EPQ MODELS WITH MIXTURE OF WEIBULL PRODUCTION EXPONENTIAL DECAY AND CONSTANT DEMAND

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Abstract

This paper deals with an economic production quantity (EPQ) model in which production is random and having heterogeneous units of production. The production process is characterized by mixture of Weibull distribution. It is assumed that the demand is constant and the lifetime of the commodity is random and follows an exponential distribution. Assuming that the shortages are allowed and fully backlogged the instantaneous state of inventory in the production unit is derived. The minimizing the expected total production cost, the optimal production quantity, the production uptime and downtime are derived. Through sensitivity analysis it is observed that the random production with mixture distribution have a significant influence on the optimal production schedules and production quantity. It is also observed that the rate of deterioration can tremendously influence the optimal operating policies of the system. This model also includes some of the earlier models as particular cases. The model is extended to the case of without shortages. A comparison of the two models reveals that allowing shortages will reduce expected total cost of the model.

Keywords: Stochastic production, Mixture of Weibull Distribution, Exponential decay, Production Schedules, Sensitivity analysis.

I. Introduction

In production quantity models much emphasis is given for the lifetime of the commodity. In many production processes the lifetime of the commodity is random and can be characterized by a probability distribution. The literature on inventory models for deteriorating items are reviewed by Pentico and Drake (2011), Ruxian Lie et al (2010), Goyal and Giri (2001), Raafat (1991) and Nahmias (1982). The exponential decay models of inventory are studied by Ghare and Schrader (1963), Shah and Jaiswal (1977), Cohen (1977), Aggarwal (1978), Dave and Shah (1982), Pal (1990), Kalpakam and Sapna (1996), Giri and Chaudhari (1999). The exponential decay is used when the rate of deterioration is constant which coincide with the deterioration of several perishable items such as medicine, sea foods, vegetable oils, cement and paints. Hence it is reasonable to assume exponential decay of the product.

Another important consideration in EPQ models is the rate of production and it is studied by several authors Perumal and Arivarignan (2002), Pal and Mandal (1997), Sen and Chakrabarthy (2007), Lin and Gong(2006), Maity et al(2007), Hu and Liu(2010), Uma Maheswararao et al (2010), Venkata Subbaiah et al (2011), Essey and Srinivasa Rao (2012), Ardak and Borade (2017), Anindya Mandal, Brojeswar Pal and Kripasindhu Chaudhuri (2020), Sunit Kumar, Sushil Kumar and

Rachna Kumari (2021). In all these papers they assumed that the production is deterministic and having finite rate. However, in many production processes the production is not deterministic and random.

Stochastic production is a reality in the modern technological industrial developments. One of the major consideration for scheduling the production and determining the optimal production quantity lies on several factors such as availability of raw material, power supply, man power skill level, machine tool wear which are governed by laws of chance and become stochastic. Because of the stochastic factors the production process in many industries is random and can be characterized by a probability distribution.

Recently Sridevi et al. (2010), Srinivasa Rao et al. (2010), Laxmana Rao et al. (2015), Srinivasa Rao et al. (2017), Madhulatha et al. (2017), Punyavathi et al. (2020) have developed and analyzed production quantity models with random production. In all these papers they assumed that the production is homogeneous even though governed by stochastic nature i.e., all the production is done in one unit or in a single machine. But in practice several of the products are produced by different machines or in different units which are operated under different conditions. Hence these heterogeneous production processes can be characterized by mixes of probability distributions. It is also observed that in each unit the production rate may be increasing/decreasing/remains constant. This type of variable rate of production can be represented by Weibull probability distribution. Hence in this paper we develop and analyze stochastic production quantity models assuming that the production is random and follows a two component Weibull mixture distribution. It is also further assumed that the demand is constant and in the production backorders are allowed and fully backlogged.

Using the differential equations the production quantity at a given time is derived. With suitable costs the total expected production cost is derived. By minimizing the total expected production cost the optimal production schedules, the production quantities are derived. Through sensitivity analysis the effect of the change in parameters and cost on optimal production schedules and production quantity is discussed. This model is extended to the case of without shortages.

II. Assumptions

For developing the model the following assumptions are made:

- The demand rate is constant say k. (1)
- The production is random and follows a mixture of two-parameter Weibull distribution. The instantaneous rate of production is:

$$R(t) = \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}; \alpha_1, \alpha_2 > 0, \beta_1, \beta_2 > 0, 0 \le p \le 1 \qquad (2)$$

- Lead time is zero.
- Cycle length is T. It is known and fixed.
- Shortages are allowed and fully backlogged.
- A deteriorated unit is lost.
- The lifetime of the item is random and follows a exponential distribution with probability density function:

$$f(t) = \theta e^{-\theta t};\ \theta > 0, t > 0$$

Therefore the instantaneous rate of deterioration is

$$h(t) = \theta;\ \theta > 0 \qquad (3)$$

The following notations are used for developing the model.
Q is the production quantity
A is setup cost
C is cost per unit
h Inventory holding cost per unit per unit time
π Shortages cost per unit per unit time

III. EPQ Model with Shortages

Consider a production system in which the stock level is zero at time $t = 0$. The stock level increases during the period $(0, t_1)$, due to production after fulfilling the demand and deterioration. The production stops at time t_1 when stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, t_2). At time t_2 the inventory reaches zero and backorders accumulate during the period (t_2, t_3). At time t_3 the production again starts and fulfills the backlog after satisfying the demand. During (t_3, T) the backorders are fulfilled and inventory level reaches zero at the end of cycle T. The Schematic diagram representing the inventory level is given in Figure 1.

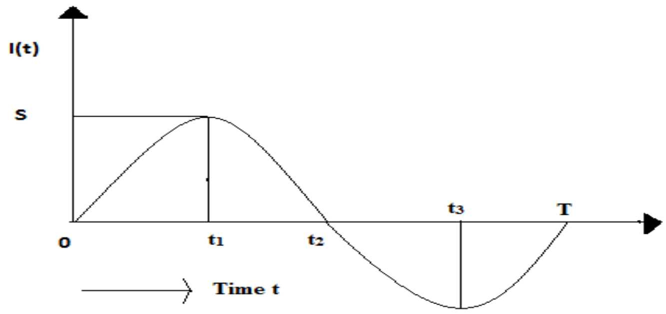


Figure 1: *Schematic Diagram representing the inventory level*

Let *I(t)* be the inventory level of the system at time '*t*' (*0 ≤ t ≤ T*). The differential equations governing the instantaneous state of *I(t)* over the cycle of length are:

$$\frac{d}{dt}I(t) + h(t)I(t) = \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}} - k; 0 \leq t \leq t_1 \quad (4)$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -k;\ t_1 \leq t \leq t_2 \quad (5)$$

$$\frac{d}{dt}I(t) = -k;\ t_2 \leq t \leq t_3 \quad (6)$$

$$\frac{d}{dt}I(t) = \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}} - k;\ t_3 \leq t \leq T \quad (7)$$

Where, $h(t)$ is as given in equation (3), with the initial conditions $I(0) = 0$, $I(t_1) = S$, $I(t_2) = 0$ and $I(T) = 0$
Solving the differential equations, the on hand inventory at time 't' is obtained as:

$$I(t) = Se^{\theta(t_1-t)} - e^{-t\theta}\int_t^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}} - k\right]e^{u\theta}du\,; 0 \le t \le t_1 \quad (8)$$

$$I(t) = Se^{\theta(t_1-t)} - ke^{-t\theta}\int_{t_1}^{t} e^{u\theta}\,du;\ t_1 \le t \le t_2 \quad (9)$$

$$I(t) = k(t_2 - t); t_2 \le t \le t_3 \quad (10)$$

$$I(t) = \int_t^T \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}dt + k(T-t); t_3 \le t \le T \quad (11)$$

Production quantity Q in the cycle of length T is:

$$Q = \int_0^{t_1} R(t)dt + \int_{t_3}^{T} R(t)dt$$

$$= \int_0^{t_1} \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}dt + \int_{t_3}^{T} \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}dt \quad (12)$$

From equation (8) and using the initial condition *I(0) = 0*, we obtain the value of 'S' as:

$$S = e^{-\theta t_1}\int_0^{t_1}\left(\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}}\right)e^{u\theta}du - \frac{k}{\theta}\left(1 - e^{-\theta t_1}\right) \quad (13)$$

When $t = t_3$, then equations (10) and (11) become:

$$I(t_3) = k(t_2 - t_3) \quad (14)$$

and

$$I(t_3) = k(T - t_3) - \int_{t_3}^{T}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}}du \quad (15)$$

Equating the equations (14) and (15) and on simplification one can get:

$$t_2 = \frac{1}{k}\int_{t_3}^{T}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}}\right]du + T = x(t_3)\ say \quad (16)$$

Let $K(t_{1,}t_2, t_3)$ be the total production cost per unit time. Since the total production cost is the sum of the set up cost, cost of the units, the inventory holding cost. Hence the total production cost per unit time become:

$$K(t_{1,}t_2, t_3) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt\right] + \frac{\pi}{T}\left[\int_{t_2}^{t_3} -I(t)dt + \int_{t_3}^{T} -I(t)dt\right] \quad (17)$$

Substituting the values of *I(t)* given in equations (8), (9), (10) and (11) and Q given in equation (12) in equation (17) one can obtain $K(t_{1,}t_2, t_3)$ as:

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{C}{T}\left[\int_0^{t_1}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt + \int_{t_3}^{T}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt\right]$$

$$+\frac{h}{T}\left[\int_0^{t_1}\left[Se^{\theta(t_1-t)}-e^{-t\theta}\int_t^{t_1}\left(\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}-k\right)e^{u\theta}du\right]dt + \int_{t_1}^{t_2}\left[Se^{\theta(t_1-t)}-ke^{-t\theta}\int_{t_1}^{t}e^{u\theta}\,du\right]dt\right]$$

$$-\frac{\pi}{T}\left[k\int_{t_2}^{t_3}(t_2-t)dt + \int_{t_3}^{T}\left[\left(\int_t^{T}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}du\right)dt\right]+k\int_{t_3}^{T}(T-t)\right] \quad (18)$$

Substituting the value of S given in equation (13) in the total production cost equation (18), we obtain:

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{C}{T}\left[\int_0^{t_1}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt + \int_{t_3}^{T}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt\right]$$

$$+\frac{h}{T}\left[\frac{1-e^{-\theta t_2}}{\theta}\int_0^{t_1}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}e^{u\theta}du+\frac{k}{\theta^2}\left(1-e^{-t_2\theta}\right) - \frac{k}{\theta}t_2-\int_0^{t_1}e^{-\theta t}\left(\int_t^{t_1}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}e^{u\theta}du\right)dt\right]$$

$$-\frac{\pi}{T}\left[\frac{k}{2}[(T-t_3)^2-(t_2-t_3)^2] + \int_{t_3}^{T}\left(\int_t^{T}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}du\right)dt\right] \quad (19)$$

Substituting the value of $'t_2'$ given in equation (16) in the total production cost equation (19), we obtain:

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{C}{T}\left[\int_0^{t_1}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt + \int_{t_3}^{T}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}dt\right]$$

$$+\frac{h}{T}\left[\frac{1-e^{-\theta x(t_3)}}{\theta}\int_0^{t_1}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}e^{u\theta}\,du+\frac{k}{\theta^2}\left(1-e^{-\theta x(t_3)}\right)\right.$$

$$-\frac{k}{\theta}x(t_3) - \int_0^{t_1} e^{-\theta t}\left(\int_t^{t_1} \frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}} e^{u\theta} du\right) dt\Bigg]$$

$$-\frac{\pi}{T}\Bigg[\frac{k}{2}[(T-t_3)^2 - (x(t_3)-t_3)^2]$$

$$+\int_{t_3}^{T}\left(\int_t^{T} \frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}} du\right) dt\Bigg] \quad (20)$$

IV. Optimal Production Schedules of the Model

In this section we obtain the optimal policies of the system under study. To find the optimal values of t_1 and t_3, we obtain the first order partial derivatives of $K(t_1,t_3)$ given in equation with respect to t_1 and t_3 and equate them to zero. The condition for minimization of $K(t_1,t_3)$ is

Where D is the Hessian matrix

$$D = \begin{vmatrix} \frac{\partial^2 K(t_1,t_3)}{\partial {t_1}^2} & \frac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} \\ \frac{\partial^2 K(t_1,t_3)}{\partial t_1 \partial t_3} & \frac{\partial^2 K(t_1,t_3)}{\partial {t_3}^2} \end{vmatrix} > 0$$

Differentiating $K(t_1, t_3)$ given in equation (20) with respect to t_1 and equating to zero, we get

$$\Bigg\{\frac{C}{T}\left[\frac{p\alpha_1\beta_1 {t_1}^{\beta_1-1}e^{-\alpha_1 {t_1}^{\beta_1}} + (1-p)\alpha_2\beta_2 {t_1}^{\beta_2-1}e^{-\alpha_2 {t_1}^{\beta_2}}}{pe^{-\alpha_1 {t_1}^{\beta_1}} + (1-p)e^{-\alpha_2 {t_1}^{\beta_2}}}\right]$$

$$+\frac{h}{T}\left[\frac{1-e^{-x(t_3)\theta}}{\theta}\left[\frac{p\alpha_1\beta_1 {t_1}^{\beta_1-1}e^{-\alpha_1 {t_1}^{\beta_1}} + (1-p)\alpha_2\beta_2 {t_1}^{\beta_2-1}e^{-\alpha_2 {t_1}^{\beta_2}}}{pe^{-\alpha_1 {t_1}^{\beta_1}} + (1-p)e^{-\alpha_2 {t_1}^{\beta_2}}}\right]e^{t_1\theta}\right]\Bigg\} = 0 \quad (21)$$

Differentiating $K(t_1, t_3)$ given in equation (20) with respect to t_3 and equating to zero, we get

$$\Bigg\{-\frac{C}{T}\left[\frac{p\alpha_1\beta_1 {t_3}^{\beta_1-1}e^{-\alpha_1 {t_3}^{\beta_1}} + (1-p)\alpha_2\beta_2 {t_3}^{\beta_2-1}e^{-\alpha_2 {t_3}^{\beta_2}}}{pe^{-\alpha_1 {t_3}^{\beta_1}} + (1-p)e^{-\alpha_2 {t_3}^{\beta_2}}}\right]$$

$$+\frac{h}{T}\Bigg[\frac{1}{\theta}\left[\frac{p\alpha_1\beta_1 {t_3}^{\beta_1-1}e^{-\alpha_1 {t_3}^{\beta_1}} + (1-p)\alpha_2\beta_2 {t_3}^{\beta_2-1}e^{-\alpha_2 {t_3}^{\beta_2}}}{pe^{-\alpha_1 {t_3}^{\beta_1}} + (1-p)e^{-\alpha_2 {t_3}^{\beta_2}}}\right]\left(1-e^{-\theta x(t_3)}\right)$$

$$-\frac{e^{-\theta x(t_3)}}{k}\left[\frac{p\alpha_1\beta_1 {t_3}^{\beta_1-1}e^{-\alpha_1 {t_3}^{\beta_1}} + (1-p)\alpha_2\beta_2 {t_3}^{\beta_2-1}e^{-\alpha_2 {t_3}^{\beta_2}}}{pe^{-\alpha_1 {t_3}^{\beta_1}} + (1-p)e^{-\alpha_2 {t_3}^{\beta_2}}}\right]\int_0^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}}\right]e^{u\theta}\, du\Bigg]$$

$$-\frac{\pi}{T}\left[k(t_3-T) + (x(t_3)-t_3)\left[\left[\frac{p\alpha_1\beta_1 {t_3}^{\beta_1-1}e^{-\alpha_1 {t_3}^{\beta_1}} + (1-p)\alpha_2\beta_2 {t_3}^{\beta_2-1}e^{-\alpha_2 {t_3}^{\beta_2}}}{pe^{-\alpha_1 {t_3}^{\beta_1}} + (1-p)e^{-\alpha_2 {t_3}^{\beta_2}}}\right] + k\right]\right.$$

$$-\int_{t_3}^{T}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}du\Bigg]\Bigg\}=0 \tag{22}$$

Solving the equations (21) and (22) simultaneously, we obtain the optimal time at which production is stopped t_1^* of t_1 and the optimal time t_3^* of t_3 at which the production is restarted after accumulation of backorders.

The optimum production quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1^*, t_3^* in equation (12).

V. Numerical Illustration

In this section we discuss the solution procedure of the model through a numerical illustration by obtaining the production uptime, production downtime, optimum production quantity and the total production cost of an inventory system. Here, it is assumed that the production is of deteriorating nature and shortages are allowed and fully backlogged. For demonstrating the solution procedure of the model the parameters are considered as A = Rs.300/-, C = Rs.10\-, h = Re.0.2\-, π = Rs.3.3\-, T = 12 months. For the assigned values of production parameters ($\alpha_1, \alpha_2, \beta_1, \beta_2, p$) = (11, 15, 0.55, 2, 0.5), deterioration parameter θ = 3, demand rate k = 3.3. The values of parameters above are varied further to observe the trend in optimal policies and the results are obtained are shown in Table 1. Substituting these values the optimal production quantity Q^*, the production uptime, production downtime and total production cost are computed and presented in Table 1.

From Table 1 it is observed that the deterioration parameter and production parameters have a tremendous influence on the optimal values of production times, production quantity and total production cost.

When the ordering cost 'A' increases from 300 to 345, the optimal production quantity Q^* decreases from 33.867 to 33.863, the optimal production down time ${t_1}^*$ remains constant, the optimum production uptime ${t_3}^*$ increases from 3.685 to 3,686, the total production cost per unit time K^* increases from 80.793 to 84.529. As the cost parameter 'C' increases from 10 to 11.5, the optimal production quantity Q^* increases from 33.867 to 33.872, the optimal production down time ${t_1}^*$ and optimal production uptime ${t_3}^*$ remains constant, the total production cost per unit time K^* increases from 80.793 to 82.451. As the holding cost 'h' increases from 0.2 to 0.23, the optimal production quantity Q^*, the optimal production down time ${t_1}^*$, the optimal production uptime ${t_3}^*$ remains constant, the total production cost per unit time K^* decreases from 80.793 to 80.755. As the shortage cost 'π' increases from 3.3 to 3.795, the optimal production quantity Q^* increases from 33.867 to 33.966, the optimal production down time ${t_1}^*$ remains constant, the optimal production uptime ${t_3}^*$ decreases from 3.685 to 3.655, the total production cost per unit time K^* increases from 80.793 to 87.753.

As the production parameter 'α_1' varies from 11 to 12.65, the optimal production quantity Q^* increases from 33.867 to 39.086, the optimal production down time ${t_1}^*$ increases from 1.274 to 1.277, the optimal production uptime ${t_3}^*$ decreases from 3.685 to 3.628, the total production cost per unit time K^* increases from 80.793 to 93.146.As the production parameter 'α_2'varies from 15 to 17.25, the optimal production quantity Q^*, the optimal production down time ${t_1}^*$, the optimal production uptime ${t_3}^*$, the total production cost per unit time K^* remains constant.

Table 1: *Numerical Illustration*

A	C	h	π	T	α_1	α_2	β_1	β_2	θ	k	p	t_1^*	t_3^*	Q^*	K^*
300	10	0.2	3.3	12	11	15	0.55	2	3	3.3	0.5	1.274	3.685	33.867	80.793
315												1.274	3.685	33.866	82.039
330												1.274	3.686	33.865	83.284
345												1.274	3.686	33.863	84.529
	10.5											1.274	3.685	33.869	81.346
	11											1.274	3.685	33.87	81.898
	11.5											1.274	3.685	33.872	82.451
		0.21										1.274	3.685	33.867	80.781
		0.22										1.274	3.685	33.867	80.768
		0.23										1.274	3.685	33.867	80.755
			3.465									1.274	3.675	33.9	83.1
			3.63									1.274	3.665	33.933	85.42
			3.795									1.274	3.655	33.966	87.753
					11.55							1.275	3.666	35.599	84.778
					12.1							1.276	3.647	37.338	88.895
					12.65							1.277	3.628	39.086	93.146
						15.75						1.274	3.685	33.867	80.793
						16.5						1.274	3.685	33.867	80.793
						17.25						1.274	3.685	33.867	80.793
							0.578					1.275	3.648	36.366	88.423
							0.605					1.276	3.609	38.996	97.002
							0.633					1.277	3.565	41.973	107.394
								2.1				1.274	3.685	33.867	80.793
								2.2				1.274	3.685	33.867	80.793
								2.3				1.274	3.685	33.867	80.793
									3.15			1.274	3.685	33.867	80.805
									3.3			1.274	3.685	33.867	80.816
									3.45			1.274	3.685	33.867	80.826
										3.465		1.274	3.689	33.853	79.886
										3.63		1.274	3.693	33.841	79.062
										3.795		1.274	3.696	33.83	78.31
											0.525	1.274	3.685	33.818	80.753
											0.55	1.274	3.685	33.772	80.714
											0.575	1.274	3.685	33.728	80.677

As the production parameter 'β_1 ' varies from 0.55 to 0.633, the optimal production quantity Q^* increases from 33.867 to 41.973, the optimal production down time $t_1{}^*$ increases from 1.274 to 1.277, the optimal production uptime $t_3{}^*$ decreases from 3.685 to 3.565, the total production cost per unit time K^* increases from 80.793 to 107.394. As the production parameter 'β_2' varies from 2 to 2.3, the optimal production quantity Q*, the optimal production down time $t_1{}^*$, the optimal production uptime $t_3{}^*$ and the total production cost per unit time K^* remains constant. As the production parameter 'p ' varies from 0.5 to 0.575, the optimal production quantity Q^* decreases from 33.867 to 33.728, the optimal production down time $t_1{}^*$ and the optimal production uptime $t_3{}^*$ remains constant, the total production cost per unit time K^* decreases from 80.793 to 80.677.

As the deterioration parameter 'θ' varies from 3 to 3.45, the optimal production quantity Q^*, the optimal production down time $t_1{}^*$ and the optimal production uptime $t_3{}^*$ remains constant, the total production cost per unit time K^* increases from 80.793 to 80.826.

As the demand rate parameter 'k ' increases from 3.3 to 3.795 the optimal production quantity Q^* decreases from 33.867 to 33.83, the optimal production down time $t_1{}^*$ remains constant, the optimal production uptime $t_3{}^*$ increases from 3.685 to 3.696, the total production cost per unit time K^* decreases from 80.793 to 78.31.

VI. Sensitivity Analysis of the Model

Sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 2. The relationships between the parameters and the optimal values of the production schedule are shown in Figure 2.

Table 2: *Sensitivity Analysis of the Model - With Shortages*

Variation Parameters	Optimal Policies	-15%	-10%	-5%	0%	5%	10%	15%
A	$t_1{}^*$	1.274	1.274	1.274	1.274	1.274	1.274	1.274
	$t_3{}^*$	3.684	3.684	3.685	3.685	3.685	3.686	3.686
	Q^*	33.871	33.869	33.868	33.867	33.866	33.865	33.863
	K^*	77.058	78.303	79.548	80.793	82.039	83.284	84.529
C	$t_1{}^*$	1.274	1.274	1.274	1.274	1.274	1.274	1.274
	$t_3{}^*$	3.684	3.685	3.685	3.685	3.685	3.685	3.685
	Q^*	33.862	33.864	33.866	33.867	33.869	33.87	33.872
	K^*	79.138	79.689	80.241	80.793	81.346	81.898	82.451
h	$t_1{}^*$	1.274	1.274	1.274	1.274	1.274	1.274	1.274
	$t_3{}^*$	3.685	3.685	3.685	3.685	3.685	3.685	3.685
	Q^*	33.867	33.867	33.867	33.867	33.867	33.867	33.867
	K^*	80.832	80.819	80.806	80.793	80.781	80.768	80.755
π	$t_1{}^*$	1.274	1.274	1.274	1.274	1.274	1.274	1.274
	$t_3{}^*$	3.715	3.705	3.695	3.685	3.675	3.665	3.655
	Q^*	33.766	33.8	33.834	33.867	33.9	33.933	33.966
	K^*	73.95	76.218	78.499	80.793	83.1	85.42	87.753
α_1	$t_1{}^*$	1.271	1.272	1.273	1.274	1.275	1.276	1.277
	$t_3{}^*$	3.74	3.722	3.704	3.685	3.666	3.647	3.628
	Q^*	28.718	30.427	32.143	33.867	35.599	37.338	39.086
	K^*	69.61	73.212	76.939	80.793	84.778	88.895	93.146

Variation Parameters	Optimal Policies	-15%	-10%	-5%	0%	5%	10%	15%
α_2	t_1^*	1.274	1.274	1.274	1.274	1.274	1.274	1.274
	t_3^*	3.685	3.685	3.685	3.685	3.685	3.685	3.685
	Q^*	33.867	33.867	33.867	33.867	33.867	33.867	33.867
	K^*	80.793	80.793	80.793	80.793	80.793	80.793	80.793
β_1	t_1^*	1.27	1.272	1.273	1.274	1.275	1.276	1.277
	t_3^*	3.776	3.748	3.719	3.685	3.648	3.609	3.565
	Q^*	27.639	29.56	31.58	33.867	36.366	38.996	41.973
	K^*	63.978	68.832	74.257	80.793	88.423	97.002	107.394
β_2	t_1^*	1.274	1.274	1.274	1.274	1.274	1.274	1.274
	t_3^*	3.685	3.685	3.685	3.685	3.685	3.685	3.685
	Q^*	33.867	33.867	33.867	33.867	33.867	33.867	33.867
	K^*	80.793	80.793	80.793	80.793	80.793	80.793	80.793
θ	t_1^*	1.274	1.274	1.274	1.274	1.274	1.274	1.274
	t_3^*	3.685	3.685	3.685	3.685	3.685	3.685	3.685
	Q^*	33.867	33.867	33.867	33.867	33.867	33.867	33.867
	K^*	80.749	80.766	80.78	80.793	80.805	80.816	80.826
k	t_1^*	1.274	1.274	1.274	1.274	1.274	1.274	1.274
	t_3^*	3.67	3.676	3.68	3.685	3.689	3.693	3.696
	Q^*	33.917	33.899	33.882	33.867	33.853	33.841	33.83
	K^*	84.168	82.916	81.798	80.793	79.886	79.062	78.31
p	t_1^*	1.274	1.274	1.274	1.274	1.274	1.274	1.274
	t_3^*	3.685	3.685	3.685	3.685	3.685	3.685	3.685
	Q^*	34.029	33.972	33.918	33.867	33.818	33.772	33.728
	K^*	80.929	80.881	80.836	80.793	80.753	80.714	80.677

VII. Observations

The major observations drawn from the numerical study are:

- t_1^* and t_3^* are less sensitive, Q^* is slightly sensitive and K^* is moderately sensitive to changes of ordering cost 'A'.
- t_1^* and t_3^* are less sensitive, Q^* is slightly sensitive and K^* is moderately sensitive to changes of cost per unit 'C'.
- t_1^*, t_3^* and Q^* are less sensitive, K^* is slightly sensitive to changes of holding cost 'h'.
- t_1^* is less sensitive, t_3^* and Q^* are slightly sensitive and K^* is highly sensitive to change in parameter 'π'.
- t_1^* and t_3^* are slightly sensitive, Q^* and K^* are highly sensitive to change in the production parameter 'α_1'.
- t_1^*, t_3^*, Q^* and K^* are less sensitive to change in the production parameter 'α_2'.
- t_1^* and t_3^* are slightly sensitive, Q^* and K^* are highly sensitive to change in the production parameter 'β_1'.
- t_1^*, t_3^*, Q^* and K^* are less sensitive to change in the production parameter 'β_2'.
- t_1^* and t_3^* are less sensitive, Q^* and K^* are slightly sensitive to change in the production parameter 'p'.
- t_1^*, t_3^* and Q^* are less sensitive, K^* is slightly sensitive to change in the deterioration parameter 'θ'.
- t_1^* is less sensitive, t_3^* and Q^* are slightly sensitive and K^* is highly sensitive to change in the demand parameter 'k'.

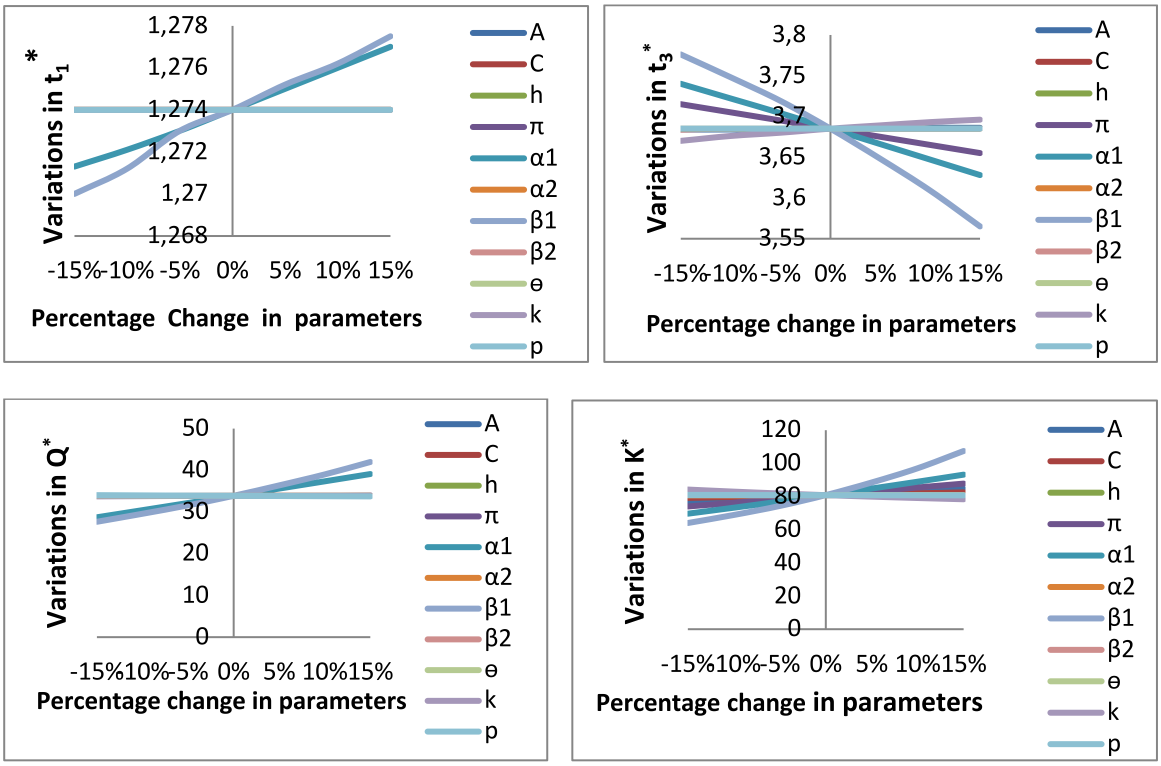


Figure 2: *Relationship between parameters and optimal values with shortages*

VIII. EPQ Model without Shortages

In this section the inventory model for deteriorating items without shortages is developed and analyzed. Here, it is assumed that shortages are not allowed and the stock level is zero at time $t=0$. The stock level increases during the period $(0, t_1)$ due to excess production after fulfilling the demand and deterioration. The production stops at time t_1 when the stock level reaches S. The inventory decreases gradually due to demand and deterioration in the interval (t_1, T) . At time T the inventory reaches zero. The schematic diagram representing the instantaneous state of inventory is given in Figure 3.

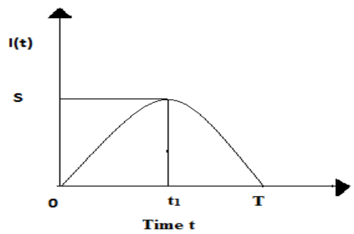


Figure 3: *Schematic diagram representing the inventory level*

Let $I(t)$ be the inventory level of the system at time 't' $(0 \le t \le T)$. Then the differential equations governing the instantaneous state of $I(t)$ over the cycle of length T are:

$$\frac{d}{dt}I(t) + h(t)I(t) = \frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}} - k;\ 0 \le t \le t_1 \tag{23}$$

$$\frac{d}{dt}I(t) + h(t)I(t) = -k;\ t_1 \le t \le T \tag{24}$$

Where, *h(t)* is as given in equation (3), with the initial conditions *I(0) = 0*, $I(t_1) = S$ and *I(T) =0*. Substituting *h(t)* given in equation (3) in equations (23) and (24) and solving the differential equations, the on hand inventory at time 't' is obtained as:

$$I(t) = Se^{\theta(t_1-t)} - e^{-t\theta}\int_t^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}} - k\right]e^{u\theta}du\,; 0 \le t \le t_1 \tag{25}$$

$$I(t) = Se^{\theta(t_1-t)} - ke^{-t\theta}\int_{t_1}^{t} e^{u\theta}\,du; t_1 \le t \le T \tag{26}$$

Production quantity Q in the cycle of length T is

$$Q = \int_0^{t_1} R(t)dt$$

$$= \int_0^{t_1}\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}dt \tag{27}$$

From equation (25) and using the initial conditions *I(0) = 0*, we obtain the value of 'S' as

$$S = e^{-\theta t_1}\int_0^{t_1}\left(\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}}\right)e^{u\theta}du - \frac{k}{\theta}\left(1 - e^{-\theta t_1}\right) \tag{28}$$

Let $K(t_1)$ be the total production cost per unit time. Since the total production cost is the sum of the set up cost, cost of the units, the inventory holding cost. Therefore the total production cost per unit time becomes

$$K(t_1) = \frac{A}{T} + \frac{CQ}{T} + \frac{h}{T}\left[\int_0^{t_1} I(t)dt + \int_{t_1}^{T} I(t)dt\right] \tag{29}$$

Substituting the values of *I(t)* and Q from equations ,(25), (26) and (27) in equation (29), we obtain *K(t₁)* as

$$K(t_1) = \frac{A}{T} + \frac{C}{T}\int_0^{t_1}\left[\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}} + (1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}} + (1-p)e^{-\alpha_2 t^{\beta_2}}}\right]dt$$

$$+\frac{h}{T}\left[\int_0^{t_1}\left[Se^{\theta(t_1-t)} - e^{-t\theta}\int_t^{t_1}\left[\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}} + (1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}} + (1-p)e^{-\alpha_2 u^{\beta_2}}}du - k\right]e^{u\theta}du\right]dt\right.$$

$$\left.+\int_{t_1}^{T}\left[Se^{\theta(t_1-t)} - ke^{-t\theta}\int_{t_1}^{t} e^{u\theta}\,du\right]dt\right] \tag{30}$$

Substituting the value of S given in equation (28) in the total cost equation (30), we obtain

$$K(t_1)=\frac{A}{T}+\frac{C}{T}\int_0^{t_1}\left[\frac{p\alpha_1\beta_1 t^{\beta_1-1}e^{-\alpha_1 t^{\beta_1}}+(1-p)\alpha_2\beta_2 t^{\beta_2-1}e^{-\alpha_2 t^{\beta_2}}}{pe^{-\alpha_1 t^{\beta_1}}+(1-p)e^{-\alpha_2 t^{\beta_2}}}\right]dt$$

$$+\frac{h}{T}\left[\left(1-e^{-\theta T}\right)\left[\frac{1}{\theta}\int_0^{t_1}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}e^{u\theta}du+\frac{k}{\theta^2}\right]\right.$$

$$\left.-\frac{k}{\theta}T-\int_0^{t_1}e^{-t\theta}\left(\int_t^{t_1}\frac{p\alpha_1\beta_1 u^{\beta_1-1}e^{-\alpha_1 u^{\beta_1}}+(1-p)\alpha_2\beta_2 u^{\beta_2-1}e^{-\alpha_2 u^{\beta_2}}}{pe^{-\alpha_1 u^{\beta_1}}+(1-p)e^{-\alpha_2 u^{\beta_2}}}e^{u\theta}du\right)dt\right] \tag{31}$$

IX. Optimal Production Schedules of the Model

In this section we obtain the optimal policies of the inventory system under study. To find the optimal values of t_1, we equate the first order partial derivatives of $K(t_1)$ with respect to t_1 equate them to zero. The condition for minimum of $K(t_1)$ is

$$\frac{\partial^2 K(t_1)}{\partial {t_1}^2}>0$$

Differentiating $K(t_1)$ with respect to t_1 and equating to zero, we get

$$\left\{\frac{C}{T}\left[\frac{p\alpha_1\beta_1 {t_1}^{\beta_1-1}e^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)\alpha_2\beta_2 {t_1}^{\beta_2-1}e^{-\alpha_2 {t_1}^{\beta_2}}}{pe^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)e^{-\alpha_2 {t_1}^{\beta_2}}}\right]\right.$$

$$\left.+\frac{h}{T}\left[\frac{\left(1-e^{-\theta T}\right)e^{t_1\theta}}{\theta}\left[\frac{p\alpha_1\beta_1 {t_1}^{\beta_1-1}e^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)\alpha_2\beta_2 {t_1}^{\beta_2-1}e^{-\alpha_2 {t_1}^{\beta_2}}}{pe^{-\alpha_1 {t_1}^{\beta_1}}+(1-p)e^{-\alpha_2 {t_1}^{\beta_2}}}\right]\right]\right\}=0 \tag{32}$$

Solving the equation (32), we obtain the optimal time t_1^* of $t_1$at which the production is to be stopped.

The optimal production quantity Q^* of Q in the cycle of length T is obtained by substituting the optimal values of t_1 in equation (27).

X. Numerical Illustration

In this section we discuss the solution procedure of the model through a numerical illustration by obtaining the production time, optimum production quantity and the total production cost of an inventory system. For demonstrating the solution procedure of the model the parameters are considered as A = Rs.310\-, C = Rs.15\-, h = Re.0.2\-, (α_1, α_2, β_1, β_2, p) = (11, 14, 0.55, 3, 0.5), θ= 3, k=3.3 and T=12 months. The values of parameters above are varied further to observe the trend in optimal policies and the results are obtained are shown in Table 3. Substituting these values the optimal production quantity Q^*, the production time and total production cost are computed and presented in Table 3

From Table 3 it is observed that the deterioration parameters and production parameters have a tremendous influence on the optimal values of the model.

Table 3: *Numerical Illustration*

A	C	h	T	α_1	α_2	β_1	β_2	θ	k	p	t_1^*	Q^*	K^*
310	15	0.2	12	11	14	0.55	3	3	3.3	0.5	5.495	28.771	61.871
325.5											5.495	28.771	63.163
341											5.495	28.771	64.454
356.5											5.495	28.771	65.746
	15.75										5.496	28.775	63.698
	16.5										5.497	28.778	65.477
	17.25										5.499	28.782	67.281
		0.21									5.495	28.772	61.875
		0.22									5.495	28.772	61.879
		0.23									5.495	28.772	61.883
				11.55							5.5	30.217	63.686
				12.1							5.501	31.598	65.402
				12.65							5.503	33.01	67.143
					14.7						5.493	28.767	61.859
					15.4						5.492	28.763	61.825
					16.1						5.491	28.761	61.788
						0.578					5.499	30.157	63.607
						0.605					5.504	31.56	65.366
						0.633					5.509	33.088	67.281
							3.15				5.496	28.773	61.89
							3.3				5.496	28.775	61.907
							3.45				5.497	28.777	61.923
								3.15			5.495	28.772	61.886
								3.3			5.495	28.773	61.901
								3.45			5.496	28.773	61.92
									3.465		5.495	28.771	61.86
									3.63		5.495	28.771	61.85
									3.795		5.495	28.771	61.839
										0.525	5.495	28.722	61.806
										0.55	5.494	28.675	61.743
										0.575	5.494	28.629	61.681

When the ordering cost 'A' increases from 310 to 356.5, the optimal production quantity Q^* and the optimal production down time t_1^* remains constant, the total production cost per unit time K^* increases from 61.871 to 65.746. As the cost parameter 'C' increases from 15 to 17.25, the optimal production quantity Q^* increases from 28.771 to 28.782, the optimal production down time t_1 increases from 5.495 to 5.499, the total production cost per unit time K^* increases from 61.871 to 67.281. As the inventory holding cost 'h' increases from 0.2 to 0.23, the optimal production quantity Q^* increases from 28.771 to 28.772, the optimal production down time t_1^* remains constant, the total production cost per unit time K^* increases from 61.871 to 61.883.

As the production parameter 'α_1' varies from 11 to 12.65, the optimal production quantity Q^* increases from 28.771 to 33.01, the optimal production down time t_1^* increases from 5.495 to 5.503, the total production cost per unit time K^* increases from 61.871 to 67.143. As the production parameter 'α_2' varies from 14 to 16.1, the optimal production quantity Q^* decreases from 28.771 to 28.761, the optimal production down time t_1^* decreases from 5.495 to 5.491, the total production cost per unit time K^* decreases from 61.871 to 61.788. As the production parameter 'β_1' varies from 0.55 to 0.633, the optimal production quantity Q^* increases from 28.771 to 33.088, the optimal production down time t_1^* increases from 5.495 to 5.509, the total production cost per unit time K^* increases from 61.871 to 67.281. As the production parameter 'β_2' varies from 3 to 3.45, the optimal production quantity Q^* increases from 28.771 to 28.773, the optimal production down time t_1^* increases from 5.495 to 5.497, the total production cost per unit time K^* increases from 61.871 to 61.923. As the production parameter 'p' varies from 0.5 to 0.575 the total production quantity Q^* decreases from 28.771 to 28.629, the optimal production down time t_1^* decreases from 5.495 to 5.494, the total production cost per unit time K^* decreases from 61.871 to 61.681.

As the deterioration parameter 'θ' varies from 3 to 3.45, the optimal production quantity Q^* increases from 28.771 to 28.773, the optimal production down time t_1^* increases from 5.495 to 5.496, the total production cost per unit time K^* increases from 61.871 to 61.92.

As the demand parameter 'k' varies from 3.3 to 3.795, the total production quantity Q^* remains constant, the optimal production down time t_1^* remains constant, the total production cost per unit time K^* decreases from 61.871 to 61.839.

XI. Sensitivity Analysis of the Model

The sensitivity analysis is carried to explore the effect of changes in model parameters and costs on the optimal policies, by varying each parameter (-15%, -10%, -5%, 0%, 5%, 10%, 15%) at a time for the model under study. The results are presented in Table 4. The relationship between the parameters and the optimal values of the production schedule is shown in Figure 4.

Table 4: *Sensitivity analysis of the model - Without Shortages*

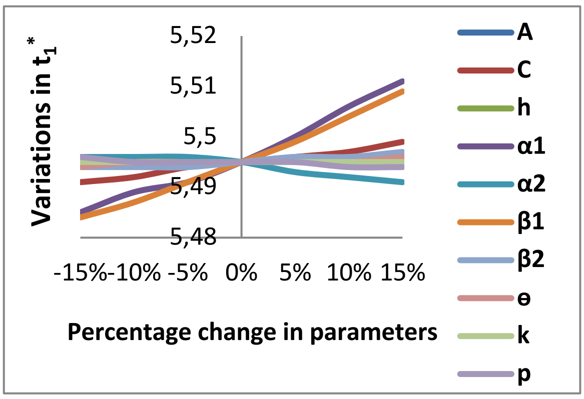
Variation Parameters	Optimal Policies	-15%	-10%	-5%	0%	5%	10%	15%
A	t_1^*	5.495	5.495	5.495	5.495	5.495	5.495	5.495
	Q^*	28.772	28.772	28.771	28.771	28.771	28.771	28.771
	K^*	57.996	59.288	60.579	61.871	63.163	64.454	65.746
C	t_1^*	5.491	5.492	5.494	5.495	5.496	5.497	5.499
	Q^*	28.761	28.765	28.768	28.771	28.775	28.778	28.782
	K^*	56.465	58.266	60.068	61.871	63.698	65.477	67.281
h	t_1^*	5.495	5.495	5.495	5.495	5.495	5.495	5.495
	Q^*	28.771	28.771	28.771	28.771	28.772	28.772	28.772
	K^*	61.859	61.863	61.867	61.871	61.875	61.879	61.883
α_1	t_1^*	5.485	5.489	5.491	5.495	5.5	5.501	5.503
	Q^*	24.537	26.203	27.358	28.771	30.217	31.598	33.01
	K^*	56.463	58.599	60.076	61.871	63.686	65.402	67.143
α_2	t_1^*	5.496	5.496	5.496	5.495	5.493	5.492	5.491
	Q^*	28.775	28.775	28.774	28.771	28.767	28.763	28.761
	K^*	61.901	61.885	61.880	61.871	61.859	61.825	61.788

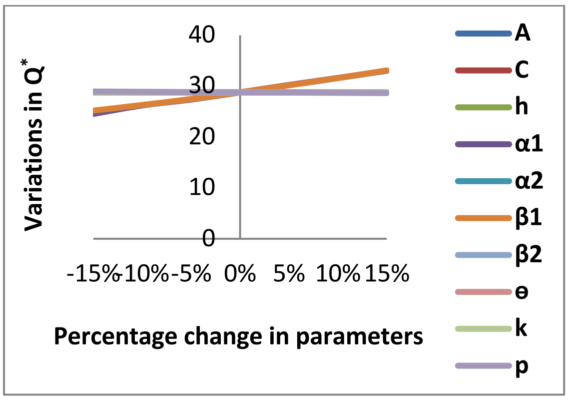
$\boldsymbol{\beta_1}$	t_1^*	5.484	5.487	5.491	5.495	5.499	5.504	5.509
	Q^*	25.171	26.286	27.453	28.771	30.157	31.56	33.088
	K^*	57.359	58.756	60.219	61.871	63.607	65.366	67.281
$\boldsymbol{\beta_2}$	t_1^*	5.494	5.494	5.494	5.495	5.496	5.496	5.497
	Q^*	28.768	28.769	28.77	28.771	28.773	28.775	28.777
	K^*	61.811	61.831	61.851	61.871	61.89	61.907	61.923
$\boldsymbol{\theta}$	t_1^*	5.494	5.495	5.495	5.495	5.495	5.495	5.496
	Q^*	28.77	28.771	28.771	28.771	28.772	28.773	28.773
	K^*	61.832	61.845	61.857	61.871	61.886	61.901	61.92
$\boldsymbol{k}$	t_1^*	5.495	5.495	5.495	5.495	5.495	5.495	5.495
	Q^*	28.771	28.771	28.771	28.771	28.771	28.771	28.771
	K^*	61.903	61.892	61.882	61.871	61.86	61.85	61.839
$\boldsymbol{p}$	t_1^*	5.496	5.495	5.495	5.495	5.495	5.494	5.494
	Q^*	28.936	28.878	28.823	28.771	28.722	28.675	28.629
	K^*	62.082	62.009	61.939	61.871	61.806	61.743	61.681

XII. Observations

The major observations drawn from the numerical study are:

- t_1^* is less sensitive, Q^* is slightly sensitive and K^* is moderately sensitive to the changes in ordering cost 'A'.
- t_1^* and Q^* are slightly sensitive and K^* is moderately sensitive to the changes in cost per unit 'C'.
- t_1^* is less sensitive, Q^* and K^* are slightly sensitive to the changes in holding cost 'h'.
- t_1^* is slightly sensitive, Q^* and K^* are highly sensitive to the change in the production parameter 'α_1'.
- t_1^*, Q^* and K^* are slightly sensitive to the change in the production parameter 'α_2'.
- t_1^* is slightly sensitive, Q^* and K^* are moderately sensitive to the change in the production parameter 'β_1'.
- t_1^*, Q^* and K^* are slightly sensitive to the change in the production parameter 'β_2'.
- t_1^*, Q^* and K^* are slightly sensitive to the change in the production parameter 'p'.
- t_1^*, Q^* and K^* are slightly sensitive to the change in the deterioration parameter 'θ'.
- t_1^* and Q^* are less sensitive, K^* is slightly sensitive to change the demand parameter 'k'.





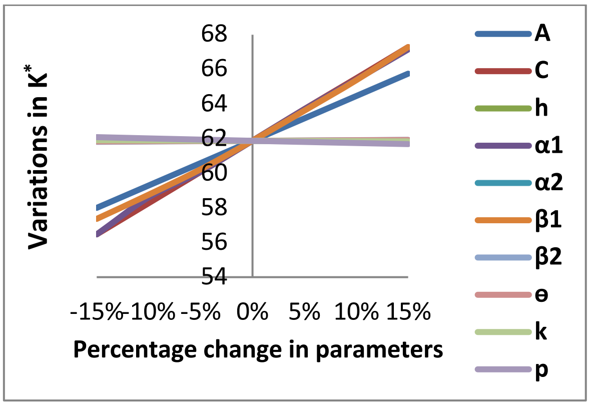


Figure 4: *Relationship between parameters and optimal values without shortages*

XIII. Conclusions

This paper introduces a new EPQ model with random production having mixture of two component Weibull production rate and exponential decay having constant demand. The mixture of two parameter Weibull distribution characterises the heterogeneous process more close to reality. By using the historical data we can estimate the replenishment and deterioration distribution parameters. The production manager can estimate the optimal production downtime and uptime with the distributional data of production and deterioration parameters. The Weibull rate of production can include increase/ decrease/constant rates for different values of parameters. Sensitivity analysis is used to understand the change in the parameters of Weibull rates of production and exponential deterioration. It is observed that random production and deterioration have significant influence on optimal values of the production schedule and production quantity. This model also includes some of the earlier models as particular cases. This model can be used to analyse production processes where the production is done in two different units/ machines and rate of deterioration is constant. It is possible to extend this model with other types of demand functions such as stock dependent demand, time and selling price dependent demand which will be taken up elsewhere. This paper is useful for analyzing optimal production schedules for the industries dealing with deteriorating items such as sea foods and edible oil. This model also includes some of the earlier EPQ models as particular cases for specific values of the parameters.

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Declaration of Conflicting Interests

The Authors declare that there is no conflict of interest.

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