On Discrete Scheduled Replacement Model of a Single Device Unit

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Abstract

This paper considered a device which is subjected to three types of failures (category I, category II and category III). Also the paper tries to combine discrete age replacement model with minimal repair, where it dealt with a discrete scheduled replacement policy. Category I failure is an un-repairable failure, which occurs, suddenly, and if it occurs, the device is replaced completely, while category II and category III failures are repairable failures, which occurs, due to time and usage, and the two failures are rectified with minimal repair. To investigate the characteristics of the model constructed and determine optimum replacement number (N^*) *of the device, a numerical example is provided, where it is assumed that the rate of the three categories of failures follow Weibull distribution.*

Keywords: category, discrete, number, optimal, replacement, scheduled

I. Introduction

Most systems deteriorate and subsequently fail due to age and usage. These deficiencies have a detrimental impact on sales, the production of faulty goods and the delay in the provision of customer services. For these reasons, many optimal replacement policies have been built by several researchers to minimize excessively high running costs and prevent sudden failure of systems. For certain purposes, such as shortage of spare units, lack of money or staff, or inconvenience of time needed to complete replacement, an operating unit may often not be replaced at the exact optimum replacement times, but in idle periods, a unit can be replaced instead. Aven and Castro [1] constructed a minimal replacement policy for a system subject to two types of failures, which determined optimal replacement time for the system. Briš *et al.* [2] presented a new approach for optimizing a complex system's maintenance strategy that respects a given reliability constraint. Chang [3] considered a device that faces two types of failures (repairable and non-repairable) based on a random mechanism. Coria *et al*. [4] introduced a method of analytical optimization for preventive maintenance policy with historical failure time data. Enogwe *et al.* [5] used the distribution of the probability of failure times and come up with a replacement model for items that fails un-notice. Fallahnezhad and Najafian [6] investigated the number of spare parts and installations for a unit and parallel systems, so as cut down the average cost per unit time. Jain and Gupta [7] studied optimal replacement policy for a repairable system with multiple vacation and imperfect coverage. Lim *et al.* [8] studied the characteristics of some age substitution policies. Liu *et al.* [9] developed mathematical models of uncertain reliability of some multi-component systems.

Malki *et al.* [10] analyzed age replacement policies of a parallel system with stochastic dependency. Murthy and Hwang [11] presented that, the failures can be reduced through effective maintenance actions (in a probabilistic sense), and such maintenance actions can occur either at discrete time instants or continuously over time. Nakagawa [12] modified the continuous standard age replacement for a unit, and come up with a discrete replacement model for the unit. Nakagawa *et al.* [13] presented the advantages of some replacement policies. Safaei *et al.* [14] studied the optimal preventive maintenance action for a system based on some conditions. Sudheesh *et al.* [15] studied age replacement policy in discrete approach. Tsoukalas and Agrafiotis [16] presented a new replacement policy warrant for a system with correlated failure and usage time. Waziri and Yusuf [17] constructed an age replacement model for a parallel-series system based on some proposed policies. Xie *et al.* [18] assessed the effects of safety barriers on the prevention of cascading failures. Yaun and Xu [19] studies a cold standby repairable system with two different components and one repairman who can take multiple vacations, where they assumed that, if there is a component which fails and the repairman is on vacation, the failed component will wait for repair until the repairman is available. Yusuf and Ali [20] constructed a minimal repair replacement model for two parallel units in which both units operate simultaneously, such that, the two components are two types of failures. Yusuf *et al.* [21] analyzed the characteristics of reliability and availability of certain number of devices. Zhao *et al.* [22] proved that age replacement policy is optimal among all replacement policies.

The key contribution of this study is to come up with a discrete scheduled replacement model for a device that is exposed to three categories of failures, in order to (i) provide opportunity to replace a system at the ideal time (ii) provide the ability to skip such special hours to avoid, and (iii) investigate those aspects of the discrete model of scheduled replacement requiring limited repair.

II. Methods

Reliability measures namely reliability function and failure rates are used to obtain the expressions of discrete scheduled replacement model involving minimal repair based on some assumptions. A numerical example was given for the purpose of investigating the characteristics of the model constructed.

III. Notations

- C_2 : cost of repair due to failure of category II.
- C_3 : cost of repair due to failure of category III.
- C_p : cost of scheduled replacement at NT, for $N = 1, 2, 3$ ....
- C_r : cost of unscheduled replacement due to failure of category I.
- $C(N)$: replacement cost rate in one replacement cycle.
- N^*: the device's optimum discrete scheduled replacement time.
- $r_1(t)$: rate of category I failure.
- $r_2(t)$: rate of category II failure .
- $r_3(t)$: rate of category III failure.
- $R_1(t)$: reliability function due to category I failure.

IV. Description of the System

Consider a device which is subjected to three independent types of failures, which are named as category I, category II and category III, such that, all the three failures arrives according to non-homogeneous Poisson process. It is assumed that, category I failure is unrepairable one, while category II and category III failures are repairable failures. The device is replace with new one whenever it reaches scheduled replacement time NT $(N = 1, 2, 3, \dots)$ for a fixed T or at category I failure, whichever occurs first.

V. Discrete Scheduled Replacement Model

This section considers a fundamental discrete scheduled replacement model involving minimal repair.

Assumptions for this model:

1. Category I failure is un-repairable one, while category II and category III failures are repairable failure.
2. Category I, Category II and Category III failures arrives according to a non-homogeneous Poisson process with failure intensity $r_1(t)$, $r_2(t)$ and $r_3(t)$, respectively, such that: $r_3(t) \geq r_2(t) \geq r_1(t)$.
3. The cost of replacement/minimal repair follows the order : $C_r > C_p > C_2 > C_3$.
4. All three failures are detected instantaneously.
5. When needed, all the resources required are available.
6. If the device fails with respect to category I failure, the device will be replaced completely.
7. If the device fails with respect to category II or category II failure, the device is minimally restored back to operation.
8. The device is replaced completely at planned time $NT(N = 1, 2, 3 \dots)$ for a fixed T or at category I failure, whichever arrives first.

Based on the assumptions, the probability of the device being replaced before category I failure occurs at the scheduled time T is

$$R_1(NT) = e^{-\int_0^{NT} r_1(t)dt}, \quad (1)$$

where $N = 1, 2, 3 \dots$ and T is fixed.

Based on the assumptions, the cost of unscheduled replacement of the device in one replacement cycle is

$$C_r(NT)\big(1 - R_1(NT)\big), \quad (2)$$

where $N = 1, 2, 3 \dots$ and T is fixed.

Based on the assumptions, the cost of scheduled replacement of the device at time NT in one replacement cycle is

$$C_p R_1(NT), \quad (3)$$

where $N = 1, 2, 3 \dots$ and T is fixed.

Based on the assumptions, the cost of minimal repair of the device due to category II failure in one replacement cycle is

$$\int_0^{NT} C_2 r_2(t) R_1(t)dt, \quad (4)$$

where $N = 1, 2, 3 \dots$ and T is fixed.

Based on the assumptions, the cost of minimal repair of the device due to category III failure in one replacement cycle is

$$\int_0^{NT} C_3 r_3(t) R_1(t) dt, \quad (5)$$

where $N = 1, 2, 3 \ldots$ and T is fixed.

Based on the assumptions, the mean of one replacement cycle is

$$\int_0^{NT} R_1(t) dt, \quad (6)$$

where $N = 1, 2, 3 \ldots$ and T is fixed.

Adding up equation (1) to equation (6), the device's cost rate in one replacement cycle is

$$C(N) = \frac{C_r(1 - R_1(NT)) + C_p R_1(NT) + \int_0^{NT} R_1(t) K(t) dt}{\int_0^{NT} R_1(t) dt}, \quad (7)$$

where

$$K(t) = C_2 r_2(t) + C_3 r_3(t). \quad (8)$$

Noting the following:

1. If the value of T is taking as one (that is, T = 1), then $C(N)$ will be a continuous standard age replacement model with minimal repair.
2. $C(N)$ is adopted as an objective function of an optimization problem, and the main goal is to obtain an optimal discrete scheduled replacement time N^* that minimizes $C(N)$.

VI. Numerical Example

In this section, we will give two numerical example, so as to illustrate the characteristics of the constructed discrete scheduled replacement model.

Let the rate of arrival of category I, category II and category III failures follows the Weibull distribution:

$$r_i(t) = \lambda_i \propto_i t^{\propto_i - 1}, \quad \text{for } i = 1, 2, 3, \quad (9)$$

where $\propto_i > 1$ and $t \geq 0$.

Let the collection of parameters and repair/replacement costs be used in this specific example:

1. $\propto_1 = 2$, $\propto_2 = 3$, $\propto_3 = 3$.
2. $\lambda_1 = 0.0002$, $\lambda_1 = 0.04$, $\lambda_3 = 0.02$.
3. $C_r = 50$, $C_p = 40$, $C_2 = 3$, $C_3 = 1.5$.

By substituting the parameters in equation (9), the category I, category II and category III failure rates were obtained as follows:

$$r_1(t) = 0.0004t, \quad (10)$$

$$r_2(t) = 0.12t^2, \quad (11)$$

$$r_3(t) = 0.06t^3. \quad (12)$$

Table 1 below is obtained, by substituting the assumed cost of replacement/repair ($C_r = 50$, $C_p = 40$, $C_2 = 3$, $C_3 = 1.5$) and rates of category I, category II and category III failures obtained above (equations (10), (11) and (12)) in equation (7), so as to evaluate the device's optimal discrete scheduled replacement time. When obtaining table 1, the value of $T = 1, T = 2, T = 3, T = 4, T = 5$ and $T = 6$ are considered so as to investigate the properties of the device's optimal discrete scheduled replacement time. Figure 1 is the graph of $C(N)$ against N, as $T = 1$. Figure 2 is the graph of $C(N)$ against N, as $T = 2$. Figure 3 is the graph of $C(N)$ against N, as $T = 3$. Figure 4 is the graph of $C(N)$ against N, as $T = 4$. Figure 5 is the graph of $C(N)$ against N, as $T = 5$. Figure 6 is the graph of $C(N)$ against N, as $T = 6$.

Table 1: *Values of C(N) for T = 1, T = 2, T = 3, T = 4, T = 5 and T =6, versus N* (1, 2, 3 …)

N	C(N) as T=1	C(N) as T=2	C(N) as T=3	C(N) as T=4	C(N) as T=5	C(N) as T=6
1	400.82	203.18	140.39	112.46	99.38	94.47
2	203.18	112.46	94.47	99.13	116.34	142.63
3	140.39	94.47	106.45	142.63	195.82	263.13
4	112.46	99.13	142.63	216.79	314.97	433.39
5	99.38	116.34	195.82	314.97	465.78	641.83
6	94.47	142.63	263.13	433.39	641.83	876.92
7	94.86	176.41	342.80	568.80	836.42	1125.87
8	99.13	216.79	433.39	717.84	1042.19	1374.67
9	106.45	263.13	533.53	876.92	1251.20	1608.65
10	116.34	314.97	641.83	1042.19	1455.03	1813.40
11	116.34	371.85	756.81	1209.54	1645.18	1975.90
12	128.47	433.39	876.92	1374.67	1813.40	2085.74
13	142.63	499.18	1000.52	1533.20	1952.23	2136.30
14	158.65	568.80	1125.87	1680.83	2055.48	2125.54
15	176.41	641.83	1251.20	1813.40	2118.67	2056.29

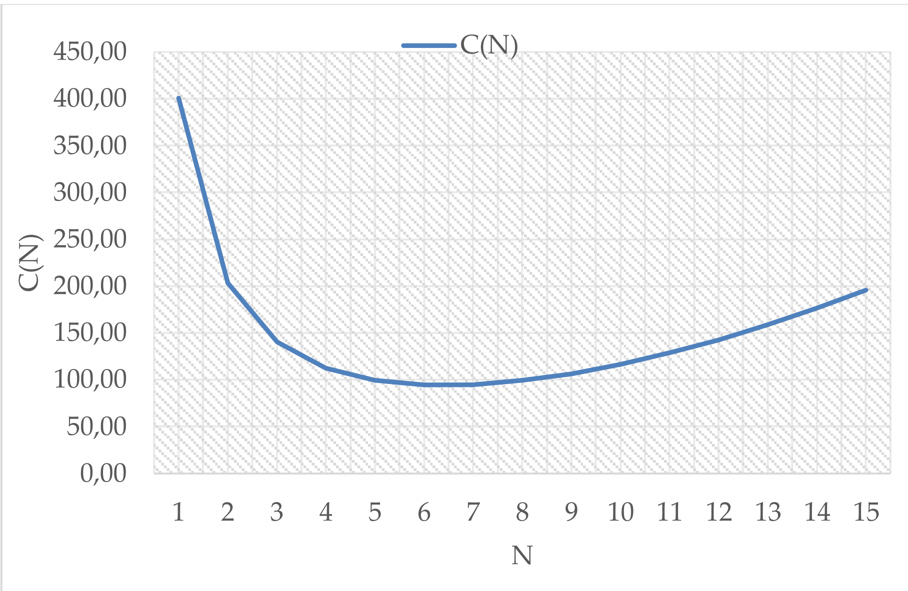

Figure 1: *C(N) against N, as T=*

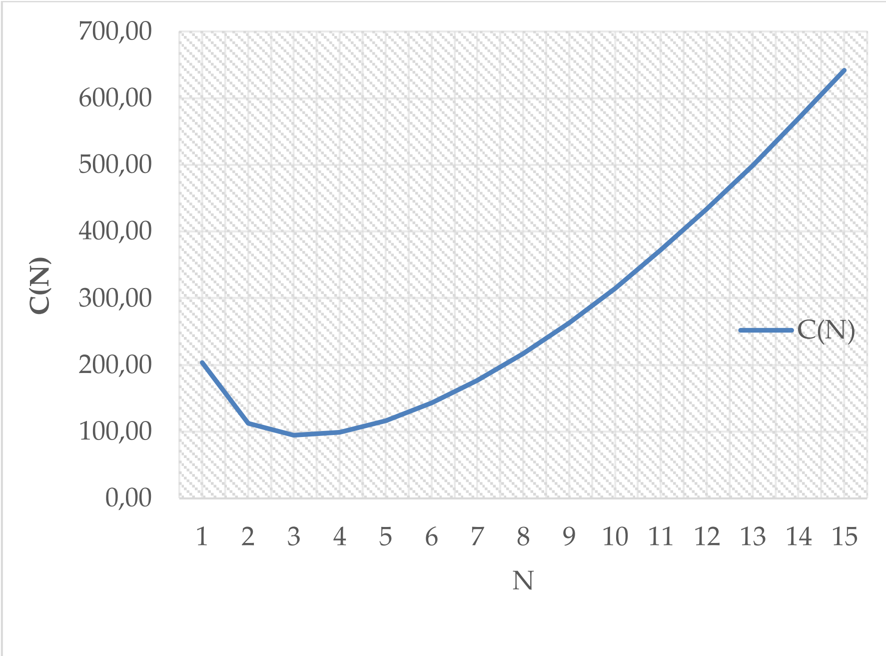

Figure 2: *C(N) against N, as T=2*

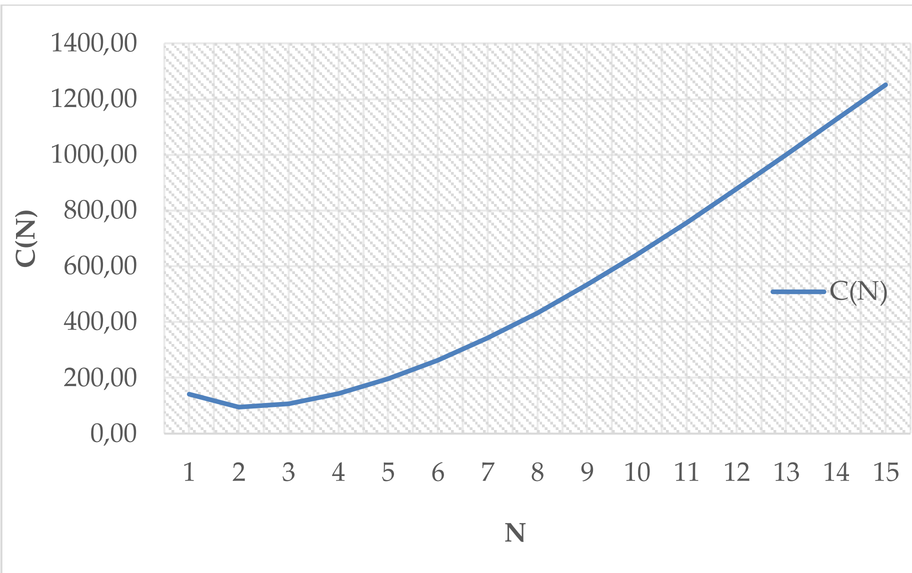

Figure 3: *C(N) against N, as T=3*

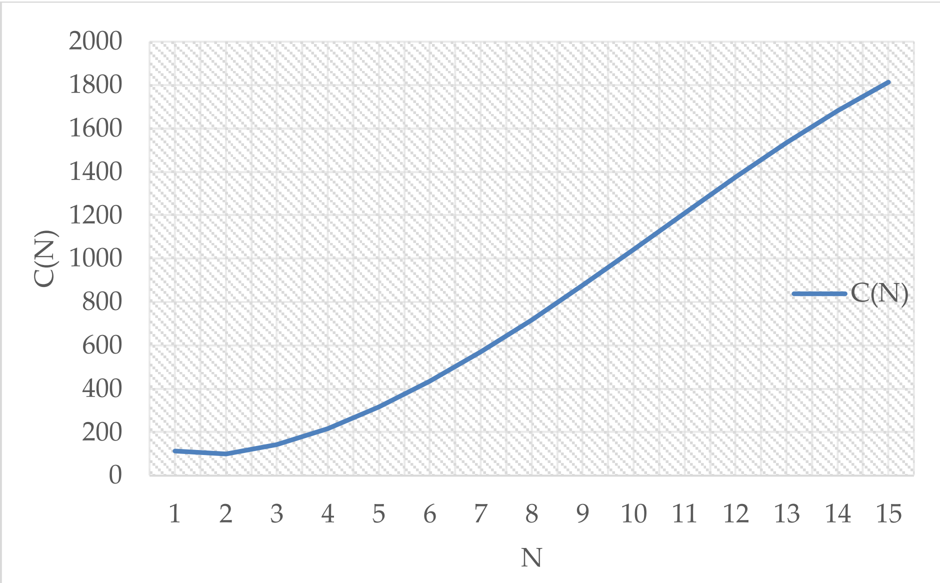

Figure 4: *C(N) against N, as T=4*

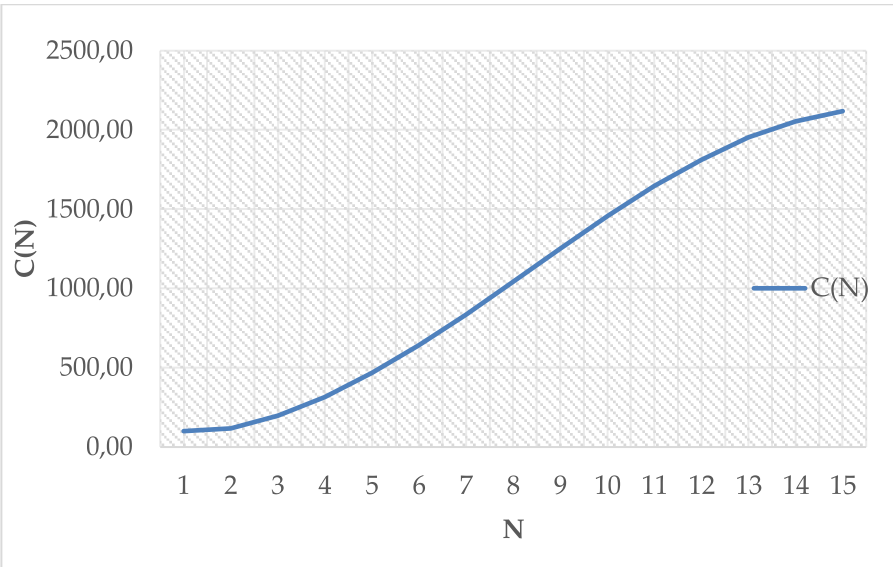

Figure 5: *C(N) against N, as T=5*

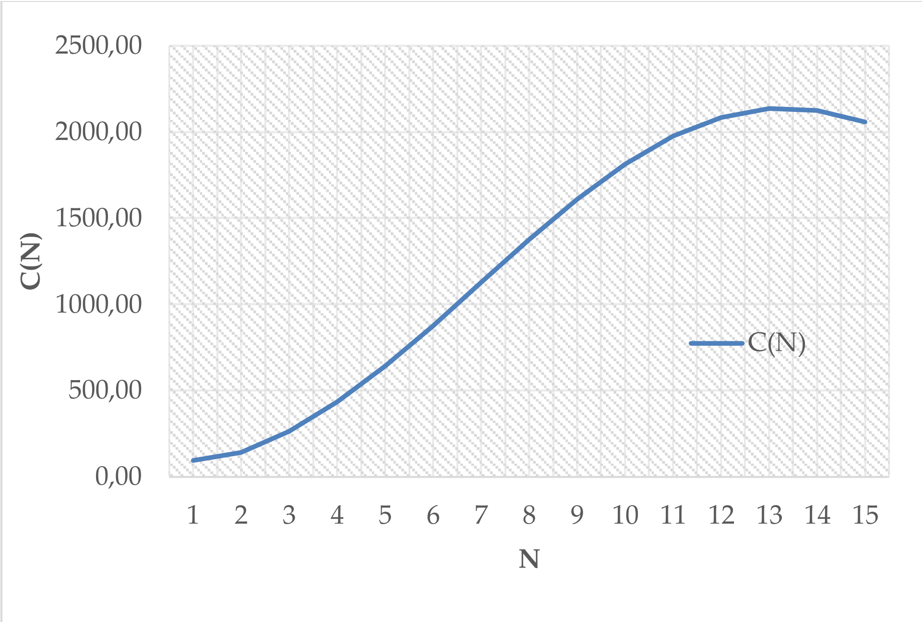

Figure 6: *C(N) against N, as T=6*

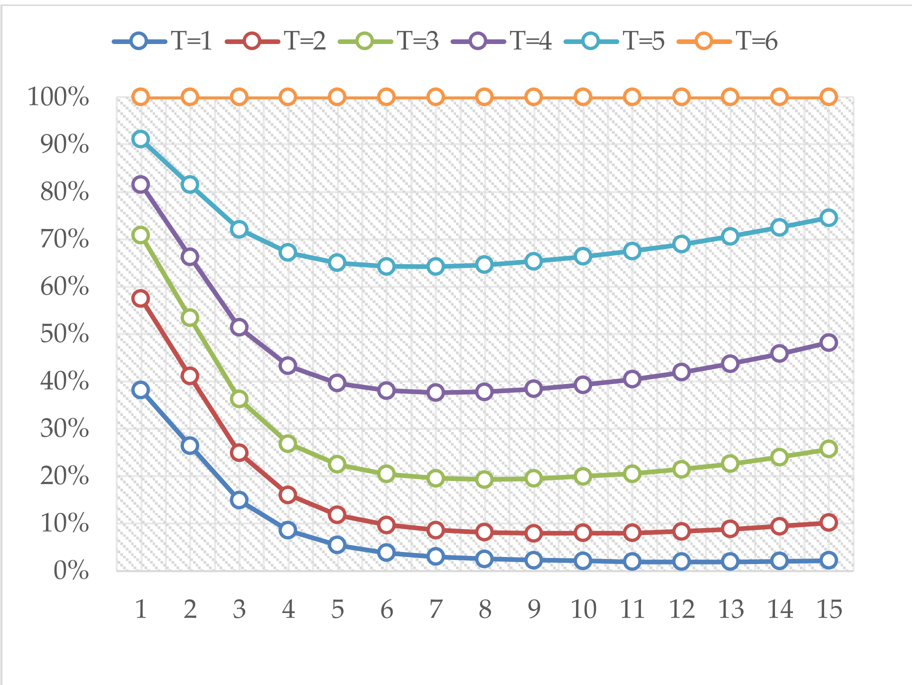

Figure 6: *Comparing C(N) for T=1, 2, 3, 4, 5, 6*

Some observations from the results obtained are as follows:

1. Observe from table 1, the optimum discrete scheduled replacement time is 6, when $T = 1$. That is, $N^* = 6$, with $C(N^* = 6) = 94.47$, when $T = 1$. See figure 1 below for the plot of $C(N)$ against N.
2. Observe from table 1, the optimum discrete scheduled replacement time is 3, when $T = 2$. That is, $N^* = 3$, with $C(N^* = 3) = 94.47$, when $T = 2$. See figure 2 below for the plot of $C(N)$ against N.
3. Observe from table 1, the optimum discrete scheduled replacement time is 2, when $T = 3$. That is, $N^* = 2$, with $C(N^* = 2) = 94.47$, when $T = 3$. See figure 3 below for the plot of $C(N)$ against N.
4. Observe from table 1, the optimum discrete scheduled replacement time is 2, when $T = 4$. That is, $N^* = 2$, with $C(N^* = 2) = 99.13$, when $T = 4$. See figure 4 below for the plot of $C(N)$ against N.
5. Observe from table 1, the optimum discrete scheduled replacement time is 1, when $T = 5$. That is, $N^* = 1$, with $C(N^* = 1) = 99.38$, when $T = 5$. See figure 5 below for the plot of $C(N)$ against N.
6. Observe from table 1, the optimum discrete scheduled replacement time is 1, when $T = 6$. That is, $N^* = 1$, with $C(N^* = 1) = 99.47$, when $T = 6$. See figure 6 below for the plot of $C(N)$ against N.
7. Observe from figure 7, we have : $(C(N),\ T = 1) < (C(N),\ T = 2) < (C(N),\ T = 3) < (C(N),\ T = 4) < (C(N),\ T = 5) < (C(N),\ T = 6)$.
8. Observe from figure 1, figure 2, figure 3 and figure 4 are all in convex shaped, which corresponded to $T = 1, T = 2,\ T = 3$ and $T = 4$, respectively.
9. Observe from figure 5 and figure 6 are in s-shaped, which are corresponded to $T = 5$ and $T = 6$, respectively.
10. Observe from table 1, as the value of T is increasing, the optimum discrete scheduled replacement time decreases.

VII. Conclusion and recommendations

This paper developed a discrete scheduled model for a device that is exposed to three categories of failures. category I failure is an un-repairable one, which occurs suddenly, and if it occurs, the device is replaced completely, while category II and category III failures are repairable failures, which occurs due to time and usage, and the two failures are minimally repaired. A numerical example was provided to test the constructed model so as to investigate the characteristics of the discrete scheduled model constructed and determine the optimum replacement number (N^*) of the device. A numerical example was provided for simple illustrations. From the results obtained, it is discovered or verified that, the value of T have an effect on the discrete scheduled replacement model, because of the following reasons:

1. as the value of T decreases, the optimal discrete replacement time (N^*) increases, while as the value of T increases, the optimal discrete replacement time (N^*) decreases.
2. as the value of T increases, $C(N)$ increases, while as the value of T decreases, $C(N)$ increases decreases.

With such reasons above, it can be easily seen that, continuous scheduled replacement model (continuous age replacement model) is better than discrete scheduled replacement model (discrete age replacement model). This paper is important to engineers, maintenance managers and plant management in maintaining multi-component systems at idle times, such as weekend, month-end or year-end.

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