Performance of a Single Server Batch Queueing Model with Second Optional Service under Transient and Steady State Domain

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Abstract

The aim of this paper is to investigate the performance of a single server batch queueing model with second optional service under transient and steady state domain. It is assumed that the customers arrive in groups as per compound Poisson process and the server gives two types of services, First Essential Service (FES), which is mandatory for all arriving customers and Second Optional Service (SOS), which is given to some customers those who request it. Both FES and SOS are provided in batches of maximum b capacity. The transient and steady state probabilities of the model are obtained by using probability generating function and Laplace transform techniques. Finally, some numerical examples are presented to study the effect of the parameters on the system performance measures.

Keywords: Batch Queueing Model, First Essential Service, Second Optional Service, Transient State, Steady State

I. Introduction

In real-life situations, one encounter numerous examples of queueing models wherein a server gives FES to all arriving customers, and a few of them may only demand the auxiliary service after the completion of the essential service. For instance, all arriving ships at a harbor may need unloading service on arrival but only a few of them may demand re-loading service immediately after the unloading. The concept of SOS was first introduced by [8] where numerous practical applications of SOS were given. [8] presented an $M/G/1$ queue with SOS, whereby the service time distribution of the FES is general and the SOS is exponentially distributed. Later on, [9] generalized the concept of [8] in which the service time for both FES and SOS are independent having a general distribution. [16] studied the SOS in correlated reneging with working vacations. They use matric geometric method to obtain the steady state probabilities distribution of the queueing system size.

Queueing models with bulk input have broad applications in manufacturing, computer networks, communication systems, etc., where the arrivals at a service point (e.g., a switch) may occur in bunches of distinctive sizes. The notation of batch arrival appeared in the queueing theory in the work of [10] who considered the single server queue with fixed size batch Poisson arrivals in transient domain. Similar work of batch arrival has been carried out in [19]. They presented a bulk input queueing model with single working vacation and they obtained the stationary queue length distribution using the matrix analysis method and probability generating function. [14] and [15] analyzed a bulk arrival queueing model with variant working vacations. The probability generating functions are derived in the stationary state and achieved the expressions of the model when the server is operating in various states. Related studies on the analysis of queueing model of bulk arrival are found in [3], [7], [12], [17], etc.,

Batch service queues have a motivation on numerous applications such as in group testing of blood samples for detecting corona/HIV viruses, in mobile crowd-sourcing app for smart cities, eliminate defective items in manufacturing system, etc. The batch service queueing models has been analyzed by many authors. [11] investigated the batch service queueing model with servers' variant vacations and obtained the steady state solutions using shifting operator and recursive technique. [6] discussed a single server queue with additional optional service in batches and server vacation. They have applied probability generating function method to obtain the queue length in stationary state. The analysis of bulk service queueing system with two heterogeneous servers in a discrete time has been presented in [5] with the help of displacement operator method and obtained closed form expressions for the limiting probabilities at arbitrary epoch.

In this model, we consider the transient state due to its importance especially in manufacturing system with regular beginning up periods and transportation frameworks with time fluctuating interest; for instance, airport terminal runway activities in major airports [4]. The analytical solutions of the transient behavior of queueing systems are very rare due to the complexity of getting analytical solutions. However, there are few works carried out in transient states such as [10], [2], [1], etc.

At the moment, most of the studies including [3], [7], [12], [13], [18] and many other are devoted to a single server batch queueing model with SOS in steady state, whereby customers arrive in groups as per Poisson process and served with general service distribution for both FES and SOS. However, in this paper, we consider a batch queueing model by involving the concept of SOS and investigated in both transient and steady state domains. We computed the probabilities and expected queue lengths when the server is busy in FES or SOS using probability generating function with the help of Laplace transform techniques. The advantage of expressions in Laplace transform is that it can be easily used for numerically transforming into time domain.

The remainder of this paper is structured as follows. In section 2, we present the model description and mathematical formulation. In section 3, we discuss the transient state equations and solving using probability generating function on the Laplace transforms equations. The steady state analysis is obtained by applying the Tauberian property in section 4. Measures of performance are discussed in section 5. Numerical analysis and discussions are presented in section 6 and in section 7, we conclude the paper.

II. Model Description and Mathematical Formula

We consider an $M^X/M^{[b]}/1$ queueing model with FES and SOS. Customers arrive in batches with rate $\lambda > 0$ conforming to a compound Poisson process. Let X be a batch size random variable and $X_1, X_2, \ldots$, are corresponding batch sizes of arriving customers which are independently and identically distributed (i.i.d.) random variables, with probability mass function $P\{X_i = k\} = C_k, k = 1, 2, 3, \ldots$. The service time distribution of both FES and SOS are exponential with rate μ_1 and μ_2, respectively and the services are given in batches of size not more than b such that if the server finds the customers less or equal to b in the waiting queue, the server takes all of them in the batch for service, but if the server finds the customers more than b waiting in the queue, then she or he takes a batch of size b while others remain waiting in the queue. The FES is required by all arriving customers and after completing FES, they may opt SOS with probability r or may depart from the system with probability $1-r$. Figure 1 below shows the transition rate diagram of various transition states of the model.

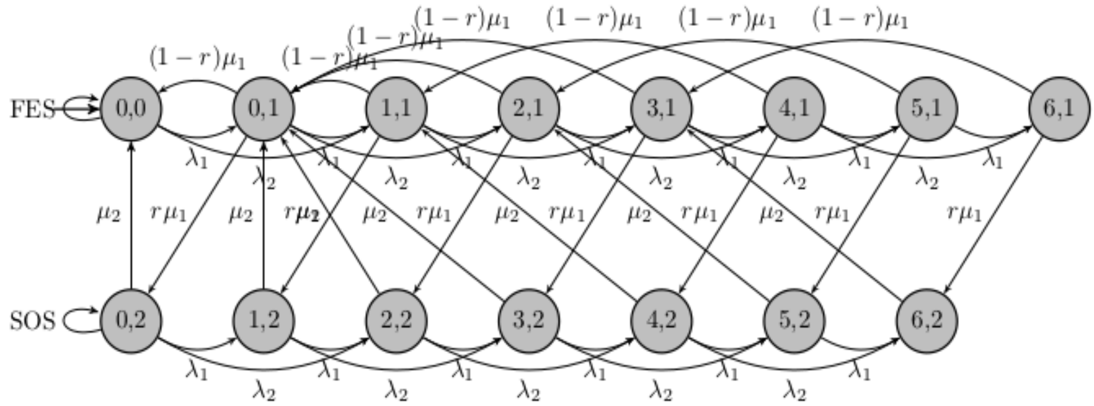

Figure 1: *Transition rate diagram for* $n = 6,\ b = 3,\ k = 2$.

I. Formulation of Mathematical Model

Suppose $L(t)$ be the length of the queue at time t, and $J(t)$ is the server state with

$$J(t) = \begin{cases} 1, & \text{if the server is providing FES} \\ 2, & \text{if the server is providing SOS.} \end{cases}$$

The stochastic process $\{(L(t),\ J(t));\ t \geq 0\}$ is a two-dimensional Markov Chain with the state space:

$$\Omega = \{(n,i);\ n \geq 0;\ i = 1,2\}.$$

Further, let the transient probabilities are defined as

$$P_{n,i}(t) = Pr\{L(t) = n, \quad J(t) = i\};\ n \geq 0;\ i = 1,2.$$

Here, $P_{n,i}(t)$ is the transient probability that there are n units in the queue at time t and the server is providing FES and SOS service, and $Q(t)$ is the probability when the queue is empty and the server is idle at time t. Using Markov theory, the differential-difference equations of the model are as follows:

$$Q'(t) = -\lambda Q(t) + (1-r)\mu_1 P_{0,1}(t) + \mu_2 P_{0,2}(t), \tag{1}$$

$$P'_{0,1}(t) = -(\lambda+\mu_1)P_{0,1}(t) + \lambda Q(t) + (1-r)\mu_1 \sum_{i=1}^{b} P_{i,1}(t) + \mu_2 \sum_{i=1}^{b} P_{i,2}(t), \tag{2}$$

$$P'_{n,1}(t) = -(\lambda+\mu_1)P_{n,1}(t) + \lambda \sum_{i=1}^{n} P_{n-k,1}(t)\, C_k + (1-r)\mu_1 P_{n+b,1}(t) + \mu_2 P_{n+b,2}(t), \qquad n \geq 1, \tag{3}$$

$$P'_{0,2}(t) = -(\lambda+\mu_2)P_{0,2}(t) + r\mu_1 P_{0,1}(t), \tag{4}$$

$$P'_{n,2}(t) = -(\lambda+\mu_2)P_{n,2}(t) + \lambda \sum_{i=1}^{n} P_{n-k,2}(t)\, C_k + r\mu_1 P_{n,1}(t), \qquad n \geq 1. \tag{5}$$

III. Transient Solution of the Model

In this section, the transient system size probability of the expected queue length when the server is idle and busy are presented by using Laplace transform (L.T) and probability generating functions. Let us assume that time is figured from the moment the server has taken a batch for service, leaving none in the queue. i.e., $P_{0,1}(0) = 1$. Let $Q^*(s)$, $P^*_{n,i}(s)$ denote the L.T of $Q(t)$, $P_{n,i}(t)$, $i = 1, 2$, respectively. Taking L.T of equations from equation (1) to (5), we get

$$(s+\lambda)Q^*(s) = (1-r)\mu_1 P^*_{0,1}(s) + \mu_2 P^*_{0,2}(s), \tag{6}$$

$$(s+\lambda+\mu_1)P^*_{0,1}(s) = 1 + \lambda Q^*(s) + (1-r)\mu_1 \sum_{i=1}^{b} P^*_{i,1}(s) + \mu_2 \sum_{i=1}^{b} P^*_{i,2}(s), \tag{7}$$

$$(s+\lambda+\mu_1)P^*_{n,1}(s) = \lambda \sum_{i=1}^{n} P^*_{n-k,1}(s)C_k + (1-r)\mu_1 P^*_{n+b,1}(s) + \mu_2 P^*_{n+b,2}(s), \ \ n \geq 1, \tag{8}$$

$$(s+\lambda+\mu_2)P^*_{0,2}(s) = r\mu_1 P^*_{0,1}(s), \tag{9}$$

$$(s+\lambda+\mu_2)P^*_{n,2}(s) = \lambda \sum_{i=1}^{n} P^*_{n-k,2}(s)C_k + r\mu_1 P^*_{n,1}(s), \ \ n \geq 1. \tag{10}$$

Let us define the probability generating functions as:

$$P_1(s,z) = \sum_{n=0}^{\infty} P^*_{n,1}(s) z^n, \qquad P_2(s,z) = \sum_{n=0}^{\infty} P^*_{n,2}(s) z^n$$

The probability generating function of arrival batch size X is defined as:

$$C(z) = \sum_{k=1}^{n} C_k z^k \ ; \ |z| \leq 1; \ \ k = 1,2,3 \ldots \tag{11}$$

Multiplying equations (7) and (8) by z^n and taking summation from $n = 0$ to $n = \infty$ then, adding to (6) and after simplification, we have

$$P_1(s,z) = \frac{z^b(sQ^*(s)-1) + (1-z^b)A(s,z) - \mu_2 P_2(s,z)}{\lambda C(z) z^b - (s+\lambda+\mu_1)z^b + (1-r)\mu_1}, \tag{12}$$

where

$$A(s,z) = \left((1-r)\mu_1 \sum_{n=0}^{b-1} P^*_{n,1}(s) z^n + \mu_2 \sum_{n=0}^{b-1} P^*_{n,2}(s) z^n \right).$$

Similarly, from equation (9) and (10), we get

$$P_2(s,z) = \frac{-r\mu_1 P_1(s,z)}{\lambda C(z) - (s+\lambda+\mu_2)}. \tag{13}$$

Substituting equation (13) in (12), we obtain

$$P_1(s,z) = \frac{(\lambda C(z) - (s+\lambda+\mu_2))[z^b(sQ^*(s)-1) + (1-z^b)A(s,z)]}{(\lambda C(z))^2 z^b - \lambda C(z)(2s+2\lambda+\mu_1+\mu_2)z^b + B}, \tag{14}$$

where

$$B = (s+\lambda+\mu_1)(s+\lambda+\mu_2)z^b + \lambda C(z)(1-r)\mu_1 z - (s+\lambda+\mu_2)(1-r)\mu_1 - r\mu_1\mu_2.$$

We assume that arrival batch size X follows a geometric distribution with parameter q as given by.

$$P(X = k) = C_k = (1-q)^{k-1}q; \quad 0 \le q \le 1; \quad k = 1,2,3 \dots \tag{15}$$

Using (11) and (15), we obtain

$$C(\mathrm{z}) = \frac{qz}{1 - z + qz}. \tag{16}$$

Substitute (16) into (14), we obtain

$$P_1(\mathrm{s,z}) = \frac{B_1(1-z+qz)[z^b(sQ^*(s)-1)+(1-z^b)A(s,z)]}{(\lambda q)^2 z^{b+2} - \lambda q(1-z+qz)(2s+2\lambda+\mu_1+\mu_2)z^{b+1}+B_2}, \tag{17}$$

where

$$B_1 = (\lambda qz - (s+\lambda+\mu_2)(1-z+qz),$$
$$B_2 = (1-z+qz)^2(s+\lambda+\mu_1)\,(s+\lambda+\mu_2)z^b + \lambda qz(1-z+qz)(1-r)\mu_1$$
$$-(1-z+qz)^2\,(s+\lambda+\mu_2)(1-r)\mu_1 - (1-z+qz)^2\, r\mu_1\mu_2.$$

We notice that the denominator of $P_1(s,z)$ has $b+2$ zeros. Using Rouche's theorem to the denominator, it follows that b of these roots lie on or inside the unit circle. One zero of the denominator is $z = 1$ and other $b-1$ zeros lie within and should harmonize with those of numerator for $P_1(s,z)$ to converge, so that when a zero shows up in the denominator, it is dropped by one in the numerator. The remaining two zeros of the denominator lie outside the unit circle. Let the roots be z_0 and z_1, we have

$$P_1(\mathrm{s,z}) = \frac{(1-z+qz)[\lambda qz-(s+\lambda+\mu_2)(1-z+qz)](1-z^b)D(s)}{(z-1)(z-z_0)(z-z_1)}, \tag{18}$$

where $D(s)$ is a function independent of z.
For $z = 1$ in (13) and using L'Hospital rule at $z = 1$ in (18), we get

$$P_1(\mathrm{s},1) = \frac{q^2(s+\mu_2)bD(s)}{(1-z_0)(1-z_1)}, \tag{19}$$

$$P_2(\mathrm{s},1) = \frac{r\mu_1 P_1(s,1)}{(s+\mu_2)}. \tag{20}$$

Using the normalization condition $P_1(s,1) + P_2(s,1) + Q^*(s) = \frac{1}{s}$, we have

$$P_1(\mathrm{s},1) = \frac{\big(1-sQ^*(s)\big)(s+\mu_2)}{s(s+r\mu_1+\mu_2)}. \tag{21}$$

Using (19) and (21) one can determine the function of $D(s)$ as

$$P_1(\mathrm{s},1) = \frac{\big(1-sQ^*(s)\big)(1-z_0)(1-z_1)}{s(s+r\mu_1+\mu_2)q^2 b}. \tag{22}$$

Substitute (22) into (18), we get

$$P_1(\mathrm{s,z}) = \frac{(1-z+qz)B_1(1-z^b)\big(1-sQ^*(s)\big)(1-z_0)(1-z_1)}{s(s+r\mu_1+\mu_2)q^2 b(z-1)(z-z_0)(z-z_1)}. \tag{23}$$

When $z = 0$, equation (23) and (13), respectively becomes

$$P_{0,1}^{*}(s) = \frac{\left(1 - sQ^{*}(s)\right)(s + \lambda + \mu_2)(r_0 - 1)(r_1 - 1)}{s(s + r\mu_1 + \mu_2)q^2 b}, \tag{24}$$

$$P_{0,2}^{*}(s) = \frac{r\mu_1\left(1 - sQ^{*}(s)\right)(r_0 - 1)(r_1 - 1)}{s(s + r\mu_1 + \mu_2)q^2 b}, \tag{25}$$

where $z_0 = 1/r_0$, $z_1 = 1/r_1$.
From equation (6), we can determine the value of $Q^{*}(s)$ by using (24) and (25), we have

$$Q^{*}(s) = \frac{B_3\,(s)}{s[(s + \lambda)(s + r\mu_1 + \mu_2)q^2 b + B_3\,(s)]}, \tag{26}$$

where

$$B_3(s) = [(1 - r)\mu_1(s + \lambda + \mu_2) + r\mu_1\mu_2](r_0 - 1)(r_1 - 1).$$

Equation (26) represents the L.T of the state probability that the queue is empty and the server is idle. It is obtained from the equation (6) by using the equations (24) and (25). In the following section, we obtain the stationary probabilities by using the Tauberian property.

IV. Steady State Solution of the Model

In this part, we obtain the closed form solutions of the limiting state probabilities for the length of the queue size when the server is idle or busy in FES and SOS by using the Tauberian property as defined below:

$$Q = \lim_{t\to\infty} Q(t) = \lim_{s\to 0} sQ^{*}(s), \tag{27}$$

$$P_{n,1} = \lim_{t\to\infty} P_{n,1}(t) = \lim_{s\to 0} sP_{n,1}^{*}(s), \tag{28}$$

$$P_{n,2} = \lim_{t\to\infty} P_{n,2}(t) = \lim_{s\to 0} sP_{n,2}^{*}(s). \tag{29}$$

If the limit exists, the steady state probabilities of (24), (25) and (26) are:

$$P_{0,1} = \frac{(1 - Q)(\lambda + \mu_2)(r_0 - 1)(r_1 - 1)}{(r\mu_1 + \mu_2)q^2 b}. \tag{30}$$

$$P_{0,2} = \frac{r\mu_1(1 - Q)(r_0 - 1)(r_1 - 1)}{(r\mu_1 + \mu_2)q^2 b}, \tag{31}$$

$$Q = \frac{B_3}{\lambda(r\mu_1 + \mu_2)q^2 b + B_3}, \tag{32}$$

where

$$B_3 = [(1 - r)\mu_1(\lambda + \mu_2) + r\mu_1\mu_2](r_0 - 1)(r_1 - 1).$$

V. Performance Measures

Practical applicability of any mathematical model can be accessed in terms of its measures of performance. In this paper different execution measures of the queue are calculated such as probability that the server is active and the expected queue size when the server is active in FES or SOS. The performance measures are carried out in both transient and steady state as follows:

I. Performance Measures in Transient State

The busy probability in FES is given by:

$$P[FES](s)=\sum_{n=0}^{\infty}P_{n,1}^{*}(s).$$

The busy probability of the server in FES is obtained by setting $z = 1$ in equation (23) and applying L'Hospital rule, we get

$$P[FES](s)=\sum_{n=0}^{\infty}P_{n,1}^{*}(s)=\frac{\big(1-sQ^{*}(s)\big)(s+\mu_2)}{s(s+r\mu_1+\mu_2)}. \tag{33}$$

The busy probability in SOS is given by

$$P[SOS](s)=\sum_{n=0}^{\infty}P_{n,2}^{*}(s).$$

The busy probability in SOS is obtained by setting $z = 1$ in equation (13) and using (33), we get

$$P[SOS](s)=\sum_{n=0}^{\infty}P_{n,2}^{*}(s)=\frac{r\mu_1\big(1-sQ^{*}(s)\big)}{s(s+r\mu_1+\mu_2)}. \tag{34}$$

The anticipated length of the queue size when the server is busy in FES

$$L[FES](s)=\sum_{n=0}^{\infty}nP_{n,1}^{*}(s).$$

This is obtained by taking derivative of equation (23) with respect to z, setting $z = 1$ and using L'Hospital rule. Thus we get

$$\sum_{n=0}^{\infty}nP_{n,1}^{*}(s)=\frac{\big(1-sQ^{*}(s)\big)\big[(r_0-1)(r_1-1)B_4(s)-[q(s+\mu_2)(4r_0r_1-2(r_0+r_1))]\big]}{2q(s(s+r\mu_1+\mu_2)(r_0-1)(r_1-1))}, \tag{35}$$

where $B_4(s)=\big[q(s+\mu_2)(b-1)-2[\lambda+(s+\mu_2)(2-2q)]\big]$.

The anticipated length of the queue size when the server is busy in SOS

$$L[SOS](s)=\sum_{n=0}^{\infty}nP_{n,2}^{*}(s).$$

This is obtained by taking derivative of equation (13) with respect to z and using (35) by setting $z = 1$, we get

$$\sum_{n=0}^{\infty}nP_{n,2}^{*}(s)=\frac{r\mu_1\big(1-sQ^{*}(s)\big)\big[(r_0-1)(r_1-1)B_4(s)-[q(s+\mu_2)(4r_0r_1-2(r_0+r_1))]\big]}{2q(s(s+r\mu_1+\mu_2)(s+\mu_2)(r_0-1)(r_1-1))}+\frac{\lambda r\mu_1\big(1-sQ^{*}(s)\big)}{qs(s+\mu_2)(s+r\mu_1+\mu_2)}. \tag{36}$$

The overall queue length is

$$L_q(s) = \sum_{n=0}^{\infty} nP_{n,1}^*(s) + \sum_{n=0}^{\infty} nP_{n,2}^*(s). \tag{37}$$

The anticipated waiting time in the queue is

$$W_q(s) = \frac{q \times L_q(s)}{\lambda}. \tag{38}$$

II. Performance Measures in Steady State

Assuming that the limit of the equations (27), (28) and (29) exist, the steady state equations corresponding to the equations (33) to (38), respectively are given by

$$P[FES] = \sum_{n=0}^{\infty} P_{n,1} = \frac{(1-Q)\mu_2}{r\mu_1 + \mu_2},$$
$$P[SOS] = \sum_{n=0}^{\infty} P_{n,2} = \frac{r\mu_1(1-Q)}{(r\mu_1 + \mu_2)},$$

$$\sum_{n=0}^{\infty} nP_{n,1} = \frac{(1-Q)\big[(r_0-1)(r_1-1)B_4 - [q\mu_2(4r_0r_1 - 2(r_0+r_1))]\big]}{2q(r\mu_1+\mu_2)(r_0-1)(r_1-1)},$$
$$\sum_{n=0}^{\infty} nP_{n,2} = \frac{r\mu_1(1-Q)\big[(r_0-1)(r_1-1)B_4 - [q\mu_2(4r_0r_1 - 2(r_0+r_1))]\big]}{2q(r\mu_1+\mu_2)\mu_2(r_0-1)(r_1-1)} + \frac{\lambda r\mu_1(1-Q)}{q\mu_2(r\mu_1+\mu_2)},$$
$$L_q = \sum_{n=0}^{\infty} nP_{n,1} + \sum_{n=0}^{\infty} nP_{n,2}$$

and

$$W_q = \frac{q \times L_q}{\lambda},$$

where $B_4 = \big[q\mu_2(b-1) - 2[\lambda + \mu_2(2-2q)]\big]$.

VI. Numerical Investigation

In this part, we perform the transient and steady state numerical analysis of the model. In transient state, the Laplace transform expressions given in section 5.1 are inverted into time domain using a software package of Mathematica. Furthermore, we study the parameters impact on the model performance and discussion on numerical results by taking the model parameters as: $b = 5$, $\lambda = 3$, $\mu_1 = 3.5$, $\mu_2 = 3$, $r = 0.45$, $q = 0.4$, $r_0 = 0.9039$, and $r_1 = 0.7686$, unless their values are mentioned in the respective places.

Figures 2 and 3 show the time dependent probability of FES and SOS with variation of time points. We observe that the probability values in FES (Figure 2) decrease rapidly in the beginning from point one up to a certain value where it reaches the steady state with increasing of time while the probability values in SOS (Figure 3) increase progressively from zero initially up to a certain value and it attains the steady state with increasing of time. In addition, it is noticed that the probabilities of both FES and SOS increase as the arrival rate λ increases. Figure 4 plots the transient state probability of empty queue and idle server versus time for different values of arrival rate. In this graph, we observe that the idleness probability decreases as the rate of arrivals increases.

Figure 5 demonstrates the variation of arrival rate λ on the expected queue size L_q with respect to time. It is noticed that expected queue size increases when arrival rate increases. This is due to the fact that when arrival rate increases, more customers join the queue and leads to an increase in the length of the queue. Figure 6 shows the impact of r on the expected waiting time in queue (W_q) and it is observed that as r increases, both $W[FES]$ and $W[SOS]$ increase. In addition, it reaches a point where the waiting time in SOS is more compared to FES as r increases. This is coherent with the fact that the service rate in FES is greater than that in SOS $i.e.$, $\mu_1 > \mu_2$.

Figures 7 and 8 show the effect of arrival rate λ on the expected queue size L_q for different batch size parameter q (Figure 7) and different batch service size b (Figure 8). It is obvious that the anticipated queue length increases with the increase in arrival rate λ (Figure 7). For a particular λ L_q increases as q decreases, this is on the grounds that the mean batch size $(1/q)$ positively influences the number of customers in the queue. Hence, the mean queue size increases. While in Figure 8 we observe that the expected queue size decreases with increase of batch service size b and increases with increasing of arrival rate λ.

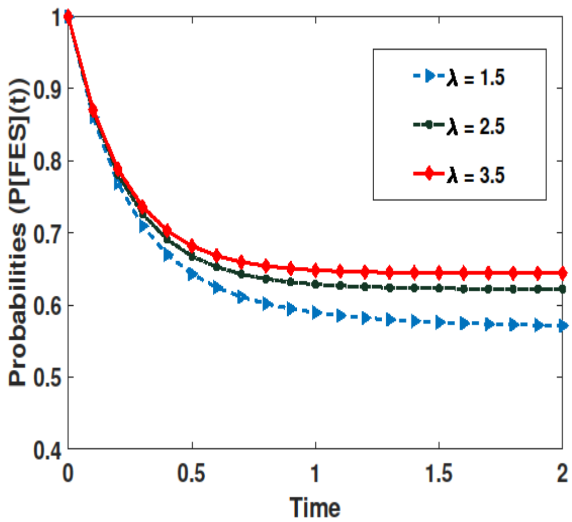

Figure 2: *The probability that the server is busy in FES versus time*

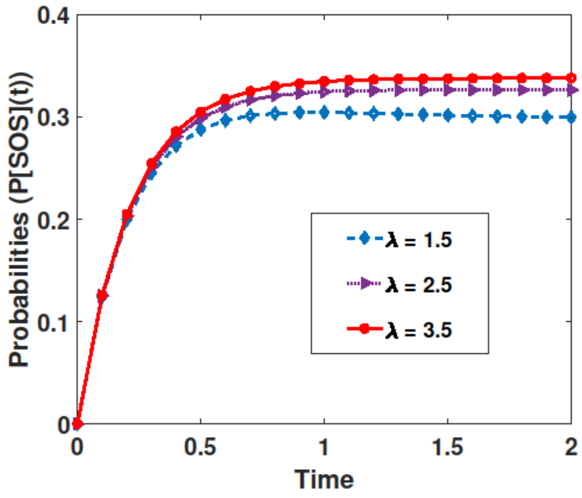

Figure 3: *The probability that the server is busy in SOS versus time*

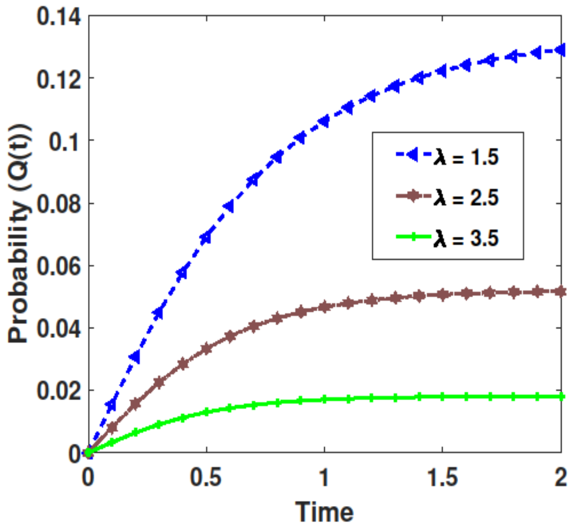

Figure 4: *The transient state probability of empty queue and idle server versus time*

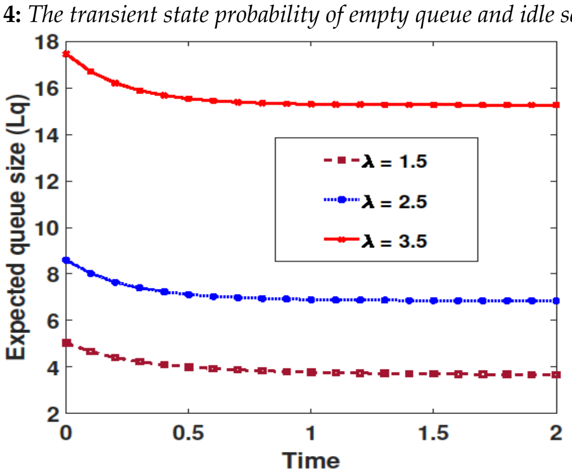

Figure 5: *Effect of variation of λ on L_q with respect to time*

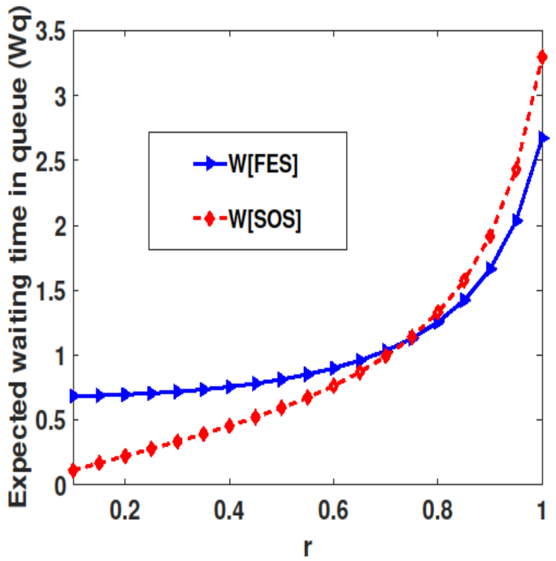

Figure 6: *Effect of r on the expected waiting time in queue (W_q)*

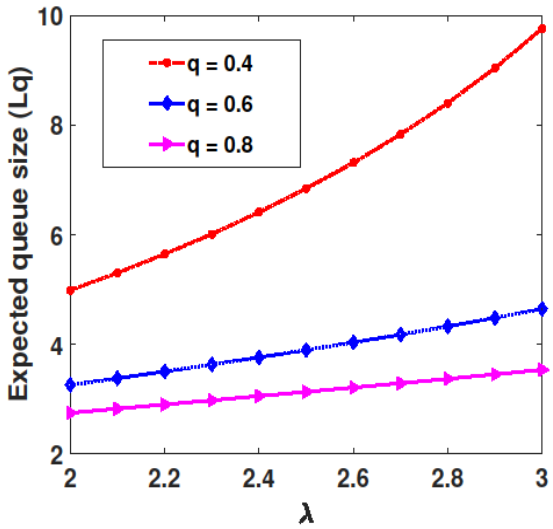

Figure 7: *Effect of variation of λ and q on the expected queue length (L_q)*

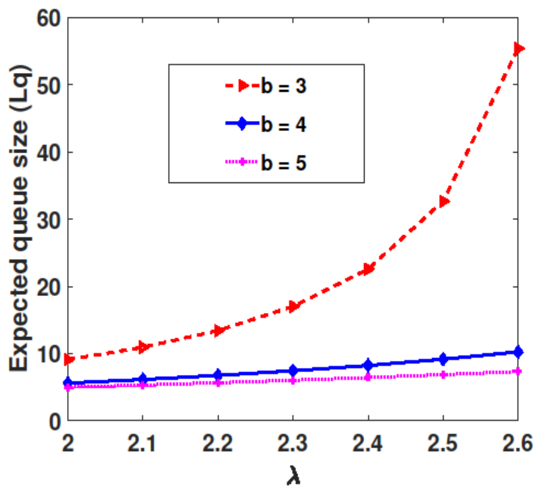

Figure 8: *Effect of variation in λ and b on the expected queue length (L_q)*

VII. Conclusion

In this article, we studied a single server batch queueing model with SOS under transient and steady state domain. We derived the transient and steady state probabilities when the server is busy in FES or SOS. Furthermore, we have studied the impact of various parameters on the performance measures of the model and discussed the results in the form of graphs. In addition, the analysis of the model will motivate a useful performance evaluation tool in practical applications such as telecommunication network through packet switching, in group testing of blood samples for detecting Corona / HIV viruses, package delivery, etc. Finally, the present work might be extended to multi-server multi-arrival system with reneging and vacations.

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