STOCHASTIC ANALYSIS OF COMPLEX REDUNDANT SYSTEM HAVING PROBLEM OF WAITING LINE IN REPAIR USING COPULA METHODOLOGY

Surabhi Sengar[1], Mangey Ram[2], Yigit Kazancoglu[3]

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[1]Pimpri Chinchwad College of Engineering, Pune, India.
[2]Department of Mathematics; Computer Science & Engineering,

Graphic Era Deemed to be University, Dehradun, Uttarakhand, India

[3]Department of International Logistics Management,Yasar University, Izmir, Turkey.
[1]sursengar@gmail.com, [2]drmrswami@yahoo.com, [3]yigit.kazancoglu@yasar.edu.tr

Abstract

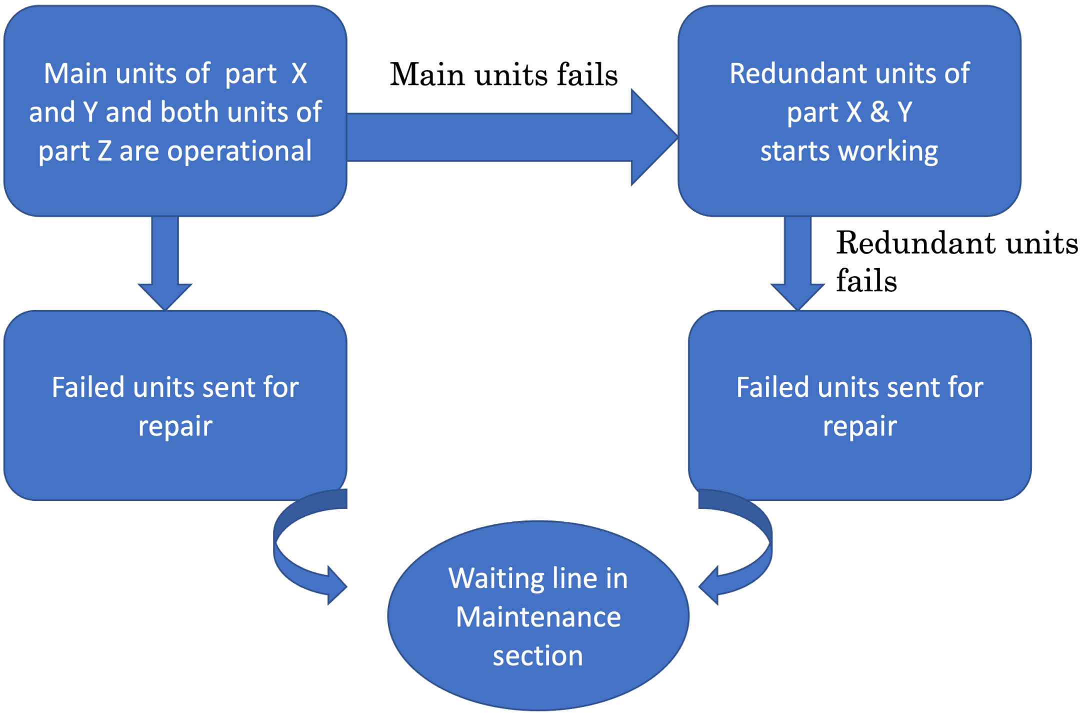
This paper investigates the stochastic behavior of a redundant system having problem of waiting line in the maintenance section in terms of various aspects such as reliability, availability, sensitivity etc. The system under consideration has three parts X, Y and Z connected in series. Each part has two units. Out of which part X has one main unit and other applied redundant unit. Similarly part Y has one main unit and other cold redundant unit to support the system. When main units of both the part have failed, then redundant units start automatically. While part Z has two units connected in parallel configuration. Here a realistic situation is discussed that when main units and redundant units of part X and Y are failed and arrived for repair then due to unavailability of repair men a line is generated there and its affect on systems reliability. So the focus of the study is to investigate the nature of the system using supplementary variable technique with the application of copula methodology under the condition when all four units are in line for repair.

Keywords: Stochastic analysis, Supplementary variable methodology availability analysis etc.

I. Introduction

Redundancy is a very general technique used to improve the performance and reliability of the system. The word redundancy is generally addressed to forecast the replacement of a unit or Part by another unit or Part in case of failure. Redundancy functions in two ways: Applied redundancy and Passive redundancy. Both redundancies prevent system performance from un predicted failure and downfall without human interference. Applied redundancy monitors the performance of individual devices so it reduces the performance decline, when a condition occurs in the system with a number of failures. The system of electricity supply is a good example of applied redundancy as huge number of electrical lines is connected to generate facility with the consumer as well as each electrical line monitors that detect overload and circuit breakers. On the other hand passive redundancy provides extra competence or capacity to avoid or to decrease the impact of system failure. This extra competency permits the failure of some parts without system failure. For example in structural engineering, the additional cables and struts that are used in construction of overpass allow some parts to fail without the whole structure fall down. Also, queuing problem is an area of mathematics which deals with the models and situation that arises due to waiting line in maintenance or service. When we talk about analysis of repair services, queue analysis plays an important role. These models have been used by various repair systems like communication and manufacturing, networking and simulation for calculating the behavior of the system and various reliability aspects. Previously, many researchers predict the system performance by assuming failure free service. But later different types of failures have been considered while evaluating a systems performance.

On the basis of above study and related facts, here in this paper we have discussed the stochastic behavior of a redundant system having three-Parts X, Y and Z, connected in series. Part X and Y have two main units and at same time two redundant units, one is applied redundant and another is Cold redundant. Part Z has two units which are connected in parallel configuration. Due to failure of any of the part, the system can result in complete failure [3]. It is assumed that when system starts operating, all the main units except redundant units are fully operational. Also, When the main units of X and Y fails, redundant units are switched on automatically and failed units are sent for repair to repairing section. The condition that has been taken into the consideration of the study is when all four units main units as well as supporting units are failed and sent for repair to maintenance section but because of unavailability of repairmen a line is generated in this section [4]. The system working process is described by the figure. Transition state diagram is shown by Figures 1. Table 1 shows the state specification of the system.



II. Assumptions

- In the beginning the system is in good operating state.
- All Parts are connected in series.
- System has two states only good and failed not degraded.
- Catastrophic failure is also responsible for system failure in the study also they require constant and exponential repair. So, copula technique is used for finding probability distribution [5].
- Repair facility which follows general time distribution is there for the service of both the Parts of unit 3 and also failure are exponential in both cases.
- For the Parts 1 and 2 failure and repairs both are exponential.

Table 1: *State specification of the system*

States	Description	System State
S0	The system is in good working state	G
S1	The system is in working state when key unit is failed.	G
S2	The system is in failed state because of failure of superfluous unit.	F
S3	When all four units are in waiting at repair section, system is in failed state.	F
S4	The system is in working state when superfluous unit of Part X is failed.	G
S5	The system is in failed state due to the failure of key unit of Part X.	F
S6	The system is in working condition when key unit Part Y is failed.	G
S7	The system is in failed state when superfluous unit of Part Y is failed.	F
S8	The system is in operable condition when key unit of Part Z failed.	G
S9	The system is in failed state from the state S8 due to failure of superfluous unit of Part Z.	FR
S10	The system is in operable condition when superfluous unit of Part Z is failed.	G
S11	The system is in failed state from the state S10 due to failure of key unit of Part Z.	FR
S12	The system is in failed state from the state S1 due to failure of Part Z.	FR
S13	The system is in failed state from the state S6 due to failure of Part Z.	FR
S14	System is failed state because of catastrophic failure.	FR

G: Good state; F: Failed State; FR= Failed state and under repair.

III. Notations

Pr Probability

$P_0(t)$ Pr (at time t system is in good state S_0)

$P_i(t)$ Pr {the system is in failed state due to the failure of the i^th Part at time t}, where i=2, 5, 7, 14.

λ_i Failure rates of Parts, where I = x1, x2, y1, y2, z1, z2, csf.

ψ Arrival rate of all four units of Parts X and Y to the repair section named as x1, x2, y1, y2.

μ Repair rate of unit's x1, x2, y1, y2.

$\phi_i(k)$ General repair rate of ith system in the time interval (k, k+◎), where i= z1, z2,(names for the units of Part Z) csf and k=v, g, r, l.

$P_3(t)$ Pr (at time t there is a queue (x1, x2, y1, y2) in the maintenance section due to servicing of some other unit and all four machines are waiting for repair.

$P_i(j,k,t)$ Pr (at time t system is in failed state due to the failure of jth unit when kth unit has been already failed, where i=9, 11. j=g, v. and k=v, g.

K1, K2 Profit cost and service cost per unit time respectively.

Let $u_1 = e^l$ and $u_2 = \phi_{csf}(l)$ then the expression for joint probability according to Gumbel-Hougaard family of copula is given as $\phi_{csf}(l) = \exp[l^\theta + (\log \phi_{csf}(l))^\theta)^{1/\theta}]$

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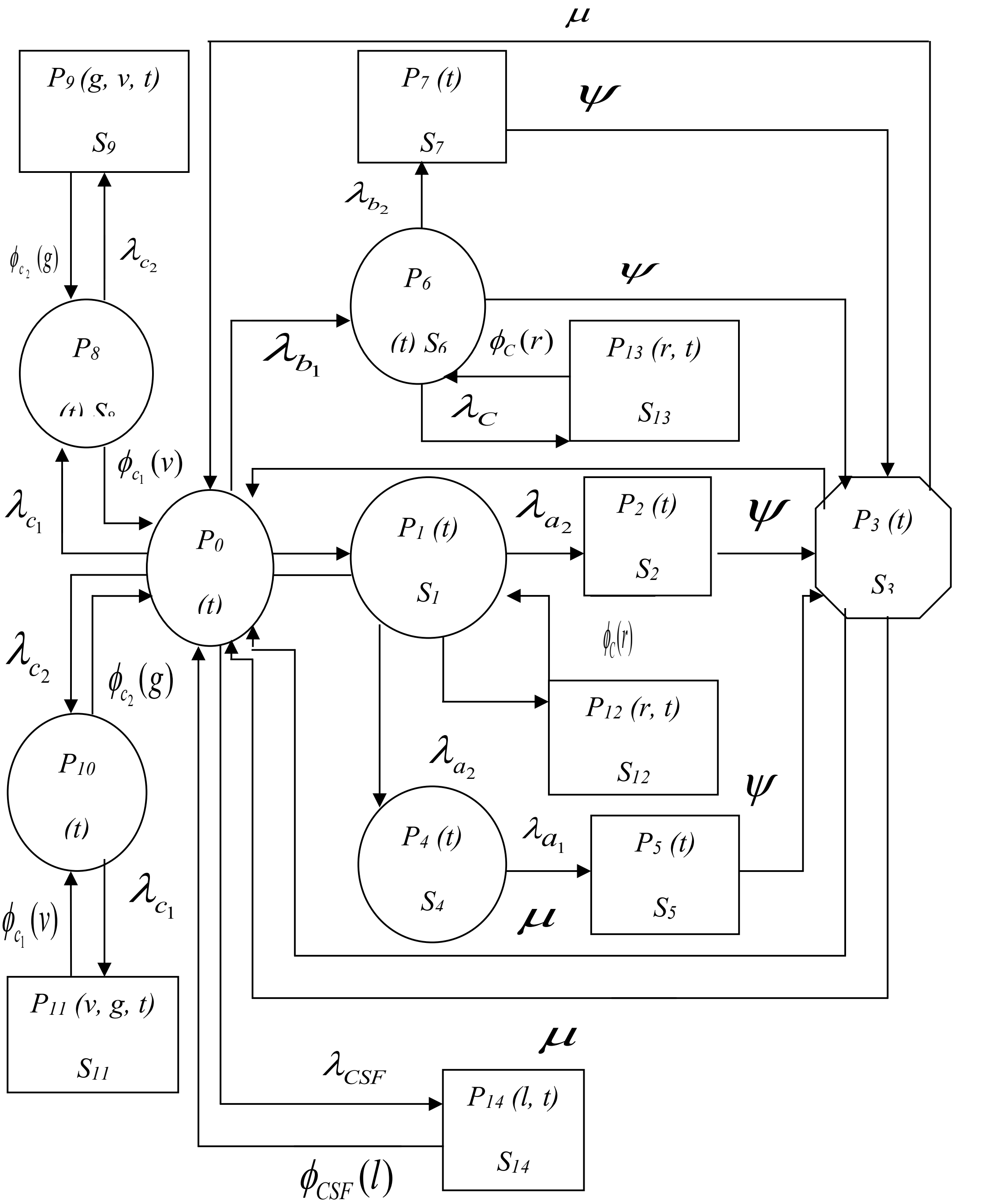


Figure 1: *Transition state diagram*

IV. Formulation of the mathematical model

The following differential equations have been obtained by considering limiting procedures and different probability constraints which satisfying the model:

$$\left[\frac{d}{dt}+\lambda_{x_1}+\lambda_{x_2}+\lambda_{y_1}+\lambda_{y_2}+\lambda_{z_1}+\lambda_{z_2}+\lambda_{csf}\right]P_0(t)=\int_0^\infty \mu\ \ (i)P_3(t)\,di+\varphi_{z_1}P_8(t)+\varphi_{z_2}P_{10}(t) \quad \text{... (1)}$$

$$\left[\frac{\partial}{\partial t}+\lambda_{x_2}+\lambda_z\right]P_1(t)=\lambda_{a_1}P_0(t)+\int_0^\infty \varphi_z(r)P_{12}(r,t)\,dr \quad \text{… (2)}$$

$$\left[\frac{\partial}{\partial t}+\psi\right]P_2(t)=\lambda_{x_2}P_1(t) \quad \text{… (3)}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial i}+(\mu+\psi)\right]P_3(t)=\psi[P_2(t)+P_5(t)+P_6(t)+P_7(t)]+\frac{(\psi t)^3e^{-\psi t}}{6} \quad \text{… (4)}$$

$$\left[\frac{\partial}{\partial t}+\lambda_{x_1}\right]P_4(t)=\lambda_{x_2}P_0(t) \quad \text{… (5)}$$

$$\left[\frac{\partial}{\partial t}+\psi\right]P_5(t)=\lambda_{x_1}P_4(t) \quad \ldots (6)$$

$$\left[\frac{\partial}{\partial t}+\lambda_{y_2}+\psi\right]P_6(t)=\lambda_{y_1}P_0(t)+\int_0^{\infty}\varphi_Z(r)P_{13}(r,t)dr \quad \ldots (7)$$

$$\left[\frac{\partial}{\partial t}+\psi\right]P_7(t)=\lambda_{y_2}P_6(t) \quad \ldots (8)$$

$$\left[\frac{\partial}{\partial t}+\varphi_{z_1}(v)+\lambda_{z_2}\right]P_8(t)=\lambda_{z_1}P_0(t)+\int_0^{\infty}\varphi_{z_2}(g)P_{11}(g,v,t)dg \quad \ldots (9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial g}+\phi_{z_2}(g)\right]P_9(g,v,t)=0 \quad \ldots (10)$$

$$\left[\frac{\partial}{\partial t}+\phi_{z_2}(g)+\lambda_{z_1}\right]P_{10}(t)=\lambda_{z_2}P_0(t)+\int_0^{\infty}\phi_{z_1}(v)P_{11}(v,g,t)dv \quad \ldots (11)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial v}+\phi_{z_1}(v)\right]P_{11}(v,g,t)=0 \quad \ldots (12)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\phi_Z(r)\right]P_{12}(r,t)=0 \quad \ldots (13)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\phi_z(r)\right]P_{13}(r,t)=0 \quad \ldots (14)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial l}+\phi_{csf}(l)\right]P_{14}(l,t)=0 \quad \ldots (15)$$

Boundary Conditions:

$$P_3(i=0,t)=\psi[P_2(t)+P_3(t)+P_6(t)+P_7(t)] \quad \ldots (16)$$

$$P_8(0,t)=\lambda_{z_1}P_0(t) \quad \ldots (17)$$

$$P_9(0,v,t)=\lambda_{z_2}P_8(t) \quad \ldots (18)$$

$$P_{10}(0,t)=\lambda_{z_2}P_0(t) \quad \ldots (19)$$

$$P_{11}(0,g,t)=\lambda_{z_1}P_{10}(t) \quad \ldots (20)$$

$$P_{12}(0,t)=\lambda_z P_1(t) \quad \ldots (21)$$

$$P_{13}(0,t)=\lambda_z P_6(t) \quad \ldots (22)$$

$$P_{14}(0,t)=\lambda_{csf}P_0(t) \quad \ldots (23)$$

Initial condition:

$P_0(0)=1$, otherwise zero.

Solving equations (1) through (15) by taking Laplace transform and by using initial and boundary conditions we obtained following probabilities of system is in up and down states at time t,

$$\overline{P}up=\overline{P}_0(s)+\overline{P}_1(S)+\overline{P}_4(s)+\overline{P}_6(s)+\overline{P}_8(s)+\overline{P}_{10}(s)$$

$$=\frac{1}{K(s)}\left[1+\frac{\lambda_{x_1}}{[s+\lambda_{x_2}+\lambda_z-\lambda_z\overline{S}_{\phi_z}(s)]}+\frac{\lambda_{x_2}}{[s+\lambda_{x_1}]}+\frac{\lambda_{x_1}}{[s+\lambda_{y_2}+\psi-\lambda_z\overline{S}_{\phi_z}(s)]}+\frac{\lambda_{z_1}}{[s+\lambda_{z_2}+\phi_{z_1}(v)-\lambda_{z_2}\overline{S}_{\phi_{z2}}(s)]}+\frac{\lambda_{c_2}}{[s+\lambda_{c_1}+\phi_{c_2}(g)-\lambda_{c_1}\overline{S}_{\phi_{c1}}(s)]}\right] \quad \dots (24)$$

$$\overline{P}_{down}=\overline{P}_2(s)+\overline{P}_5(s)+\overline{P}_7(s)+\overline{P}_9(s)+\overline{P}_{11}(s)+\overline{P}_{12}(s)+\overline{P}_{13}(s)+\overline{P}_{14}(s)$$

$$=\frac{\lambda_{x_1}\lambda_{x_2}}{[s+\psi][s+\lambda_{x_2}+\lambda_z-\lambda_z\overline{S}_{\phi_z}(s)]}\frac{1}{K(s)}+\frac{\lambda_{x_1}\lambda_{x_2}}{[s+\psi][s+\lambda_{x_1}]}\frac{1}{K(s)}+\frac{\lambda_{y_1}\lambda_{y_2}}{[s+\psi][s+\lambda_{y_2}+\psi-\lambda_z\overline{S}_{\phi_z}(s)]}\frac{1}{K(s)}+\frac{\lambda_{c_1}\lambda_{c_2}D_{\phi_{c2}}(s)}{[s+\lambda_{c_2}+\phi_{c_1}(v)-\lambda_{c_2}\overline{S}_{\phi_{c_2}}(s)]}\frac{1}{K(s)}+\frac{\lambda_{z_1}\lambda_{z_2}D_{\phi_{z1}}(s)}{[s+\lambda_{z_1}+\phi_{z_2}(g)-\lambda_{z_1}\overline{S}_{\phi_{z1}}(s)]}\frac{1}{K(s)}+\frac{\lambda_z\lambda_{x_1}D_{\phi_z}(s)}{[s+\lambda_{x_2}+\lambda_z-\lambda_z\overline{S}_{\phi_z}(s)]}\frac{1}{K(s)}+\frac{\lambda_z\lambda_{y_1}D_{\phi_z}(s)}{[s+\lambda_{y_2}+\psi-\lambda_z\overline{S}_{\phi_z}(s)]}\frac{1}{K(s)}+\frac{\lambda_{csf}D_{\phi_{csf}}(s)}{K(s)} \quad \dots (25)$$

where,

$$K(s)=s+\lambda_{x_1}+\lambda_{x_2}+\lambda_{y_1}+\lambda_{y_2}+\lambda_{z_1}+\lambda_{z_2}+\lambda_{csf}-\psi\{[\frac{\lambda_{x_1}\lambda_{x_2}}{[s+\psi][s+\lambda_{x_2}+\lambda_z-\lambda_z\overline{S}_{\phi_z}(s)]}+\frac{\lambda_{x_1}\lambda_{x_2}}{[s+\psi][s+\lambda_{x_1}]}+\frac{\lambda_{y_1}}{[s+\lambda_{y_2}+\psi-\lambda_z\overline{S}_{\phi_z}(s)]}+\frac{\lambda_{y_1}\lambda_{y_2}}{[s+\psi][s+\lambda_{y_2}+\psi-\lambda_z\overline{S}_{\phi_z}(s)]}]D_\mu(s)+\frac{\psi^3}{(s+\psi)^4}\}-\frac{\lambda_{z_1}\phi_{z_1}(v)}{[s+\lambda_{z_2}+\phi_{z_1}(v)-\lambda_{z_2}\overline{S}_{\phi_{z_2}}(s)]}-\frac{\lambda_{z_2}\phi_{z_2}(g)}{[s+\lambda_{z_1}+\phi_{z_2}(g)-\lambda_{z_1}\overline{S}_{\phi_{z1}}(s)]}-\lambda_{csf}\overline{S}_{\phi_{csf}}(s)$$

$$M(s)=\psi\{[\frac{\lambda_{x_1}\lambda_{x_2}}{[s+\psi][s+\lambda_{x_2}+\lambda_z-\lambda_z\overline{S}_{\phi_z}(s)]}+\frac{\lambda_{x_1}\lambda_{x_2}}{[s+\psi][s+\lambda_{x_1}]}+\frac{\lambda_{y_1}}{[s+\lambda_{y_2}+\psi-\lambda_Z\overline{S}_{\phi_Z}(s)]}+\frac{\lambda_{y_1}\lambda_{y_2}}{[s+\psi][s+\lambda_{y_2}+\psi-\lambda_z\overline{S}_{\phi_z}(s)]}]D_\mu(s)++\frac{\psi^3}{(s+\psi)^4}\} \quad \dots (26)$$

$$D_\mu(s)=\frac{1-\overline{S}_\mu(s)}{s+\psi} \quad \dots (27)$$

$$\phi_{csf}(l)=\exp[l^\theta+(\log\phi_{csf}(l))^\theta)^{1/\theta}] \quad \dots (28)$$

Also,

$$\overline{P}_{up}(s)+\overline{P}_{down}(s)=\frac{1}{s} \quad \dots (29)$$

Steady state behavior of the system By Abel's lemma we have,

$$\lim_{s\to 0}\{s\overline{F}(s)\}=\lim_{t\to\infty}F(t)$$

In equations (24) and (25) we get,

$$\overline{P}_{up}(s) = \frac{1}{K(0)}[1 + \frac{\lambda_{a_1}}{\psi\lambda_{a_2}} + \frac{\lambda_{a_2}}{\lambda_{a_1}} + \frac{\lambda_{b_1}}{\lambda_{b_2} + \psi - \lambda_c} + \frac{\lambda_{c_1}}{\phi_{c_1}(v)} + \frac{\lambda_{c_2}}{\phi_{c_2}(g)}] \quad \dots (30)$$

$$\overline{P}_{down} = \frac{1}{K(0)}[\frac{\lambda_{a_1}}{\psi} + M(0) + \frac{\lambda_{a_2}}{\psi} + \frac{\lambda_{b_1}\lambda_{b_2}}{\psi(\lambda_{b_2} + \psi - \lambda_C)} + \frac{\lambda_{c_1}\lambda_{c_2}M_{\phi_{c2}}}{\phi_{c_1}(v)} + \frac{\lambda_{c_1}\lambda_{c_2}M_{\phi_{c1}}}{\phi_{c_2}(g)} + \frac{\lambda_c\lambda_{a_1}M_{\phi_C}}{\lambda_{a_2}} + \frac{\lambda_C\lambda_{b_1}M_{\phi_C}}{\lambda_{b_2} + \psi - \lambda_C} + \lambda_{CSF}M_{\phi_{CSF}}] \quad \dots (31)$$

where,

$$M(0) = \lim_{s \to 0} M(s) \quad \dots (32)$$

$$M_{\phi_i} = \lim_{s \to 0} \frac{1 - \overline{S}_{\phi_i}(s)}{s} \quad \dots (33)$$

$$\mathrm{S}_{\phi_i}(s) = \frac{\phi_i}{\mathrm{s} + \phi_i} \quad \dots (34)$$

IV. Discussion

In this paper, A common problem of waiting line, which generally occurs in the manufacturing industries is discussed through the reliability, availability, Mean time to failure and cost analysis of the considered system by using supplementary variable technique and copula methodology. Also, we have analyzed the steady state behaviour to improve the practical utility of the system. One can easily observe from figure 2 that reliability of the system decreases rapidly with the transitions in time when all failures follow exponential time distribution. The reason for this decrease is waiting line in the repair section because of which the system is in non operational state for a long time. Figure 3 gives an idea about the availability of the system that decreases approximately in a constant manner with the increment in time.
Here, we have also done the analysis of effect of various parameters on mean time to failure of the system. Figures 4 represents decreases in MTTF with the increases failure rate (λ_{x_1}) of main unit of part X. Similarly, figures 5 and 6 shows the MTTF decreases of the system with the increase of failure rates λ_{z_1}, λ_{z_2} and λ_{csf}. A common phenomenon can be observed from the graphs of all the parameters that initially because of waiting line in the repair section the MTTF is negative and gradually it becomes positive.

At last cost function G(t) analysis, for different values of K_1 and K_2 with respect to time is done in figure 7. This analysis reveals that expected profit decreases as the service cost of the system increases.

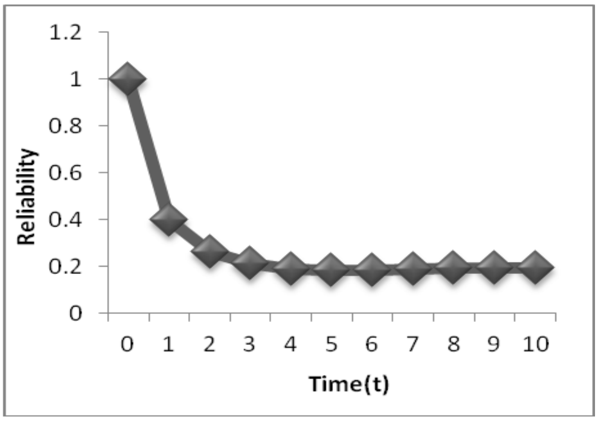


Figure 2: *Reliability against time*

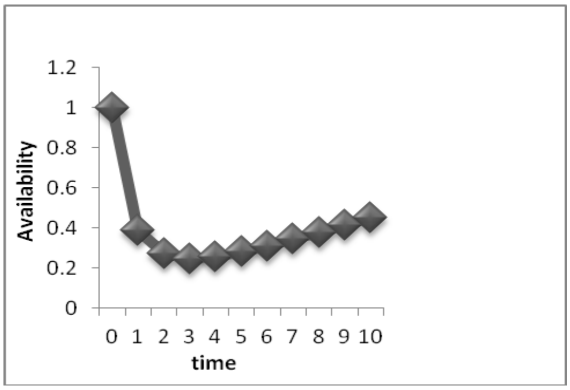


Figure 3: *Availability against time*

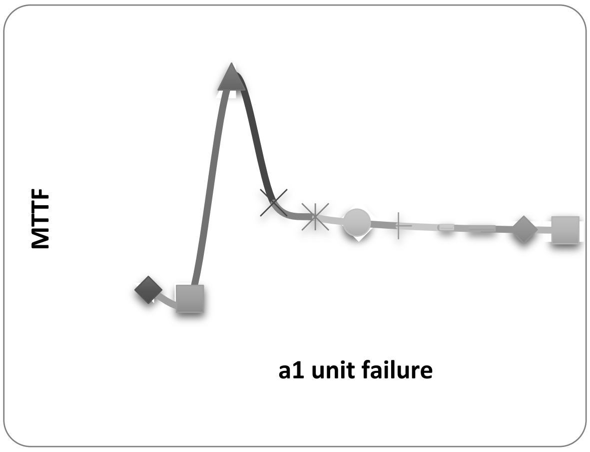


Figure 4: *MTTF Vs* λ_a

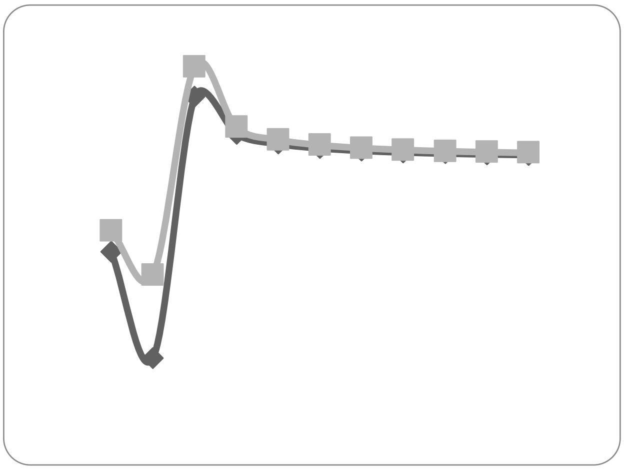
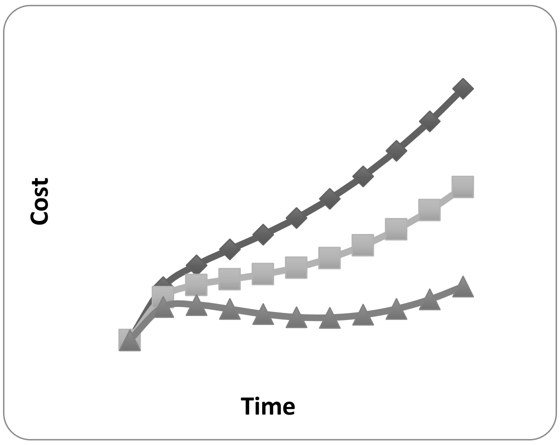


Figure 5: *MTTF Vs* λ_{c_1} *and* λ_{c_2}



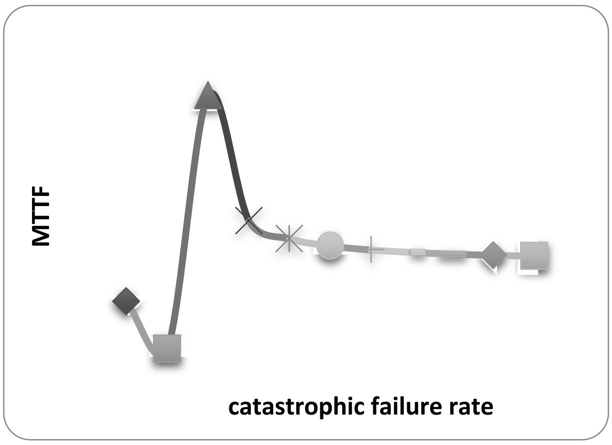


Figure 6: *MTTF Vs* λ_{CSF}

Figure 8: *Cost Vs time*

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