = (q_1 + q_2)T; \\ &1 \rightarrow \Delta Q_1 = q_1 T; && 5 \rightarrow \Delta Q_5 = (q_1 + q_2)T; \\ &2 \rightarrow \Delta Q_2 = q_2 T; && 6 \rightarrow \Delta Q_6 = (q_1 + q_2)T; \\ &3 \rightarrow \Delta Q_3 = (q_1 + q_2)T; && 7 \rightarrow \Delta Q_7 = (q_1 + q_2)T; \end{aligned} \tag{4}$$

where T is the interval of time for which the discharged sewage volume Q_d is to be estimated.

For the sake of concreteness, we assume that $q_1 = 0.4$ m^3/s, $q_2 = 0.6$ m^3/s and $T = 1$ yr = $31.536 \cdot 10^6$ s. Then ΔQ_0= 0, ΔQ_1= $126.144 \cdot 10^5$m^3, ΔQ_2= $189.216 \cdot 10^5$ m^3, $\Delta Q_3 = \Delta Q_4 = \Delta Q_5 = \Delta Q_6 = \Delta Q_7$= $315.360 \cdot 10^5$ m^3. The raw sewage volume is calculated as expectation of the random variable

$$Q_d = \sum_{i=0}^{7} \Delta Q_i p_i = 1.978 \cdot 10^5 \text{ m}^3 \tag{5}$$

that is 0.63 % of the total volume of sewage $(q_1 + q_2)T$= $315.360 \cdot 10^5$ m^3 that entered the inlets of the considered network during the year.

Thus, the problem formulated for the sewer network, shown in Fig. 1, is solved.

A more realistic and much used approach proceeds from the fact that the probabilities of simultaneous failure of two or more sewers are extremely low. This fact is easy to verify by analyzing the results of numerical calculations cited above. Taking this into account and assuming that these probabilities are equal to zero, it can be seen that for the network fragment shown in Fig. 1 only four possible states are available, namely:

0: (sewers 1, 2 and 3 up) - p_0;

1: (sewer 1 down, sewers 2 and 3 up) - p_1;

2: (sewer 2 down, sewers 1 and 3 up) - p_2;

3: (sewer 3 down, sewers 1 and 2 up) - p_3.

The corresponding state space graph is shown in Fig. 3,a.

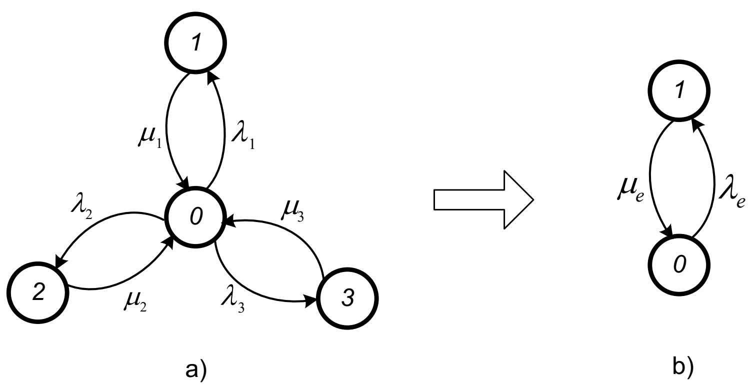

Figure. 3: *Simplified state space graph for the Y-like fragment a) and its transformation b).*

The set of equations written with respect to the stationary probabilities takes the form:

$$\begin{cases} p_0(\lambda_1+\lambda_2+\lambda_3)=\mu_1 p_1+\mu_2 p_2+\mu_3 p_3; \\ p_1\mu_1=\lambda_1 p_0; \\ p_2\mu_2=\lambda_2 p_0; \\ p_3\mu_3=\lambda_3 p_0. \end{cases} \tag{6}$$

It is possible to solve this set of equations analytically. Granting that , we get:

$$\begin{aligned} p_0&=\frac{1}{1+\gamma_1+\gamma_2+\gamma_3}; \\ p_1&=\frac{\gamma_1}{1+\gamma_1+\gamma_2+\gamma_3}; \\ p_2&=\frac{\gamma_2}{1+\gamma_1+\gamma_2+\gamma_3}; \\ p_3&=\frac{\gamma_3}{1+\gamma_1+\gamma_2+\gamma_3}, \end{aligned} \tag{7}$$

where dimensionless parameters characterizing the rate of the "failure-repair" process for each sewer of Y-like network fragment are introduced.

By analogy with (4) we can write for volumes :

$$\begin{aligned} &0\to\Delta Q_0=0; \quad 2\to\Delta Q_2=q_2 T; \\ &1\to\Delta Q_1=q_1 T; \, 3\to\Delta Q_3=(q_1+q_2)T, \end{aligned} \tag{8}$$

and to make an estimate of the raw sewage discharge Q_d as:

$$Q_d=\sum_{i=0}^{3}\Delta Q_i p_i=\frac{(\gamma_1+\gamma_3)q_1+(\gamma_2+\gamma_3)q_2}{1+\gamma_1+\gamma_2+\gamma_3}T. \tag{9}$$

For data used in this numerical example, the calculation by (9) yields: $Q_d = 1.969\cdot 10^5\,\mathrm{m}^3$, that is coincident practically with the result (5) obtained above.

IV. Equivalenting of the network fragment

Difficulties emerge when we estimate the raw sewage discharge resulting from sewer network failures for a sufficiently branched, multicomponent sewer network. The problem is that the number of the possible states rapidly increases with number n of network elements (sewers), and equals 2^n. For example, for $n = 15$ we have 32768 possible states. The high order of the problem presents difficulties in solving an associated set of equations, equals to the number of possible states, in actual practice. Below is proposed an approach that provides a way of simplifying the procedure of estimating the discharged sewage volume for a sufficiently branched sewer network.

First of all, we recall that the mean relative time of the system residing in the inoperable state having only two possible states (up and down), is numerically equal [16]:

$$p=\frac{\lambda}{\lambda+\mu}=\frac{\gamma}{1+\gamma} \tag{10}$$

where $\gamma=\lambda/\mu$.

Return again to Fig. 1 and imagine a fictitious sewer 123 with unknown, for now, failure (λ_e) and repair (μ_e) rates, at the inlet of which the sewage flow rate (q_1+q_2) is the case, that substitute, in some sense, the Y-like network fragment shown in Fig. 1.

Schematically, such substitution is represented in Fig. 4.

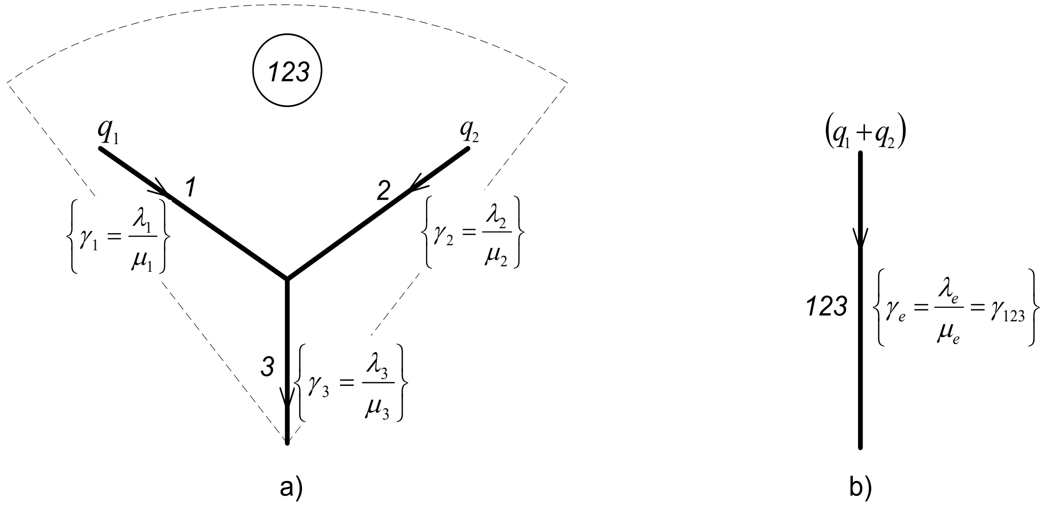

Figure. 4: *Three-component network a) and its equivalent b).*

The state space graph corresponding to Fig. 4b) is shown in Fig. 3b).

It is easy to verify that the volume of raw sewage $(Q_d)_{123}$ discharged from this sewer for time T, is:

$$(Q_d)_{123} = \frac{\gamma_e}{1+\gamma_e}(q_1 + q_2)T \tag{11}$$

We call attention to the fact that, at given flow rates q_1 and q_2 at the inlets of Y-like network fragment, the volume of discharged sewage for time T is dependent on the dimensionless parameter γ_e of fictitious sewer only. In this case, under equivalenting of Y-like network fragment, is no need to find λ_e and μ_e separately, but their ratio only.

We find γ_e leading to the same output for the same input. To this end we equate (11) to (9) and solve the equation obtained for γ_e This leads to

$$\gamma_e = \frac{(\gamma_1 + \gamma_3)q_1 + (\gamma_2 + \gamma_3)q_2}{(1+\gamma_2)q_1 + (1+\gamma_1)q_2} \tag{12}$$

Usually, in actual practice the mean time to failure is far in excess of mean time to repair, that $\gamma << 1$, and, then, Eq. (12) can be written as:

$$\gamma_e \approx \frac{(\gamma_1 + \gamma_3)q_1 + (\gamma_2 + \gamma_3)q_2}{q_1 + q_2} \tag{13}$$

Thus, the Y-like sewer system shown in Fig. 4a) is superseded formally with an equivalent fictitious sewer 123, having the dimensionless parameter $\gamma_e = \gamma_{123}$ and sewage flow rate at the inlet $(q_1 + q_2)$ (see Fig. 4b)).

Sometimes, the cases occur when at one point of network more than two (generally, k) sewers are connected (Fig. 5a)).

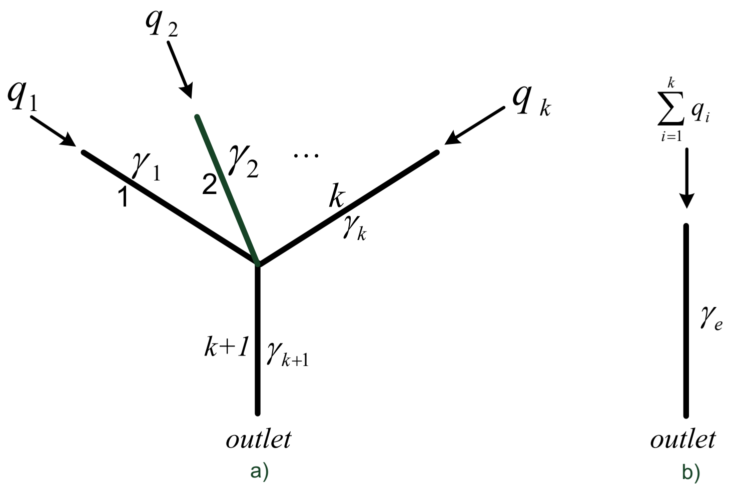

Figure. 5:. *Extension of an equivalenting procedure*

In this case γ_e must be calculated by formula [16]:

$$\gamma_e = \gamma_{k+1} + \frac{\sum_{i=1}^{k} \gamma_i q_i}{\sum_{i=1}^{k} q_i} \tag{14}$$

and, then, the system depicted in Fig. 5a) can be superseded by one sewer as shown in Fig. 5b).

Now we find out a physical meaning of the dimensionless parameter γ_e.

Because $\gamma_e << 1$, the Eq. (11) may be rewritten as $(Q_d)_{123} \approx \gamma_e (q_1 + q_2) T$. It is evident that cofactor $(q_1 + q_2)T = Q$ in the right-hand side of this expression is a total volume of raw sewage that entered the inlets of the considered network at time T . Then, γ_e is a part of Q that is not conveyed to the network outlet, i. e. is discharged to the environment. When multiplied by 100, physically shows the raw sewage discharge resulting from sewer network failures, expressed as a percentage of total sewage volume entered to its inlets. By virtue of the fact that γ_e is varied from 0 (absolutely reliable network) to 1 (theoretically, completely inoperable network), the parameter γ_e, in our opinion, may be used as an objective, single-valued measure of the sewer network reliability.

We emphasize that the sewer network fragment of Fig. 4a) (or Fig.5a)) is a structure-forming component in the sense that any arbitrary complicated dendritic sewer network may be thought of as a composition of such components that substantially reduces and simplifies a body of calculations in estimating raw sewage discharged from the network. Below we give a technique of how to apply this approach.

V. Decomposition-equivalence technique

We shall call this procedure as the "decomposition-equivalence technique". It is more convenient to demonstrate this technique by the following example.

Consider the network in Fig. 6a) consisting of seven sewers, each determined by the values

λ_i and μ_i, and, hence, by the value $\gamma_i = \lambda_i / \mu_i \; (i = 1 \div 7)$. In addition, the sewage flow rate at the network inlets (q_1, q_2, q_3, q_4) will be considered to have constant values.

It is necessary to estimate the raw sewage volume discharged from the network throughout the year as a consequence of possible failures.

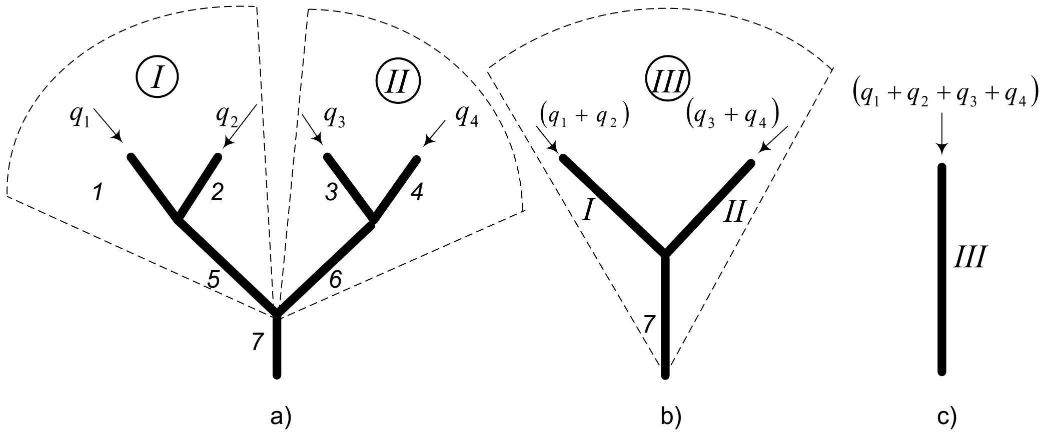

Figure. 6:*.Decomposition-equivalence technique.*

First we consider the contours I and II in Fig. 6a). Either contour includes the Y-like system, and, consequently, can be substituted by one equivalent sewer with its associated value of parameter γ calculated according to the method proposed above. Using Eq. (13), we have γ_I for contour I. Similarly, with assigned notations, for contour II we have γ_{II}.

The results obtained enable one to present the initial network in the form shown in Fig. 6b). But this is an Y-like system (contour III) again. Using Eq. (13), we have finally the parameter $\gamma_{III} = \gamma_e$ of one equivalent sewer substituting the initial network (see Fig. 6c)). Thus, the problem is solved.

As may be seen from this example, unlike the state-enumeration method here, there is no need to solve an unwieldy set of equations. The problem reduces to a sequence of simple computations using, at every stage, the results of a preceding step.

Although this methodology has been applied to a comparatively simple case, it can be extended easily to multicomponent networks.

VI. Applications

The method developed in this paper may be used to solve many practical problems. Some of these, in the form of numerical examples, are considered below in a deliberately simplified but well realistic statement.

6.1. Problem 1.The sewer network shown in Fig. 7a) is given.

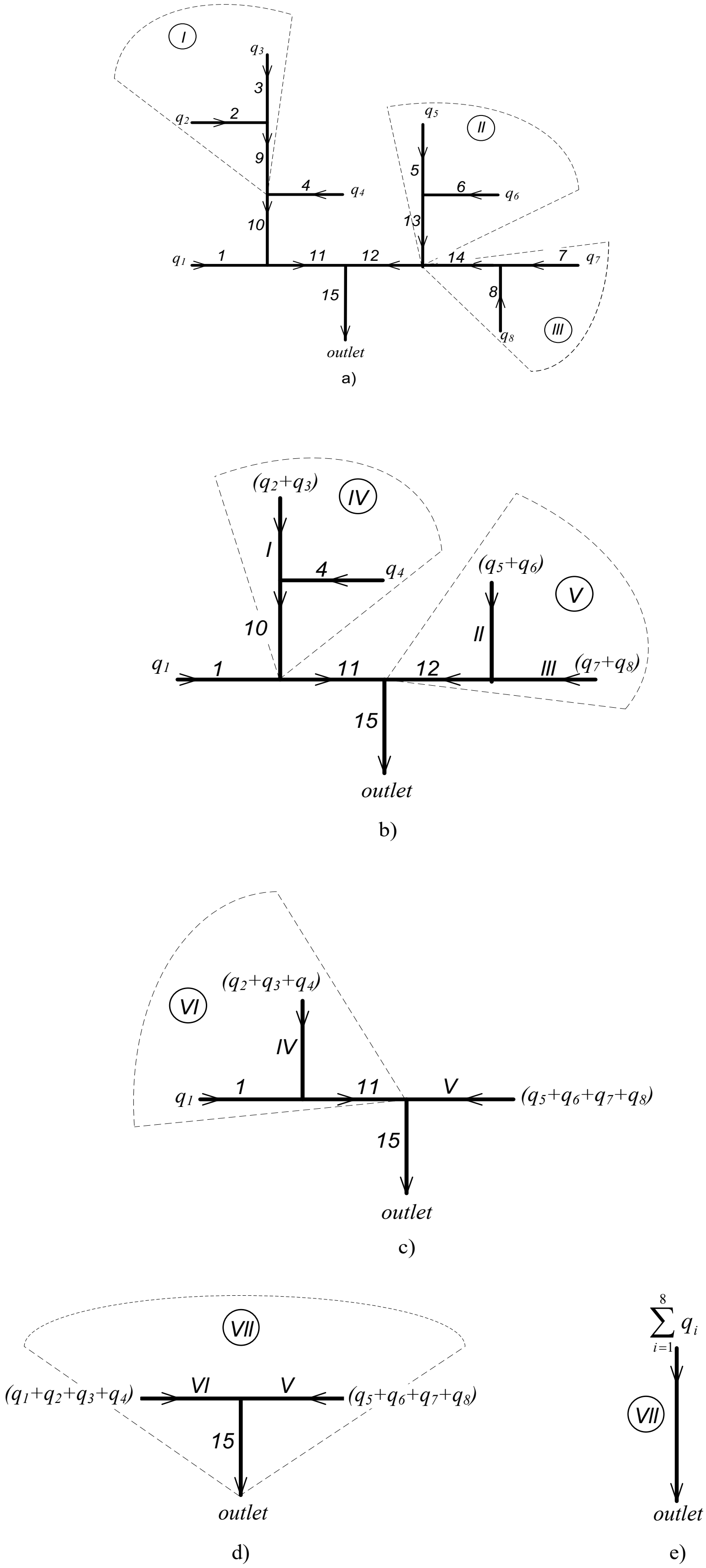

Figure 7: *Initial sewer network a) and its sequential transformations b), c), d), e).*

The network consists of 15 enumerated sewer sections; the number of inlets is equal to 8. The direction of the sewage flow through an each sewer is shown by the arrow. There is a need to estimate a reliability level of this network (in the sense of the proposed criterion).

To carry out the calculations we need some data. Such data are represented in Table 1.

Table 1: *Input data for calculations*

Section, i	1	2	3	4	5	6	7	8	9	10
Failure rate λ_i, (1/yr)	0.52	0.68	0.79	0.91	1.34	0.83	0.75	0.03	0.85	0.62
Repair rate μ_i, (1/yr)	220	220	220	220	220	220	220	220	200	150
Parameter $\gamma_i = \lambda_i / \mu_i \, (\times 10^3)$	2.36	3.09	3.59	4.14	6.09	3.77	3.41	0.14	4.25	4.13
	11	**12**	**13**	**14**	**15**					
	0.84	1.10	0.03	0.50	0.05					
	120	120	200	200	90					
	7.00	9.17	0.15	2.50	0.56					

Besides, the inlets sewage flow rates in Table 2 are shown.

Table 1: *Network inlets sewage flow rate.*

Inlet, i	1	2	3	4	5	6	7	8
Sewage flow rate $q_i (\times 10^2)$, (m^3/s)	3	9	6	4	5	1	5	7

In addition, without loss of generality, we assume that the length of each sewer section is equal to 1 km. We note also that all values are hypothetical, convenient for calculations only.

First we consider the contours I, II and III (Fig. 7a)) at the network periphery. Either contour includes the Y-like system, and, consequently, can be substituted by one equivalent sewer with its associated value of parameter γ calculated according to the method proposed above. Using Eq. (13) where now, taking account of the new notations, and the data from Table 1 and Table 2 we have for contour I:

$$\gamma_I = \frac{(\gamma_2 + \gamma_9)q_2 + (\gamma_3 + \gamma_9)q_3}{q_2 + q_3} = 7.54 \cdot 10^{-3}$$

Similarly, for contour II:

$$\gamma_{II} = \frac{(\gamma_5 + \gamma_{13})q_5 + (\gamma_6 + \gamma_{13})q_6}{q_5 + q_6} = 5.85 \cdot 10^{-3}$$

and for contour III:

$$\gamma_{III} = \frac{(\gamma_7 + \gamma_{14})q_7 + (\gamma_8 + \gamma_{14})q_8}{q_7 + q_8} = 4.00 \cdot 10^{-3}$$

The results obtained enable one to present the initial network in the form shown in Fig. 7b). But here are the Y-like systems (contours IV and V) again. Using Eq. (13) we have the parameter γ_{IV} for contour IV:

$$\gamma_{IV} = \frac{(\gamma_1 + \gamma_{10})(q_2 + q_3) + (\gamma_4 + \gamma_{10})q_4}{q_2 + q_3 + q_4} = 10.95 \cdot 10^{-3}$$

and for contour V:

$$\gamma_V = \frac{(\gamma_{II} + \gamma_{12})(q_5 + q_6) + (\gamma_{III} + \gamma_{12})(q_7 + q_8)}{q_5 + q_6 + q_7 + q_8} = 13.79 \cdot 10^{-3}$$

As a result, the structure shown in Fig. 7b) substitutes by the structure depicted in Fig. 7c) where the Y-like sub-system (contour VI) may be selected. Equivalenting this contour again by one sewer section with the parameter

$$\gamma_{VI} = \frac{(\gamma_1 + \gamma_{11})q_1 + (\gamma_{IV} + \gamma_{11})(q_2 + q_3 + q_4)}{q_1 + q_2 + q_3 + q_4} = 16.78 \cdot 10^{-3}$$

we are going to the Fig. 7d).

But the structure shown in Fig. 7d) is the Y-like fragment (contour VII) in itself that may be substituted by one sewer (see Fig. 7e)). Thus, finally we have the parameter γ_{VII} of one equivalent sewer substituting the initial network depicted in Fig. 7a):

$$\gamma_{VII} = \frac{(\gamma_{VI} + \gamma_{15})(q_1 + q_2 + q_3 + q_4) + (\gamma_V + \gamma_{15})(q_5 + q_6 + q_7 + q_8)}{q_1 + q_2 + q_3 + q_4 + q_5 + q_6 + q_7 + q_8} = 16.00 \cdot 10^{-3}$$

The sequence of "decomposition-equivalence" operations is completed. Hence, in this case, $\gamma_e = \gamma_{VII} = 0.016$. This means that, on the average, 1.6 % of the total sewage volume that entered the network inlets during time T , discharges from the sewer network to the environment arising from the network component failures. The accuracy of this measure increases as T increase, that is characteristic for probabilistic problems at all.

6.2. Problem 2.Let us assume that specialists analyzing the results obtained in preceding Problem 1 come to the conclusion that the raw sewage discharge from the sewer network (Fig. 7a)) is much too large, and, consequently, the network reliability needs to be increased. The question concerning replacement of some components by a new sewer pipe is discussed, but it is possible to replace only one sewer because available funds are limited. On the present evidence, it may be argued that the failure rate for a new sewer (manufacturer's data) is $\lambda_n = 0.02$ 1/yr; the repair rate μ_n is taken to be equal to 200. It is desired to identify the preferential alternative.

First of all, we compute $\lambda_n = \lambda_n / \mu_n = 0.1 \cdot 10^{-3}$. As before, we will take the discharged sewage volume as an efficiency index of the alternative to be accepted. Calculate this quantity assuming that the replacement of sewer section 1 in the initial network (Fig. 7a)) has just been made. For this purpose, we substitute the input data (associated with the sewer 1) of the Problem 1, for one another (corresponding to the new sewer), namely $\gamma_n = 0.1 \cdot 10^{-3}$. Carrying out the relevant calculations, we obtain the discharged sewage volume expressed as a percentage of the total sewage entered the network: 1.582 %. By repeating the similar calculations with respect to each network section we come to the results represented in Table 3.

Table 3: *Example table Result of calculations.*

Section to be replaced Relative sewage volume discharge from network,%	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
	1.582	1.532	1.547	1.559	1.524	1.590	1.558	1.559
	9	**10**	**11**	**12**	**13**	**14**	**15**	
	1.444	1.408	1.220	1.191	1.599	1.527	1.523	

Referring to Table 3, it is seen that the smallest volume of sewage to be discharged from the network occurs when the network's section 12 is replaced (in Table 3 this is highlighted in bold print). It is obvious that, under otherwise equal conditions, this alternative is preferable from the viewpoint of the reliability index accepted in this work.

The problems considered are simple as well; for this reason, the results seem to be trivial. Note, however, that the simplicity of the examples makes it possible to see the potential of the proposed method for practical use.

VII. Conclusion

Although sewer reliability depicts a fairly complete reliability measure of the sewer network, it is convenient to use a single index to represent the composite effect of the component reliabilities. We propose to assess sewer network reliability as a whole by a volume of raw sewage discharged from the system because of failures of its components for an appreciable length of time. The traditional method for solving such problems is the so-called state-enumeration method. But, for the multicomponent networks, this generates a need to solve a set of equations having very high order, which renders the method unsuitable for many practical applications. The approach proposed in this work makes it possible to circumvent these difficulties by using the concept of equivalent sewer. As a result, the problem reduces to a sequential consideration of elementary sub-problems the solution of which is easily accomplished.

As the methodology is applicable for sewer networks, each component of which can be either up (operable) or down(failed), additional research is need for extending the method for more complex cases.

In our view, similar problems exist also in the course of maintenance of oil, gas and other pipeline systems. Such a setting and solving of problems may also be of interest for specialists working with general reliability issues.

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